TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

EDINBURGH.

VOL. XXVII.

EDINBURGH:
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LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH,

AS REVISED 31ST OCTOBER 1871.

LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5.), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

Title.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

II.

The fees of Ordinary Fellows residing in Scotland.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.*

III.

Payment to cease after 25 years.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from farther payment.

IV.

Fees of Non-Resident Ordinary Fellows.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any farther payment. In the case of any Resident Fellow ceasing to reside

Case of Fellows becoming Non-Resident.

* At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s., to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

A modification of this rule, in certain cases, was agreed to 3d January 1831.

in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of election, and shall lie upon the table during that time.

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall

* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

XI.

Royal Personages.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X.

XII.

Recommendation of Honorary Fellows.

Mode of Election.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election.

XIII.

Election of Ordinary Fellows.

The election of Ordinary Fellows shall only take place at the first Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote.

XIV.

Ordinary Meetings.

The Ordinary Meetings shall be held on the first and third Mondays of every month from November to June inclusively. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make.

XV.

The Transactions.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which

* We hereby recommend_____
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

_____

_____

_____

To the President and Council of the Royal Society of Edinburgh.

they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

XVI.

How Published. The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council. There shall be elected annually, for conducting the publications and regulating the private business of the Society, a Council, consisting of a President; Six Vice-Presidents, two at least of whom shall be resident; Twelve Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors. Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers. An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

XX.

Special Meetings; how called. Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties. The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor. At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the procedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXV.

Curator of Museum and Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXVI.

Use of Museum and Library.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint.

XXVII.

Register Book.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

ROYAL SOCIETY OF EDINBURGH.

THE KEITH, BRISBANE, AND NEILL PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1874–75, for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1873–74 and 1874–75." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1874–75, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 1st June 1874.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the Paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration nay scientific papers presented to the Society during the Sessions 1872–73 and 1873–74, whether they may have been given in with a view to the Prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1874–75.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 1st May 1874,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

LIST OF THE ORDINARY FELLOWS OF THE SOCIETY.

N.B.—*Those marked *are Annual Contributors.*

1846 Alex. J. Adie, Esq., Rockville, Linlithgow
1871 *Stair Agnew, Esq, 2 Buckingham Terrace
1868 *Rev. Dr David Aitken, 4 Charlotte Square
1866 *Major-General Sir James E. Alexander of Westerton, Bridge of Allan
1867 *Rev. Dr W. Lindsay Alexander, Pinkie Burn, Musselburgh
1848 Dr James Allan, Inspector of Hospitals, Portsmouth
1856 Dr G. J. Allman, Emeritus Professor of Natural History, Wimbledon, London
1849 David Anderson, Esq., Moredun, Edinburgh
1872 John Anderson, LL.D., 32 Victoria Road, Charlton, Kent
1845 Dr Thomas Anderson, Professor of Chemistry, University, Glasgow 10
1823 Warren Hastings Anderson, Esq., Isle of Wight
1867 *Thomas Annandale, Esq., 34 Charlotte Square
1862 *T. C. Archer, Esq., Director of the Museum of Science and Art, 5 West Newington Terrace
1849 His Grace the Duke of Argyll, K.T., (HON. VICE-PRESIDENT), Inverary Castle
1871 *John Auld, Esq., 18 Grosvenor Crescent

1843 David Balfour, Esq., Trenaby
1835 Dr J. H. Balfour (GENERAL SECRETARY), Professor of Medicine and Botany, 7 Inverleith Row
1870 *Dr Thomas A. G. Balfour, 51 George Square
1867 *George F. Barbour, Esq., 11 George Square
1872 *George Barclay, Esq., 17 Coates Crescent 20
1858 Edmund C. Batten, M.A., Lincoln's Inn, London
1870 *Dr James Warburton Begbie, 16 Great Stuart Street
1843 Dr Bennett, Professor of Institutes of Medicine, 1 Glenfinlas Street
1861 *George Berry, Esq., 2 Windsor Terrace, Portobello
1866 *Adam Black, Esq., 38 Drummond Place
1850 *Hugh Blackburn, Esq., Prof. Mathematics, University, Glasgow
1863 *Professor Blackie, 24 Hill Street
1857 *John Blackwood, Esq., 3 Randolph Crescent
1862 *Rev. Dr W. G. Blaikie, 9 Palmerston Road
1854 Ernest Bonar, Esq. 30
1872 *James Thomson Bottomley, Esq., Taunton School College.
1869 *Robert Henry Bow, Esq., C.E., 7 South Gray Street
1871 *Thomas J. Boyd, Esq., 41 Moray Place
1873 *William Boyd, Esq., Peterhead
1864 *Dr Alex. Crum Brown, Prof. of Chemistry, 8 Belgrave Crescent
1859 *Dr John Brown, 23 Rutland Street
1861 *Rev. Thomas Brown, 16 Carlton Street
1835 William Brown, Esq., 25 Dublin Street
1870 Dr James Crichton Browne, Wakefield
1867 *A. H. Bryce, D.C.L., LL.D., 42 Moray Place 40
1856 *David Bryce, Esq., Architect, 131 George Street
1833 His Grace the Duke of Buccleuch, K.G., Dalkeith Palace
1869 *Alexander Buchan, A.M., 72 Northumberland Street
1870 *John Young Buchanan, Esq., 10 Moray Place
1857 *Dr W. M. Buchanan, 3 Carlton Terrace
1847 J. H. Burton, LL.D., Advocate, Craig House

1869 *Rev. Henry Calderwood, LL.D., Professor of Moral Philosophy, Craigrowan, Napier Road, Merchiston
1865 *Alfred R. Catton, B.A.
1866 *David Chalmers, Esq., Kate's Mill, Slateford
1860 *William Chambers, Esq. of Glenormiston, 13 Chester Street 50
1872 Dr Thomas B. Christie, Royal India Asylum, Ealing, London
1823 Sir Robert Christison, Bart., D.C.L., Professor of Materia Medica (PRESIDENT), 40 Moray Place
1863 Dr H. F. C. Cleghorn, Stravithy, St Andrews
1856 *Thomas Cleghorn, Esq., Advocate, 26 Queen Street
1844 Dr Thomas R. Colledge, Lauriston House, Cheltenham
1829 The Right Honourable Lord Colonsay, London
1829 A. Colyar, Esq.

1850 *Dr James Scarth Combe, 36 York Place
1872 *Archibald Constable, Esq., 11 Thistle Street
1843 Sir John Rose Cormack, M.D., 7 Rue d'Aguesseau, Paris 60
1872 *The Right Rev. Bishop Cotterill, 1 Athole Place.
1843 Andrew Coventry, Esq., Advocate, 29 Moray Place
1863 *Charles Cowan, Esq., Westerlea, Murrayfield
1854 *Sir James Coxe, M.D., Kinellan
1830 J. T. Gibson-Craig, Esq., W.S., 24 York Place
1829 Sir William Gibson-Craig, Bart., Riccarton
1871 *Rev. Dr Crawford, Professor of Divinity, 13 Great King Street
1873 *Donald Crawford, Esq., Advocate, 18 Melville Street.
1853 Rev. John Cumming, D.D., London
1852 *James Cunningham, Esq., W.S., 50 Queen Street 70
1871 *Dr R. J. Blair Cunyninghame, 6 Walker Street
1823 Liscombe J. Curtis, Esq., Ingsdown House, Devonshire

1851 *E. W. Dallas, Esq., 34 Hanover Street
1841 James Dalmahoy, Esq., 9 Forres Street
1867 *David Davidson, Esq., Bank of Scotland
1848 Henry Davidson, Esq., Muirhouse
1870 *St John Vincent Day, Esq., C.E., 4 Hamilton Park Terrace, Glasgow
1867 *Francis Deas, LL.B., Advocate, 9 St Colme Street
1869 *James Dewar, Esq., 15 Gilmore Place
1869 *Alexander Dickson, M.D., Professor of Botany, University of Glasgow, 11 Royal Circus 80
1869 *William Dickson, Esq., 38 York Place
1867 Henry Dircks, LL.D., C.E., London
1863 *W. Dittmar, Esq., Owens College, Manchester
1867 *James Donaldson, LL.D., 20 Great King Street
1866 *David Douglas, Esq., 41 Castle Street
1839 Francis Brown Douglas, Esq., Advocate, 21 Moray Place.
1868 *Rev. D. T. K. Drummond, B.A., 6 Montpelier
1867 *G. Stirling Home Drummond, Esq., Blair-Drummond
1860 *Patrick Dudgeon, Esq. of Cargen
1863 *Dr J. Matthews Duncan, 30 Charlotte Square 90
1870 *Dr John Duncan, 8 Ainslie Place
1851 *Sir David Dundas, Bart. of Dunira
1859 *Rev. Dr John Duns, 4 Mansion-House Road, Grange
1866 *Dr James Dunsmure, 53 Queen Street

1869 *George Elder, Esq., Knock Castle, Wemyss Bay
1856 *W. Mitchell Ellis, Esq., Wellington Lodge, Portobello
1855 Robert Etheridge, Esq., Clifton, Bristol
1866 *William Euing, Esq., Glasgow
1863 J. D. Everett, LL.D., Prof. Nat. Phil., Queen's College, Belfast

1866 *James Falshaw, Esq., C.E., 14 Belgrave Crescent 100
1859 Dr Fayrer, Professor of Surgery, Calcutta
1868 *Robert M. Ferguson, Ph.D., 12 Moray Place
1858 Frederick Field, Esq., Chili
1852 Dr Andrew Fleming, H.M.I.S., 3 Napier Road
1872 *Dr J. G. Fleming, B.A., 155 Bath Street, Glasgow
1872 *George Forbes, Esq., Lecturer on Natural Philosophy, Anderson Institution, Glasgow, 4 Coates Crescent
1859 Major James George Forlong, Bombay
1828 John Forster, Esq., Liverpool
1858 *Professor Fraser, M.A., 20 Chester Street
1867 *Dr Thomas R. Fraser, 3 Grosvenor Street 110
1867 *Frederick Fuller, Esq., Professor of Mathematics, University, Aberdeen

1867 Dr Charles Gayner, Oxford
1867 *Arthur Gamgee, Professor Owens College, Manchester
1868 J. Samson Gamgee, Esq., Birmingham
1861 *A. Geikie, Esq., Professor of Geology, Geological Survey Office, India Buildings, George IV. Bridge
1871 *James Geikie, Esq., 16 Duncan Street, Newington
1870 *Hon. Lord Gifford, Granton House
1868 *Rev. Joseph Taylor Goodsir, 11 Danube Street

1846 L. D. B. Gordon, Esq., C.E., London
1850 *Lieut.-Col. W. D. Gosset, R.E., Portsmouth 120
1867 *Dr Andrew Graham, R.N., 35 Melville Street.
1869 *Principal Sir Alex. Grant, Bart., (VICE-PRESIDENT), 21 Lansdowne Crescent
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1862 *Dr James Hector, Wellington, New Zealand 140
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1855 *The Right Hon. John Inglis, D.C.L., LL.D., Lord Justice-General, 30 Abercromby Place 150
1858 *Professor Innes, M.A., Inverleith House

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1863 William Jameson, Esq., Surgeon-Major, Saharunpore
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1865 *Charles Jenner, Esq., Easter Duddingston Lodge
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1869 Dr John Wilson Johnston, Bengal
1867 *T. B. Johnston, Esq., 9 Claremont Crescent 160

1867 *William Keddie, Esq., 5 India Street, Glasgow
1866 *Dr Alexander Keiller, 21 Queen Street
1839 Rev. Prof. Kelland, M.A., 20 Clarendon Crescent
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1872 *Thomas Knox, Esq., 2 Dick Place

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1865 *Charles Lawson, jun., Esq., 35 George Square
1856 *Dr Laycock, Professor of the Practice of Medicine, 13 Walker Street 170
1872 *Alexander H. Lee, Esq., C.E., 45 Moray Place
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1858 *James Leslie, Esq., C.E., 2 Charlotte Square
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1861 *Dr W. Lauder Lindsay, Gilgal, Perth

1864 *William Lindsay, Esq., Hermitage-Hill House, Leith
1870 *Professor Lister, 9 Charlotte Square
1871 *Dr Cosmo Garden Logie, Surgeon Major, Royal Horse Guards
1857 Thomas Login, Esq., C.E., India 180
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1855 *Dr Stevenson Macadam, 11 East Brighton Crescent, Portobello
1861 *Dr James M'Bain, R.N., Logie Villa, York Road, Trinity
1868 *Dr Thomas Smith Maccall, Polmont, Falkirk
1867 *John M. M'Candlish, Esq., 4 Doune Terrace
1866 *John M'Culloch, Esq., Banker, 11 Duke Street
1871 *Dr Angus Macdonald, 41 Northumberland Street
1820 Dr Wm. Macdonald, Prof. Civ. and Nat. Hist., St Andrews 190
1847 W. Macdonald Macdonald, Esq., St Martins
1869 *David MacGibbon, Esq., Architect, 89 George Street
1840 John Mackenzie, Esq., 11 Abercromby Place
1870 *Hon. Lord Mackenzie, 12 Great Stuart Street
1843 Dr Maclagan (CURATOR), Prof. of Medical Jurisprudence, 28 Heriot Row
1872 *David Maclagan, Esq., C.A., 9 Royal Circus
1853 Lieut.-Col. R. Maclagan, Royal Engineers, Bengal
1869 *Dr R. Craig Maclagan, 5 Coates Crescent
1870 *Dr G. H. B. Macleod, Professor of Surgery, University, Glasgow
1869 *Dr William C. M'Intosh, Murthly 200
1873 *Dr John G. M'Kendrick, 29 Castle Terrace
1864 *Peter M'Lagan, Esq. of Pumpherston, M.P.
1869 *John M'Laren, Esq., Advocate, 5 Rutland Square
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1853 *Græme Reid Mercer, Esq., Ceylon Civil Service
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1869 *Oliver G. Miller, Esq., Panmure House, Forfarshire
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1833 Admiral Sir Alexander Milne, G.C.B., Inveresk
1866 *Dr Arthur Mitchell, 5 East Claremont Street
1843 Joseph Mitchell, Esq., C.E., Viewill, Inverness 220
1865 *Dr John Moir, 52 Castle Street
1870 *The Right Hon. James Moncreiff, Lord Justice-Clerk, 15 Great Stuart Street
1871 *Rev. William Scott Moncrieff, of Fossaway, 12 Eton Terrace
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1866 *Dr Charles Morehead, 11 North Manor Place
1873 *M. M. Pattison Muir, Esq., Anderson's Institution, Glasgow.
1861 *John Muir, D.C.L., LL.D., 10 Merchiston Avenue
1870 *David Munn, Esq., 11 Gayfield Square
1857 Dr John Ivor Murray, The Knowle, near Tunbridge Wells

1856 *Hon. Lord Neaves, LL.D. (VICE-PRESIDENT), 7 Charlotte Square 230
1866 *Thomas Nelson, Esq., Arthursley
1870 *Dr Henry Alleyne Nicholson, Prof. of Nat. Hist., Toronto
1847 James Nicol, Esq., Prof. Nat. Hist., Aberdeen
1863 *Hon. Lord Ormidale, 14 Moray Place
1837 Dr Richard Parnell, Melrose
1863 *Dr Alexander Peddie, 15 Rutland Street
1868 *John Dick Peddie, Esq., Architect, 33 Buckingham Ter.
1869 John Pender, Esq., Manchester
1873 *Dr J. Bell Pettigrew, 57 Queen Street.

1849 W. Pirrie, Esq., Professor of Surgery, Marischal College, Aberdeen 240
1859 *Lyon Playfair, C.B., LL.D., M.P., 4 Queensberry Place, South Kensington, London
1834 Mungo Ponton, Esq., W.S., Clifton, Bristol
1852 Eyre B. Powell, Esq., Director of Public Instruction, Madras
1865 *James Powrie, Esq., Reswallie, Forfar
1849 Hon. B. F. Primrose, 22 Moray Place
1873 Andrew Pritchard, Esq., 87 St Paul's Road, Highbury, London.

1868 *Samuel Raleigh, Esq., Park House, Dick Place
1869 Rev. Thos. Melville Raven, M.A., Crakehall, Bedale
1865 *Rev. Francis Redford, M.A., Rectory, Silloth
1836 David Rhind, Esq., Architect, 54 Great King Street 250
1872 Major F. Ignacio Rickard, Government Inspector-General of Mines, Argentine Republic, Buenos Ayres, South America
1840 Martyn J. Roberts, Esq., Crickhowell, South Wales
1872 *Dr D. Argyll Robertson, 40 Queen Street
1859 *George Robertson, Esq., C.E., 47 Albany Street
1832 Dr Montgomery Robertson, Mortlake, Surrey
1860 *Dr William Robertson, 28 Albany Street
1862 *Dr E. Ronalds, Bonnington Road
1870 *Alexander Russel, Esq., 9 Chester Street
1852 *Alex. James Russell, Esq., C.S., 9 Shandwick Place
1837 J. Scott Russell, Esq., 5 Westminster Chambers, London 260
1869 *Dr William Rutherford, Professor of Physiology, King's College, London

1870 *Dr William R. Sanders, Prof. General Pathology, 11 Walker Street
1863 *James Sanderson, Esq., Surg.-Major, 41 Manor Place
1864 *Rev. D. F. Sandford, 19 Rutland Street
1849 Edward Sang, Esq., 2 George Street
1846 Dr Schmitz, International Institution, London
1853 *Hugh Scott, Esq. of Gala, Galashiels
1864 *Professor Sellar, LL.D., 15 Buckingham Terrace
1872 *George Seton, Esq., Advocate, 42 Greenhill Gardens
1870 Dr Edward James Shearman, Moorgate, Rotherham, Yorkshire 270
1834 Dr Sharpey, Prof. Anatomy, Univ. Coll., London
1872 *Dr John Sibbald, 16 Dalrymple Crescent
1870 *James Sime, Esq., Craigmount House, Dick Place
1871 *Dr A. R. Simpson, Prof. of Midwifery, 52 Queen St.
1829 Ven. Archdeacon Sinclair, Kensington
1859 *William F. Skene, LL.D., W.S., 20 Inverleith Row
1868 *Adam Gillies-Smith, Esq., C.A., 5 Lennox Street
1839 David Smith, Esq., W.S. (TREASURER), 10 Eton Ter.
1863 *Dr John Alexander Smith, 7 West Maitland Street
1866 *Dr John Smith, F.R.C.P.E., 20 Charlotte Square 280
1871 *Dr John Smith, F.R.C.S.E., 11 Wemyss Place
1855 *R. M. Smith, Esq., 4 Bellevue Crescent
1871 *Rev. W. R. Smith, Free Church Coll., Aberdeen
1846 Professor Piazzi Smyth, 15 Royal Terrace
1866 *Professor Spence, 21 Ainslie Place
1850 *Dr James Stark, 21 Rutland Street
1843 Henry Stephens, Esq., Red Braes Cottage, Bonnington
1844 David Stevenson, Esq., C.E., 45 Melville Street
1868 John J. Stevenson, Esq., Hyde Park, London
1848 Thomas Stevenson, Esq., C.E., 17 Heriot Row 290
1868 Major J. H. M. Shaw Stewart, R. Engineers, Madras
1869 *John L. Douglas Stewart, Esq. of Glenogil, 7 Grosvenor Crescent
1866 *Dr T. Grainger Stewart, 19 Charlotte Square
1873 *Walter Stewart, Esq., 76 Haymarket Terrace

1848 Patrick J. Stirling, Esq., LL.D., Kippendavie House
1823 Captain T. D. Stuart, H.M.I.S.
1870 *Patrick D. Swan, Esq., Kirkcaldy
1848 William Swan, Esq., Professor of Natural Philosophy, St Andrews
1844 Archibald Campbell Swinton, Esq., Kimmerghame, Dunse

1872 Rev. Andrew Tait, Rector of Kilkerrin, Ireland 300
1861 *Professor P. Guthrie Tait, M.A. (SECRETARY), 38 George Square
1870 *Robert R. Tatlock, Esq., 151 George Street, Glasgow
1846 Dr Taylor, Pau, France
1872 *Rev. Charles R. Teape, 15 Findhorn Place
1873 *Robert Tennent, Esq., 21 Lynedoch Place
1843 Dr Allen Thomson, Prof. Anatomy, Univ., Glasgow
1870 *Rev. Dr Andrew Thomson, 63 Northumberland Street
1842 James Thomson, Esq., C.E., Norfolk Square, Hyde Park, London
1863 *Dr Murray Thomson, Roorkee, East Indies
1870 *Spencer C. Thomson, Esq., 10 Chester Street 310
1847 Sir William Thomson, Prof. Nat. Phil. (VICE-PRESIDENT), Glasgow
1870 *William Burns Thomson, Esq., 1 St John Street
1849 William Thomas Thomson, Esq., Bonaly
1855 *Wyville Thomson, LL.D., Prof. Nat. Hist., 20 Palmerston Place
1871 *Thomas E. Thorpe, Ph.D., Lecturer on Chemistry, Anderson Institution, Glasgow
1822 Sir W. C. Trevelyan, Bart., Wallington, Morpeth
1867 *William Turnbull, Esq., 14 Lansdowne Crescent
1861 *Professor Turner, M.B. (SECRETARY), 6 Eton Terrace
1849 Most Noble the Marquis of Tweeddale, K.T., Yester House, Haddington

1867 *Peter Waddell, Esq., 5 Claremont Park, Leith 320
1869 *Viscount Walden, Yester House, Haddington
1829 James Walker, Esq., W.S., Tunbridge Wells
1873 *Robert Walker, Esq., 3 Adelphi, Aberdeen
1864 *William Wallace, Ph.D., Glasgow
1853 Dr James Watson, Bath
1870 *James Watson, Esq., 45 Charlotte Square
1866 *John K. Watson, Esq., 14 Blackford Road
1866 *Dr Patrick Heron Watson, 16 Charlotte Square
1862 *Rev. Robt. Boog Watson, Madeira, 4 Bruntsfield Place, Edinburgh
1873 *Dr Morrison Watson, 28 Ann Street 330
1873 Major Welsh, Bengal Artillery
1840 Allan A. Maconochie Welwood, Esq. of Meadowbank and Pitliver
1869 *Captain T. P. White, Royal Engineers, 1 Drummond Place
1868 *W. Williams, Esq., Gayfield House
1858 *Dr Thomas Williamson, 28 Charlotte Street, Leith
1834 Dr Isaac Wilson
1847 Professor John Wilson, College
1863 *Dr J. G. Wilson, 9 Woodside Crescent, Glasgow
1873 Robert Wilson, Esq., Engineer, Patricroft, Manchester
1870 John Winzer, Esq., Assistant Surveyor, Civil Service, Ceylon 340
1864 *Dr Alexander Wood, 36 Moray Place
1864 *Dr Andrew Wood, 9 Darnaway Street
1855 Dr Wright, Cheltenham
1864 *Robert S. Wyld, Esq., W.S., 19 Inverleith Row

1861 *James Young, Esq., of Kelly, Wemyss Bay
1863 *Dr John Young, Professor of Natural History, Glasgow 346

Fellows elected between the commencement of the Session and the 1st January of the following year are entered under the latter date, by which their Subscriptions are regulated:— Thus, Fellows elected in December 1871 have the date of 1872 prefixed to their names.

NON-RESIDENT MEMBER,

ELECTED UNDER THE OLD LAWS.

Sir Richard Griffiths, Bart., Dublin.

LIST OF HONORARY FELLOWS.

His Royal Highness the Prince of Wales.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X.)

	J. B. A. L. Léonce Elie de Beaumont,	*Paris.*
	Claude Bernard,	*Do.*
	Adolphe Théodore Brongniart,	*Paris.*
	Robert Wilhelm Bunsen,	*Heidelberg.*
5	Michael Eugene Chevreul,	*Paris.*
	James D. Dana, LL.D.,	*Newhaven, Connecticut.*
	Jean Baptiste Dumas,	*Paris.*
	Charles Dupin,	*Do.*
	Christian Gottfried Ehrenberg,	*Berlin.*
10	Elias Fries,	*Upsala.*
	François Pierre Guillaume Guizot,	*Paris.*
	Christopher Hansteen,	*Christiania.*
	Herman Helmholtz,	*Heidelberg.*
	Gustav Robert Kirchhoff,	*Do.*
15	Albert Kölliker,	*Wurzburg.*
	Johann von Lamont,	*Munich.*
	Richard Lepsius,	*Berlin.*
	Rudolph Leuckart,	*Leipzig.*
	Urbain Jean Joseph Leverrier,	*Paris.*
20	Henry Milne-Edwards,	*Do.*
	Theodore Mommsen,	*Berlin.*
	Louis Pasteur,	*Paris.*
	Prof. Benjamin Peirce,	*United States Surrey.*
	Adolphe Pictet,	*Geneva.*
25	M. Le Comte De Ramusat,	*Paris.*

	Henry Victor Regnault,	*Paris.*
	Angelo Secchi,	*Rome.*
	Karl Theodor von Siebold,	*Munich.*
	Bernard Studer,	*Berne.*
30	Otto Torell,	*Lund.*
	Rudolph Virchow,	*Berlin.*
	Wilhelm Eduard Weber,	*Gottingen.*
33	Friedrich Wohler,	*Do.*

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X.)

	John Couch Adams, Esq.,	*Cambridge.*
	Sir George Biddell Airy,	*Greenwich.*
	Thomas Andrews, M.D.,	*Belfast (Queen's College).*
	Thomas Carlyle, Esq.,	*London.*
5	Arthur Cayley, Esq.,	*Cambridge.*
	Charles Darwin, Esq.,	*Down, Bromley, Kent.*
	John Anthony Froude, Esq.,	*London.*
	James Prescott Joule, LL.D.	*Cliffpoint, Higher Broughton, Manchester.*
	William Lassell, Esq.,	*Liverpool.*
10	Rev. Dr Humphrey Lloyd,	*Dublin.*
	Sir William E. Logan,	*London.*
	Sir Charles Lyell, Bart.,	*Do.*
	William Hallowes Miller, LL.D.,	*Cambridge.*
	Richard Owen, Esq.,	*London.*
15	Lieut.-General Edward Sabine, R.A.,	*Do.*
	George Gabriel Stokes, Esq.,	*Cambridge.*
	James Joseph Sylvester, LL.D.,	*London.*
	William Henry Fox Talbot, Esq.,	*Lacock Abbey, Wiltshire.*
	Alfred Tennyson, Esq.,	*Freshwater, Isle of Wight.*
20	Sir Charles Wheatstone, D.C.L.,	*London.*

The following Public Institutions and Individuals are entitled to receive Copies of the Transactions and Proceedings of the Royal Society of Edinburgh:—

ENGLAND.

British Museum.
Bodleian Library, Oxford.
University Library, Cambridge.

Royal Society.
Linnean Society,
Society for the Encouragement of Arts.
Geological Society.
Royal Astronomical Society.
Royal Asiatic Society.
Zoological Society.
Royal Society of Literature.
Royal Horticultural Society.
Royal Institution.
Royal Geographical Society.
Statistical Society.
Institution of Civil Engineers.
Hydrographic Office, Admiralty.
Medico-Chirurgical Society.
Athenæum Club.
Cambridge Philosophical Society.
Manchester Literary and Philosophical Society.
Yorkshire Philosophical Society.
Chemical Society of London.
Museum of Economic Geology.
United Service Institution.
Royal Observatory, Greenwich.
Leeds Philosophical and Literary Society.
Historic Society of Lancashire and Cheshire.
Royal College of Surgeons of England.

SCOTLAND.

Edinburgh, University Library.
... Advocates' Library.
... College of Physicians.
... Highland and Agricultural Society.
... Royal Medical Society.
... Royal Physical Society.
... Royal Scottish Society of Arts.
Glasgow, University Library.
St Andrews, University Library.
Aberdeen, University Library.

IRELAND.

Library of Trinity College, Dublin.
Royal Irish Academy.

COLONIES, &c.

Asiatic Society of Calcutta.
Library of Geological Survey, Calcutta.
Literary and Historical Society of Toronto.
University of Sydney.
New Zealand Institute.

CONTINENT OF EUROPE.

Amsterdam, Royal Institute of Holland.
Basle, Natural History Society.
Berlin, Royal Academy of Sciences.
... Physical Society.
Berne, Society of Swiss Naturalists.
Bologna, Academy of Sciences.
Bonn, Cæsarean Academy of Naturalists.
Bourdeaux, Society of Physical and Natural Sciences.
Brussels, Royal Academy of Sciences.
Buda, Literary Society of Hungary.
Christiania University.
Copenhagen, Royal Academy of Sciences.
Erlangen University.
Frankfort, the Senkenbergian Museum.
Geneva, Natural History Society.
Giessen, University Library.
Göttingen, University Library.
Haarlem, Natural History Society.
Jena, Prof. Carl Gegenbaur, Editor of Zeitschrift Medicinisch-Physikalisch Gesellschaft.
Leipzig, Royal Saxon Academy.
Lille, Royal Society of Sciences.
Lisbon, Royal Academy of Sciences.
Lyons, Agricultural Society.
Milan, Royal Institute.
Moscow, Imperial Academy of Naturalists.

Munich, Royal Academy of Sciences of Bavaria (2 copies).
Neufchatel, Museum of Natural History.
Naples, Dr Anton Dohrn, Palazzo Toslonia.
Paris, Royal Academy of Sciences.
... Geographical Society.
... Royal Society of Agriculture.
... Society for Encouragement of Industry.
... Ecole des Mines.
... Marine Depôt.
... Museum of Jardin des Plantes.
... Société Mathématique.
Rotterdam, Batavian Society of Experimental Philosophy.
St Petersburg, Imperial Academy of Sciences.
... Archæological Society.
... Pulkowa Observatory.
Stockholm, Royal Academy of Sciences.
Turin, Royal Academy of Sciences.
... M. Michelotti.
Upsala, Society of Sciences.
Venice, Royal Institute.
Vienna, Imperial Academy of Sciences.
... Geologische Reichsanstalt.
... Geological-Botanical Society.

UNITED STATES OF AMERICA.

Boston, the Bowditch Library.
... Academy of Arts and Sciences.
... Society of Natural History.
Cambridge, Mass. U.S., Harvard University.
New York, State Library.
Philadelphia, American Philosophical Society.
... Academy of Natural Sciences.
United States, Naval Observatory.
Washington, the Smithsonian Institution.
Yale College, United States.

SOUTH AMERICA.

Buenos Ayres, Public Museum, per Dr Burmeister.

(*All the Honorary and Ordinary Fellows of the Society are entitled to the Transactions and Proceedings.*)

The following Institutions and Individuals receive the Proceedings only:—

ENGLAND.

Scarborough Philosophical Society.
Whitby Philosophical Society.
Newcastle Philosophical Society.
Geological Society of Cornwall.
Ashmolean Society of Oxford.
Literary and Philosophical Society of Liverpool.
Meteorological Office, 116 Victoria Street, London.
Editor of Nature, London.

SCOTLAND.

Philosophical Society of Glasgow.
Botanical Society of Edinburgh.
Geological Society of Edinburgh.
Meteorological Society of Edinburgh.

IRELAND.

Natural History Society of Dublin.

COLONIES.

Literary and Philosophical Society of Quebec.
Library of the Geological Survey, Canada.
Literary Society of Madras.
China Branch of Asiatic Society, Hongkong.
North China Branch of the Royal Asiatic Society, Shanghae.
Royal Society of Victoria.

CONTINENT OF EUROPE.

Utrecht, the Literary and Philosophical Society.
Baris, Editor of L'Institut.
Abbé Moigno, Paris.
Em. Alglave, Directeur de la Revue des Cours Litteraires et Scientifiques, Paris.
Cherbourg, Society of Natural Sciences.
Belgium, the University of Ghent.
Sicily, Catania, Academia Gionia de Scienze Naturali.

UNITED STATES.

Peadody Academy of Science, Salam, Massachusetts.

LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH,

AS REVISED 31ST OCTOBER 1871.

LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5.), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

Title.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

II.

The fees of Ordinary Fellows residing in Scotland.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.*

III.

Payment to cease after 25 years.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from farther payment.

IV.

Fees of Non-Resident Ordinary Fellows.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any farther payment. In the case of any Resident Fellow ceasing to reside

Case of Fellows becoming Non-Resident.

* At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s., to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

A modification of this rule, in certain cases, was agreed to 3d January 1831.

in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of election, and shall lie upon the table during that time.

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall

* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

XI.

Royal Personages.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X.

XII.

Recommendation of Honorary Fellows.

Mode of Election.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election.

XIII.

Election of Ordinary Fellows.

The election of Ordinary Fellows shall only take place at the first Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote.

XIV.

Ordinary Meetings.

The Ordinary Meetings shall be held on the first and third Mondays of every month from November to June inclusively. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make.

XV.

The Transactions.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which

* We hereby recommend_____
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

_____

_____

_____

To the President and Council of the Royal Society of Edinburgh.

they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

XVI.

How Published. The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council. There shall be elected annually, for conducting the publications and regulating the private business of the Society, a Council, consisting of a President; Six Vice-Presidents, two at least of whom shall be resident; Twelve Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors. Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers. An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

XX.

Special Meetings; how called. Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties. The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor. At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the procedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXV.

Curator of Museum and Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXVI.

Use of Museum and Library.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint.

XXVII.

Register Book.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

ROYAL SOCIETY OF EDINBURGH.

THE KEITH, BRISBANE, AND NEILL PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1874–75, for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1873–74 and 1874–75." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1874–75, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 1st June 1874.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the Paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented to the Society during the Sessions 1872–73 and 1873–74, whether they may have been given in with a view to the Prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1874–75.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 1st May 1874,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

LIST OF THE ORDINARY FELLOWS OF THE SOCIETY.

N.B.—*Those marked * are Annual Contributors.*

1846 Alex. J. Adie, Esq., Rockville, Linlithgow
1871 *Stair Agnew, Esq. 22 Buckingham Terrace
1868 *Rev. Dr David Aitken, 4 Charlotte Square
1866 *Major-General Sir James E. Alexander of Westerton, Bridge of Allan
1867 *Rev. Dr W. Lindsay Alexander (VICE-PRESIDENT), Pinkie Burn, Musselburgh
1848 Dr James Allan, Inspector of Hospitals, Portsmouth
1856 Dr G. J. Allman, Emeritus Professor of Natural History, Wimbledon, London
1849 David Anderson, Esq., Moredun, Edinburgh
1872 John Anderson, LL.D., 32 Victoria Road, Charlton, Kent
1874 Dr John Anderson, Professor of Comparative Anatomy, Medical College, Calcutta. 10
1823 Warren Hastings Anderson, Esq., Isle of Wight
1867 *Thomas Annandale, Esq., 34 Charlotte Square
1862 *T. C. Archer, Esq., Director of the Museum of Science and Art, 5 West Newington Terrace
1849 His Grace the Duke of Argyll, K.T., (HON. VICE-PRESIDENT), Inverary Castle
1871 *John Auld, Esq., 18 Grosvenor Crescent

1843 David Balfour, Esq., Trenaby
1835 Dr J. H. Balfour (GENERAL SECRETARY), Professor of Medicine and Botany, 27 Inverleith Row
1870 *Dr Thomas A. G. Balfour, 51 George Square
1867 *George F. Barbour, Esq., 11 George Square
1872 *George Barclay, Esq., 17 Coates Crescent 20
1874 W. F. Barrett, Esq., Royal College of Science, Dublin
1858 Edmund C. Batten, M.A., Lincoln's Inn, London
1870 *Dr James Warburton Begbie, 16 Great Stuart Street
1874 *Dr Joseph Bell, 20 Melville Street
1843 Dr Bennett, Emeritus Professor of Institutes of Medicine, Nice
1850 Hugh Blackburn, Esq., Prof. Mathematics, University, Glasgow
1863 *Professor Blackie, 24 Hill Street
1857 *John Blackwood, Esq., 3 Randolph Crescent
1862 *Rev. Dr W. G. Blaikie, 9 Palmerston Road
1854 Ernest Bonar, Esq. 30
1872 *James Thomson Bottomley, Esq., University, Glasgow
1869 *Robert Henry Bow, Esq., C.E., 7 South Gray Street
1871 *Thomas J. Boyd, Esq., 41 Moray Place
1873 *William Boyd, Esq., Peterhead
1864 *Dr Alex. Crum Brown, Prof. of Chemistry, 8 Belgrave Crescent
1859 *Dr John Brown, 23 Rutland Street
1861 *Rev. Thomas Brown, 16 Carlton Street
1835 William Brown, Esq., 25 Dublin Street
1870 Dr James Crichton Browne, Wakefield
1867 *A. H. Bryce, D.C.L., LL.D., 42 Moray Place 40
1856 *David Bryce, Esq., Architect, 131 George Street
1833 His Grace the Duke of Buccleuch, K.G., Dalkeith Palace
1869 *Alexander Buchan, A.M., 72 Northumberland Street
1870 *John Young Buchanan, Esq., 10 Moray Place
1847 J. H. Burton, LL.D., Advocate, Craig House, 19 St Giles Street

1869 *Rev. Henry Calderwood, LL.D., Professor of Moral Philosophy, Craigrowan, Napier Road, Merchiston
1874 Dr Benjamin Carrington, Eccles, Lancashire
1866 *David Chalmers, Esq., Kate's Mill, Slateford
1860 *William Chambers, Esq. of Glenormiston, 13 Chester Street
1874 *Dr John Chiene, 21 Ainslie Place 50
1872 Dr Thomas B. Christie, Royal India Asylum, Ealing, London
1823 Sir Robert Christison, Bart., D.C.L., Professor of Materia Medica (HON. VICE-PRESIDENT), 40 Moray Place
1863 Dr H. F. C. Cleghorn, Stravithy, St Andrews
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LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH,

AS REVISED 31ST OCTOBER 1871.

LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5.), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

Title.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

II.

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III.

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The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any farther payment. In the case of any Resident Fellow ceasing to reside

Case of Fellows becoming Non-Resident.

* At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s., to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

A modification of this rule, in certain cases, was agreed to 3d January 1831.

in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of election, and shall lie upon the table during that time.

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall

* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

XI.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X. Royal Personages.

XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election. Recommendation of Honorary Fellows. Mode of Election.

XIII.

The election of Ordinary Fellows shall only take place at the first Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote. Election of Ordinary Fellows.

XIV.

The Ordinary Meetings shall be held on the first and third Mondays of every month from November to June inclusively. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make. Ordinary Meetings.

XV.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which The Transactions.

* We hereby recommend_____
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

_____

_____

_____

To the President and Council of the Royal Society of Edinburgh.

they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

XVI.

How Published. The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council. There shall be elected annually, for conducting the publications and regulating the private business of the Society, a Council, consisting of a President; Six Vice-Presidents, two at least of whom shall be resident; Twelve Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors. Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers. An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

XX.

Special Meetings: how called. Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties. The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor. At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the procedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXV.

Curator of Museum and Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXVI.

Use of Museum and Library.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint.

XXVII.

Register Book

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

ROYAL SOCIETY OF EDINBURGH.

THE KEITH, BRISBANE, AND NEILL PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1875–76, for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1873–74 and 1874–75." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1876–77, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 1st June 1876.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the Paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented to the Society during the Sessions 1874–75 and 1875–76, whether they may have been given in with a view to the Prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1877–78.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 1st May 1877,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

AWARD OF THE KEITH PRIZE.

The Council awarded the Keith Prize for the 23d Biennial Period, 1871–73, to Professor Tait for his Paper, entitled "First Approximation to a Thermo-Electric Diagram."

AWARD OF THE MAKDOUGALL BRISBANE PRIZE.

The Council awarded the Makdougall Brisbane Prize for the 8th Biennial Period, 1872–74, to Professor Lister for his Paper "On the Germ Theory of Putrefaction and other Fermentative Changes."

AWARD OF THE NEILL PRIZE.

The Council awarded the Neill Prize for the 6th Triennial Period, 1871–74, to Mr Charles William Peach, for his contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

LIST OF THE ORDINARY FELLOWS OF THE SOCIETY.

N.B.—*Those marked * are Annual Contributors.*

Alex. J. Adie, Esq., Rockville, Linlithgow
*Stair Agnew, Esq. 22 Buckingham Terrace
*John Aitken, Esq., Darroch, Falkirk
*Major-General Sir James E. Alexander of Westerton, Bridge of Allan
*Rev. Dr W. Lindsay Alexander (VICE-PRESIDENT), Pinkie Burn, Musselburgh
Dr James Allan, Inspector of Hospitals, Portsmouth
Dr George J. Allman, Emeritus Professor of Natural History, Wimbledon, London
David Anderson, Esq., Moredun, Edinburgh
John Anderson, LL.D., 32 Victoria Road, Charlton, Kent
Dr John Anderson, Professor of Comparative Anatomy, Medical College, Calcutta 10
Warren Hastings Anderson, Esq., Isle of Wight
*Thomas Annandale, Esq., 34 Charlotte Square
*T. C. Archer, Esq., Director of the Museum of Science and Art, 5 West Newington Terrace
His Grace the Duke of Argyll, K.T., (HON. VICE-PRESIDENT), Inverary Castle

*James Bain, Esq., 3 Park Terrace, Glasgow
David Balfour, Esq., Trenaby
Dr J. H. Balfour (GENERAL SECRETARY), Professor of Medicine and Botany, 27 Inverleith Row
*Dr Thomas A. G. Balfour, 51 George Square
*George F. Barbour, Esq., 11 George Square
*George Barclay, Esq., 17 Coates Crescent 20
W. F. Barrett, Esq., Royal College of Science, Dublin
Edmund C. Batten, M.A., Lincoln's Inn, London
*Dr James Warburton Begbie, 16 Great Stuart Street
*Dr Joseph Bell, 20 Melville Street
Dr Ludwik Stanthorpe Bernstein, Queensland
Hugh Blackburn, Esq., Prof. Mathematics, University, Glasgow
*Professor John S. Blackie, 24 Hill Street
*John Blackwood, Esq., 3 Randolph Crescent
*Rev. Dr W. G. Blaikie, 9 Palmerston Road
Ernest Bonar, Esq. 30
*James Thomson Bottomley, Esq., University, Glasgow
*Robert Henry Bow, Esq., C.E., 7 South Gray Street
*Thomas J. Boyd, Esq., 41 Moray Place
*William Boyd, Esq., Peterhead
*Dr Alex. Crum Brown, Prof. of Chemistry, 8 Belgrave Crescent
*Dr John Brown, 23 Rutland Street
*Rev. Thomas Brown, 16 Carlton Street
William Brown, Esq., 25 Dublin Street
Dr James Crichton Browne, Wakefield
*A. H. Bryce, D.C.L., LL.D., 42 Moray Place 40
*David Bryce, Esq., Architect, 131 George Street
*James Bryce, Esq., M.A., LL.D., 25 Lauder Road
His Grace the Duke of Buccleuch, K.G., Dalkeith Palace
*Alexander Buchan, A.M., 72 Northumberland Street
*John Young Buchanan, Esq., 10 Moray Place
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*Rev. Henry Calderwood, LL.D., Professor of Moral Philosophy, Craigrowan, Napier Road, Merchiston
Dr Benjamin Carrington, Eccles, Lancashire
*David Chalmers, Esq., Kate's Mill, Slateford
*William Chambers, Esq. of Glenormiston, LL.D., 13 Chester Street 50
*John Chiene, Esq., 21 Ainslie Place
*John Christie, Esq., Cowden, Dollar
Dr Thomas B. Christie, Royal India Asylum, Ealing, London
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*Robert Clark, Esq., Barnton House
Dr H. F. C. Cleghorn, Stravithy, St Andrews
*Dr T. S. Clouston, Tipperlin House, Morningside
Dr Thomas R. Colledge, Lauriston House, Cheltenham
A. Colyar, Esq.
Dr James Scarth Combe, 36 York Place 60

1872 *Archibald Constable, Esq., 11 Thistle Street
1843 Sir John Rose Cormack, M.D., 7 Rue d'Agueseau, Paris
1872 *The Right Rev. Bishop Cotterill (VICE-PRESIDENT), 1 Atholl Place.
1843 Andrew Coventry, Esq., Advocate, 29 Moray Place
1863 *Charles Cowan, Esq., Westerlea, Murrayfield
1854 *Sir James Coxe, M.D., Kinellan
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1829 Sir William Gibson-Craig, Bart., Riccarton
1875 *Dr William Craig, 7 Lothian Road
1873 *Donald Crawford, Esq., Advocate, 18 Melville Street 70
1853 Rev. John Cumming, D.D., London
1852 *James Cunningham, Esq., W.S., 50 Queen Street
1871 *Dr R. J. Blair Cunyninghame, 6 Walker Street
1823 Liscombe J. Curtis, Esq., Ingsdown House, Devonshire

1851 E. W. Dallas, Esq., 34 Hanover Street
1841 James Dalmahoy, Esq., 9 Forres Street
1867 *David Davidson, Esq., Bank of Scotland
1848 Henry Davidson, Esq., Muirhouse
1870 *St John Vincent Day, Esq., C.E., Garthomlock House, Shettleston, Glasgow
1869 *James Dewar, Jacksonian Prof. of Natural Experimental Philosophy, University of Cambridge 80
1869 *Alexander Dickson, M.D., Professor of Botany, University of Glasgow, 11 Royal Circus
1869 *William Dickson, Esq., 38 York Place
1863 *W. Dittmar, Esq., Anderson Institution, Glasgow
1867 *James Donaldson, LL.D., 20 Great King Street
1866 *David Douglas, Esq., 41 Castle Street
1839 Francis Brown Douglas, Esq., Advocate, 21 Moray Place.
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1870 *Dr John Duncan, 8 Ainslie Place
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1859 *Rev. Dr John Duns, 4 Mansion-House Road, Grange
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1855 Robert Etheridge, Esq., Royal School of Mines, London
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1875 Thomas Fairley, Esq., Lecturer on Chemistry, Leeds
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1868 *Robert M. Ferguson, Ph.D., 12 Moray Place
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1852 Dr Andrew Fleming, H.M.I.S., 3 Napier Road
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1859 Major James G. Forlong, Chief Engineer, Lucknow 110
1828 John Forster, Esq., Liverpool
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1867 *Dr Thomas R. Fraser, The Lodge, Knutsford, Cheshire
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1871 *James Geikie, Esq., 16 Duncan Street, Newington
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1868 *Rev. Joseph Taylor Goodsir, 11 Danube Street
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1854 Professor Robert Harkness, Queen's College, Cork
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1870 *Thomas Harvey, Esq., LL.D., 32 George Square
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1872 *Captain Charles Hunter, Glencarse, Junior United Service Club, London 150
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Guards 874
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Crescent
1869 *Maurice Lothian, Esq. of St Catherine's, 54 Queen St... 865
1849 Dr W. H. Lowe 875

1855 *Dr Stevenson Macadam, 11 East Brighton Crescent, Por... 849
bello 873
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1847 W. Macdonald Macdonald, Esq., St Martins 1836
1869 *David MacGibbon, Esq., Architect, 89 George Street 1875
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1843 Dr Douglas Maclagan (CURATOR), Prof. of Medical Ju...
prudence, 28 Heriot Row
1872 *David Maclagan, Esq., C.A., 9 Royal Circus 1840
1853 Major-General R. Maclagan, Royal Engineers, Bengal 1872
1869 *Dr R. Craig Maclagan, 5 Coates Crescent 1859
1870 *Dr G. H. B. Macleod, Professor of Surgery, Univers... 1832
Glasgow 1860
1869 *Dr William C. M'Intosh, Murthly 1862
1873 *Dr John G. M'Kendrick, 2 Chester Street 1870
1864 *Peter M'Lagan, Esq. of Pumpherston, M.P. 1852
1869 *John M'Laren, Esq., Advocate, 5 Rutland Square 1837
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race, Glasgow 1869
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1840 Sir John M'Neill, G.C.B., Burnhead, Liberton 1870
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1869 Dr Henry Marshall, Clifton, Bristol 1863
1864 *J. D. Marwick, Esq., Glasgow 1864
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1843 Joseph Mitchell, Esq., C.E., Viewill, Inverness
1865 *Dr John Moir, 52 Castle Street
1870 *The Right Hon. Lord Moncreiff, Lord Justice-Clerk, 1870
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1871 *Rev. William Scott Moncrieff, of Fossaway, 12 B... 1859
Terrace 1868
1868 *Very Rev. Dean Montgomery, 17 Atholl Crescent 1839
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1861 *John Muir, D.C.L., LL.D., 10 Merchiston Avenue 1871
1873 *M. M. Pattison Muir, Esq., Owens College, Manches... 1863
1874 *Thomas Muir, Esq., High School, Glasgow 1855
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1857 Dr John Ivor Murray, The Knowle, near Tunbridge W... 1846
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1874 *James Napier, Esq., Caproun House, Partick 1874
1856 *Hon. Lord Neaves, LL.D., 7 Charlotte Square 1850
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1870 *Dr Henry Alleyne Nicholson, Professor of Civil ... 1868
Natural History, St Andrews 1848
1847 James Nicol, Esq., Prof. Nat. Hist., Aberdeen 1868
1866
1863 *Hon. Lord Ormidale, 14 Moray Place 1873
1848
1837 Dr Richard Parnell, Melrose, 17 Merchiston Avenue 1823
1863 *Dr Alexander Peddie, 15 Rutland Street
1868 *John Dick Peddie, Esq., Architect, 33 Buckingham...
1869 John Pender, Esq., Manchester
1873 *Dr J. Bell Pettigrew, Professor of Medicine and Anat...
St Andrews
1849 W. Pirrie, Esq., Professor of Surgery, Marischal Col...
Aberdeen
1859 *The Right Hon. Lyon Playfair, C.B., LL.D., M.P.
Queensberry Place, South Kensington, London

1834 Mungo Ponton, Esq., W.S., Clifton, Bristol
1874 Baden Henry Baden-Powell, Esq., Forest Department, India
1852 Eyre B. Powell, Esq., Director of Public Instruction, Madras
1865 *James Powrie, Esq., Reswallie, Forfar
1875 E. W. Prevost, Esq., Ph.D., Lecturer on Chemistry at Queenswood College, Hants 250
1849 Hon. B. F. Primrose, 22 Moray Place
1873 Andrew Pritchard, Esq., 87 St Paul's Road, Highbury, London.

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1869 Rev. Thos. Melville Raven, M.A., Crakehall, Bedale
1865 *Rev. Francis Redford, M.A., Rectory, Silloth
1836 David Rhind, Esq., Architect, 54 Great King Street
1875 *Ralph Richardson, Esq., 16 Coates Crescent
1872 Major F. Ignacio Rickard, Government Inspector-General of Mines, Argentine Republic, Buenos Ayres, South America
1840 Martyn J. Roberts, Esq., Crickhowell, South Wales
1872 *Dr D. Argyll Robertson, 40 Queen Street 260
1859 *George Robertson, Esq., C.E., 47 Albany Street
1832 Dr Montgomery Robertson, Mortlake, Surrey
1860 *Dr William Robertson, 28 Albany Street
1862 *Dr E. Ronalds, Bonnington House, Bonnington Road
1870 *Alexander Russel, Esq., 9 Chester Street
1852 *Alex. James Russell, Esq., C.S., 9 Shandwick Place
1837 John Scott Russell, Esq., 5 Westminster Chambers, London
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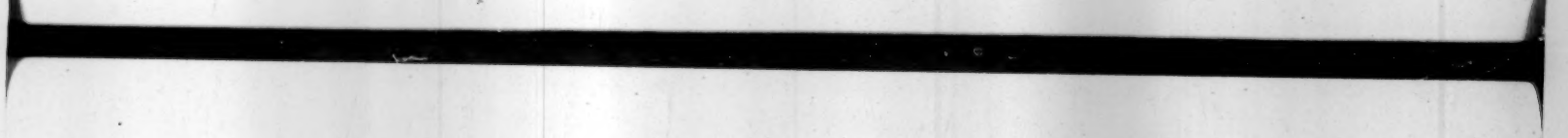
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TRANSACTIONS.

I.—*On the Philological Genius and Character of the Neo-Hellenic Dialect of the Greek Language.* By Professor Blackie.

(Read 16th December 1872.)

Proposition I.—All spoken language is a growth, subject, like the creature who uses it, to a constant course of mutation; it is a living organism, developed according to certain laws, partly inherent, partly superinduced; and, though it is liable to decay, disintegration, and death, this disintegration, except in special cases of extermination, becomes the soil of a new growth, and this death the cradle of a new life. The historical action of this process of mutation is to produce either new varieties or dialects of one language, or new species of one family of languages.

Proposition II.—Though no living language is capable of an absolute stoppage, and, according to the Heraclitan doctrine of πάντα ῥεῖ, must either go on growing or be exterminated, yet there are certain influences at work in the constitution of human society that may retard the process of change to an indefinite period, creating a more or less fixed type, from which deviations are few and far between. These influences are of two kinds, internal and external, or, as we may say, intellectual and political: intellectual, proceeding from the predominant and authoritative force of great creative intellects, such as Homer and Dante; political, proceeding from the unifying effect of a stable form of government, and a permanent type of social order. In other words, the changes that naturally go on in language, as in everything vital, will be impeded and retarded by the traditions of the past so long as these retain a firm hold on the national habit of thought and expression. And the duration of the type of any ancient language will be in the direct ratio of the force of the controlling influences, internal and external.

PROPOSITION III.—In the case of the Greek language, while the internal conservative influences were peculiarly strong, the external were loose and variable. An absolute political cohesion in the Roman style the Greeks never had. Variety by expansion, and dispersion, and consolidation round a number of special social nuclei, was during their most brilliant period the law of their external growth; but during this period, the influence, first, of an Ionic minstrelsy in Asia Minor, and then of an Attic culture in south-eastern Europe, was so strong that it controlled in a very imperial fashion the separative and particularising forces of independent political centres; and afterwards, when a strong central government was established, and continued for many centuries at Constantinople, this unifying influence, acting with the double power of Church and State, though disturbed at first by the intrusion of a strong Roman vein, combined, with an unexampled weight of intellectual and moral tradition, to retard and impede, or practically to ignore, the changes which, by a process of nature, were naturally going on in the Greek language, in an increasing ratio, from the overthrow of the political and intellectual supremacy of Athens by the Macedonians, to the taking of Constantinople by the Turks, and from that time by natural propagation, though with diminished force, up to the present hour.

PROPOSITION IV.—These retarding forces, however, being in a manner artificial, and acting contrary to the natural law of variation by growth, are necessarily limited in their operation, and can, of course, act only where they are felt; that is to say, in those classes of society which are kept constantly under the moulding and controlling influence of the inherited traditions of the past; or, in common language, in the well-educated classes of the community. The uneducated classes, on the contrary, by whom the controlling power of this traditional culture is not felt, or felt only indirectly and with greatly diminished force, go on, partly breaking down old forms of speech, partly sending forth new shoots, so as to form what becomes a distinctly marked dialect of their own; and in this way the language of a whole people in a state of imperfect and inadequate culture may be propagated in two distinct parallel lines, like an upper and a lower stratum in geology, without coalescing into any common type.

PROPOSITION V.—This bistratified condition of a spoken language is exactly what we find realised in the capital of the Byzantine Empire at the time of the Crusades. For here, while a remarkably strong and unbroken chain of literary and ecclesiastical tradition had preserved, with very trivial alterations, the Catholic dialect of the Greek language, of which Attic is the most finished type, the gradual disintegration of an ill-governed empire had combined with

various influences, topographical, and commercial, and social, and political, to shake the language of the great mass of the illiterate masses loose from all precedent, and to favour the growth of a corrupt and hybrid dialect, which, with the aid of favourable circumstances, might in due season shape itself, like the barbarous Latin of the middle ages, into a new language. Of this we have happily the most distinct and clear evidence in the two short poems of the monk THEODORE PTOCHOPRODROMUS, written in the popular dialect, and addressed to the Emperor MANUEL, who came to the throne in the year 1143. These poems are composed, not only, like the Annals of CONSTANTINE MANASSES, who wrote about the same period, with a total disregard of the old classical rhythmical laws, but with a phraseology, and in a style so corrupt and so hybrid, that, even after the lights thrown on the work by DU CANGE, KORAES, and other scholars, not a few passages still remain obscure, and would be much more so were it not that the Latin, which forms one of the chief corrupting elements, is a language with which the readers of Byzantine Greek are generally familiar.

PROPOSITION VI.—We must not suppose, however, from the fact of THEODORE, or any other stray writer of the Byzantine period, having taken it into his head to write a book or two of verses in the corrupt popular dialect, that this dialect had at that time asserted for itself a place, and received a certain recognition in the world of books. Quite the contrary. In those dreary days, there arose no popular genius to stamp the popular dialect with a certain character of limited classicality; but even had the Byzantines of those times had strength to produce a Burns, the traditional Greek of the court, the Church, and all educated intellects, was too strong to allow a mere lyrical variety, fostered in the hotbed of barbarism and corruption, to claim for itself more than a very little corner in a very large vineyard. The consequence was, that while the lower stratum of the spoken language was ripening from day to day into the well-marked form of a new dialect, or even a new language, it does not appear to have advanced a single step out of its ignored position as a literary organ, from the time of THEODORE down to the taking of Constantinople by the Turks in 1453. Byzantine Greek was classical Greek from beginning to end, with only such insignificant changes as altered circumstances, combined with the law of its original genius, naturally produced.

PROPOSITION VII.—By the fall of the last PALÆOLOGUS, the bond of amity which held the motley provinces of the Byzantine Empire together was broken; one of the two strong external links that connected the degraded present with the glorious past was snapped; and with the ruin of the Greek Empire, if the example of the Western Empire was to be a precedent, the death of the Greek language might naturally be expected to follow. But this result did not follow,

and that principally from the action of THREE VERY POWERFUL FORCES. The system of government introduced by the conquering Turks was not such as to render a fusion of the dominant and subject races possible; here was the first element of repulsion; in the domain of religion the repellent force on the side of the vanquished was even stronger; and if we add to these two influences the fact, that the accumulated intellectual forces of ages were all on the same side, we shall have no difficulty in perceiving how the taking of Byzantium by the Turks could have no such effect on the language of the Greeks, as the Lombard reign in Italy had on that of Rome, or the Norman invasion of England, in a much more decided way, on the speech of the Anglo-Saxons. Nor were matters much different in the south-western division of the Greek Empire, where the Venetians and other Franks had parcelled among themselves, in governments of greater or less permanency, the dismembered inheritance of the Byzantine Cæsars; for the Greeks hated the Pope, who had on various occasions endeavoured to deprive them of their ecclesiastical liberties, scarcely with less intensity than they did the Turks, who had deprived them of all liberty; and thus, in Frankish Greece also, the new forces introduced by external conquest were not strong enough to effect the disintegration of the old linguistic inheritance, and the construction of a new language, or even the general recognition of a new dialect.

PROPOSITION VIII.—But in spite of the strong and long-continued action of these retarding forces, nature would have her way; a process of growth was slowly going on, which could not but issue in the formation either of an entirely new language, or of a well-marked species of an old language; and under the continued action of the strong conservative force indicated, the latter was the only result possible. The matter was brought to a practical decision, like so many other significant events in modern times, by the invention of printing and the diffusion of books. By means of these powerful engines, the great storehouses of knowledge were no longer confined to the few, but gradually, as by a well-organised system of irrigation, the refreshing waters were brought down from the far hills, and dispersed through the plains; and an essential part of such a machinery, of course, was the adoption of a language understood by the great mass of the people. If the Greek people were to be raised from the state in which they were kept by their political oppressors, the great preparatory instrument, till an opportunity for physical resistance should present itself, was popular education; and popular education remained impossible so long as the learned wrote in a dialect artificially fed from reservoirs of dead tradition not beating with the living pulses of the present. Under the influence of this patriotic necessity, books of various kinds, especially theological and ecclesiastic, had been issued from the Greek press in a popular

dialect, somewhat similar to, but not nearly so corrupt as that used by Prodromus; and books originally written in classical Greek, like the well-known Church History of Meletius, Bishop of Athens (ob. 1714), were translated into Romaic or modern Greek, just as we modernise Chaucer for the benefit of the million. These patriotic exertions for elevating the popular intellect were brought to a distinctly marked and generally recognised climax by the learned Adamantine Koraes (nat. 1748), a Smyrniote Greek of great learning, philological talent, and ardent patriotism. This distinguished man, living under the inspiring influence of the great French Revolution, showed his countrymen, by precept and example, how it was possible to use the popular dialect according to its own now fully formed type, preserving a well-balanced medium between the classical norm familiar to scholars and the gross barbarisms practised in the most remote districts, and by the rudest portion of the community. This wise and patriotic example, followed generally by a succession of accomplished men, has issued in planting modern Greek, or Neo-Hellenic, as it is now generally called in its perfect form, as one of the recognised types of the great Greek language, on the same platform with the Ionic of Homer and the Doric of Theocritus.

Proposition IX.—In attempting now to state scientifically the specific characteristic differences between the Neo-Hellenic dialect and what we are accustomed to call ancient Greek in all its extent, two important questions occur on the threshold. *First*, what do we mean by a *dialect of a language*, as distinguished from a new language formed from old materials; and from what sources, as a standard, are we to make our inductions with regard to the real philological character of modern Greek? The first question is one which, in theory, it may be very difficult to answer; but practically we may say, that whenever the old materials of a language are so modified as that only a very few words remain in their original form, and that more accidentally than systematically, and when the obscurity arising from this source is increased by the admixture, in larger or smaller quantity, of foreign materials, in this case, as in the examples of *Spanish* and *Italian*, a new language has been created.* But whenever the changes induced on the old materials are comparatively slight and more sporadic than penetrating and pervading in their character, with only a very spare admixture

* The lines in Italian—

"In mare irato, in subita procella,
Invoco te, nostra benigna stella,"

often quoted (C. Lewis' "Romanic Languages," 2d edit., p. 246) to prove the nearness of Italian to Latin, are no proof of the rule in that language, but are altogether exceptive, as any one may perceive by taking a stanza of *Ottava rima* either in Tasso or Ariosto, and counting how very few words in the eight lines have retained the unaltered form of the Latin from which they are derived.

of foreign materials, in this case we shall have only a *new dialect*—not a new *language*. The second question, as to what we may take as a fair standard of modern Greek, can be answered as a matter of fact only by hitting a judicious medium between the two extremes of gross corruption, and that greater or less approximation to the standard of classical Greek, which the practice of some writers presents. Topographical and political causes conspired to create different shades or grades, or types of modification in the popular Greek dialect, which had more or less of a local character. The Byzantine Greek of THEODORUS, the Albanian Greek of the Epirotic KLEPHTS, and the Cretan Greek of CORNARO, who wrote the romance of Erotocritus, in the first half of the eighteenth century (first edition, Venice, 1737), are in some characteristic points essentially different. These constitute what, according to a botanical analogy, we might call local varieties of a common species; and such varieties, as a rule, present a greater amount of deviation from the normal classical type than the floating mass of modern Greek common to all the existing race. On the other hand, since the time of KORAES, there has commenced a process of purification and restoration which tends to remove from the modern language some of those peculiarities which are its most distinctive characteristics. In judging of the language as a whole, therefore, it is wise to take some work or book of an essentially popular character written for general circulation in the last century, before the appearance of KORAES; and I have used for this purpose a translation of the "Arabian Nights" into modern Greek, published at Venice by the well-known house of GLYCYS, in the year 1792. This choice, however, was dictated purely with a view to the conclusions of philological science; for practical purposes, it is manifest that the best type of the Greek actually now spoken in Greece is contained in the Greek newspapers destined for general circulation. But neither can the philologer, though he refuses to accept local varieties, as part of the general norm of the dialect, overlook them as a fact. They are part either of the disintegration of the old type, or of the growth of the new, of which he is bound to take cognisance in all its stages; the more that the phenomena of linguistic change, which are the most interesting to him, present themselves more strikingly in the more corrupt than in the less corrupt forms of the language.

PROPOSITION X. — In examining the processes of modification through which the Neo-Hellenic language has attained its present type, the most obvious helps are, of course, the dictionaries of mediæval Greek by DU CANGE and MEURSIUS, the learned commentaries of KORAES on Theodorus, the dictionary of Byzantine Greek by SOPHOCLES, the dictionaries of modern Greek by GERASIMUS, BENTOTES, KIND, DE HEQUE, BYZANTIUS, and others, with the grammars from THOMAS downwards to that of SOPHOCLES and MULLACH, which is the most

complete. Besides the grammar, MULLACH has edited the "Batrachomyomachia" of DEMETRIOS ZENUS, from which the student will reap benefit; and with him should be taken the collection of mediæval Greek chronicles and poems by ELLISEN. Great zeal has been shown in the same department of Hellenic study by the French, of which the "Collection de Monuments pour servir à l'Etude de la langue Neo-Hellenique," by Monsieur LE GRAND," and the Paris association for the same object, of which Monsieur D'ECHTHAL is the moving genius, furnish ample evidence.* English scholars as a rule have paid little attention to the subject. PASHLEY, TOZER, and the late Viscount STRANGFORD, and the late Professor FELTON in America, were the only English names known to me in connection with this branch of scholarship, till the publication last year of the highly original and ingenious work of M. GELDART; in Scotland special praise is due to Dr CLYDE, and after him to DONALDSON.† The late CORNEWALL LEWIS, unfortunately, was altogether ignorant of this branch of Philology; otherwise, as is evident from a note in his Essay on the Romanic Languages (p. 237, second edit.), he was prepared to have made an admirable use of it.

* (1.) KORAES, ἄτακτα.
(2.) SOPHOCLES' Glossary of Byzantine Greek. London, 1860.
(3.) GERASIMI Thesaurus quatuor Linguarum. Venet. 1723.
(4.) KIND, Handbuch der Neu-grechischen Sprache. Leipzig, 1841.
(5.) DE HEQUE. Paris, 1825.
(6.) Λεξικὸν Ἑλληνικὸν καὶ Γαλλικόν. By BYZANTIUS. Athens, 1846.
(7.) Λεξικὸν τρίγλωσσον. By BENTOTES and BLANTE. Venice, 1820.
(8.) Nova Methodus Linguæ Græcæ vulgaris; auctore THOMA. Paris, 1709.
(9.) Methode pour etudier la langue Grecque moderne, par DAVID. Paris, 1821.
(10.) Grammatica linguæ Græcæ recentioris. Franz, Romæ, 1837.
(11.) SOPHOCLES' Modern Greek Grammar. Hartford, 1842.
(12.) Grammatik der Griechischen Vulgar sprache. Mullach. Berlin, 1856.
(13.) Analekten der Mittel und Neu Griechischen Litteratur, Ellisen. Leipzig, 1855.
(14.) DEMETRII ZENI; Paraphrasis Batrachomyomachia, Mullachius. Berolini, 1837.
(15.) Collection de monuments pour servir à l'Etude de la langue Neo-Hellenique, par EMILE LE GRAND. Paris, 1869.
(16.) Mediæval Greek texts; the Philological Society's extra volume. By W. WAGNER. London, 1870.
(17.) Association pour l'encouragement des Etudes Grecques en France; ANNUAIRES. Paris, 1868-71.

† (1.) PASHLEY; Travels in Crete. London, 1837.
(2.) TOZER; Researches in the Highlands of Turkey. London, 1869.
(3.) Viscount STRANGFORD on the Cretan Dialect in SPRATT's travels in Crete. London, 1865. See also collected works of Viscount STRANGFORD. London, 1871.
(4.) CLYDE; Romaic and Modern Greek compared. Edinburgh, 1855.
(5.) DONALDSON; Modern Greek Grammar. Edinburgh, 1853.
(6.) GELDART; The Modern Greek Language in its Relation to Ancient Greek. Oxford, 1870.
(7.) Professor FELTON published a collection of pieces in modern Greek, which was once in my possession, but of which I cannot now recover the title.

PROPOSITION XI.—When scholars talk of a type of language, such as the strange events of long centuries have produced in modern Greece, they are apt to talk of it altogether as a *corruption*, and as the pure result of phonetic decay. But this is only a part, and a small part of the truth. A language is corrupt when it abandons its own natural analogies, or adopts foreign ones, which do not harmonise with its original type, or when it is defaced and disfigured in various ways by sheer ignorance and carelessness. In this sense it is quite correct in Italian, for instance, to say, that *donna* is a corruption of *domina*, *avuto* of *habitum;* and in the same way, in Neo-Hellenic, to say that ἐβγάλλω is a corruption of ἐκβάλλω, and μαθαίνω of μανθάνω; and of such corruptions, no doubt, a large part of Italian, and a certain much less considerable part of modern Greek is composed. But, on the other hand, it is no corruption when, in the progress of time, an old word comes to be used in a modified, or perhaps, altogether different sense; as when κάμνω, in modern Greek, takes the place of ποιῶ, when σηκόω, which in Plutarch signifies to *weigh*, in modern Greek signifies to *raise*, when φθάνω is used generally for ἀφικνέομαι, to arrive, or παιδεύω, for μαστιγόω, as they are not only in modern Greek, but in the New Testament. Such progressions and transmutations of meaning are always going on in all living languages, of which ample illustrations could be produced from English and every spoken language, if it were necessary. As little can it be called a corruption, when new forms blossom out from old roots, so long as these new forms follow the fair analogies of the language. No one, for instance, supposes that the English language is corrupted when we bring into currency such words as *solidarity*, *complicity*, *utilise*, and similar French formations from Latin roots long ago acknowledged in both languages. In the same way, χρήσιμεύω, νοστιμεύομαι, and other such words, are perfectly legitimate formations, even though it be true that the Attics were not in the habit of affixing the termination εύω to verbals in μος; for the Athenians at all times had their peculiar local idioms, just as London has its Cockneyisms; and the mere Attic usage could prescribe the law to the common Greek tongue, only so long as Athens remained the political and literary capital of Greece, which it ceased to be exclusively when the centre of intellectual activity was transferred first to Alexandria and then to Byzantium. To write pure Attic Greek, after the Athenian literary dictatorship had ceased, was an affectation of which only pedantry or a false fastidiousness could be guilty. The barbarism to which, in a general way, a certain class of scholars would consign such words, is more justly regarded as an overgrowth of lusty vitality. Such new coined words may not indeed be necessary; the old words might have served all purposes equally well; but they are the natural and legitimate product of the right of a living organism to put forth new shoots according to its type. If it be said that a word is always barbarous till it be stamped by the authority of a great classic of the language,

I answer this may be a very healthy limitation which a writer at a certain period of the adult growth of a language may choose to put on his choice of vocables; but it is to the eye of the philologer an arbitrary procedure of which science can take no account. To him *claritudo*, and *gratitudo*, and *beatitudo* are perfectly good Latin words, whether CICERO happens to use them or not; and in such words what a nice Ciceronian of the Bembo school would brand as a departure from the norm of Latin purity, a philologer may often have cause to recognise a natural extension and fair enrichment of a meagre and inadequate medium of expression. If, therefore, HOMER uses not only γλυκύς, but γλυκερός, there is no reason why a scholar should condemn as barbarous varieties of a similar description in the existing Greek tongue; and if the ancients, in their exuberant play of terminational affixes, chose to say ἀληθωνός as well as ἀληθής, shall it be forbidden to the modern Greek to say ταχωνός as well as ταχύς, and βρωμερός as well as βρωμώδης? So in English one writer may say *kingliness* and another *kinglihood* (TENNYSON), with equal right. But further, even in the case of corruptions properly so-called, that is not a new growth of the language, but a breaking down of the old structure from sheer carelessness, or the intrusion of some extraneous element, there is a great and vital distinction to be made between those corruptions, which, to the eye of a philologer, look like a scar or a patch, and such as easily take their place and fit into the old structure of the language. Thus the word γεμάτο, from γέμω, to be full, is a gross barbarism, for it attaches to a Greek root a Latin participial termination, which is at once recognised as something foreign and incongruous. It is as if, instead of the word *obesity*, we should talk of the *fattosity* of a corpulent person, a combination at once felt to be barbarous, though certainly our English tongue, through its loss of native terminations, has an excuse for incongruities of this kind, which could not be pleaded by the Greek. On the other hand, the modern Greek habit of substituting the diminutive for the simple word, and then cutting off the final or accented syllable, as when παιδίον is used for παῖς, and then παιδί for παιδίον, is an offence against the ancient tradition readily condoned; for by the emphasis of the accent on the accented syllable, the ear is already accustomed to the sound, which becomes final when the short ultimate syllable is dropped. In the same manner, the favourite diminutive termination άκι, appearing in γεροντάκι, κοντάκιον, δενδράκι, and scores of other words, where analogically it is not the diminutive termination pertaining to such roots, passes lightly into the habit of the ear from the analogy of μειράκιον, and other such words, where the ακ is part of the root. Of superficial and false analogies of this kind, both in flexion and in syntax, the structure of all languages gives sufficient instances.

PROPOSITION XII.—Following out the principles just stated, the first notice-

able characteristic that strikes us in the Neo-Hellenic dialect, is the remarkable change and extension that has taken place in the usage of certain words, giving rise to significations either altogether different from classical usage, or deriving from that usage only their late and scarcely apparent germ. Of these I shall here set down some of the more remarkable.

	Modern.	Ancient.
σηκόω,	to raise,	weigh, balance.
βαστάζω,	to refrain, restrain,	carry.
καμαρώνω (όω),	to give one's self airs, to admire,	vault or arch.
τάσσω,	to promise,	arrange.
συντρέχω,	to assist,	run together.
ἰδίωμα,	manners,	a peculiarity.
λαλῶ,	sing,	prate.
χώνω,	conceal,	heap up.
σώνω (σώζω),	suffice,	save.
κύριος,	a father,	a master.
κομβόω,	deceive,	to bind up, gird.
χωνεύω,	digest,	cast metal.
ψοφάω,	to die (of an animal),	to make a slight noise.
βραδύ,	the evening,	slow.
πρόξενος,	a match-maker,	patronus (in politics).
πλακόω,	oppress, overwhelm,	lay with flat plates.
ἐξωτική,	a fairy,	extraneous, foreign.
ἀψυχῶ,	to spare life,	to die.
μετεώρισμα,	an amusing tale,	floating in the air.
βασιλεύω,	to set (of the sun),	reign.
ποδαπός,	a person of no account,	from whence.
χάνω,	lose,	gape.
αὐγή,	morning,	brightness.
βάνω (βαίνω),	place, set,	go.
φθάνω,	to arrive,	to get before.
ἄνοιξις,	the spring,	opening.
πλάτη,	shoulder,	flat part of an oar.
ὑποκείμενον,	a person,	a subject.
ἀράσσω,	to land,	to crash.
σκιάζομαι,	to be afraid,	to shade one's self.
ἀδειάζω,	to fire a gun,	to give an amnesty.
πνευματικό,	a clergyman,	windy.
ταχύ,	the morning,	swift.

	Modern.	Ancient.
γεῦμα,	dinner,	taste, smack.
τὰ σωστά,	sense, wits,	things saved.
συμπάθεια,	pardon,	sympathy.
σκοτίζω,	stun, astonish,	darken.

Proposition XIII.—But these changes of meaning and application in old words, however strange, and however important to be noticed as presenting a practical difficulty to a student passing from the classical to the modern dialect, are only what must take place in any language, which for the space of more than a thousand years has been allowed to float freely on the surface of the popular mind without any central authoritative control. More characteristic of the peculiar genius of the Greek tongue, is the luxuriant growth of new terminations to old roots, especially verbal, which the living dialect presents. This habit of luxuriating in a variety of verbal forms differing nothing in signification, but displaying only, to adopt a botanical phrase, a greater breadth of terminal blossom, was characteristic of Hellenic speech from the earliest times. Thus, from the adjective ὅμαλος we have the three equivalent forms ὁμαλίζω, ὁμαλύνω, and ὁμαλόω, for which our meagre English is only too happy to find that it possesses the single *equalise*; and in the same way, from the root σεβ, we have the verbal forms σέβομαι, σεβάζομαι, σεβίζω, and later σεβασμιάζω (in Zonaras), all perfectly identical in signification. And though, no doubt, it may seem that άζω, as in ἑλκυστάζω, and ύζω as in ἑρπύζω, had originally a frequentative meaning, yet this distinctive feature was lost in the growth of the language; and between στένω and στενάζω there is only a difference of sound, as, in Latin, between *claritas* and *claritudo*. Following out this instinct of rich terminational ramification and effloresence, the current Greek vocabulary presen such varieties as the following:—

Modern.	Ancient.
ἀρχίζω and ἀρχινίζω,	ἀρχομαι.
δακρύζω,	δακρύω.
πασχίζω,	πάσχω.
φοβερίζω,	φοβέω.
τρομ ΄ζω,	τρομέω.
ξυραφίζω,	ξύρω.
ἀραχνιάζω,	ἀραχνιόω.
λαβόω, to catch, hit, wound,	λαμβάνω.
λογιάζω,	λογίζομαι.
μελανιάζω,	μελανίζομαι.
σκοτεινιάζω,	σκοτίζομαι.
χορταίνω,	χορτάζω.

All these are legitimate formations, being either parallel forms from the original root, or secondary verbs from the substantive of a primary verb, or the adjective connected with the substantive. Not unfrequently this luxuriance of terminational blossom shoots out into pregnant new formations, which have no parallel in the classical tongue, as in

πλαγιάζω,	to fling one's self down across the mattress and sleep.
ξεκαρδίζομαι,	my heart leaps out of my body, as with violent joy.
ἀρκούδίζω,	to go on all fours like a bear, as children do.

A similar lustihood of terminational vitality—a point where our English tongue is so remarkably feeble—displays itself in the terminations of adjectives and substantives, some of which are authorised by ancient analogy, as in βρωμερός, for δυσώδης, and others are a separate creation of the modern linguistic instinct, as in φρονιμάδα, νοστιμάδα, and the whole family of abstract nouns in άδα.

Proposition XIV.—But it is not only in terminational variety that the ample vitality of the living Greek tongue asserts itself. As in the ancient, so in the modern dialect, the tendency to a florid growth of ever new compound words is irresistible. It is in the domain of the verbs again where this tendency exhibits itself most strongly. Here, of course, the compounding elements are often prepositions; but other elements are not excluded; and not a few very expressive compounds are formed from elements of which, for this purpose, the ancients made little or no use. Let the following serve as examples:—

ἀνακατόνω (όω),	to turn upside down.
στρεφογυρίζω,	to turn round and round.
ἀποκαμαρόνω,	to contemplate with admiration.
ξανθογυρομάλλος,	having red and curly hair.
πολυσκουριάζω,	to cover with rust.
χειλημουκτρίζω,	to neigh.
ἐκκοκκινίζω,	to turn pale, *lit.*, to go out of the red.
καλοκυττάζω,	to look favourably on.
γλυκυφιλῶ,	to give sweet kisses.
γλυκυχαράζω,	to break sweetly, as of the dawn.
γλυκυκυττάζω,	to look sweetly on.
κακοκαρδίζω,	to displease.
κακοκυττάζω,	to look with an envious eye on.
κακοφαίνεται,	it displeases me.
ἀστροπελεκίζω,	to lighten, *lit.*, to send down meteor-axes.
χαμογέλασμα,	a slight smile.

χαροκόπος,	a bon vivant, a man of pleasure, a voluptuary.
μοσχομυρίζομαι, μοσχοβολῶ,	to be fragrant, odorous.
κλωθογυρίζω,	to spin round and round and lose one's way.
ὀφθαλμοφανῶς,	for ἐναργῶς, *i.e.*, clearly, distinctly, bodily.
ἀναποδιάζω,	to do a thing perversely, awkwardly.
ψυχοπαιδίον,	an adopted son, son of my soul.
ψυχοπονέω,	to sympathise.
μονοπάτιον,	a footpath.
παλαιογουρνομύτης,	a swine-snouted old sot.
κοσμογυριστής,	a world-perambulator, said of the sun.
ἀναποδογυρίζω,	to turn upside down.
συμποσόω,	to amount to.
βρεφουργῶ,	to make incarnate.
σπαταλοκρομάδης,	an onion-gourmand.
βομβοκτυπίζω,	to din the ears, *obtundo*.
μοσχοκάρυον,	nutmeg.
ἐποφθαλμιῶ,	to cast an envious eye on.
ἐκμυστηριάζομαι,	to reveal.
κρασοπατέρας	a wine-bibber.

These examples, and a host of others that might be adduced, show how absurd it is to class under the head of corruption those changes in a long-lived language, which indicate a buoyant juvenescence rather than a withered decrepitude of expression. Let us now turn our attention to those changes in the form of the language, which fall distinctly under the category of LOSS or ABATEMENT, though by no means necessarily under that of DEFACEMENT and DISFIGUREMENT.

PROPOSITION XV.—As language, in long ages of neglect and semi-barbarism, is used by the masses principally for purposes of convenience, it is plain that considerations of an æsthetical nature, such as influence men of genius and high culture in their use of language, will be subordinated ; and the purely scientific considerations, on which the philologer puts a high value, will not influence the popular mind at all. Carelessness, convenience, custom, and sometimes mere freak, fashion, and the itch of novelty, will produce important changes in the structure of a language which a delicate sense of harmony, and a scientific perception of organic completeness, would alike repudiate. Among the phenomena of this kind which continually tend to break down the classical form of words, those known to grammarians under the heads of *aphæresis*, *apocope*, and *syncope*, are the most frequent. But, not to encumber a plain

subject with learned phraseology, we shall say simply, that all cultivated languages, when used merely for convenience, without the continued check of a higher aim, are liable to have their vocabulary changed by a process of CURTAILMENT, which makes a part of the word serve for the whole. Thus in America, from the rattling haste in which the people delight, "an acute man" is called "a 'cute man;" from the same careless instinct, the ignorant English peasant, or the sharp London street boy, talks of the "varsity" instead of the "university;" and the familiar words, *bus* and *cab*, are only the tail and head respectively of two polysyllabic words, borrowed the one from the Latin, the other from the French. That this is a corruption, not of a very elegant kind, no person will deny; for even when the original form of the word may have died out from the popular memory, it requires only a little bookish culture to make one feel that a segment has been cut from the full sound of the thing, the absence of which makes itself felt. Curtailments of this kind are obvious everywhere in French and Italian, and in not a few German words also derived from Latin and Greek, as in *probst* for *propositus*, *pfingst* for πεντηκοστή, and such like. Now, it is manifest that these corruptions may be made in various ways. Sometimes only a weak initial or final letter or short syllable may be dropt, which leaves the word not much the worse; sometimes a half, and that not the most important half, may be left, after the amputation, and in such a fashion it may be, that only a scientific eye can recognise its original identity. One of the commonest forms of curtailment in Greek is that of an initial short vowel, of which the following list contains some of the most common:—

λίγος,	for	ὀλίγος.	μπορῶ,	for	ἐμπορῶ.
μαλιά,	,,	ἅμιλλα.	τέρι,	,,	ἑταῖρος.
μιλῶ,	,,	ὁμιλῶ.	πωρικά,	,,	ὀπωρικά.
πές,	,,	εἰπές (ἐιπέ).	βρίσκω,	,,	εὑρίσκω.
νός,	,,	ἑνός.	βλόγια,	,,	εὐλόγια.
ψηλός,	,,	ὑψηλός.*	λημέρι,	,,	ὁλημέρι.
μέρα,	,,	ἡμέρα.	βαγγέλιον,	,,	εὐαγγέλιον.
μερόνω,	,,	ἡμερόω.	πεθυμιά,	,,	ἐπιθυμία.
διόμα,	,,	ἰδίωμα.	γούμενος,	,,	ἡγούμενος.
πανδρεύομαι,	,,	ὑπανδρεύομαι.	σφαλίζω,	,,	ἀσφαλίζω.
σκύπτω,	,,	εἰσκύπτω.	παντυχαίνω,	,,	ἀπαντυχαίνω.
γιαλος,	,,	αἰγιαλός.	φετεινός,	,,	ἐφέτινος.
ῥέγομαι,	,,	ὄρέγομαι.	ματώνω,	,,	αἱματόω.
σάζω,	,,	ἰσάζω.	ῥημοκλῆσι,	,,	ἔρημα ἐκκλησία.
σώθικα,	,,	ἐσώθικα, *i.e.*, ἐντόσθια.	πετραχήλι,	,,	ἐπιτραχήλιον.
μβάινω,	,,	ἐμβαίνω.	γειά,	,,	ὑγίεια.

* The highest peak of Mount Ida in Crete is now called *Psiloriti*, or High-mount.

πίσω	for ὀπίσω.	λάμνω	for ἐλαύνω.
μᾶς	,, ἡμᾶς.	γλυτόνω	,, ἐκλυτόω = ἐκλύω from λυτός.

In these cases of initial curtailment it will be seen that the vowel which falls off, as in the case of the American "'cute," is generally a very feeble one, such as in rapid pronunciation is little missed. Sometimes, however, it is a diphthong, though never an accented one, as in ματόω for αἱματόω, δὲν for οὐδὲν, γιαλός for αἰγιαλός, and some others. It may even be a whole syllable, like the Italian *scuro* for *obscuro*, as in δάσκαλος for διδάσκαλος.*

PROPOSITION XVI.—But the end of the word presents to a hasty speaker even greater facilities than the beginning for a popular amputation, of which tendency the final *m* and *s* of the ancient Roman poets afford familiar examples. And here we find a remarkable analogy pervading old Roman, Italian, and modern Greek. As the Roman dropt his final *m*, producing the Italian *domino*, from the perfect form *dominum*, so the modern Greek regularly drops the final ν, which with him corresponds to the Latin *m*, and says καλό for καλόν; and not only so, but in diminutives he drops regularly the complete last syllable ον, so that all diminutives which are paroxytones in classical Greek, come out with a sharp acute accent on the final syllable. Thus παιδίον becomes παιδί, κρασίον, κρασί,† just as in Italian *amavit* becomes *amò*, *potestatem*, *podestà*, and so forth. When the original word has the oxytone accent, as in ποταμός, κεφαλή, the accent of the curtailed diminutive lies on the penult

* The following examples from Italian illustrate the same tendency to initial curtailment in languages set free from the control of strict literary aristocracy :—

bieco	for	obliquum.
scuro	,,	obscurum.
sciocco	,,	exscuccus.
badia	,,	abbadia.
cagionè	,,	occasione.
stivali	,,	æstivale.
ricciò	,,	cirricius.
Lamagna	,,	Alemannia,
rame	,,	æramen.
nemico	,,	inimico.
stimare	,,	æstimare.
state	,,	æstate.
sperto	,,	esperto.
spietato	,,	dispietato.
sbarcare	,,	disbarcare, and other compounds with *dis*.
romita	,,	eremita.
stra (French *très*)	,,	extra, as in straordinario, and other compounds.
mentre	,,	dum-interim (Diez).

† The historic steps in the process were παιδίον, παιδίν, παιδί, the intermediate form appearing generally in THEODORUS.

ποτάμι, κεφάλι, and many others. Similarly, a final oxytone in prepositions may fall off, as in ἀπ for ἀπό.

PROPOSITION XVII.—It is manifest that both these kinds of amputation, by head and by tail, may be exercised upon the same word; and then there arises sometimes a new word consisting only of the middle syllable, or two middle syllables of the amputated diminutive, which it requires an exercised eye to detect. Of this double curtailment the following are a few familiar examples:—

shore,	γίαλι,	from	αἰγίαλιον,	αἰγιαλός.
eye,	μάτι,	,,	ὀμμάτιον,	ὄμμα.
fish,	ψάρι,	,,	ὀψάριον,	ὄψον.
companies,	τέρι,	,,	ἑταῖρος,	
oil,	λάδι,	,,	ἐλάδιον,	ἔλαιον.
vinegar,	ξίδι,	,,	ὀξύδιον,	ὀξύ.
house,	σπίτι,	,,	ὀσπίτιον, (Lat.) hospitium.	

A perfect analogy to this system of double amputation occurs in many English words, when their present form is compared with the original German word. Thus, our "*fought*," is "*foughten*" in old English, and *gefochten* in German.

PROPOSITION XVIII.—As the Greek verb is the part of speech in which the formative instinct of that rich language blossoms out most luxuriantly, we should expect that in this domain the pernicious, or it may be in this case perhaps, the beneficial effects of amputation will best appear. And so in fact it is. For not only have individual verbal affixes and prefixes been lopped off, but whole tenses and moods totally disappear, to supply whose place auxiliary verbs after the fashion of the modern languages are freely used. But as this is a matter that affects largely the syntax of the language, I shall reserve what is to be said on these organic losses for another section. In this place it will be sufficient to say, that of verbal curtailments falling under the heads of aphæresis and apocope, and not affecting the syntax of the language, the three following seem to be the principal:—(1.) The very irregular use and frequent omission of the augment. In this, however, it is superfluous to mention that the moderns follow the example of Homer, and that the loss is mostly as unimportant as that of the reduplicated second aorist of the poet was to the Athenians. (2.) The reduplication before the perfect participle passive is omitted—γραμμένος for γεγραμμένος. This is a change precisely analogous to that which the German has suffered in passing into English, as in *given* for *gegeben*, and so forth. (3.) The infinitive of the present infinitive active, after cutting off the terminative ν, becomes ει, as γράφει for γράφειν, and that of the first aorist passive,

in the same way after cutting of the terminal *ναι*, becomes *ῆ*, as *ἐλευθερωθῆ*, for *ἐλευθερωθῆναι*. This again forms a perfect analogy to the process by which the *ν*, which originally belonged to the full form of the infinitive, as *ἔμμεναι*, *ἔμμεν*, *εἶναι*, and which appears in the German, *loben*, *geben* is dropt in English, so that only the monosyllables *love* and *give* remain.

Proposition XIX.—A very peculiar and characteristic species of initial curtailment is that which takes place when a preposition precedes the definite article, and being absorbed by it, is pronounced as one word. Thus, *εἰς τὴν πόλιν*, *to the city*, by curtailment of the initial diphthong and absorption of the remaining consonant by the following article, become *στην πόλιν*, or *σταμπόλιν*, where the Dorism of *α* prevails, whence the vulgar *Stamboul* arising from a misunderstanding of the Franks. In the same way the ancient Cos was supposed to have been metamorphosed into *Stanco* (*ἐς τὰν Κω*); and in Crete, according to Spratt,* they have actually *Νήδα* from *ἔις τὰν Ιδα*. A similar error, arising from the ignorance of the Lowlanders, occurs in the Highlands, where *Loch Nell*, for instance, near Oban, receives its barbarous and unintelligible Saxon designation from the absorption of the definite article *na* by the substantive *eala*, signifying a *swan*, as is the case also with *Loch Ness*, *Moness*, and other such names, occurring not unfrequently in the topography of the Highlands, where the Gaelic *ais*, signifying a waterfall, attaching to itself the *n* of the definite article, presents to the uninstructed the false appearance of an independent word *ness*. In the same way the substantive verb vanishes into the following noun: as *smellum so* = *is maith leum so*, *this is good with me*, *I like*; and *spik orm*, for *is beag orm*, *this is little on me*, *I dislike*. It is remarkable that in German it is not the preposition in such cases, but the article that is absorbed. Thus *zu der* becomes *zur*, *zu dem*, *zum*, *in das*, *in's*, and so forth. In Italian sometimes both the article and the preposition are curtailed, as in *pel* for *per lo*, *nel* for *in illo*.

Proposition XX.—Of *syncope*, or the dropping of consonants in the middle of a word, and of *synizesis*, or the slurring of two consecutive vowels into one syllable, there are not wanting examples in Romaic; but, as they are neither so frequent as the initial and final curtailment, nor occasion the same difficulty to the student, they may be slightly mentioned here. As in Homer we have *κάββαλε* for *κατέλαβε*, and such like, so in Romaic we have *συβάζω* for *συμβιβάζω*, *σπίθη* for *σπυθήρ*, *κάνω* for *κάμνω*, *νύφη* for *νύμφη*, *φάιτο* for *φάγετο*, *λάιι* for *λαγένιον*, *πεντήντα* for *πεντήκοντα*, and with a large initial curtailment *σαράντα* for *τεσσαράκοντα*. Two of the most familiar examples of this medial curtailment are those which take place on the verbs *λέγω* and *ὑπάγω*. When I

* Travels in Crete, by Captain Spratt. London 1865.

was living a lodger in Athens, about twenty years ago, a little girl, the landlord's daughter, as I was going out, used to say to me πού πάτε. When I heard this first I was considerably puzzled, and began at once to think of πατέω and *pattens;* but this scent led me far astray, and on inquiry I found that the mystical dissyllable was only a curtailed form of ὑπάγετε, as that word is used in the New Testament (Matthew xvi. 23). The same habit of dropping the medial gamma, I afterwards found, led to the forms λές, λέτε, λέν, familiarly used for λέγεις, λέγετε, and λέγουν ; and in like manner φᾶν stands for φάγουν. (So in CHAUCER *han* for *haben* = *have.*) As to *synizesis,* or the slurring of two vowels into one syllable, which the readers of the Attic plays are quite familiar with, the modern Greek makes a systematic use of this figure in words of the first declension, such as σοφία, φωτία, καρδία, accompanying the slur of the slender vowel with the transference of the accent to the strong final vowel καρδιά. Sometimes in such cases the slight vowel is omitted altogether, as κυρά for κυρία, *lady, mother.*

Closely connected with *synizesis* is the practice of *crasis,* so perplexing to the tyro in Sanscrit, and familiar to the ancient Greek in such cases as ἀνήρ for ὁ ἀνήρ, ὦνερ for ὦ ἄνερ, and others. The most common phenomena of this kind in modern Greek, and on which a classical scholar will be apt to stumble, are such forms as—

νᾶσαι,	for	ἴνα εἶσαι,
νἄλθῃ,	,,	ἵνα ἔλθῃ,
νᾶχει,	,,	ἵνα ἔχει,
τώχει,	,,	τὸ ἔχει,
πώρχεται,	,,	ὅπου ἔρχεται,
τὠπα,	,,	τὸ εἶπα,

and others of a similar description, which occur frequently in the Erotocritus.

PROPOSITION XXI.—These remarks taken singly, might naturally lead to the idea that modern Greek is a sort of amputated ancient Greek, as the Saxon half of English is a sort of amputated German—a meagre dialect in which every dissyllable has been cropped down into a monosyllable, and every trisyllable into a dissyllable. But this is by no means the case. Miserable and meagre, in point of vocal swell and syllabic roll, as our truncated Saxon tongue would have been, had it not been reinforced by the strong intrusion of the sonorous element from the classical languages, the tongue of the ancient Hellenes has suffered no such loss as to produce any bald disfigurement of this kind. The explanation of this is obvious. The words of the Greek language are so exuberantly polysyllabic, that the abstraction of a single syllable from each word would leave the great body of the language still distinctly polysyllabic ; thus, ᾽πιθυμῶ falls upon the ear with pretty much the same amplitude

as the full form ἐπιθυμῶ. But further, the fact is that the curtailments of which we have spoken affect only a limited class of words; and, singularly enough, the largest class to which apocope is applied contains a compensation which leaves the apocopated word as many syllables, or perhaps more than the original word of the classical tongue. This apparent paradox arises out of that substitution of diminutives for the original word, which we have already noted as characteristic of the modern dialect. For the diminutive in Greek, and all the Aryan languages, while it lessens the idea, increases the magnitude of the word, as in βρέφος, βρεφύλλιον, μεῖραξ, μειράκιον, σάρξ, σαρκάριον, and so forth, from which the consequence follows, that where a diminutive is systematically substituted for the simple word, and afterwards apocopated, the syllabic magnitude of the word is not diminished. Sometimes it may even be increased, as παιδί, παιδίον, παῖς; ποτάμι, ποτάμιον, ποταμός; γεροντάκι, γεροντάκιον, γερόντιον γέρων. If to these considerations we add the exuberant, amplificative and expansive tendency of the Greek language, and its delight in the formation of new compounds (p. 12, above), we shall be prepared to believe that the existing Greek language is no wise inferior to its classical prototype in point of syllabic luxuriance, and the student of ARISTOPHANES, whose ear swells with pleasure at the σφραγιδονυχαργοκομήτης, and other sesquipedalian luxuries of the jovial Attic comedian, will not be surprised at encountering χαβιαροκατελυτος, σκουμπροπαλαμιδοπαστός, ἐγγραυλοπαστοφάγος, and such like, in his meagre Byzantine follower.

PROPOSITION XXII.—The practice of substituting the *diminutive* regularly for the simple word, which we have just mentioned, deserves further the special remark, that besides being a characteristic of Italian, as in *sorella, fratello, uccello,* &c., it had its origin in the earliest classical times, as we see in the Latin *oculus,* from the old root ὄκκος (HESYCH), identical with ἀνγή, German *auge,* Sanscrit *akshi,* in *auricula,* Italian *orecchia,* for *auris,* and in the classical Greek θηρίονν, πεδίον, and a few others. In ARISTOPHANES the frequent use of the diminutives must strike every reader; and we see here, as in most other instances, that what we call modern Greek is merely the natural development and full growth of tendencies deeply rooted in the heart, if not always visible on the surface of the classical tongue.

PROPOSITION XXIII.—The chapter of curtailment on which we have been dilating, so common in the progress of all languages, suggests, as its natural complement and counterpart, the chapter of *addition* or *increment,* whether that addition be real, that is, an appendage posteriorly added to the root, or only a part of the root, by change of circumstance brought to the surface after a long period of concealment, as happens to seeds sometimes by the process of

deep trenching. It is a well-known fact in the classical Greek language, that familiar words sometimes appear in two forms, the one differing from the other only by the addition of an appended letter or syllable,—an appendage which, when it appears in the front of the word, is technically called *prosthetic*, and when at the end *paragogic*. Thus we have χθές and ἐχθές, κεῖνος and ἐκεῖνος, μαυρόω and ἀμαυρόω, and not a few others. The same phenomenon appears largely in the comparison of different languages of the same family, as in *dent*, ὀδοντ, ὀφρύς and our *brow*, ἀστήρ (Gaelic, *bruach*), *star*, *tara*, τείρεα (Hom.) Now, though in many of these cases it is quite plain that curtailment has taken place, as when *pater* becomes *athar* in Gaelic, and *plenus lan*, it seems pretty certain that in others the taste or fashion of some particular time and place has added a letter to the original root. For this adhibition there may be various causes, demonstrative emphasis perhaps, as in the *celui ci* and *celui la* of the French, or mere euphony, as in the favourite habit of the Italian ear, which leads them to avoid a consonantal ending; thus *sunt* becomes not *son*, but *sono*. To determine the history and significance of these prosthetic and paragogic letters, is in many cases one of the most difficult tasks of scholarship; and on some of these problems the masculine erudition of one of the greatest of German scholars has been not unworthily spent;* but to enter into such discussions here would lead too far from the main purpose of this discourse; so I shall merely set down a few of the more notable of these enlargements of the old Greek form which the existing popular dialect presents, without speculating curiously on their origin. In the first place, we have sometimes, though far from regularly, a vowel appended to the third person plural of the indicative mood, λέγουνε for λέγουν, ἐλέγανε for ἔλεγαν, quite in the Italian fashion. Similarly, among the pronouns we meet frequently with τόνε for τὸν, just as the Athenians said οὐτοσί for οὗτος, and ἐκεινοσί for ἐκεῖνος; then for ἀυτοῦ we have αὐτουνοῦ, for ἀυτῆς, αὐτηνῆς, and αὐτηνῆς τῆς. Among the particles we have ἀυτίς for ἀυτί, and τότες for τότε. This final ς appears in the imperative of certain verbs, as in εἰπές, by apocope 'πές, following the ancient analogy of δός and θές. The accusative of the pronouns of the first and second persons presents the lengthened forms of ἐμένα and ἐσένα. The demonstrative, on the other hand, is lengthened only in the front, and τοῦυτο becomes ἐτοῦυτο. Τοῖος is subject to a reduplication, and becomes τέτοιος. A prosthetic ι before two initial consonants, of which the first is σ, is familiar to the student of the Romanic languages, and appears in the modern Greek ἴσκιον for σκία.† This initial σ itself is added in some words, as in σκόνη

* Pathologia Græci Sermonis. Lobeck, Konigsberg, 1853.

† This prosthetic *i* appears regularly in Italian when the previous word ends in a consonant, as *con isdegno* for *con sdegno*. But this is only the occasional form of the word for the sake of euphony; the rule is, that while in Italian the initial *e* or *i* in such cases is never added, but regularly rejected, in Provencal, Spanish, and early popular French, it is always prefixed, even where no traces of it are found in Latin. Cornewall Lewis on the "Romanic Languages," 2d edit., p. 107.

for κόνις, just as in the classical dialect μικρὸς and σμικρὸς appear in friendly company. The letter γ, which appears sometimes as a prosthetic letter, seems to be a modification of the rough breathing, as in γυιός for ὑιός, γιεράκι for ἱεράκιον, from ἴεραξ. But this γ has a tendency to show itself, not only in the front of words, but specially in the middle between two vowels, by the figure known to the old grammarians by the name of epenthesis. Thus we have—

σαλένγω,	for	σαλεύω.
γυρεύγω,	,,	γυρεύω.
ἀγοῦρος,	,,	ἄωρος.
καβαλλικεύγω,	,,	καβαλλικεύω.
αὔγον,	,,	ᾠόν.
θηργιό,	,,	θηριό = θηρίον.
λάγουτα,	,,	λαοῦτα (lute).
πλέγω,	,,	πλέω.
ῥαγίζω,	,,	ῥαΐζω.
στέργιος,	,,	στέρεος.
βογγάω,	,,	βοάω.
ἀγναντεύω,	from	ἐναντίος.

That this epenthetic γ is a relic of the famous digamma, which plays such a redoubtable part in the criticism of the Homeric text, is a favourite notion with scholars. That it actually is so in some cases seems probable; scarcely in all, I should think, or in the majority of cases.

One of the most notable epenthetic lengthenings of classical words occurs in the present indicative of certain classes of verbs, by the insertion of ν. Thus, in all liquid verbs, for στέλλω, στέλνω, for φέρω, φέρνω, κολνάω for κολλάω, a fashion clearly traceable in the New Testament. There we have δέρνω for δείρω, ὀπτάνομαι for ὄπτομαι, χύνω for χέω, and others. In fact, this peculiarity is as old as HOMER, who has θύνω and δύνω for θύω and δύω, and the lengthened form of ἀγωνέω for ἄγω. We need not, therefore, be surprised at its great prevalence in modern Greek. From the Alexandrians downwards, the people have always been in the habit of inserting a ν before the final vowel of the large class of verbs in όω; as in the common σκοτόνω, to *darken one's day-lights*—to *kill*, φανερόνω, καμαρόνω, and many others. But this ν is found also in other and less marked cases, as in ἀφίνω from ἀφίημι, σώνω for σώζω, βάνω for βάζω, χαρίνω for χαρίζω, ψένω for ἑψω, ψάχνω for ψαύω. This is a peculiarity indeed, to which every one must tune his ear who wishes to read modern Greek with any comfort.*

* Mr GELDART (c. 4) remarks that, "in ancient Greek we may regard αίνω as a strengthened form of έω, and άνω, as a strengthened form of άω." This may be quite true; only in such verbs as λευκαίνω, the ν is retained throughout all the parts of the verb, whereas in the modern σκοτόνω, and such like, it belongs only to the present indicative.

The remarkable paragoge of κα, which appears in the vulgar form of the first aorist passive, ἐγράφθηκα for ἐγράφθην, appears to have arisen from an illiterate confusion of the terminations of the past forms of the active and passive voice.

PROPOSITION XXIV.—The process of partial disintegration and reconstruction to which modern Greek, like Italian and French, owes its birth, is governed by a law, which seems to belong naturally to all human speech under similar circumstances, what I may call the LAW of SIMPLIFICATION and REGULARISATION. All language, by the natural action of the human mind, is constructed originally on the principle of likeness and analogy; but when, by the mixing of diverse tribes, or from other causes, irregularities and anomalies have once got a place in it, which disregard or contradict its characteristic analogies, the establishment of a classical norm of speech, by a succession of authoritative writers, may stereotype such irregularities as a recognised part of the language for an indefinite period. Thus in English, the irregular plurals—fragments, by the way, of an old regularity—*men*, *women*, *oxen*, and a few others, remain fixed; and so in Greek, the aoristic forms in κα, ἔθηκα, ἔδωκα, ἦκα, contrary to the general rule of verbal formation. But so soon as the firm control of intellectual authority is weakened, as by the removal of some artificial constraint, the popular ear falls back on the familiar analogy, and abolishes the anomaly; and thus in modern Greek, ἔθηκα becomes ἔθεσα, ἔδωκα, ἔδωσα, ἀνέγνων, ἀνέγνωσα (also in ancient Ionic), and so forth. A familiar example of this tendency is the *seèd* for *seen*, *coomed* for *come*, &c., of the common people in England. Similarly we find in Greek ἐφέρθην from φέρω, βαλμένος for βεβλημένος, καλεσμένος for κεκλημένος. To the same category belongs the total abolition of all verbs in μι, the relic, as is well known, of the oldest form of the verb in the Aryan languages. Thus, as early as the New Testament epoch, from the general use of ἔστηκα as a present, the new regular form στέκω had been produced, which is the only form of the old ἵστημι now used in the Greek tongue. So for δίδωμι we have now δίδω, for τίθημι, a secondary verb, θέτω, formed from the verbal adjective θετός, like ἀστατέω (1 Cor. iv. 11) from ἄστατος, and στατός, for ἀφίημι, ἀφίνω, and for παρίστημι, παριστάνω (Rom. xii. 1). So much for the verbs. Among the nouns this tendency displays itself most strikingly, as in Italian, in the habit of taking the objective case of nouns of the third declension, and turning it into a new nominative, to be declined after the fashion of the first or second declension.* Thus,—

ἡ μητέρα,	from	μήτηρ.
ἡ πεδιάδα,	,,	πεδιάς.
ἡ ἀγελάδα,	,,	ἀγελάς.

* CORNEWALL LEWIS, in his essay on the Romanic Languages (2d edit., 1862, p. 91), while tracing this tendency to prefer the accusative case through all the Romanic languages, says that "he is unable to

ἡ φλέβα, from φλέψ.
ἡ φτέρα, „ πτερίς.
ἡ φοράδα, „ φοράς.

And this termination άδα, when it had once caught the popular ear, became a favourite norm for the formation of abstract substantives even when there was no objective case of the third declension from which to make the transfer. Thus,—

For λαμπρότης, λαμπυράδα.
For φρόνησις, φρονημάδα.
From νόστιμος, νοστιμάδα.

In the same way with masculines,—

For ἔρως, ἔρωτας.
„ πατήρ, πατέρας.
„ ἀήρ, ἀέρας.
„ βασιλεύς, βασιλιάς.
„ Rex (*Lat.*) Ρήγας.

For, in fact, ς in non Attic Greek was the favourite termination of masculine agents, as we see in *Cosmas, Ducas,* and other proper names of the Middle Ages. So for the Attic ἀρτοποιός they preferred the shorter form ἀρτάς, for ὡρολογοποιός, ὡρολογάς, and many others. The prevalence of this termination is specially marked by the systematic use of the form βλέπωντας for βλέπων as the nominative of the participle, and that in both genders. The termination ος of the second declension is sometimes used in the same way, as γέρος for γέρων, δράκος for δράκων; of which confusion we have examples also in the classic authors, as ἀλάστορος (HOMER, ÆSCHYL., and SOPH.) for ἀλάστωρ, μάρτυρος for μάρτυρ. To the same category of regularisation belong, of course, the forms καλήτερος, πολλότατος, μεγαλήτερος, μεγαλότατος, for the well-known irregular forms of the classical grammar.

PROPOSITION XXV.—The previous observations relate exclusively to the changes that time has wrought on individual words, apart from their relation to one another. Now, the elements of language which indicate the method of the connection of one word with another, or of one sentence with another, are

suggest any very satisfactory explanation" of the phenomenon. The explanation seems simply to be that, in the case of most nouns, which are not agents, we think of them regularly as *objects* of our thoughts and feelings, that is, grammatically, we generally use the objective case; and even in the case of agents, we feel more strongly, and have to express more frequently, our action on them than their action on us, as, *I like* HIM, *I hate* HIM, *I* TELL HIM, *I order* HIM, *I forbid* HIM, &c.

chiefly the cases of nouns, the tenses and moods of verbs and prepositions and conjunctions. Every change, indeed, made on a verb by diminution or increase, does not mark a change in the law of the dependence of words on each other, or of sentences on sentences; but for the sake of a complete view (in addition to what was said above), it will be convenient to note the principal changes made on the verb in this place. Now, every one who bears in mind the prodigal luxuriance of form in the Greek verb, will be prepared to find that the change here falls almost exclusively under the category of loss. Thus the common third person plural of the present indicative in *ουν*, sometimes *ουνε*, is a manifest curtailment of the old Doric *οντι*, Latin *unt*. This *οῦν* of the indicative is then transferred to the subjunctive, and we have *πράξουν* for *πράξωσι*, and similar forms. The next thing that strikes us is the total disappearance of that double form of the aorist, which gives so much annoyance to young students of the classical tongue. The first aorist with the *α* as its final syllable has gained a decided victory over the *ο* form in the active voice; and in the passive the aspirated form which appears in *ἐστάλθην*, is preferred to the Attic *ἐστάλην*. Thus, even when the second aorist is retained, because no first aorist existed, it assumes the *α*, which is the characteristic of the first aorist. So *ἔλαβαν* for *ἔλαβον*, a peculiarity already prominent in the septuagint. But the two most striking amputations which the verb has suffered, and which most prominently affect the syntax, are those of the optative and the infinitive mood, both changes for which the way was fully prepared in ancient times, as the student of the New Testament must be aware. The loss of the *optative* as the natural and symmetric form, of the conjunctive after a past tense in the leading clause, is no doubt in an æsthetical point of view to be lamented; its effect in modern Greek is the same as if in English we should say, "*I* GAVE *you this property, that you* MAY *enjoy yourself on it*; practically, however, it occasions no ambiguity, and accordingly we find that in the language of the New Testament the place of the optative in such dependent clauses is almost always taken by the subjunctive; and not in the New Testament only, but frequently in Plutarch, and not seldom even in Thucydides. It may be said, therefore, with perfect truth that the dropping of the optative is in the end an improvement, rather than a corruption of the language; as it is better that a person should be dead altogether, than that being alive, his occasional presence should serve only to remind us with the more acute pain of his habitual absence. But the loss of the *infinitive mood* is something to which the thorough-bred classical scholar will feel it much more difficult to reconcile his ear. If there is one thing more than another that distinguishes the flexible grace of Greek syntax from the somewhat formal dignity of the Roman, it is the frequent use of the infinitive mood. And one cannot but express a wonder at first blush, that a form of expression so con-

venient and so flexible as the infinitive certainly is, should have given place to the lumbering form of a conjunction with the subjunctive or indicative mood. But so it is; for even the Greeks in their best days said οἶδα ὡς or ὅτι, instead of the less circuitous infinitive or participle; and the Romans who, for a special class of cases well-known to schoolboys, had prescribed the accusative before the infinitive as the only form, before the time of St Augustine began pretty generally to say, *Scio quod Petrus est vir doctus*, or even *quia*, which is the mother of the French *que* and the Italian *che*. Whence the habit now so characteristic of modern Greek took its rise of using να (for ἵνα), with the subjunctive instead of the old infinitive, it seems useless to inquire. I have sometimes thought it might be by a contagion caught from the Roman syntax; but the relation of the two languages was of such a kind as to create a current of contagion from Greek upon Latin syntax (as indeed we see in Tacitus), rather than the reverse. I shall say, therefore, it was only a change of fashion; certain it is that the partiality for ἵνα with the subjunctive mood, appears already largely developed in the New Testament in cases where a classical ear feels the want of the familiar infinitive. But custom, which exercises a despotic authority in such matters, soon reconciles us to the change; which indeed, when considered apart from the habit of the ear, is anything but an important one, and quite in harmony with the commonest grammatical phenomena, both in ancient and modern languages. When I say in English, for instance, *It is too bad that you should do so and so*, I am merely using the modern Greek syntax of δεινὸν νὰ τὰ τοιαῦτα κάμῃς for the classical δεινὸν τὸ τὰ τοιαῦτα σέγε πράττειν; and the apparently more awkward syntax, διὰ τὸ να πραχθῶσι ταῦτα, for πραχθῆναι ταῦτα, is again only the English *on account of the fact that*, and the Latin *propterea quod*. There is only one other amputation in the Greek classical verb which must be mentioned, for it is certainly the most grievous of all; I mean the loss of the future and the pluperfect, with the substitution therefor of the auxiliary verbs, θέλω or θά, and ἔχω. Now, it is no doubt true that the particle ἄν, so familiar in the classical dialect, and κε in HOMER, could have been nothing in their origin but auxiliary verbs; and so θα may plead a classical precedent. True also it is that the verb ἔχω is not unfrequently in the tragic writers joined with the past participle in a way that has some analogy to the function of an auxiliary verb; but it is in reality very different; and as there is nothing in modern Greek that so offends the polite ear of the elegant scholar as the presence of these auxiliary verbs, it is matter of congratulation that the best modern writers know so rarely and so dexterously to use them, that the offence is practically reduced to a minimum, and with some writers appears to cease altogether. There is another characteristic of Neo-Hellenic prose closely allied to such essentially modern syntactic combinations; I mean the use of such modern turns of speech as βάλλω εἰς πρᾶξιν, *to put into execu-*

tion—χάνω τὴν στράταν, to lose the way—κάμνω πανία, to make sail, &c.; and in respect of these also, it is comfortable to remark, that even in the lowest phase of the language they are much fewer than might have been expected, and in the more cultivated forms are becoming fewer and less prominent every day.

Another severe syntactic loss which the modern dialect has suffered, is the disappearance of the *dative case*, and the confounding of that case, not only with the accusative, as in our English pronoun *him*, but with the genitive. The loss of the cases, as is well known, takes place naturally from the relations originally expressed by the terminational affixes having become obscured to the popular ear; a process which was sometimes precipitated by a confusion in the pronunciation of cognate diphthongs, as when τοῦ was pronounced τῷ, or the contrary. This loss is repaired, in the first place, by the use of *prepositions* along with case-affixes, in which conjunction they are in fact tautological, as in the Homeric phrase ἐξ ἐμέθεν. Afterwards, when the removal of all authoritative control, and the weakening of precedent, have allowed the affixes to be dropt or confounded, the prepositions take their place as the alone significant element, and attach themselves prominently or exclusively to the accusative as the dominant case. Thus, in modern Greek, ἀπό takes the accusative, and εἰς, usurping the function of ἐν, is joined with the same case, whether its signification be *in* or *into*. This is the case in Scotch also, as we say "*a head wi' a muckle lot in till't*," i.e., *in it;* and both in Latin and in German there is only one form for εἰς and ἐν.

In reference to the connection of clauses, there are only two observations more to be made: one, that the relative ὅς is in the modern dialect frequently replaced by ὁ ὁποῖος (the Italian, *il quale*), or simply by the indeclinable ὁποῦ, somewhat as in English we say, *whereof* and *wherefrom*, instead of *of which* and *from which*. In German the adverb *so* is used in the same way; and among ourselves, with the uneducated, *as* often serves the same purpose. The other point is the confounding of ὡς ἄν or σάν with ὡς; and combined with this the formation of some new conjunctions, of which the most common are,—

ἀγκαλά,	though.
ἀμή,	but.
ὡς τόσον,	meanwhile.
εἰς τόσον ὁποῦ,	the while—while that.
μὲ ὅλον τοῦτο,	nevertheless.

Some new prepositions and adverbs, or old forms curtailed, may also here be noticed, as,—

συμμά,	near.
μέ,	with.

μαζί,	with
μέσα,	in, into, within.
ὁλόγυρα,	for περὶ.
δίχως,	for χωρίς, ἄνευ.
τὰ ισῖα (*soeben*, German),	for αὐτίκα.

This phenomenon appears also in the Romanic dialects,* and seems to belong naturally to a language in a case of nascent or complete disintegration.

PROPOSITION XXVI.—Not the least important element in the new phasis of an old language is that which either does not appear at all, or is only partially represented in the Dictionary—viz.: the *pronunciation*. This is a matter with regard to which, from the Reformation downwards, very fierce battles have been fought between the living Greeks and the great majority of classical scholars; but after three centuries of ink shed, and now that in the great German school an accurate philology has been added to a large philosophy, we feel warranted in asserting that the great salient points of the question have been planted before the thinking scholar in their right aspect. No person who has examined the subject will now deny that, while correct Greek speakers pronounced their prose orations both according to accent and quantity, in prose the accent was the dominant element, while quantity prescribed the law to poetry.† Then as to the sounds of the individual vowels and diphthongs, while on the one hand no sensible person will suppose or attempt to prove that the present vocalisation of the Greek tongue has remained in all respects unchanged from HOMER downwards, on the other hand, such a person will regard it as no less certain that the comparative tenuity, the so-called *itacism* of the modern dialect, is a peculiarity of very ancient date in the language, being in fact clearly noted by QUINCTILIAN in the marked contrast which he draws between the *strong* Roman language and the *slender* Hellenic.‡ As little will any well-informed scholar in these days be bold enough to assume the advocacy of that altogether arbitrary pronunciation of Greek which has obtained currency in this country—a pronunciation which both corrupts vocalisation by the insular anomalies of John Bull, and travesties intonation at almost every step by the arbitrary substitution of the Latin for the Greek accent. On this basis, and emancipated formally from the evil habit of our English school utterance, the position of the modern Greek with regard to *accent, quantity, and vocalisation*, may be simply stated thus. The accent, with a very few exceptions, has asserted its supre-

* See CORNEWALL LEWIS, ch. v., for an analysis of the French, Italian, and Provencal prepositions, adverbs, and conjunctions.

† For the detailed proof and illustration of this, I must refer to my paper on the *Power and Place of Accent in Language*, Transactions of Royal Society, Edinburgh, March 1871.

‡ *Non possumus esse tam* GRACILES? *Simus* FORTIORES.—INSTITUT. ORAT. xii. 10.

macy on those very syllables of the word where its emphasis was marked by the grammarian, ARISTOPHANES of Byzantium, in the early days of the Ptolemies; and the circumstance that, wherever a word is curtailed by dropping an initial, or final syllable, or both, the accented syllable always remains, as containing, to use DIOMEDE's phrase, "the soul of the word;"* this circumstance of itself were sufficient proof, though all others were wanting, that emphasis or stress, in the modern sense, and not mere elevation, as some English scholars hastily assume, was the essential element in that affection of articulate speech which the Greeks called τόνος (*stress* or *strain*, from τείνω). When we reflect what extensive changes in English accentuation have taken place with us since the time of CHAUCER, we shall consider this persistency of the same element in the Greek, during a period of more than two thousand years, a phenomenon not a little remarkable, and we shall rejoice to think that the great Byzantine grammarian, if not in the general practice of English scholars, certainly in the living tradition of his people, and in the practice of the national Church, has received the reward which he deserved. But the accent, like other strong forces, having lost the salutary control of a hereditary school of music, has not only maintained its position, but invaded the domain of quantity; so that with the modern Greek the word ἄνθρωπος, for instance, is pronounced not like the English word *lándhōlder*, as it was by DEMOSTHENES, but like the English word *abbacy*, or *atrophy*, that is, with the middle syllable curtailed of its natural volume of sound. The fact of the matter is, that there is in all language a popular tendency to cheat the unaccented syllable of its full quantity, especially when it comes immediately after an accented syllable; and to this tendency Latin yielded at an early period (as we see from the short final *ŏ*, in Martial) and Greek probably about the same time. Along with this abuse, there crept in also in Greek that other one of, in many cases, dwelling upon the accented syllable in such a manner as to change its natural quantity from short to long; just as the Scotch, who as a rule speak slower than the English, draw out the accented syllable in many words, such as *mājesty*, *nātional*, which the English pronounce short. The consequence of this excessive emphasis of the accent is, that in their rhymed poetry, the Greeks find no offence in echoing αἶνα by ἔνα, μένος by μῆνος, and so forth; which is just the same as if we were to abolish the difference between *hare* and *her*, *mane* and *men*, *pope* and *pop*. Then again as to the vowel sounds. In the face of the distinct gamut of the vowels given by DIONYSIUS of Halicarnassus (*De Structurâ Orationis*, xiv.), it is impossible to maintain that the itacism now universally prevalent was considered correct speaking (though it might perhaps have been the vulgar fashion) in the days of Augustus Cæsar. We must say, therefore, that the present fashion of pronouncing η like *ee* is a corruption and an enfeeblement of the classic speech, in so far as it

* *Accentus est anima vocis.*—DIOMEDE. "Pulsch. Gr. Lat. Auct.," p. 425.

substitutes a weak and feminine for a strong and masculine sound. The like verdict must be pronounced on the itacising of the delicate sound of υ, which was equivalent to the German *ü* in *Brüder*, and the Scotch *ui* in *bluid*, *guid*—a vulgarising of a fine sound to which the descent, when corruption once sets in, is very prone; as we find even now in Saxony, where *Brüder* is commonly pronounced *breeder*, and in Aberdeen, where the south country *guid* is squeezed out into *gweed*.

Proposition XXVII.—It is another question altogether, how far the euphony of the Greek language, considered as an æsthetical product, has been affected by these corruptions. Latin was corrupted in a similar fashion, and came out of the process, not only not less, but, as some think, considerably more euphonious, in the shape of Italian and Spanish. The mere change from quantitative to accented poetry implies no absolute cacophony; it is merely the introduction of a new rhythmical law, and the transference of the musical weight of a word in verse from one syllable to another. This the beautiful hymnology of the Latin Church sufficiently attests; and no man of taste, who knows how to read these compositions, will speak less favourably of the unrhymed accented chants of the Byzantine Church, except, of course, in so far as he may feel the want of the pleasant recurrent sweetness of rhyme.

χαῖρε ἀστὴρ, ἐμφαίνων τὸν ἥλιον.
χαῖρε γαστὴρ ἐνθέου σαρκώσεως·
χαῖρε, δι ἧς νεουργεῖται ἡ κτίσις
χαῖρε δὶ ἧς βρεφουργεῖται ὁ κτίστης
χαῖρε, νύμφη ἀνύμφευτε.*

Then, as to the effect of the excessive frequency of the feeble sound of ι, expressed by the term itacism, we must bear in mind that, though the modern pronunciation applied absolutely to certain picked lines of the ancient classics might produce a very petty vocal effect, it does not at all follow that the same result will be produced when the modern language is used by those who know how to handle it. Such a sequence of weak identical sounds, for instance, as occurs when the law of itacism is applied to a word like πληθυνθείη, does not exist in the Neo-Hellenic verb; for the optative is obsolete. But further; it seems to be quite certain, that if certain lines in classical Greek have their music marred by the application of the modern itacism, the harmony of the whole language is destroyed by the barbarous English pronunciation of the diphthong ου, in which the rude canine *ow* is substituted for the soft and velvety oo. And with regard to this beautiful ου, which the English so pervert, it is a noticeable fact, that not only is it one of the most prominent sounds

* Sergii, ὕμνος ἀκάθιστος τῆς Θεοτόκου. Anthol. Græca Carminum Christianorum. *Christ et Paranikas.* Lips. 1871, p. 140.

in classical Greek, but it has extended its sphere in the modern dialect so as to produce some new and sonorous terminations. How beautiful, for instance, are the diminutives in οῦλα, as—

ῥαχοῦλα,	for	ῥάχη.
μανοῦλα,	,,	μάνη.
αὐγοῦλα,	,,	αὐγή.
φωνοῦλα,	,,	φωνή.
περδικοῦλα,	,,	περδίκιον.
παλαιὰ κλησοῦλα,	,,	παλαιὰ ἐκκλησία.
καρδοῦλα,	,,	καρδία.
λαλοῦλα,	,,	λάλημα.
βρυσοῦλα,	,,	βρύσις.

No man with an ear will deny that in these cases,—and they are abundant enough to give a marked character to the modern dialect,—the classical type of the word has been corrupted into a decided improvement. A similar euphony strikes the ear in the words with the less common termination οῦδα, as in ἀρκοῦδα and καλιακοῦδα. The same favourite diphthong appears in βροντοῦνε for βροντῶσι, in ζοῦνε for ζῶσι, and others of the same kind, which may be picked out of the Klephtic ballads. The ranks of the same favourite diphthong are further swelled by desertions from the ancient υ, as in

φουσκόνω,	from	φύσκοω.
κουδούνιον,	,,	κώδων.
βρουχίζω,	,,	βρύχω.
σκούζω,	,,	σκύζω.
σκούβαλον,	,,	σκύβαλον,

and others.

Nor is the sonorous sound of α, according to DIONYSIUS the most musical of the vowels, but which the English in many cases degrade into the feeble rank of η (Scotch), less prominent in the modern than in the ancient dialect. On the contrary, many beautiful new substantive forms are used with a marked preference to which this rich vowel gives the key-note, as in

σημάδι,	for	σημεῖον.
σκοτάδι,	,,	σκότος.
ποτάμι,	,,	ποταμός.
εὐμορφάδα,	,,	εὐμορφιά.
νοστιμάδα,	,,	νοστιμότης.
φρονημάδα,	,,	φρόνημα.

With the troop of favourite diminutives in άρι and άκι.

And I must say, generally, that in reading through the Erotocritus, I was more struck by the predominance of these two rich musical sounds of α and ου, than by any offensive accumulation of itacised syllables; and both in that poem and in the Klephtic ballads, my ear not seldom rested on lines of a full and masculine melody, not inferior to the best in HOMER. Thus we read—

Ὀλημεροῦλα πολεμᾷ τὸ βράδυ καραοῦλι.
"*All day we fight, and all the night we waste in sleepless watches.*"

And

Σκοτόνει τοὺς Ἀγαρηνούς, πεζοῦρα καὶ καβάλο.
"*He mows them down, the Agarenes, both foot and horse he mows them.*"

And, again,

Πολλὴ μαυρίλλα ἔρχεται, μαυρὴ σὰν καλιακοῦδι.
"*A blackness sweeps across the plain, black as a troop of ravens.*"*

And what can be more vigorous and powerful, so far as sound is concerned, than the following lines from the Erotocritus, describing the impatient steeds at a battle, eager to start at the first blast of the trumpet?

Κτυποῦν τὰ πόδια τους στὴν γῆν, τὴν σκόνη ἀνασηκόνουν,
Τὸ τρέξιμον ἀναζητοῦν, ἀφρίζουν, καὶ δριμώνουν.
Ἡ γλῶσσα μὲ τὸ στόμα τους παίζει τὸ χαλινάρι,
Τ᾽ένα καὶ τ᾽ ἄλλο ἀγριεύετο, σὰν κάνει τὸ λιοντάρι.
Τἀρθούνια τους καπνίζουσι, συχνὰ τ᾽ ἀφτιὰ σαλεύουν,
Καὶ νὰ κινήσουν βιάζονται, νὰ τρέξουσι γυρεύουν.

Or, again, take the beautiful little χελιδόνισμα, or swallow song to welcome the spring, from KIND'S collection—

Ὁ Ἀπρίλης ὁ γλυκὺς ἔφθασε, δὲν ᾽ναι μακρυά·
Τὰ πουλάκια κελαδοῦν, τὰ δενδράκια φυλλανθοῦν,
Τὰ ὀρνίθια νὰ γεννοῦν ἄρχησαν καὶ νὰ κλωσσοῦν,
Τὰ κοπάδια ἀρχινοῦν ν᾽ ἀναβαίνουν ᾽ς τὰ βουνὰ,
Τὰ καιζίκια νὰ πηδοῦν καὶ νά τρώγουν τὰ κλαδιά.

So that, upon a broad practical estimate of the whole, I scarcely think any impartial person, who has taken the trouble to train his ear to the euphony of language (which I am sorry to see even great scholars don't always do), will pass any other verdict than this, that the old language of HOMER, and PLATO, and DEMOSTHENES, whatever amputations and transmutations it may have suffered in the course of centuries, remains still among the organs of human expression one of the most vigorous and the most harmonious; as a magnificent

* PASSOW: Popularia Carmenia Græcorum, ix. 2, lxxxii. 12, xcvi. 6.
† KIND: Neugrichische Anthologie, p. 72. Leipzig. 1847.

tree with full luxuriant leafage may bear much lopping, and still remain very beautiful.

PROPOSITION XXVIII.—In connection with the orthoepy of the existing Greek language, one or two other points deserve mention. The first is that the present race, while allowing the *spiritus asper* to drop, have shown no tendency to narrow the sphere of that consonantal *aspiration* so favoured by their ancestors, but have rather increased it. For not only do φ, χ, ψ, and θ remain with their full aspirated force in the words which originally had them, but many words which had a slender consonant in classical usage, now regularly receive an aspirate. This specially happens where two slender consonants come together, in which case, in the modern practice, the former is always aspirated; thus,

For κλέπτης,	they say	κλέφτης.
βλάπτω,	,,	βλάφτω.
πίπτω,	,,	πέφτω.
δείκνυμι,	,,	δείχνω,
πτωχός,	,,	φτωχός.
πτέρυγες,	,,	φτέρουγες.
ῥίπτω,	,,	ῥίχνω.
ῥάπτω,	,,	ῥάφτω.
ἅπτομαι,	,,	ἄφτω.
πταίω,	,,	φταίω.

And many more. The same tendency to aspiration—closely connected with the sibilation so common in classical Greek—has led to the lisping of every δ, and the softening of β into our English *v*, which often produces a very pleasant effect, as when *voólomæ*, the Romaic pronunciation of βούλομαι is compared with the rude and canine English *bowlomai*. And if the modern Greek has shown no objection to the classical aspirates, as little has he felt inclined to soften down the masculine ξ—κς—after the fashion of the Italians, into a double sibilant. Besides retaining the ancient ξ in all cases where it existed, he has new created a large class of compound verbs with ἐκ or ἐξ, of which the characteristic is, that the initial vowel is omitted or transposed, so that εκ becomes ξε, and the word appears commencing with the double consonant. Of this class the following are characteristic examples:—

ξαγαπῶ,	to cease loving—grow cool.
ξακουστοσύνη,	celebrity.
ξαστερία,	clear starry brightness.
ξαφεγγαρία,	moonlessness.
ξεβάπτω,	to take out the colour.

ξεβουλλόνω,	to unseal.
ξεδοντίζω,	to draw teeth.
ξεκαρδίζω,	to dishearten.
ξεκαρφόνω,	to take out nails.
ξεμανθάνω,	to unlearn.
ξεμυαλίζω,	to put one's brain out of shape—madden.
ξεσπαθόνω,	to draw the sword.
ξετραχηλίζω,	to break one's neck.
ξεψυχέω,	give up the ghost.
ξημερόνει,	the day breaks.

PROPOSITION XXIX.—In the above analysis I have made no allusion to one element of corruption, which from the analogy of Italian one might expect to find in the modern Greek dialect. I mean the element of *adulteration* from foreign sources. In the purest modern languages, as in German, for instance, this adulteration is considerable, and, except on a principle of pedantic purism, to which utility and good taste are equally opposed, could not be dispensed with. But it is different with Greek. For not only do all nations borrow their scientific terminology from the language of ARISTOTLE and HIPPOCRATES, while the language of these great master-builders of early science borrows from no one, but the foreign elements which at different periods got into use as part of the current Hellenic, did so only by way of external attachment, so to speak, not by way of inoculation; they did not infuse themselves into the blood, or, to use a legal simile, they were not fixtures. And thus it came to pass, that with the change of masters one adopted element was readily thrown off, and another as readily assumed, and as loosely retained. The colloquial style of the Byzantine Greeks thus became superficially Latinised, or spotted rather with Latinisms; that of the Frankish chroniclers included a sprinkling of French words; that of the Cretans flirted with Italian; while that of the Acarnanian, Thessalian, and Epirotic Klephts was forced, for the sake of convenience, to tolerate a certain admixture of the Turkish element, which their most deeply rooted principles and their most powerful associations would have led them to disdain. But the number of these foreign words was at no time considerable; and as one immediate result of the grand national resurrection in 1821, the unseemly crusts and blotches of this foreign element instantly fell off like the scurf of a cutaneous disease, and the pure Hellenic came out of the caldron of a barbarous broth as clean and bright, and pure from all stain, as the god-like Ulysses out of his bath. In the current Greek newspapers which present the language in its ultimate type, not one of those Latin, Italian, Albanian, and Turkish words is to be found, which raise a not unfrequent stumblingblock in the way of the student of mediæval and Venetian

Greek; much less do those hybrid compounds occur which bristle in THEODORUS, made by the addition of the Latin terminations ἄτωρ, and ἄτος, and ἄριος, to a Greek root.* At the present time, a little girl in the street, if you ask for a flower by the Albanian word λουλούδι, may surprise you by saying that you must say ἄνθος; and if you wish a boat to shoot across from the Piræus to Ægina, you must ask in Thucydidean phrase for a λέμβος, and not for a *barchetta*.

PROPOSITION XXX.—With regard to the materials of which the modern dialect consists, as well as the physiognomy which it presents, a question has been raised, in how far it represents any of the ancient dialects—Doric, Ionic, or Æolic; for Attic it certainly is not. Now, it seems quite plain that, looking at the conditions of the case, nothing would have been more strange than that some considerable traces of these non-Attic varieties of Greek speech should not have presented themselves in the modern new formation. For Attic, we ought to remember, was a dialect originally confined to a comparatively small portion of the Greek-speaking population; and the breadth of space which it afterwards occupied was a purely literary phenomenon, leaving untouched the great popular substratum of a distinctly diverse feature, spread out in large reaches of country from Tænarus to Trebizond. Byzantium specially, the centre of non Attic Hellenism in later days, was a Doric colony; in Africa, neither Alexandria nor Cyrene could lay any claim to a native Atticism; and on the sunny shores of Asia Minor, as well as the bright isles of the Ægean, Doric, Æolic, or Ionic varieties of Greek speech were everywhere at home.† Now, in what proportions these local peculiarities might mix themselves up in a new dialect to be formed after the long and disturbed action of centuries, so as to serve for a common medium of understanding, no man could dare to prophesy; but that the traces of their existence should appear in some shape and to some extent, seemed certain. And so, in fact, it has proved. Putting out of view the local dialect of the Tzacones‡ in Eastern Laconia, as something isolated and

* Of this barbarism, the name of an illustrious Fanariot family, MAUROCORDATO—*black-hearted*, is a familiar example.

† How much of the peculiarities of the Alexandrian Greek has passed, either directly, or through the influence of the Church and the Alexandrian translation of the Old Testament, into modern Greek, the student of the Septuagint will readily convince himself. It is notable also, that not a few of the rare words used in the Septuagint, though unknown to Attic Greek, are found in POLYBIUS, HERODOTUS, and DIODORUS, as SCHLEUSNER, and other Biblical lexicographers, have been careful to note. The Biblical Greek which issued from Alexandria is, in fact, a sort of half-way house between Attic Greek and Romaic; and in this view it is certain that a familiarity with the living dialect of Greece would be of more value to our young theologians than many of the branches of philology which at present occupy their attention.

‡ On the peculiarities of the dialect of the *Tzacones*, which MULLACH identifies with the *Caucones* of the ancient topographers, see MULLACH, "Grammar," p. 94, and "Das Tzakonische von Prof. MORIZ SCHMIDT in Studien zur Griechischen und Lateinischen Grammatik, herausgegeben von GEORG CURTIUS," vol. iii. p. 347.

deserving special consideration, the general dialect of the modern Greek, from the Byzantine lingo of PTOCHOPRODROMUS, to the Cretan of CORNARO, and the Epirotic of the Klephtic ballads certainly does present certain features of an Æolo-Doric physiognomy. Of this the dominance of the broad vowel α, not only in the nominative of all masculine agents, as above mentioned (p. 23), but in a large class of verbs, as in ζητᾷ for ζητεῖ, μετρᾷ for μετρεῖ, φοβᾶται for φοβεῖται, is in itself sufficient evidence. But besides this, we have the well-known Doric peculiarity of forming all verbs whose future is in ξ from a present in ζ instead of a double σ, according to the Attic usage; thus, instead of τάσσω we have τάζω;* and in this way has been formed a large class of modern verbs, as, φωνάζω, φωνάξω, φοβερίζω, φοβερίξω, τρομάζω, τρομάξω. Of Æolism, the accusative plural of the first declension in αι instead of ᾱ, as Μούσαις for Μούσας, is a familiar example. On the other hand, it is not to be denied that there are some traces of Ionic also in modern Greek, as in the conversion of ο or ω into ου, and the use of the slender η for the broad α in certain adjectives in ρος; but I have not been able to confirm from my observation the strong remark of the late Viscount STRANGFORD on this point—one of the best authorities no doubt—to the effect that, "it would be easy to show two Ionisms for one Æolism or Dorism in modern Greek.†

A kindred point to this dialectic physiognomy of the modern tongue is brought out by the occurrence in it of certain words, or forms of words, which anciently were confined to poetic usage, but have now passed into general circulation. The following is a short list of this type which I have collected:—

σιγαλός,	(PINDAR).
βροχή,	for ὑετός.
ὀμμάτια,	ὄμματα, HOMER, for ὀφθαλμός.
τρικυμία,	storm at sea, for λαῖλαψ.
νόστιμος,	for τερπνός.
κτυπάω,	έω, HOM., for τύπτω.
κτίριον,	κτέαρ, HOM., for κτῆμα.
ἄρμενα,	tackling, riggling, for ὅπλα.
λιβάδιον,	from λιβάς, for λειμών.
δροσερός,	for ἡδύς.
τάχινός,	(CALLIM), for ταχύς.
αἴθουσα,	a hall.

The transference of such words from poetical into common usage is not a phenomenon that need excite any surprise. The popularity of a great

* AHRENS, De Dialecto Dorica.

† "On Cretan and Modern Greek," in the Appendix to Captain SPRATT's "Travels in Crete," vol. i.

national poet, or the affectation of some fashionable writer, may readily achieve this; but we must bear in mind also, that what we would justly mark as poetic in an Attic writer may, in some of the widely scattered provinces occupied by the Greek race, have been a word which was generally current in the mouth of the people. That HOMER himself, as being a popular minstrel, addressed the inhabitants of the eastern shores of the Ægean, to which country he belonged, not in a peculiarly and distinctly marked so-called poetical style, but, like our own BURNS, in a style familiar to the common people, we cannot for a moment doubt; and in this view any Homeric element in the existing Romaic may have come down by direct descent from the pre-Homeric popular dialect, not by degradation from the peculiar poetic and epic style used by literary Greece.

PROPOSITION XXXI. In conclusion, it seems not improper, in the case of a famous language of such remarkable longevity, to cast an eye into the future, and calculate the chances of its permanence. And in this divination past experience certainly entitles us to say with confidence, that a language which has survived so many changes, and resisted such a succession of destructive forces, will maintain its vitality unimpaired, so long as the moral motive power of the world is mainly Christian, and the science of the world is proud to root itself in Greek traditions. For whether the present little Greek kingdom shall have strength enough to grow into an independent political integer, or whether, which seems its more probable destiny, it shall at no distant day be attached to the great Russian Empire in the manner of an outlying principality, as Cymric Wales was attached to Saxon England, it does not appear that it will have to contend with any disintegrating or exterminating force in any way so strong as those which it successfully resisted when under Turkish and Venetian supremacy. The conservative force of the Welsh language in the south-western corner of our insular triangle, is a fact of such potency, as to have been deemed worthy of special notice and wise concession by our administrative council in London;* but if Cymric, which is no doubt as old a language as Greek, with its scanty stores of literary tradition, still flourishes in a green old age, the extinction of Greek, with its immense momentum of intellectual and moral force, not less intense in kind than extensive in operation, is not a thing to be looked for within any assignable period. If the Greek kingdom should unfortunately not be able to maintain its position against the

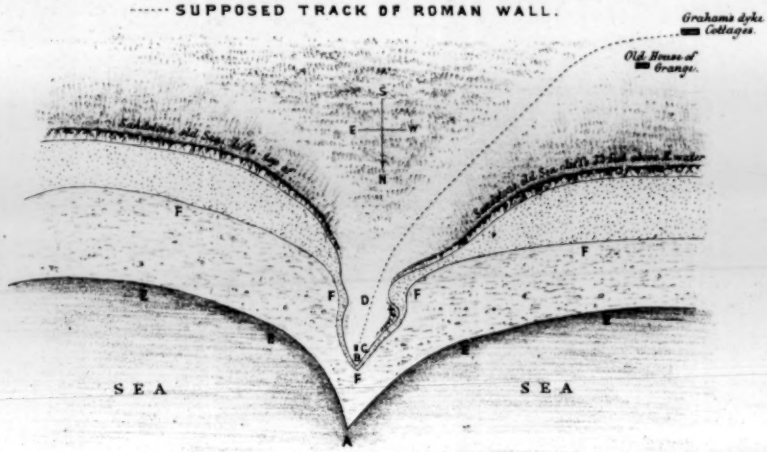
* In the winter of 1872 a representation was made to the British Government on the evils arising from the appointment to judicial situations in Wales of barristers unacquainted with the language of the natives; and the Government pledged themselves to have respect to the representation in future, in so far as it might be possible to do so with a due regard to the legal knowledge of the persons appointed, *i.e.*, that a Welsh-speaking barrister should in future be preferred, if his abilities and learning were equal.

combination of internal faction and external aggression, which may at any moment break it up, there is nothing in the antecedents of any of the great powers into whose hands a dismembered Greece might fall, to warrant the apprehension that they would employ any severe measures for the purpose of stamping out the national language. Russia certainly, which in religion is full sister to Greece, would have neither desire nor interest to look upon the language of Athens with the same jealousy that she looks on that of Warsaw. Under Russia the Greeks might readily become the founders of an enlightened Broad Church party in that country, just as under the Turks they became first the necessary interpreters, and then the wise governors of great and important provinces. As for the form in which it seems most desirable that this noble language should be transmitted to distant ages, it seems necessary, on the one hand, to warn against any forced and affected recurrence to the classical type, and on the other to invite literary men to the culture of the popular dialect as the fittest for a certain style of lyrical and essentially popular poetry. As in Scotland the language of popular song runs in its own channel, apart from the English, used as a general literary medium, so the Greek of CORAES and the newspapers might still continue to occupy a middle place between the Greek of classic tradition and the Greek of the popular ballads. Variety was one of the distinguishing features of the Greek family of languages from the beginning, and it may well remain so to the end. Whether or not the course of affairs shall ever be so ordered by Divine Providence, as that, according to the pious aspirations of Monsieur D'EICHTHAL, and other eminent French Hellenists, it may at no distant period be prepared to take the place of Latin as a catholic medium of correspondence between cultivated men of all countries, it seems in vain to speculate; but, in the form that it has now assumed, and in all likelihood will maintain for centuries, there is no reason why it should not be much more extensively studied by all classes than at any previous period. When our classical scholars shall have become ashamed of their false methods and narrow prejudices, and when a succession of intelligent travellers shall have been practically convinced that it is as easy to learn Greek in Athens, as to learn German in Berlin, or French in Paris, the sons or grandsons of Monsieur D'EICHTHAL and his French associates may behold with joy the probable advent of that kingdom of Pan-Hellenic brotherhood of which it is now only permitted to dream.

SKETCH I.

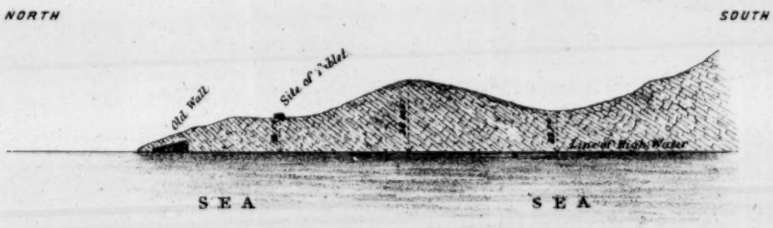
GROUND PLAN OF PROJECTING KNOLL AT BRIDGENESS

------ SUPPOSED TRACK OF ROMAN WALL.



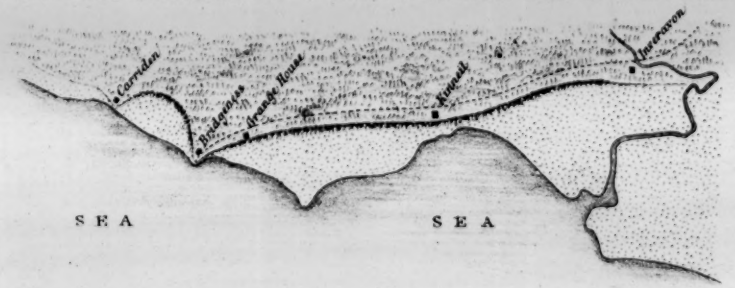
SKETCH II

SHOWING SECTION NORTH & SOUTH OF WHINSTONE KNOLL AT BRIDGENESS.



SKETCH III

GIVING BIRD EYE VIEW OF COAST, SHEWING OLD SEA CLIFF ABOUT 25 FEET ABOVE PRESENT HIGH WATER, AND BY ------ LINE, THE SUPPOSED TRACK OF ROMAN WALL.



Mc Farlane & Erskine. Lith.rs Edin.r

II.—*On the supposed Upheaval of Scotland in its Central Parts since the time of the Roman Occupation.* By David Milne Home, Esq. (Plate I.)

(Read 20th January 1873.)

Sir Charles Lyell, in the 3d edition of his "Antiquity of Man," published in 1863, makes the following statement:—

"Until lately, it was confidently assumed, that no alteration had occurred in the relative levels of land and sea, in the central district of Scotland, since the construction of the Roman or Pictish wall (the wall of Antonine), which reached from the Firth of Forth to that of the Clyde."

"But Mr Geikie has lately shown, that a depression of twenty-five feet on the Forth, would not lay the eastern extremity of the Roman wall at Carriden under water; and he was therefore desirous of knowing, whether the western end of the wall would be submerged by a similar amount of subsidence. Antiquaries have sometimes wondered, that the Romans did not carry the wall farther west than *Chapel Hill.* But Mr Geikie now suggests, in explanation, that *all the low land at present intervening between that point and the mouth of the Clyde,* was, sixteen or seventeen centuries ago, washed by the tides at high water."

"The wall of Antonine, therefore (adds Sir Charles), yields no evidence in favour of the land having remained stationary, since the time of the Romans; but, on the contrary, *appears to indicate,* that, *since its erection,* the *land has actually risen.*"

Sir Charles, after stating the facts pointed out by Professor Geikie to show this rise, proceeds thus:—

"On a review of the whole evidence, geological and archæological, afforded by the Scottish coast line, we may conclude that the last upheaval of twenty-five feet took place not only since the first human population settled in the island, but long after metallic implements had come into use; and there seems even a strong presumption in favour of the opinion, that the date of the elevation *may have been* subsequent to the Roman occupation."

The opinion expressed by Professor Geikie, and to a certain extent adopted by Sir Charles Lyell, attracted considerable attention. Several geologists went to examine the section of strata near Leith, which had led Professor Geikie to his remarkable conclusion. Amongst others, the late Mr Alexander Bryson of Edinburgh and Mr Carruthers of the British Museum wrote papers

on the subject, and stated their conviction, that Professor GEIKIE had either not correctly observed, or had not properly understood, the Leith deposit.

Professor GEIKIE was not, however, convinced of having made any mistake, for in his "Scenery and Geology of Scotland," published in 1865, he has a chapter on "Raised Beaches," to show how "the country, after its submergence, *rose again slowly*, with long intervals of rest, each of these pauses giving the sea an opportunity of cutting a notch or horizontal terrace along the margin of the land." He states that the "lowest and latest terrace, long ago described by Mr SMITH of Jordanhill and Mr MACLAREN, runs at a height of about twenty or twenty-five feet above high water. It has yielded, in several places, works of human fabrication. From the nature of these remains, and other evidence, I have been led to infer, that its formation has taken place, either in whole or in part, *since the first century in our era.*" In a foot-note, he mentions that his inference had been disputed by Mr BRYSON and Mr CARRUTHERS; but he offers no refutation, and indeed takes no notice of the grounds of their objections.

Professor GEIKIE's opinion on this subject has been adopted, without any further evidence, by the distinguished geologist who succeeded Sir RODERICK MURCHISON as Director-General of the Surveys of the United Kingdom. Professor RAMSAY, in his work quite recently published on the "Geology and Geography of Great Britain," specially refers to the "inference which has been drawn by my colleague, Professor GEIKIE, that this last elevation (of Scotland) took place at a time that is historical, and even since the Roman occupation of our island." (P. 251.) Professor RAMSAY enumerates the facts from which this inference was drawn,—one of these is "the great wall of Antoninus,"—terminating at its east end, and also at its west end, in a way which Professor RAMSAY suggests can only be accounted for by supposing, that since the wall was built, it has been lifted above the sea-level, twenty-five feet higher than before.

A few months ago, Sir CHARLES LYELL wrote to me, that in gathering materials for a new edition of his "Antiquity of Man," he had fallen in with my little book on the "Estuary of the Forth," in which book, notice is taken of this alleged recent upheaval of Scotland, and circumstances stated which were to his mind very conclusive, against the soundness of that opinion. He asked from me information on two points—1st, Whether I knew of any answer which had been given to my objections; and, 2d, Whether, since the publication of my book, any new facts bearing on the question had been discovered.

To the first inquiry, whether I knew of any answer which had been given to my objections, I replied in the negative. The only reviews of my book which I had seen were in "Nature" newspaper, and the "Athenæum," both in September 1871. The review in "Nature" (bearing the initials J. G.), whilst it took exception to other parts of my book, made no allusion to that part of it which

combated the supposed recent upheaval of Scotland. The review in the "Athenæum" stated, most incorrectly, that I had adduced evidence to support that opinion.

To the second question by Sir CHARLES LYELL, I replied that one or two facts had come to my knowledge confirmatory of the opinion which I had expressed. In particular, I referred to the discovery of a remarkable Roman-sculptured tablet, which showed that Antonine's wall terminated not at Carriden, as supposed by Professors GEIKIE and RAMSAY, but at Bridgeness, and so close to the sea, as to preclude the idea that when that wall and tablet were erected, the land could have been twenty-five feet lower than now.

Sir CHARLES LYELL, as I found by his answer, had not before heard of this tablet. I therefore referred him for an account of it to the "Transactions of the Scottish Society of Antiquaries," and informed him that, if he came to Scotland, he could inspect the tablet in the Society's Museum, and could afterwards go out to the place where it had been found.

Sir CHARLES came to Edinburgh last autumn, and inspected the tablet. He wrote to me expressing much interest in it, and great regret that his health had not permitted him to visit Bridgeness. He pointed out to me, that the account in the "Transactions of the Antiquarian Society" did not state with sufficient precision, the height above the sea, of the place where the tablet was discovered, and he urged me to take some opportunity of ascertaining all the facts bearing on the upheaval question.

With this request of Sir CHARLES LYELL I complied; and as both he and I consider the facts ascertained sufficiently interesting to deserve being made known, I now bring them before this Society.

Place where the Tablet was discovered.

The tablet was found on an elongated knoll projecting into the sea, where there is now the harbour called Bridgeness.

I understand from HENRY CADELL, Esq. of Grange, to whom the knoll and the harbour belong, that the original name was *Briech*-ness. The word so spelt occurs in old deeds and writings in Mr CADELL's possession.

On consulting Mr WILLIAM SKENE as to the probable etymology of the word, I was informed by him that, in Pictish times, the district lying between the rivers Carron on the west, and Almond on the east, was called "Briach," which means "speckled;" and he supposes that the projecting knoll received its name of Ness or Nose, as being the only, or, at all events, the largest rock of that shape in the district projecting into the sea.

The knoll, which is about 100 yards long, and about 50 yards wide, and composed of hard blue whinstone, *is*, in pointof fact, larger than any other projecting point or promontory in that part of the coast. At two other places, one

on each side, about a quarter of a mile distant, whinstone does occur on the shore; but at neither of these places, is the rock so extensive or so projecting as that at Bridgeness.

It occurred to me to suggest to Mr SKENE, whether the original name might not have been *Broch*-ness. My reason for this was, that on this projecting Ness there may have originally been a broch or castle, as there once had been at Burghhead, near Elgin, in a position somewhat similar. But I now think that Mr SKENE's explanation is the more probable.

A rough sketch of the ground may render more intelligible a farther description.

On Sketch I., a general plan of the projecting knoll is given.

Its longer axis runs about north and south, or at right angles to the coast. Its steepest side, consisting of bare rock, is on the north-west—its height there being about 38 feet above high-water mark. The east half of the knoll is flat, and formerly sustained one or two small gardens of labourers' cottages.

The letter C, on Sketch I., indicates the spot where the tablet was found—19 feet above ordinary spring-tides.

With regard to its distance from the sea, some explanation is required. The present sea-mark is not where it once was. An immense amount of rubbish has been thrown into the sea, at and near Bridgeness, from coal and ironstone workings, as well as from salt and other manufactories. The outer line, E E A E E, represents the present sea-margin; the inner line, F F F F F, represents the former sea-margin, as ascertained by Mr CADELL, he having opened out at various points, the ancient beach by means of pits 3 or 4 feet in depth, in which he showed to me beds of broken shells and sea shingle.

It will be observed, that the line F F F F F keeps pretty close along the sides of the Ness or Knoll, so that in ancient times, the knoll projected well into the sea beyond the general line of coast. The water at this place, is also deeper than elsewhere. A harbour has been formed on the east side of the Ness, which is now frequented by sloops drawing eight or ten feet of water.

The general line of the old sea cliff, when the sea stood about 20 or 25 feet above its present level, is a good way farther back. As the rocks along that cliff are ordinary coal sandstone, they yield more readily to the action of the sea; and hence between them and the sea, there is is a stripe of flat land, a few hundred yards wide. But this whinstone knoll being much harder than the sandstone of the cliffs, has not suffered in the same way. It has even protected those parts of the sandstone cliffs which adjoin it. Accordingly these sandstone cliffs form a junction with the knoll, making a bend on each side of the knoll towards it.

At the place where the tablet lay (viz., C in Sketch I.) there was a quantity of squared stones in a confused heap. Many of these bore the marks of

masons' tools; and the impression is, that they had previously formed part of a wall or building, of which they were the ruins, and in which wall the tablet had been fixed.

At the point of the knoll, close to what had been the original sea-margin, and about one or two feet above the present level of ordinary spring tides,—viz., at B, a portion of a building was discovered, a few yards in length, consisting chiefly of large whinstone boulders. This building still exists, and I examined it carefully. It is on the west side of the point of the knoll, and may originally have reached to a higher level, so as to form a vertical bulwark. The building is evidently of great antiquity. It had been covered up by rubbish, and was discovered only by accident. The line of this building points towards the place where the tablet was found, so that if the building had continued on the same line, it would have passed through or near the site of the tablet. The relative positions of this old wall and the tablet, are shown in Sketch II.

The size of the slab is 9 feet in length, 3 feet in height, and 8 inches in thickness. It is of the ordinary sandstone rock occurring in the adjoining cliffs, and weighs probably about $1\frac{1}{4}$ tons.

The stone has been sculptured, to represent three separate panels, on one of its sides. The centre panel, which is also the largest, bears an inscription; each of the side panels contains figures of men and animals. In one of the panels, a Roman soldier on horseback is galloping over the naked bodies of men, with shields of the shape used by the Britons. In the other panel, a sacrificial scene is represented at an altar, with the standard displayed of the particular Legion mentioned in the inscription. All the figures as well as the letters are in *alto relievo;* and it is important to notice, that the sculpture is in perfect preservation, there being not the slightest trace of mutilation or injury of any kind, on the figures, letters, or projecting ornamental pilasters.

In the account of the slab given in the "Transactions of the Society of Antiquaries," it is stated, that "on the top edge of the tablet, towards the back, and on each end, there are dove-tailed recesses, by which it appears to have been held up." These recesses clearly indicate, that it was intended to place the stone in a building, so as to keep it in a proper position, to show the sculpture. The tablet was lying on its sculptured face, and in three pieces, the fragments being so close to one another as to suggest the idea, that in falling forward, the tablet broke by its concussion on the ground. There was soil over the tablet to the depth of about 2 feet, or $2\frac{1}{2}$ feet;—for as long as can be remembered, garden crops had grown above it. The inscription on the tablet, when freed from its verbal contractions, reads thus:—"Imperatori Cæsari Tito Ælio Hadriano Antonino Augusto Pio, Patri Patriæ, Legio Secunda Augusta, Per millia passuum 4652 fecit." It will be observed that in this inscription it is not explained, what it was which the Legion constructed—"*fecit*" or "*per-*

fecit." But if the tablet was set up in a wall or rampart, it would have been surplusage to have stated, that the "fecit" and the 4652 paces, had reference to that work.

It appears from STEWART'S "Caledonia Romana," that along the line of this military rampart, several other tablets have been found, all in like manner indicating the number of paces constructed by different Legions. Three of these tablets were found very near the western termination of the wall on the Clyde, and on a hill or knoll in many respects similarly situated to that of Bridgeness.

The traces of the rampart are not now visible very near its eastern termination. I have seen it in Callender Park, at Inveravon, and at Kinneil. These places are indicated on Sketch III. General ROY mentions that in his time, the ditch was visible not only at Kinneil, but also to the south of the House of Grange. Close to this last-mentioned point, an old cottage was shown to me by Mr CADELL, bearing the name of "Graham's Dyke," and this point is only about half a mile distant from Bridgeness. There can be little doubt, therefore, that the rampart went forward to the projecting rocky knoll, where the tablet was found.

The knoll was in every way suitable, as a place at which the rampart should terminate, considering its object, viz., to form a barrier against the natives inhabiting districts to the north. The object of the Roman engineers evidently was, that each end of the rampart should be at the sea shore, or as near it as possible, so that an enemy might not easily get past the end of the rampart. Now Bridgeness is the first rock projecting into the sea met with coming from the west; and in like manner, Chapel Hill on the Clyde is the first projecting spur abutting on the river coming from the east. It was also probably thought that whinstone rock would afford a firmer foundation than any other kind of rock for the rampart, and for any building connected with it.

It appears to me that the discovery of this tablet on Bridgeness promontory makes it so clear that the rampart terminated there, as to render any confirmation of the opinion quite unnecessary. Nevertheless, one or two circumstances may be just mentioned, without being insisted on:—

1. The 4652, *passus* (on the assumption that a Roman *passus* contains five English feet) indicate $4\frac{2}{3}$ English miles. Now, this is almost exactly the distance of Bridgeness from Inveravon, a place where the rampart is still visible, and where an important Roman fort existed in connection with it.

2. In the year 1642, the rampart must have been traceable along the high grounds bordering on the sea, for in that year a petition was presented to the Scotch Parliament, by the inhabitants and merchants of Borrowstounness, praying that a new parish should be formed out of the parish of Kinneil, and suggesting that the south boundary of the new parish should be "*Graham's Dyke*," and that the east boundary (separating the new parish from Carriden) should run from

Graham's Dyke, by Thirlstane, to the sea. Parliament granted the application, and the Presbytery of Linlithgow accordingly designed the new parish, declaring its southern boundary to be "Graham's Dyke," and the east boundary to run by Thirlstane. Now this east boundary, separating Borrowstounness parish from Carriden parish, runs northwards, past old Grange House, viz., on the west side of it; so that this Act of Parliament, in the year 1649, and the decree of Presbytery which followed on it a year or two afterwards, is distinct evidence that the Roman wall was at that time not only distinguishable, but a well-marked and conspicuous work, as far east as old Grange House. But even at a later date, viz., in the year 1726, the rampart or dyke as far as Grange House was still visible. ALEX. GORDON, whose journey through Scotland was published in that year, writes with reference to ANTONINE'S wall thus:—"*For a mile beyond Kinneil*, a faint track of the rampart may be traced *to the House of the Grange*, above Borrowstounness, where it is yet to be seen, a little way farther eastward. But from this place I could never find a vestige of it any more." (Page 60.)

It is right to notice, that the word Kinneil, a place situated between Inveravon and Bridgeness, is supposed by Celtic scholars to signify the "end of a turf embankment." This might lead one to expect that the Roman wall should have terminated at the place now bearing that name. Perhaps this etymological difficulty may be got over by remembering that, though the wall of ANTONINE was brought as far eastward as Bridgeness, when the sculptured tablet was erected there to record the fact, the wall may not always have extended as far as that spot. In fact, about half a century previously, in the time of AGRICOLA, there had been a different barrier, though following the same general line, and it may have terminated at the place now called Kinneil. After ANTONINE'S time, there were also several changes made on this military work. Moreover, it deserves to be mentioned, that Kinneil formed a large barony and a parish, before Borrowstounness was carved out of it, and that there may have been another place farther eastward anciently called Kinneil.

This, however, is a question for the archæologists to settle; and I proceed now to consider the bearing of the tablet on the geological question, assuming that the place where it was discovered really indicates the eastern termination of the rampart.

If the land was then twenty-five feet lower than now, then the tablet, and the wall in which it was fixed, must have been six feet under the sea at every tide, and must also have been so exposed to the beating of the waves, that neither tablet nor wall could have stood many weeks. It is impossible to suppose that the tablet, with elaborate sculpturing, and bearing a dedication to the emperor, could have been set up in such a position. Moreover, the neck of land which joins the ness or knoll to the mainland, being only twenty-three

feet above high water, must have been submerged and exposed, so that any wall or rampart on that neck would soon also have succumbed to the waves. Then there is the old building at the point of the ness, which, if Roman (as it appears to be) must have been *at all times* under water, even at the lowest tide, were Professor GEIKIE's theory correct.

Perhaps it may be suggested that the tablet, where found, was not in the position where it was set up by the Romans. Any one suggesting this improbable view, must state some grounds for it. The presumptions are all against it. It was natural that the tablet, after being set up, should in the course of time fall, and fall on its face. In that position it was found. The tablet might fall from the decay of the building which supported it;—or it might have been thrown down by the Romans when they had to abandon the district, to preserve the tablet from the desecration to which it might probably be subjected by the natives;—and there is evidence, that the Romans did follow this practice in the case of altar-stones. Or, the tablet might have been discovered by the natives while still standing, and been thrown down by them. No other supposition is admissible; for it will scarcely be surmised, that the tablet, having been found by the Caledonians in some other part of the Roman rampart, was brought by them to the knoll, and thrown down there. As the tablet weighs more than a ton, it would have been exceedingly difficult for the natives to have transported it any considerable distance, at all events without injury to the sculpturing. Therefore the conclusion seems inevitable, that the place where the tablet was discovered, is the place where it was originally set up.

I venture to think that Professor GEIKIE, had he known the facts stated in this paper, would not have affirmed, as he has done, that the Roman "wall was built, *when the land was at least twenty feet lower than at present.*"—*Lond. Geolog. Soc. Journal*, vol. xviii. p. 209. One of the circumstances founded on by him in proof of that position is, that the eastern termination of the wall was (as he alleges) at Carriden, where (as he says), so far from "having any reference to the present limit of the tide, it actually stood on the summit of a steep bank overhanging the sea, above which it was elevated fully 100 feet. If," he adds, "the land here were depressed twenty-five feet, no part of the wall would be submerged."

Here, I think, has been the chief cause of the Professor's mistake. He supposed that the wall terminated at Carriden, on a steep bank overhanging the sea, at a height of fully 100 feet. Had he been aware of the wall having terminated at Bridgeness, and at a height of only nineteen feet above the sea, he would at once have seen that it could not have been built, when the land was twenty-feet lower than at present.

When I commenced this paper, I had intended to confine myself to an account of the slab found at Bridgeness; but as similar slabs have been

discovered at the western termination of the Roman wall on the Clyde, and as Professor GEIKIE founded also on some circumstances observed at that end of the wall in confirmation of his views, I have made inquiries applicable to that district also, and think it right to state the result of them in this paper.

Western Termination of the Roman Wall.

Chapel Hill, where the wall terminated on the west, is on the north bank of the Clyde, and projects from the Kilpatrick Hills. I cannot say what is the nature of the rock composing Chapel Hill—not having had an opportunity of visiting the place. Its point next the river, is about 150 yards distant from what is now the river margin; and the height of the hill, as ascertained by Ordnance surveyors, is about forty-two feet above high water.

The base of the hill is stated by Professor GEIKIE to be about twenty-five feet above the present sea-level. (*Lond. Geol. Soc. Journ.* vol. xviii. p. 229.) He considers that, when the Roman wall was built, the sea washed the base of the hill, *because* in that case (as he argues) there would be (I now quote his words) "*a peculiar fitness* in the site of its western termination." "The Chapel Hill must, in that case, have been a promontory jutting out into the stream," and "commanded the passage of the Clyde."

If this is the only ground on which it is supposed that the sea must then have reached to the base of Chapel Hill—and I can see no other stated—it appears to me to be a very slender basis for so weighty a conclusion. Chapel Hill, though it might not be peculiarly fit as a site for the termination of the wall, might have been thought sufficiently suitable; and the mere circumstance that a narrow stripe of land existed between the hill and the Clyde, certainly did not make it unsuitable. This stripe was probably not then so wide as it now is, viz., 150 yards; and, at all events, with a garrison of soldiers on the hill, it would have been easy to have opposed the passage of an enemy between the hill and the river.

I think, therefore, that the reason assigned for supposing that the sea *must* have washed the foot of the Chapel Hill, when the wall was built, is not sound. But in the course of my inquiries I have learnt one or two facts, which seem conclusive against the supposition.

(1.) Thus it is stated, in the Statistical Account of the parish by the Rev. Mr BARCLAY, that "the workmen employed in forming the canal in 1790, which passes the bottom of the Chapel Hill, found, in a subterranean recess, vases and Roman coins."

I do not know whether it is to this "*find*" that Professor GEIKIE alludes when he says in his paper, that, "in making the canal, a number of Roman antiquities were found at various depths in the alluvium. These seem to have

been part of the ruins from the fort above." He had previously mentioned, that between the Chapel Hill and the "margin of the river, lies the Forth and Clyde Canal, the surface of which is twenty feet above high-water mark, and the base of the hill five or six feet higher."

If the Roman antiquities here mentioned be the same as those described in the Statistical Account, their position is not correctly stated by Professor GEIKIE. They can in no sense be represented as having fallen from the fort above. The relics were found not (as he says) at various depths in the alluvium, but "*in a subterranean recess*," *i.e.*, in a cavity which contained them. As there were vases as well as coins, the probability is that it was a grave. Now, as this recess, when formed, must have been several feet below the surface of the ground, and as the surface of the ground is admitted to have been only twenty feet above the present high-water mark, the "recess" must have been at least seven or eight feet under the sea, if, during the Roman occupation, the land was twenty-five feet lower than now.

(2.) Another fact of the same kind is mentioned by Mr JAMES SMITH of Jordanhill—viz., that some Roman coins were found at Ferrydike (which is not far from Chapel Hill), within ten feet of the high-water level."—*Newer Pliocene Geology*, p. 15.

(3.) All along the narrow stripe of low land, which lies between Chapel Hill and Dumbarton, Roman remains of various kinds have been discovered, implying that this low land certainly could not have at that time been occupied by the sea. The public road between Chapel Hill and the town of Dumbarton runs along this plain, keeping the highest parts of it; and I see the heights above the sea marked on the Ordnance Survey maps at various points as follows:—As the figures on the map are above the medium sea-level, I have deducted eight from them to get the height above high water. Opposite to Dunbuck Hill, where the stripe of land is 320 yards wide, the height above high water is 11 feet; one quarter of a mile to the west, 11 feet; half a mile farther west, 5 feet; half a mile farther west, 8 feet; one mile farther west, 10 feet; half a mile farther west, 11 feet; one mile farther west, 10 feet; nearer Dumbarton, 4 feet. Professor GEIKIE, in the passage quoted by me at the commencement of this paper, admits that before the last rise of land, the whole district to the west of Chapel Hill was covered by the sea at high water. Now it is well known, from various authentic sources, that the Romans had a colony and a garrison at Dumbarton. In fact, it was for a time the capital, so to speak, of the Roman province which was attempted to be formed in that part of Britain. But to this town they had no access, except from the eastward; and accordingly a military way, with several forts, existed between Dumbarton and Chapel Hill.

One of these forts was at Dunbuck; and the remains of it, consisting of solid masonry, are referred to by several recent authors.

Part of the military way was discovered at Glenarbach, where the stripe of land is only about 300 yards from the Clyde, and the ground is about eight feet above high-water mark.—*Stewart, Caled. Rom.* p. 286.

It is hardly necessary to observe, that all these facts are entirely inconsistent with the possibility of the sea, during the occupation of this part of Scotland by the Romans, having stood twenty-five feet or even ten feet lower than it does at present.

Roman Roads across the Forth.

If the position of the various Roman remains, at both the east and west terminations of ANTONINE's Wall, is conclusive against the idea that during the time of the occupation of Scotland by the Romans, the land was not twenty-five feet lower than now, it seems unnecessary to advert to any other facts bearing on the subject.

But since an opportunity is afforded, I may make some corrections, though they are not very material, in the description given in the "Estuary of the Forth," of two Roman roads and fords, the position of which was there stated to be also conclusive against the same view. The corrections which I have to make, strengthen that conclusion.

The first road and ford mentioned, was one crossing the River Forth at or near a place called the Drip, about two miles N.W. of Stirling. This road led from Camelon northwards to the camp at Ardoch, and is for several miles traceable near Torwood. I have myself walked along it. The channel of the river at Drip is firm, being rocky, and the depth of water there, when the river is not flooded, is only about two feet. The tide now comes up to Craigforth, which is about half a mile below Drip, and with a fall of only four feet between the two points. If, therefore, the land was during the time of the Romans twenty-five feet lower than now, neither the Drip Ford nor any river could then have existed; for the whole country west of Stirling must have been covered by the sea even at the lowest spring tides.*

The Roman road north of the Drip Ford passed through Kincardine Moss, and was found at a depth of eight feet below the surface; of course, no such road could have been used, or could have been made, if the land here was twenty-five feet lower than at present.

Besides the road through Kincardine Moss, another Roman road was found in Flanders Moss, a few miles to the westward, which is equally inconsistent with possibility, if the whole of this district had been covered by the sea.

* Rev. Mr TAIT, in his paper on Peat Mosses in Perthshire, published in Trans. Roy. Soc. Edin. vol. iii. p. 276, mentions, that seventy yards of this road were traced through the moss, and was about twelve feet wide; see also New Stat. Account of Parish of Kincardine.

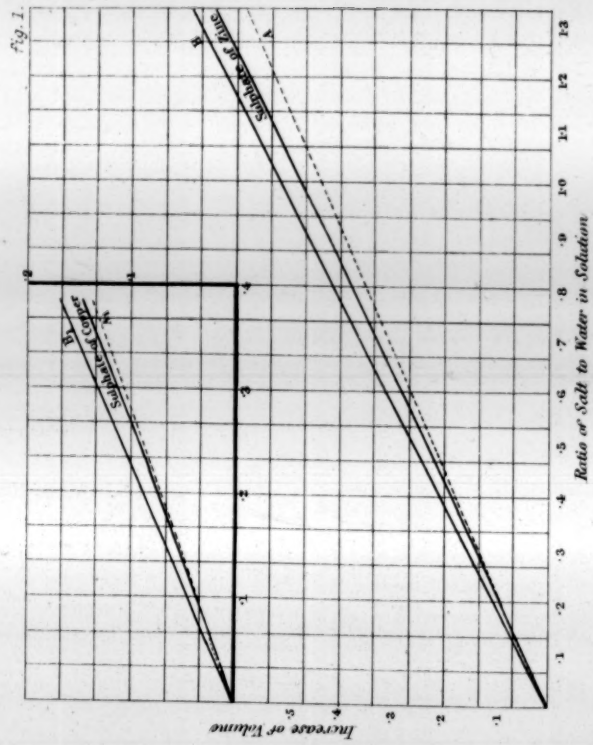
I have assumed, that the upheaval in this part of Scotland was, as Professor GEIKIE says, only twenty-five feet; but in the upper part of the estuary the upheaval must have been more than twenty-five feet, because the old sea cliff, from an examination of which, at Leith, Professor GEIKIE drew his belief, rises towards the head of the estuary, up to a height of above thirty feet, so that the extent to which these roads and fords across the Forth were submerged (according to Professor Geikie) would be six or seven feet more than I have stated.

If these facts have been correctly stated, it follows that an erroneous view must have been taken of the Roman remains at Cramond and Camelon, when they were supposed to indicate that the sea at these places, in Roman times, stood twenty-five feet higher than now. The sea could not be at one level at Bridgeness and Stirling, and at a very different level between these places on the same coast. The appearances at Cramond and Camelon, and others which have been founded on, can, I think, be well enough explained by the deposition of sediment brought down by the rivers to the sea-coast during a period of 1800 years. It is true that the surface of the land in these quarters is higher than formerly. That is due, not to any rise of land, but to the combined operation of river floods and sea waves. For example, the reason why the tide does not flow, as formerly, up to the old bridge across the Esk at Musselburgh, and up to the site of the old supposed Roman town, near Camelon, on the River Carron, is, that the sea is obstructed and dammed back, by enormous accumulations of mud, sand, and gravel brought down by these rivers.

From the facts brought forward in this paper, it appears clear, that the last change in the relative levels of sea and land, as indicated by the line of old sea cliff which fringes our coasts, not only in the central parts of Scotland, but all round our island, must be referred back to a much earlier period than the time of the Romans. That, indeed, had been the opinion of all geologists who had studied the question, until Professor GEIKIE brought forward his views on the subject in the year 1862. As his views have been adopted by such high authorities as Sir CHARLES LYELL and Professor RAMSAY, not to speak of other geologists of less eminence, I have thought it right, in a matter of considerable geological interest, to show that these views proceeded on mistake; and I have no doubt that, when Professor GEIKIE becomes acquainted with the new facts which I have brought forward in this paper, he will, in the interests of scientific truth, frankly admit that he had been misled by inaccurate information.

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Fig. 1.


Solubility of Salts
A
Sulphate of Copper
B
Increase of Volume
Ratio of Salt to Water in Solution

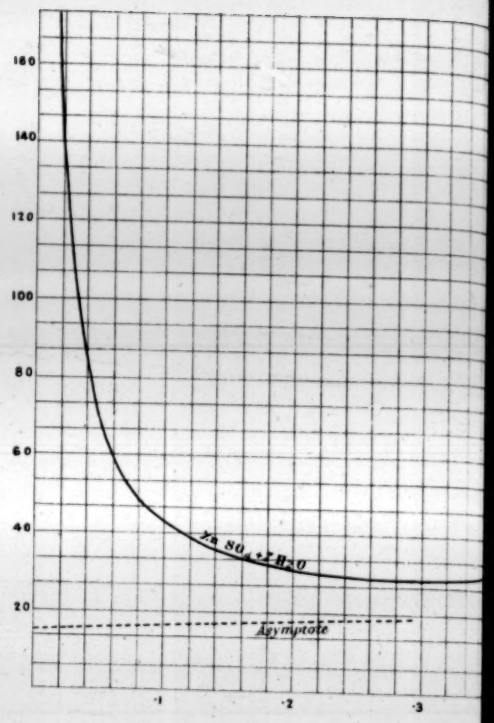
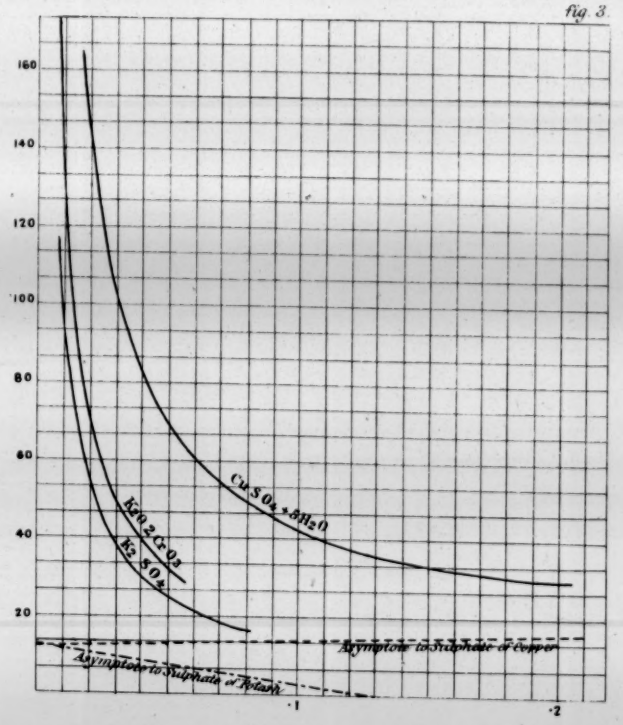
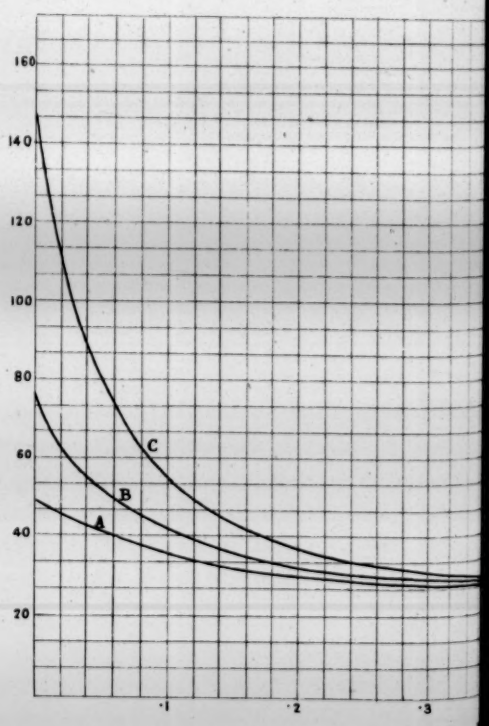

Cu SO4 + 7H2O
Asymptote

Fig. 3.


Cu SO4 + 5H2O
K2 Cr O4
K2 SO4
Asymptote to Sulphate of Copper
Asymptote to Sulphate of Potash


C
B
A

III.—*On the Electrical Conductivity of Certain Saline Solutions, with a Note on the Density.* By J. A. Ewing and J. G. MacGregor, B.A. Communicated by Professor Tait. (Plate II.)

(Read April 21, 1873.)

I. *Note on the Density.*

In preparing solutions of various salts with a view to determine their electrical conductivity, we found it of importance to know both the amount of salt in each solution, and its density. We prepared the solutions by mixing known weights of the salt under examination with known volumes of distilled water, and then measured their densities as soon as solution was complete by weighing a glass bulb in the liquid. We employed a balance which weighed to a milligramme with a kilogramme in each pan, and to give the results as great accuracy as possible we corrected for the upward pressure of the air displaced by the weights. The weight of the bulb itself was about eighty grammes. When not in use it was kept immersed in distilled water, in order to prevent change in its weight or volume being caused by impurities adhering to it. The temperature at which the densities were taken was 10° centigrade.

The following table shows the connection between the density and the composition of solutions of pure sulphate of zinc. The third column gives the volume to which unit volume of the original water was extended after the salt was dissolved.

I. Ratio of $ZnSO_4 + 7H_2O$ to water in solution.	II. Density = weight of a cubic centimeter at 10° C. Grammes.	III. Ratio of volume of solution to volume of water.	I. Ratio of $ZnSO_4 + 7H_2O$ to water in solution.	II. Density = weight of a cubic centimeter at 10° C. Grammes.	III. Ratio of volume of solution to volume of water.
1 to 40	1·0140	1·0108	1 to 2	1·2186	1·2308
1 „ 30	1·0187	1·0144	1 „ 1·5	1·2709	1·3114
1 „ 20	1·0278	1·0216	1 „ 1·361	1·2895	1·3455
1 „ 10	1·0540	1·0436	1 „ 1	1·3530	1·4782
1 „ 7	1·0760	1·0622	1 „ ·8	1·4053	1·6011
1 „ 5	1·1019	1·0893	1 „ ·752 Saturated	1·4220	1·6385
1 „ 3	1·1582	1·1512			
1 „ 2·5	1·1845	1·1819			

The increase of volume due to the dissolution of the salt, that is to say, the decimal part of the numbers in column III., is given as a curve in fig. 1, along with the ratio (expressed as a decimal) of salt to water in the solutions. It

will be seen that the line thus obtained is at first slightly curvilinear, curving upwards, but afterwards becomes as nearly as possible straight. In other words, the amount of expansion is not proportional to the quantity of salt dissolved in the case of the weaker solutions, but the ratio increases as the solutions become stronger, till it is sensibly constant.

If the expansion had been due to the water of crystallisation, and the anhydrous part of the salt had dissolved in the total water present without further increase of volume, the line would have been straight, since the amount of expansion would be directly as the amount of salt dissolved. The dotted line A shows the volumes which would have been obtained if this had been the case. The actual volumes of the strong solutions are much greater than such a hypothesis would allow them to be; but it is a singular fact that the experimental curve passes at first below this straight line, and then crosses and rises considerably above it, or, the expansion by the solution of this salt in water is *less* than is accounted for by the water of crystallisation when a small quantity of salt is dissolved, but *greater* when the quantity of salt dissolved is great. Owing to the proximity of the two lines, it is difficult to show this graphically, but the following table will make it obvious :—

Ratio of salt to water.	Actual volume of solution.	Calculated volume due to $7H_2O$.	Ratio of salt to water.	Actual volume of solution.	Calculated volume due to $7H_2O$.
1 to 40	1·0108	1·0110	1 to 7	1·0622	1·0627
1 „ 30	1·0144	1·0146	1 „ 5	1·0893	1·0878
1 „ 20	1·0216	1·0220	1 „ 3	1·1512	1·1464
1 „ 10	1·0436	1·0439	&c.	&c.	&c.

This result would seem to show that the water of crystallisation is not separated from a salt when it is dissolved in water.

The line B shows what the volumes would have been had the salt simply been introduced into the water without being dissolved. It is calculated by taking as the density of crystallised sulphate 1·931 (MILLER's Chemistry).

SULPHATE OF COPPER (PURE).

I. Ratio of $CuSO_4+5H_2O$ to water in solution.	II. Density = weight of a cubic centimeter at 10° C. Grammes.	III. Ratio of volume of solution to volume of water.	I. Ratio of $CuSO_4+5H_2O$ to water in solution.	II. Density = weight of a cubic centimeter at 10° C. Grammes.	III. Ratio of volume of solution to volume of water.
1 to 40	1·0167	1·0082	1 to 5	1·1174	1·0739
1 „ 30	1·0216	1·0115	1 „ 4	1·1432	1·0934
1 „ 20	1·0318	1·0176	1 „ 3	1·1823	1·1277
1 „ 10	1·0622	1·0356	1 „ 2·597	1·2051	1·1494
1 „ 7	1·0858	1·0526	Saturated		

Columns I. and III. are given graphically on the upper part of fig. 1. The scale in this case is double that previously employed, on account of the sparingly soluble character of this salt. As before, the line is at first curved and afterwards straight. The rate of expansion is throughout much slower than with sulphate of zinc. The dotted line A_1 indicates the expansion due to the water of crystallisation alone. With regard to this line, the fact mentioned above appears again here, namely, that this hypothetical expansion is at first greater and afterwards smaller than that which is observed. The line B_1 indicates what the increase of volume would be if the salt were in the water without being dissolved, the specific gravity of the crystals being taken as 2·254 (MILLER).

SULPHATE OF POTASH.

I. Ratio of K_2SO_4 to water in solution.	II. Density = weight of a cubic centimeter at 10° C. Grammes.	III. Ratio of volume of solution to volume of water.	I. Ratio of K_2SO_4 to water in solution.	II. Density = weight of a cubic centimeter at 10° C. Grammes.	III. Ratio of volume of solution to volume of water.
1 to 100	1·0082	1·0018	1 to 20	1·0394	1·0119
1 „ 80	1·0103	1·0022	1 „ 16	1·0482	1·0136
1 „ 40	1·0201	1·0048	1 „ 11	1·0697	1·0189
1 „ 30	1·0265	1·0067	1 „ 9·488	1·0801	1·1054
1 „ 25	1·0319	1·0078	Saturated		

This table is not given graphically, but the numbers show a slight curvature or change in the rate of expansion, similar to what was observed in the two previous salts. As sulphate of potash is anhydrous, none of the increase of volume can be traced to water of crystallisation. The same remark applies to

BICHROMATE OF POTASH.

I. Ratio of $K_2Cr_2O_7$ to water in solution.	II. Density = weight of a cubic centimeter at 10° C. Grammes.	III. Ratio of volume of solution to volume of water.	I. Ratio of $K_2Cr_2O_7$ to water in solution.	II. Density = weight of a cubic centimeter at 10° C. Grammes.	III. Ratio of volume of solution to volume of water.
1 to 100	1·0069	1·0031	1 to 25	1·0274	1·0123
1 „ 80	1·0088	1·0037	1 „ 20	1·0345	1·0150
1 „ 50	1·0137	1·0062	1 „ 15	1·0452	1·0205
1 „ 40	1·0172	1·0077	1 „ 12	1·0561	1·0261
1 „ 30	1·0231	1·0100	Saturated		

An inspection of column III. shows that in this case the rate of expansion is as nearly as possible uniform from the first.

When solutions of sulphate of zinc and sulphate of copper were mixed it appeared as if a slight contraction took place. The density of the mixture seemed generally to be slightly greater than the mean of those of the two components, equal volumes of both being taken. But to make a satisfactory series of experiments on this subject great precautions would be necessary; such as to take the densities of the components immediately before mixing them, and that of the mixture immediately after, in order to prevent error due to evaporation; also to mix the two in absolutely equal volumes. Such precautions as these we did not attempt to take, as the main object of our inquiry was the electrical resistance, and hence we cannot speak positively on this point.

II. *Electrical Conductivity.*

The electrical conductivity of saline solutions is a subject which has received the attention of numerous physicists at intervals during the last thirty-six years. The earliest observer was POUILLET, whose investigations are given in the "Comptes Rendus," vol. iv., 1837, and may also be found in his "Traité de Physique," vol. i. He does not appear to have taken any account of the polarisation of the electrodes produced by the transmission through the solution of the electric current, without which it is impossible to measure its conductivity. His results, which are few in number, are rendered untrustworthy by this circumstance alone.

The next deserving of notice is HANKEL (Pogg. Ann., lxix., 1846). He divided the current of two or three DANIELL's cells into two parts, the one passing through a rheostat, and the other through a tube containing the solution whose resistance was to be measured, and then passed them through the coils of a differential galvanometer. It will be seen that this method neglects polarisation—except in the case of sulphate of zinc, in which it was at least partially avoided by using zinc electrodes. But, though for this reason HANKEL's figures cannot be even approximately correct, his paper is interesting on account of his having made some general discoveries of importance, such as that the effect of increase of temperature on a saline solution is to diminish its resistance. It will be remarked that this is exactly the opposite of what holds in the case of metallic conductors, but is similar to what takes place with glass, india-rubber, gutta-percha, and, as FARADAY showed, with dry sulphide of silver and other substances. HANKEL also observed, that though, generally speaking, the conductivity increases as the density of solution increases, there is a solution of sulphate of zinc which conducts better than the saturated one.

In the same year (1846) E. BECQUEREL published a paper (Ann. de Chimie,

3, xvii.), in which he announced the same discoveries as HANKEL. He went into the subject much more fully than HANKEL did, and his method was also better than the previous one. It was based on a suggestion of WHEATSTONE's, and depended, like that of HANKEL, on the use of the differential galvanometer. But instead of having only one tube of the solution, BECQUEREL had a tube in each circuit. After adjusting the length of the tubes so that no deflection was produced, he then introduced a coil of known resistance into one circuit, and shortened its tube so as to bring the needle again to zero. Then the resistance of the wire was equal to that of the column of liquid by whose length he had shortened that tube. This method was an improvement on that of HANKEL, in so far as it did away with, or at least diminished the effect of polarisation—a phenomenon, however, to which BECQUEREL seems to have paid little attention, and to which he appears to refer in the expression, "la resistance au passage des solides dans les liquides." But it was open to one grave objection, namely, that the current employed was of sufficient intensity to cause a very perceptible electrolysis of the liquid during the time it was being tested. This difficulty has been overcome by the use of the mirror galvanometer in the mode of testing we have employed.

The results obtained by BECQUEREL are summed up by him as follows:—"Saline solutions may be divided into two classes with regard to conductivity. The first includes those solutions whose conducting power increases with the degree of concentration up to the point of saturation, such as sulphate of copper and chloride of sodium. The second includes the solutions of deliquescent salts, or those which are exceedingly soluble in water, and whose conductivity at first increases with the degree of concentration, soon attains a maximum, then diminishes as the concentration increases; sulphate of zinc and nitrate of copper belong to this class.

"If we represent by C the conductivity, and by q the quantity of salt dissolved in unit volume of the solution, we have the equation—

$$\frac{1}{C} \text{ (or R, the resistance)} = A + \frac{B}{q},$$

A and B being two constants for the same salt at a constant temperature."

This equation, he says, applies to the solutions of the second class only from a very weak solution up to that of maximum conductivity. BECQUEREL's results are often quoted as conclusive on the subject of the conductivity of liquids.

The next experimenter was HORSFORD (Pogg. Ann., lxx., 1847). His method was no improvement on those of his predecessors, but rather the reverse, as polarisation was not properly eliminated. His experiments were chiefly made on sulphuric acid and the metallic chlorides.

After him came WIEDEMANN (Pogg. Ann., xcix., 1856). He examined one or two salts carefully. He gives no formula.

Next comes BECKER (Ann. der Chem. und Pharm., 1850 and 1851). His experiments were chiefly on the effect of increased temperature. He gives an elaborate formula, in which the resistance appears as an expansion of the first, second, and third powers of the temperature, and also of the amount of salt in solution.

The most extensive series of experiments on sulphate of zinc were those of BEETZ (Pogg. Ann., cxvii., 1862). His only precaution against polarisation was the use of zinc electrodes, which, curiously enough, he amalgamated. His investigations of the relation between conductivity and temperature are very valuable. Unfortunately, in the other part of his work—the connection between conductivity and density—he was not careful to keep to exactly the same temperature throughout a whole series of solutions, so that his results do not admit of accurate graphic representation. He gives the conductivity in the form of an expansion of the first, second, and third powers of the amount of salt in solution, and does not appear to have arrived at any more simple relation between them.

The electromotive force caused by the polarisation of the electrodes reaches a maximum in every case, and, however great a decomposing electromotive force be used, this can never be exceeded. One form of apparatus, based on this fact, was a trough with parallel plates of platinum for electrodes, the distance between which could be varied at pleasure. The current was first measured by a tangent galvanometer, when the trough alone was in circuit along with a sufficient number of cells to produce the maximum polarisation. The distance between the plates was then reduced, a metallic resistance being introduced into the circuit and adjusted till the deflection of the galvanometer needle was the same as before. Then the resistance of the coils which had been introduced was equal to that of a column of liquid, whose cross section was the area of the plate, and whose length was the difference of the distances between the plates in the first and second experiment. The effect of polarisation, being the same in both cases, was eliminated. The electromotive force, however, which was required to produce the maximum polarisation, was so high as to cause rapid electrolysis, which not only changed the constitution of the liquid during the time occupied by the test, but also introduced an element of error due to the continually varying resistance of the bubbles of gas which formed on the plates. In order to remedy these defects, KOHLRAUSCH and NIPPOLDT (Göttingen Nachr., 1868; Jahresbericht des phys. Vereins zu Frankf., 1867–68; also Pogg. Ann., cxxxviii., 1869) employed induced currents from a magneto-electric machine, which followed each other in rapid succession in opposite directions. The electromotive force of these currents was reduced by means of a thermoelectric

pair to a very small fraction ($\frac{1}{429000}$) of a GROVE's cell. Their intensity was measured by an electrodynamometer. In this way KOHLRAUSCH and NIPPOLDT determined the resistance of solutions of sulphuric acid, and found that at a temperature of 22° C. the maximum conductivity is reached when the specific gravity is from 1·20 to 1·25, a result which agrees fairly with those of BECKER and others.

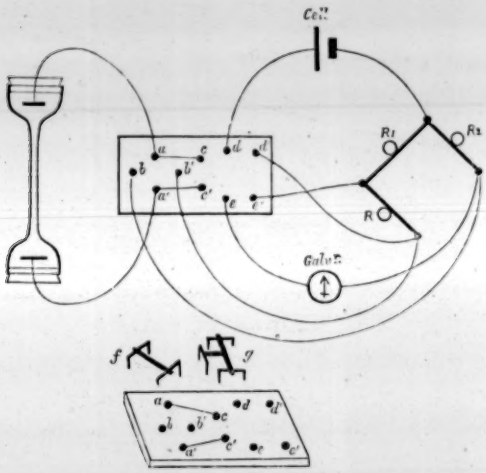
The method of PAALZOW (Berlin, Monatsbericht, 1868; also Pogg. Ann., cxxxvi., 1869) was very ingenious. He employed for electrodes two pieces of pure zinc, which were placed in the bottom of glasses filled with saturated solution of sulphate of zinc. These two glasses were connected together by a siphon filled with the liquid whose resistance was to be measured. It is well known that pure zinc electrodes do not become polarised in a solution of sulphate of zinc. Hence this method avoided polarisation, provided that none took place at the junctions of the two liquids. We are not aware that any experiments have been made to determine whether this is possible. It might form an interesting subject of inquiry. PAALZOW's method permitted that sufficiently low electromotive forces might be used to avoid electrolysis. The diffusion of the two liquids must have been a source of error, especially as the resistance of mixtures is totally different from that of their components (*v.* page 67).

These notices are sufficient to show how much importance has been attached to this subject. The various modes of definition, both of the solutions and of their resistance, and also the variety of temperatures adopted by the above experimenters, render comparison of their results extremely difficult. The attempt, however, has been made by WIEDEMANN in his "Lehre vom Galvanismus," vol. i., part 1. He finds great discrepancies in many cases, which are probably due to the disadvantages of the several forms of experiment. We think that these disadvantages are avoided in the following mode of testing, a mode which, so far as we know, is considerably different from any that have hitherto been made use of.

The electrical resistance of a substance is easily measured, if it either does not act at all as a producer of an electric current, or produces a constant one. A wire is an example of the first class; the cell of a galvanic battery is an example of the second. The resistance of the first is generally best measured by the "Wheatstone bridge;" that of the second by the electrometer, by "shunting" the current through a known metallic resistance. But the solutions under examination fulfilled neither of these conditions: not the first, because of the polarisation of the electrodes; and not the second, because the polarisation disappears (after the polarising current is stopped) far too rapidly to allow of such a measurement. We found, however, that polarisation does not attain its maximum whenever a current is made to pass through the liquid, in fact, that at the moment the circuit is completed there is *no* polarisation. It

therefore occurred to us that the Wheatstone bridge test would be applicable provided we could observe the *instantaneous* effect of a current upon the solution. This we accomplished in the following manner:—

The solutions were placed in a glass tube, narrow along the central part, and wide at the ends (see figure). The narrow part was about 15 centimetres



in length, and 0·35 square centimetres in internal cross section. The current passed between platinum plates, connected with platinum wires passing through the bungs which closed the tube. The ratio of the cross section of the wide parts to that of the narrow part was so great as to permit small variations in the positions of the plates, without perceptibly changing the electrical resistance of the solution. The resistance coils used were in the form of a Wheatstone bridge, as arranged by ELLIOTT Brothers, London. The galvanometer was an ordinary "dead beat" mirror one, of small resistance. It is essential that in this test the inertia of the mirror and needle should be very small. The electromotive force was that of one GROVE'S cell. The electrodes of the tube dipped into mercury pools a and a'. The "rockers" were arranged together on one board. By means of the commutator f, a could be connected with b or b', and a' with b' or b. In this way the alternate currents could be sent through the tube in opposite directions; this tended to diminish polarisation. The "battery circuit" was completed when d and d' were connected,—the "galvanometer circuit" when e and e' were connected; c and c' were kept in permanent connection with the electrodes of the tube by means of the fixed wires ac and $a'c'$. The battery circuit and galvanometer circuit were simultaneously closed by means of the rocker g. This consisted of a ⊣ frame of insulating material, with a cross wire at each of the three extremities, bent like an inverted U. The six

feet were so adjusted as to be capable of dipping into the six mercury pools c, c', d, d', e, e', but not all at once, for the two feet which dipped into d and e were longer than the other four, so that the rocker could oscillate about an imaginary line joining d and e as an axis. Its natural position was one inclined to the left, that is to say, with feet dipping c, c', d, and e, and the other two legs suspended in the air above d' and e'. In this position of the rocker no current was passing through the tube, but the ends of the tube were connected together through c and c'. Hence so long as the rocker remained thus, the tube was "short circuited," and depolarised itself much more rapidly than it would have done had its terminals been left insulated,—that is to say, the polarisation produced in it by previous currents disappeared much more rapidly than it would otherwise have done.

When g was rocked to the right this connection between the ends of the tube was broken by the left hand part of the rocker being lifted out of the pools c and c'; and the battery and galvanometer circuits were completed by connection being established between d and d', and e and e' respectively. This state of things lasted for an instant only, as the rocker was immediately allowed to fall back to its natural position, that is, inclined to the left. During that instant a current was passing through the tube, and it was by its instantaneous effect upon the galvanometer that the resistance of the liquid was ascertained. But this current, however short its duration, produced a sensible and even considerable polarisation in the tube. This polarisation soon reduced itself when the rocker g fell back to its natural position, in virtue of the connection between c and c', and the tube returned in a few seconds to a neutral state, in which it was ready for another rock of g.

We found, after a time, that it was best to connect e and e' by a separate key, because if by the previous method d and d' were connected, even a very small fraction of a second before e and e' (as might easily happen through an inequality in the length of the rocker's legs), polarisation went on to such an extent before the galvanometer circuit was closed as to render the measurement incorrect. In order to make the rate of polarisation as small as possible, the current was diminished by inserting 10,000 B. A. units into each of the two arms of the bridge (R_1 and R_2). The resistance of the tube itself varied from 1000 to 10,000 B. A. units. By these high resistances the current was so weakened as to make the amount of electrolysis in a short time quite inappreciable, while the rate at which the electrodes became polarised was made so slow, that the effect of a current upon the solution could be noted before polarisation had time to interfere with the result.

The mode of testing will be best understood by an example. Suppose that the resistance R of the coils is nearly equal to that of the tube. Insert f, and

rock g. The spot of light moves (say) first to the right, but immediately passes to the left of zero, the deflection to that side gradually increasing if the rocker is held down. This after-deflection to the left takes place although the rocker g was down for the smallest possible time only. The first motion to the right shows that R is too great; the next, to the left, shows that owing to the polarisation the *apparent* resistance in the tube has come to exceed R. But the first indication is the one which is to be received; so diminish R, reverse f, and give the tube some time to depolarise itself through $c\,c'$. Then rock g again. Suppose this time there is no motion whatever to the right, but one to the left only. This shows that R has now been made too small. In this way, by successive trials, a very exact value of R will be obtained, the object being to adjust R so that the motion to the left shall just, and only just be preceded by an excessively slight motion to the right. When this state of things is nearly arrived at, the motion to the right becomes a mere trembling of the spot of light, requiring great care to observe it at all.

We found it almost impossible to get rid of local currents in the tube, which seemed to be due either to impurities on the platinum plates, or to the presence of moisture at the junction of two metals in the binding-screws or rockers. They were so considerable as at first to threaten to spoil the method of testing completely. Numerous anomalous results which we got in our earlier experiments were traceable to their action. This difficulty, which at first seemed very formidable, was easily overcome in the following way. Whenever we noticed that there was a local current in the tube (which we could tell by removing g altogether, and observing whether there was any deflection when e and e' were connected), we set the commutator f, so that when g was rocked the residual polarisation in the tube would be in the opposite direction to that of the local current. By closing the battery circuit for a sufficient number of seconds, a polarisation could be induced considerably greater than the local current; then, as that gradually died away, there was a particular instant at which polarisation was exactly equal and opposite to the local current; or, in other words, for an instant the tube was in an absolutely neutral condition. If, then, the battery circuit were closed just at this instant, the test would be a fair one; and in order to effect this, it was only necessary that we should know the exact instant when the tube reached the neutral state. This could be ascertained by keeping a small rocker, which connected e and e' only, in a constant state of oscillation, and noticing when the spot of light came to rest at zero on the scale. Of course, during this time the left hand part of g, which connected c and c', had to be removed, else the galvanometer could not give any indication of the electrical state of the tube.

These various improvements on the original mode of testing were effected gradually, and as the want of them was felt. It took us about two months to

bring the method to its full efficiency. During that time the results of our experiments were untrustworthy and inconsistent. But after every difficulty had been overcome, they were obtained in a perfectly satisfactory manner. We think we need have no hesitation in saying, that the above method of measuring the resistance of liquids meets all the difficulties of the problem, and, though somewhat laborious, is capable, if proper care be taken, of giving results on which the utmost reliance may be placed.

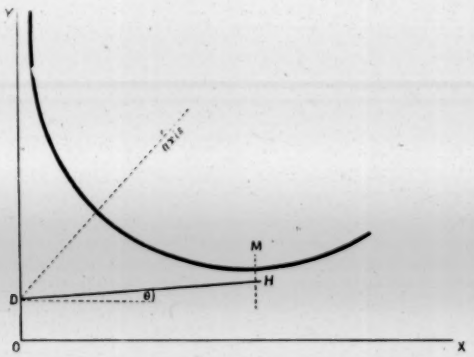
We found that the effect of temperature was so marked as to make it absolutely necessary to ascertain the resistance of all the solutions at the same temperature. We adopted 10° centigrade as our standard, and all our observations, both of density and resistance, were made at this temperature.

The salt we first examined was normal sulphate of zinc—$ZnSO_4 + 7H_2O$. Before use the salt was freed from all impurities, of which at first it was very full. Nineteen solutions in all were prepared, most of them by dissolving a known weight of salt in a known weight of distilled water, from 1 in 40 parts, and so on, down to a saturated solution. The resistance of the weak solutions was found to be very great. The rate of diminution was at first rapid, but gradually fell off as the density increased, till it became very slow as the point of minimum resistance was approached. After that the resistance slowly rose again up to the point of saturation. The following table gives our numerical results with reference to this salt. Column III. contains the resistance in B. A. units of the liquid as measured in the tube. Column IV. contains the specific resistance in B. A. units. By "specific" resistance is meant the resistance to conduction between a pair of opposite faces of a cubic centimeter of the substance. This quantity varies with the size of the cube adopted (being directly proportional to the length of the edge, and inversely proportional to its square), and has therefore no claim to be called specific; but the term is now in general use, and is convenient for purposes of calculation. In order to reduce the results of column III. to column IV., we had to make an accurate determination of the length of the tube and also of its cross-section, the latter being done by finding the weight of mercury it could hold. The ends were assumed to be parts of cones, and their resistance found by integration. The coefficient by which the figures in column III. are multiplied to give those in column IV., is for this tube ·022301.

Sulphate of Zinc.

I. Ratio of salt to water in solution.	II. Density at 10° C.	III. Resistance in tube at 10° C. B. A. Units.	IV. Specific resistance at 10° C. B. A. Units.	I. Ratio of salt to water in solution.	II. Density at 10° C.	III. Resistance in tube at 10° C. B. A. Units.	IV. Specific resistance at 10° C. B. A. Units.
1 to 40	1·0140	8200	182·9	1 to 1·5	1·270	1280	28·5
1 „ 30	1·0187	6300	140·5	1 „ 1·37	1·2891	1270	28·3 min.
1 „ 20	1·0278	4980	111·1	1 „ 1·361	1·2895	1280	28·5
1 „ 10	1·0540	2860	63·8	1 „ 1·3	1·2987	1288	28·7
1 „ 7	1·0760	2280	50·8	1 „ 1·124	1·3288	1310	29·2
1 „ 5	1·1019	1890	42·1	1 „ 1	1·3530	1390	31·0
1 „ 3	1·1582	1510	33·7	1 „ ·8	1·4053	1440	32·1
1 „ 2·5	1·1845	1440	32·1	1 „ ·763	1·4174	1500	33·4
1 „ 2	1·2186	1360	30·3	1 „ ·752	1·4220	1510	33·7
1 „ 1·626	1·2562	1310	29·2	Saturated			

It will be seen that the conductivity reaches its maximum when the density is 1·2891, a solution which may be prepared by dissolving ·735 parts of salt in 1 of water. Becquerel referred to this point, but did not determine it, as he only tested three solutions, none of which were at all near it. We have recently found that Paalzow made an approximate determination of this point (Pogg. Ann., cxxxvi., 1869). He places it near the solution of 1 part of salt to 1 of



water, a considerably different position from that to which we have assigned it. This discrepancy may be due, and probably is due, to the fact that Paalzow's measurements were made at a temperature of 23° C., a very much higher one than ours. We regret that we have not had time to investigate the influence

of temperature on the position of the point of minimum resistance. This would form an interesting subject of inquiry.

The above table is given graphically in fig. 2. Excess of density over unity is measured along the horizontal ordinates (OX in sketch); specific resistance along vertical ordinates (OY).

The curve from the axis of Y to the point of minimum resistance M, we find to be an hyperbola, but not rectangular, as BECQUEREL'S formula would make it, the lower asymptote DH not being horizontal, but inclined as shown. The vertex corresponds to a solution of density 1·0785. And further, the other part of the curve, from M to the point of saturation, is symmetrical with the first part about a vertical line passing through M. These two facts enable us to give a definite formula, connecting the conductivity and density of solutions of this salt.

If we call θ the angle of inclination of DH, h the intercept of DH on the axis of Y = OD, we have, since the curve is an hyperbola from Y to M,

$$x \sec\theta\,(y - h - x\tan\theta) = c \quad \text{(a constant)};$$

or, generally,

$$y = a + bx + \frac{c}{x},$$

a, b, and c being three constants which can be determined by substituting three sets of known values of x and y. In the case of this curve, we obtained a and b from actual measurement. In finding b, the tangent of the angle of inclination of the asymptote, in this way, care must be taken to assign to the measured lengths in fig. 2 their true values, according to the scales employed. This being done, we find that $\tan\theta$ or b is 16, and h, or a (the intercept), is 14·4. Substituting these values in the above equation, and then finding c, the only remaining constant, by means of a known pair of values of x and y, we have

$$y = 14{\cdot}4 + 16\,x + \frac{2{\cdot}66}{x}.$$

To make this formula more convenient, we may write the density instead of the excess of density over unity. Thus—

$$\mathrm{R} = 14{\cdot}4 + 16\,(\mathrm{D} - 1) + \frac{2{\cdot}66}{\mathrm{D} - 1},$$

or

$$\mathrm{R} = 16\,\mathrm{D} + \frac{2{\cdot}66}{\mathrm{D} - 1} - 1{\cdot}6,$$

where R is the specific resistance and D the density, at 10° C.

This equation, of course, applies only to the left hand part of the curve

down to the solution of maximum conductivity; density 1·2891. To shew its accuracy we give the following table:—

Specific Resistance.		Specific Resistance.	
Observed.	Calculated by above Equation.	Observed.	Calculated by above Equation.
182·9	204·6	33·7	33·7
140·5	156·9	32·1	31·8
111·1	110·5	30·3	30·1
63·8	64·6	29·2	28·9
50·8	50·6	28·5	28·5
42·1	42·1	28·3	28·2

It will be seen that the first and second points do not agree at all well with the formula. With regard to them two things should be noticed. They are at a part of the curve near the axis of y, where an excessively small alteration in the density produces an enormous change in the resistance. Hence the liability to error is very great. If the curve got by this equation, however, were plotted on the plate, it would lie very close to the experimental curve, even at these points where the divergence is greatest. Further, at these points the experimental curve would be slightly to the left of the hypothetical one,—that is, a little nearer to the axis of y than the latter. Now the above formula assumes that the resistance of pure water is infinite. This is not absolutely the case, and hence the axis of y should not be a true asymptote, but should meet the curve at a finite distance from the origin. We might, therefore, expect that the curve should incline towards the axis of y, where its form is determined by weak solutions, more than it would if it were truly hyperbolic. Possibly this consideration of the finite resistance of pure water may account for the divergence in the case of the first two points. The other points all agree with the formula very exactly, the slight deviations being quite accounted for by the difficulty of keeping the solutions absolutely at the standard temperature.

Since the curve is symmetrical about a vertical axis passing through the point of maximum conductivity, we have the means of forming an equation for the part to the right of that point. Considering the axis of y as transferred parallel to itself to the right to a distance, equal to twice the excess over unity of the density of the solution of maximum conductivity, we must write $(\cdot 578 - x)$, instead of x in the above equation.

Then
$$y = 14{\cdot}4 + 16\,(\cdot 578 - x) + \frac{2{\cdot}66}{\cdot 578 - x},$$

or
$$y = 23{\cdot}6 - 16\,x + \frac{2{\cdot}66}{\cdot 578 - x}.$$

Writing D for the density, and R for the sp. resistance, at 10° C., as before, we have

$$R = 39{\cdot}6 - 16\,D + \frac{2{\cdot}66}{1{\cdot}578 - D}.$$

This equation applies to all solutions, from that of density 1·2891 to that of saturation. The curve shews that throughout all the solutions, from that of density 1·16 to the saturated one, there is only a comparatively small variation in resistance, not greater than 15 per cent. This result is important from a practical point of view, as it proves that in a galvanic cell any one of a wide range of solutions of this salt may be employed with approximately equal advantage, so far as conductivity is concerned.

Our next salt was sulphate of copper, which was carefully purified before use. Eleven solutions were tested, containing from 1 part in 40 to 1 in 2·597, which is the ratio of saturation.

SULPHATE OF COPPER.

I. Ratio of $CuSO_4 + H_2O$ to Water in solution.	II. Density at 10° C.	III. Resistance, as measured in second tube, at 10° C. B. A. Units.	IV. Specific resistance at 10° C. B. A. Units.	I. Ratio of $CuSO_4 + 5H_2O$ to Water in solution.	II. Density at 10° C.	III. Resistance, as measured in second tube, at 10° C. B. A. Units.	IV. Specific resistance at 10° C. B. A. Units.
1 to 40	1·0167	9500	164·4	1 to 4·146	1·1386	2020	35·0
1 „ 30	1·0216	7790	134·8	1 „ 4	1·1432	1970	34·1
1 „ 20	1·0318	5700	98·7	1 „ 3·297	1·1679	1830	31·7
1 „ 10	1·0622	3410	59·0	1 „ 3	1·1823	1770	30·6
1 „ 7	1·0858	2730	47·3	1 „ 2·597	1·2051	1690	29·3
1 „ 5	1·1174	2200	38·1	Saturated			

The resistances were in this case measured in a second tube, of the same form as the first, but of slightly different dimensions. The numbers in column IV. are obtained from those in column III., by multiplying by 01731, a coefficient which was determined by a careful calibration of the tube.* The conductivity here increases steadily up to the point of saturation. It appears that a saturated solution of sulphate of copper has almost exactly the same resistance as the solution of sulphate of zinc of maximum conductivity.

This set of experiments is also presented in the form of a curve (fig. 3). As before, the excesses of density over unity are taken as the horizontal, and the resistances as the vertical ordinates. The horizontal scale (that of densities) is twice as great as that of fig. 2.

We find that in this case also the curve is an hyperbola, not rectangular, the lower asymptote being inclined upwards as before. It is considerably more

* Comparing this number with that already given for the first tube (·022301), it appears that the ratio of the resistance in the first to that in the second tube is 1 to 1·29, a ratio exactly the same as one which we obtained experimentally by measuring the resistance of several solutions in *both* tubes.

inclined than in the case of sulphate of zinc, tan θ (see fig. on page 62) being in this case 23, while h, the intercept on the axis of y, is now smaller, being only 12·2. Substituting these values in the general equation given before, and finding c from a known point, we have

$$y = 12{\cdot}2 + 23x + \frac{2{\cdot}7}{x} .$$

Curiously, c is here very nearly the same as for sulphate of zinc. If we write, as before, D for the density, and R for the specific resistance at 10° C., we have

$$R = 23\,D + \frac{2{\cdot}7}{D - 1} - 10{\cdot}8 .$$

The resistances given by this formula are compared with experimental ones in the following table :—

Specific Resistance.		Specific Resistance.	
Observed.	Calculated by above Formula.	Observed.	Calculated by above Formula.
164·4	174·2	35·0	34·9
134·8	137·7	34·1	34·3
98·7	99·1	31·7	32·1
59·0	57·0	30·6	31·2
47·3	46·7	29·3	30·0
38·1	37·9		

The remarks which follow the similar table for sulphate of zinc (page 64) apply to this table also.

The next part of our experiments consisted in testing the resistance of mixtures of the above sulphates. We selected three solutions of sulphate of copper: one pretty dense (·2875 to 1), which we may call A; the next, 1 to 7, B; and the third, a very weak one, 1 to 20, C. Five solutions of sulphate of zinc were selected: L, saturated; M, that of maximum conductivity (·735 to 1); N, one whose resistance is very nearly equal to that of the saturated solution, and the constitution of which is ·337 to 1. This solution corresponds (in density) pretty nearly to the solution of sulphate of copper, A. Also, O corresponding to B—constitution, 1 in 7; and P corresponding to C—constitution, 1 in 20. Each of the zinc solutions was mixed with each of the copper ones, equal volumes of the two solutions being in all cases taken. The following table shows the resistance of each solution separately, and the resistances of the mixtures :—

MIXED SOLUTIONS.

Solutions of $CuSO_4 + 5H_2O$.			Specificresistances at 10 °C when mixed with equal volumes of solutions of $ZnSO_4 + 7H_2O$. B. A. Units.				
Name.	Density.	Specific resistance at 10° C. B. A. U.	With L (Saturated) Density =1·422). Sp. resistance =33·2.	With M Density =1·2895. Sp. resistance =29·1 (minimum).	With N Density =1·1595. Sp. resistance =34·3.	With O Density =1·0760. Sp. resistance =51·8.	With P Density =1·0278. Sp. resistance =107·7.
A	1·1605	32·2	27·3	28·2	31·8	37·9	45·3
B	1·0858	47·3	27·5	29·6	34·3	46·7	59·3
C	1·0318	98·7	27·9	31·3	42·0	64·5	101·6

From this it appears—

1*st*, That invariably the resistance of the mixture is less than the mean resistance of the components, being in many cases less than that of either.

2*d*, That in mixtures, consisting of equal volumes of the solutions of these two salts, the maximum of conductivity is reached when a saturated solution of sulphate of zinc is mixed with a solution of sulphate of copper. What the strength of this copper solution is, appears to affect the result but little (AL, BL, and CL being of very nearly equal resistance). The least resistance of all is given when both solutions are saturated. We have also represented these results in a graphic form (fig. 4). The vertical ordinates are the specific resistances of the mixtures; the horizontal ones are the excesses over unity of the densities of the solutions of sulphate of zinc, with which the three solutions of sulphate of copper are mixed. The three curves correspond to the three solutions of sulphate of copper, and are lettered A, B, and C, after them. The points where these curves cut the axis, along which resistances are measured, of course represent the resistances got by mixing the copper solutions with equal volumes of solution of sulphate of zinc, whose density = 1, that is, with equal volumes of pure water. These points are calculated, from the curves of density and resistance already given, for cupric sulphate alone. (Figs. 1 and 3.) They are—

A, diluted with an equal volume of water, sp. res. = 49·5
B, „ „ „ = 78·7
C, „ „ „ = 147·0

The other points on the curves are determined by experiment. By the help of these curves it would be possible to determine, at least approximately, the resistance of any mixture of equal volumes of solutions of these two salts.

We endeavoured to account for the increased conductivity of the mixtures in the following manner:—We had data by which we could calculate the amount of water, of cupric sulphate, and of zincic sulphate in each mixture. Now,

supposing that each salt dissolved in the whole of the water, independently of the presence of the other salt, we could find first the density, and then the resistance of the two hypothetical solutions thus formed, by means of the curves on figs. 1, 2, and 3. We might expect that the conductivity of the mixture would be the sum of the two conductivities thus obtained. We calculated what would be the conductivity of several of the mixtures, if this supposition held good, but found that they did not agree with the experimental results, the amount of difference varying from 10 to 20 per cent. This difference is far too great to be explained as an error of experiment, and we may therefore say that the above hypothesis is erroneous, or at least imperfect.

Since these experiments were made, we have found that the subject of mixed solutions had previously been touched upon by PAALZOW (Pogg. Ann., cxxxvi., 1869), who noticed that in the case of the few solutions he examined, the resistance of the mixture was less than the mean of those of its components, as we also have found to be the case.

The next salt tested was sulphate of potash, K_2SO_4, a salt which seems never to have been examined before. We chose it with a view to mixing its solutions with those of sulphate of copper and sulphate of zinc, because it forms a double salt with each of these. But we were unable to test these mixtures owing to the extreme insolubility of the double salt, which crystallised out in both cases shortly after the mixtures were made. This took place even when the component solutions were very weak. However, sulphate of potash itself turned out to be a most interesting salt, on account of its remarkably high conductivity. With the same amount of salt in solution, its resistance is about three times less than that of zincic or cupric sulphates. In spite of its sparingly soluble character, a saturated solution has a very much higher conductivity than any solution of the other two salts, or even any mixture of them. The following table shows this :—

SULPHATE OF POTASH.

Ratio of salt to water in solution.	Density at 10° C.	Resistance in second tube at 10° C. B. A. Units.	Specific resistance at 10° C. B. A. Units.	Ratio of salt to water in solution.	Density at 10° C.	Resistance in second tube at 10° C. B. A. Units.	Specific resistance at 10° C. B. A. Units.
1 to 100	1·0082	6860	118·7	1 to 20	1·0394	1800	31·2
1 „ 80	1·0103	5540	95·9	1 „ 16	1·0482	1480	25·6
1 „ 40	1·0201	3150	54·5	1 „ 11	1·0697	1100	19·0
1 „ 30	1·0265	2420	41·9	1 „ 9·488	1·0801	960	16·6
1 „ 25	1·0319	2070	35·8	Saturated			

This table is given as a curve in fig. 3, the mode of representation being

the same as before. The scales are the same as those for sulphate of copper in the same figure.

This curve is also an hyperbola, but the lower asymptote is inclined downwards, unlike those of the two previous salts. For this reason the term involving tan θ is now negative. Substituting its value, as well as the value of the intercept h, and then finding c from a known point, we have

$$y = 13{\cdot}1 - 96\,x + \frac{{\cdot}87}{x},$$

or

$$\mathrm{R} = 109{\cdot}1 - 96\,\mathrm{D} + \frac{{\cdot}87}{\mathrm{D} - 1},$$

R and D having the same meaning as before.

From this formula we find—

Specific Resistance.		Specific Resistance.	
Observed.	Calculated.	Observed.	Calculated.
118·7	118·4	31·2	31·4
95·9	96·5	25·6	26·5
54·5	54·4	19·0	18·9
41·9	43·4	16·6	16·3
35·8	37·3		

We have seen that with some salts the lower asymptote is inclined upwards, and that with at least one other it is inclined downwards; we might therefore expect that salts exist for which it is not inclined at all, in other words, whose curves are rectangular hyperbolas. This appears to be the case with bichromate of potash. The following table gives the results of nine experiments with this salt:—

BICHROMATE OF POTASH (COMMERCIAL).

Ratio of salt to water in solution.	Density at 10° C.	Resistance in second tube at 10° C. B. A. Units.	Specific resistance at 10° C. B. A. Units.	Ratio of salt to water in solution.	Density at 10° C.	Resistance in second tube at 10° C. B. A. Units.	Specific resistance at 10° C. B. A. Units.
1 to 100	1·0069	10800	186·9	1 to 25	1·0274	3100	53·6
1 „ 80	1·0088	8680	150·2	1 „ 20	1·0345	2590	44·8
1 „ 50	1·0137	5560	96·2	1 „ 15	1·0452	2040	35·3
1 „ 40	1·0172	4600	79·6	1 „ 12	1·0561	1710	29·6
1 „ 30	1·0231	3490	60·4	Saturated			

See also curve in fig. 3.

Owing to the sparingly soluble character of this salt, the curve cannot be carried far enough to enable its form to be determined with great accuracy. As we have said, it seems to be a rectangular hyperbola, having this equation—

$$y = 6{\cdot}5 + \frac{1{\cdot}3}{x},$$

or

$$R = 6{\cdot}5 + \frac{1{\cdot}3}{D - 1}.$$

Calculating the resistances which this formula gives for the various densities, we have—

Specific Resistance.		Specific Resistance.	
Observed.	Calculated.	Observed.	Calculated.
186·9	194·9	53·6	53·9
150·2	154·4	44·8	44·2
96·2	101·3	35·3	35·3
79·6	81·5	29·6	29·6
60·4	62·7		

In conclusion, we give the following table for purposes of comparison:—

	Specific Resistance, B. A. Units.
Sulphate of zinc, saturated	33·7 at 10° C.
Do. do. (minimum)	28·3 do.
Sulphate of copper, saturated	29·3 do.
Sulphate of potash, do.	16·6 do.
Bichromate of potash, do.	29·6 do.
Mercury*	·0009563 at 0° C.

It only remains that we should express our thanks to Dr Crum Brown and his assistants for the trouble they took to provide us with pure salts, which it seemed impossible for us to obtain without their help. The above experiments were made in the Physical Laboratory of the University of Edinburgh during the winter session 1872–73. We are particularly indebted to Professor Tait for direction and advice throughout the whole course of our work.

* Report Brit. Assoc. 1864.

Fig. 2.

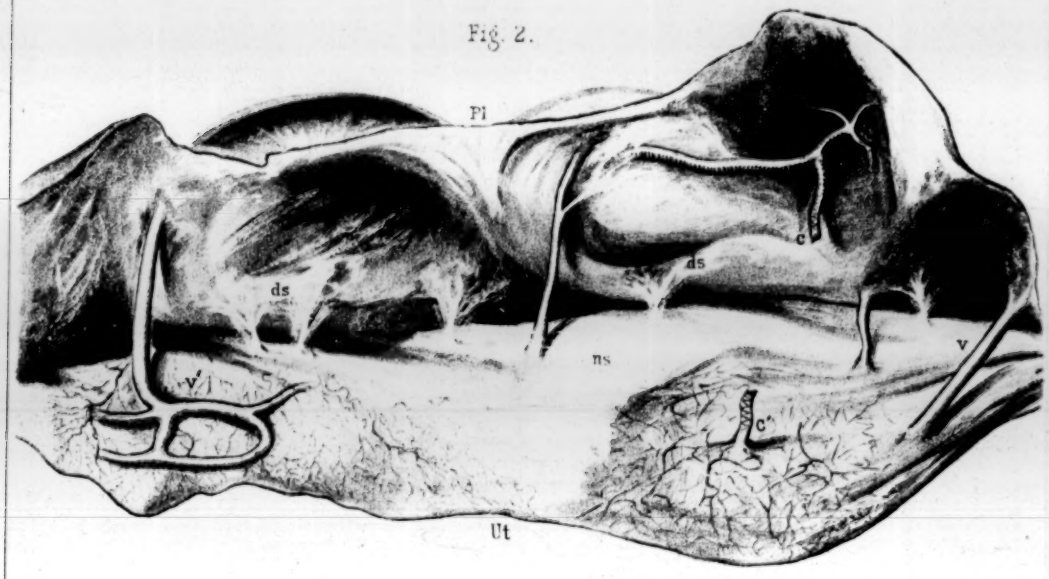
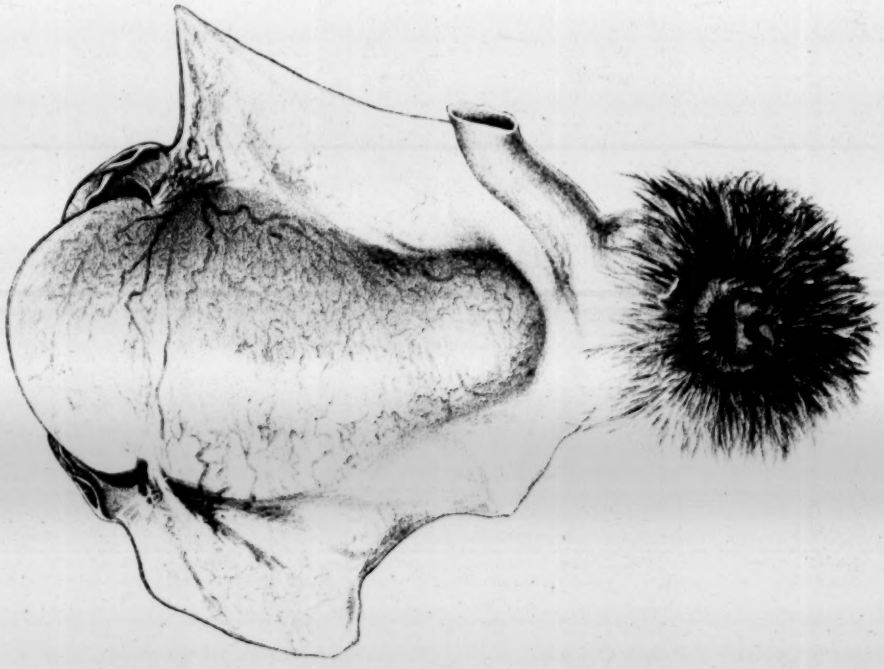


Fig. 1





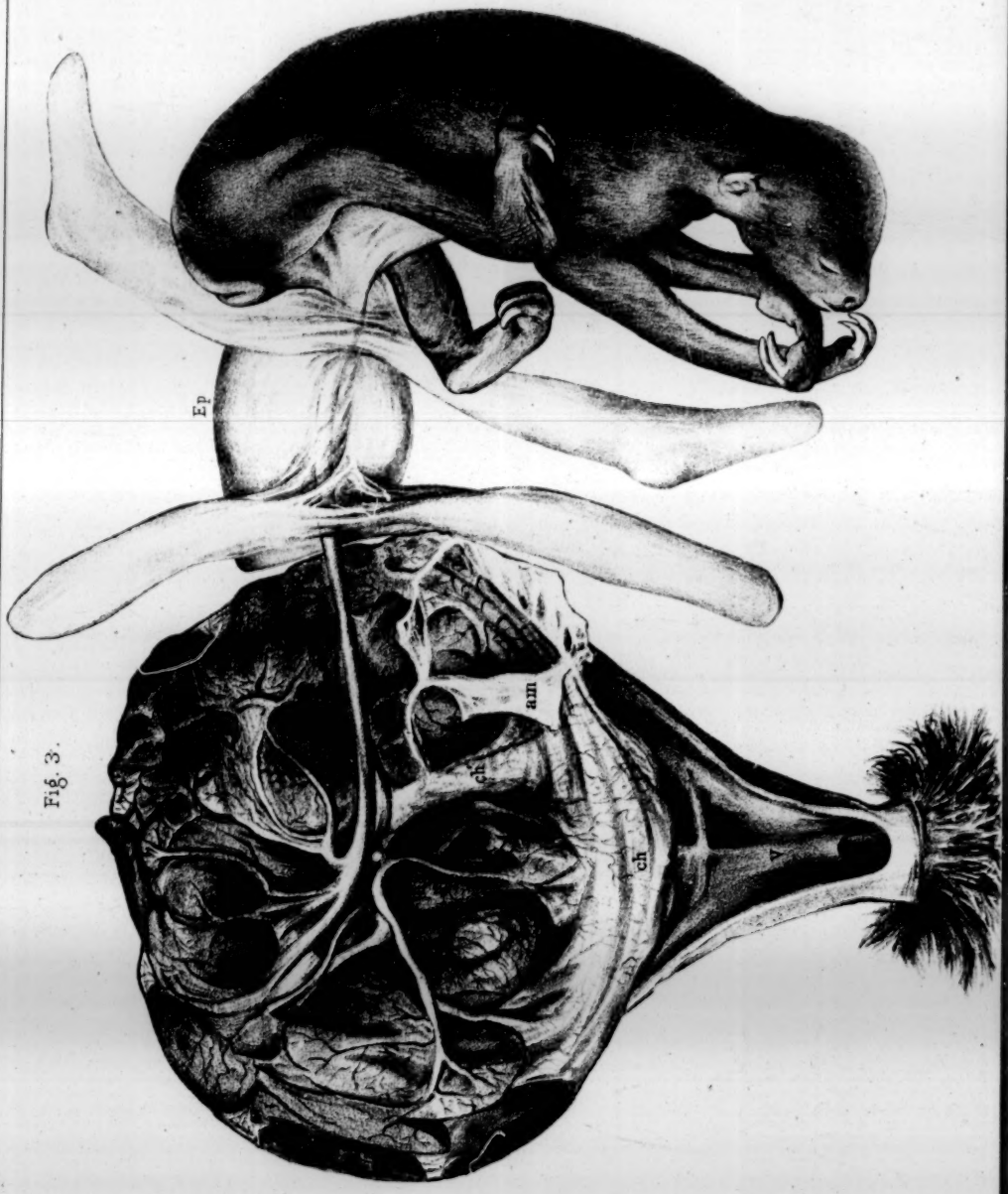


Fig. 3.

Fig. 4. Ch

Fig. 5.

Fig. 6.

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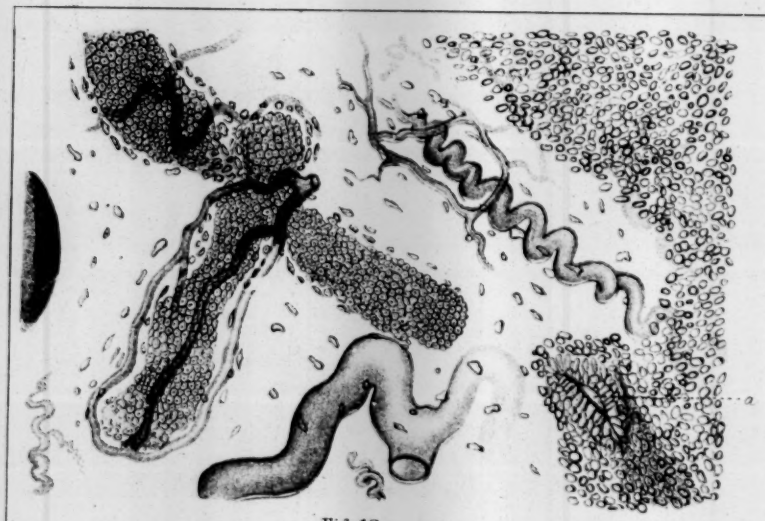


Fig. 12.

Fig. 11.

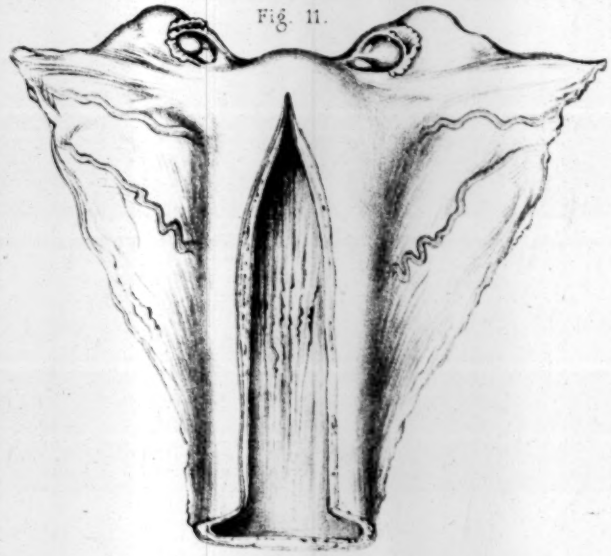


Fig. 10.



Fig. 7.

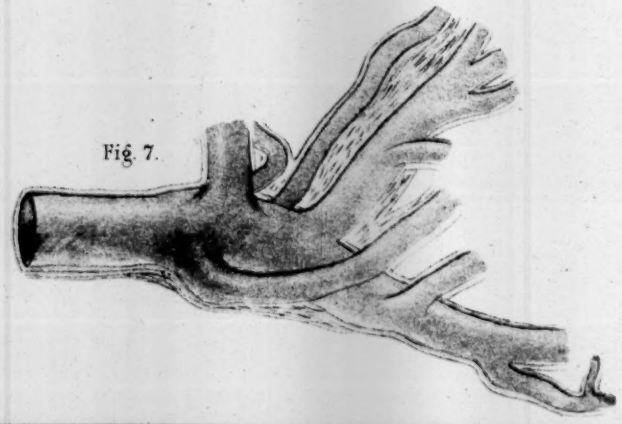


Fig. 8.



Fig. 9.

IV.—*On the Placentation of the Sloths.* (Plates III.–VI.)

By Professor Turner.

(Read May 19, 1873.)

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Comparative Anatomists have long been desirous to obtain detailed information on the Placentation of the Sloths. The only observations on this subject which appear to have been made up to this time have been recorded by Professor Rudolphi of Berlin,* and Dr C. G. Carus of Dresden.† Rudolphi in the course of some remarks on the structure of the umbilical cord in *Bradypus tridactylus* incidentally mentions that the placenta was cotyledonary as in the Ruminants. Carus figures the placenta and an almost mature fœtus of a *B. tridactylus* which came into his possession in 1830. He gives no description, however, of the specimen, but contents himself with a brief explanation of his engraved figures; in the course of which he says, that the specimen seemed to him important on account of the length of the umbilical cord, and the form of the cotyledons, which did not project, as is usual, outwards, but towards the inner face of the ovum, a peculiarity which had not yet been observed and described.

The complete absence of any description by these anatomists of the structure of the sloth's placenta, the brevity of their observations on its form, and the want of any explanation of the sense in which they use the term cotyledons, have afforded room for a considerable amount of speculation as to the character of the placentation in these animals, and have led to the expression of very diverse opinions, as may be gathered from the comments which have been made in the writings of eminent anatomists both in this country and abroad.

Von Baer states,‡ "The ovum of the Tardigrada is a remarkable intermediate form between the very heterogenic placentæ of the Apes and the Ruminants. It

* Ueber den Embryo der Affen und einiger anderen Säugethiere, in Abhandl. der Akad. der Wissensch. zu Berlin, 1828.

† Erläuterungs Tafeln zur Vergleich. Anatomie Heft III. Plate IX. Leipzig, 1831.

‡ Ueber Entwicklungs-Geschichte der Thiere, p. 263. 1837.

is a longish rounded placenta, in which, according to CARUS and RUDOLPHI, distinct cotyledons, which however lie near each other, can be recognised." M. H. MILNE-EDWARDS in his memoir *Sur la Classification Naturelle des Animaux** constructs a Table in which he places the Edentata amongst the mammals with diffused and cotyledonary placentæ. In the text of his valuable *Leçons sur la Physiologie et l'Anatomie comparée*, he states† that the structure of the placenta of the Edentata is so very imperfectly known, that it is needless to discuss it; but in some comments in a foot-note, he remarks that CARUS says nothing of the nature of the cotyledons of the sloth's placenta to make us think that their structure is analogous to that of the cotyledons of the Ruminant: "as far as one can judge from his figure, they appear to me rather to resemble the placenta of a monkey, but instead of being bilobed merely, to be subdivided into a considerable number of lobular portions."

Professor OWEN remarks‡ that the placenta of the phytophagous sloth (*Bradypus*) is almost as much subdivided as in the smaller Ruminants; but the true affinities of the sloth would be violated by transferring it to the Ruminantia on the score of mere accordance of placental form. Subsequently in his "Treatise on the Comparative Anatomy of Vertebrates,"§ after referring to CARUS' description of the placenta of *Bradypus*, he states that the placental cotyledons have no corresponding partial thickenings of the lining substance of the uterus as in the Ruminants, but "their flattened outer surface applied, with the uniting layer of chorion, to the inner surface of the uterus, may receive therefrom a medium of ramification of maternal vessels, answering to a decidua serotina. The probability indeed is that maternal deciduous substance is interblended with such allantoic lobules of the sloth, as is the case with the single thin oblong placental disc in *Dasypus*." Professor HUXLEY again, in his published Lectures, remarks,‖ "among the Edentata the sloths have presented a cotyledonary placenta;" and after relating some observations by Dr SHARPEY on the placental structure of *Manis*, which show it to be a non-deciduate placenta, he states, "and the cotyledonary form of that of the sloths leads me to entertain little doubt that it belongs to the same category." Also in his "Introduction to Classification"¶ he says, "In the Edentata the placentation appears to vary, being diffuse and non-deciduate in *Manis*, cotyledonous (and non-deciduate?) in *Bradypus*, and discoidal and deciduate in *Orycteropus*: but further investigation is needed before such variations can be

* Annales des Sciences Naturelles, p. 98, vol. i. 1844.
† Vol. ix. p. 563. Paris, 1870.
‡ Philosophical Transactions, 1857, p. 352.
§ Vol. iii. p. 731.
‖ Medical Times and Gazette, May 30, 1863, p. 555, and Elements of Comparative Anatomy, p. 111, 1864.
¶ London, 1869, p. 104.

safely admitted to exist." Dr ROLLESTON on the other hand states,* that the figure of the placenta of the sloth which is given by C. G. CARUS "does not seem to me to be so decidedly different from even the human placenta, in its naked-eye bossy outlines, as Dr SHARPEY's account of the placenta of *Manis* shows it to be from the placenta of all the Carnivora, Rodentia, Insectivora, Chiroptera, and Simiadæ which have been as yet examined. A well injected or even well preserved pregnant uterus of a sloth, would be most valuable, and would enable me to speak more confidently as to the extent of intimacy with which the maternal and fœtal blood-vessels are connected than the figures from Professor CARUS's work can do."

Lastly, those systematic zoologists who, like Professors VICTOR CARUS† and E. HAECKEL,‡ have adopted the placental system of classification, have placed the sloths as members of the Edentate order amongst the Indeciduata.

In this memoir, I hope to clear up the obscurity which has hitherto surrounded this subject, to place on a more precise and definite basis our knowledge, not only of the condition of the gravid uterus and the arrangement of the fœtal membranes in the sloths, but of the form of the placenta, its naked eye, and, so far as the examination of a single specimen can enable one, its microscopic structure; and to throw some additional light on the zoological affinities of this interesting group of mammals.

Uterus and Placenta.

On March 4th of the present year, I had the good fortune to receive the perfectly fresh carcase of a female two-toed sloth. I am indebted to my former pupil Dr DAVID RIDPATH, Surgeon in the West India Mail Packet Service, for this valuable specimen, which he procured alive at Colon on the Atlantic side of the Isthmus of Panama. It died only two days before reaching England. Dr RIDPATH packed the carcase in salt, placed it in the ice-chest, and forwarded it to me immediately on landing.§ The subsequent dissection revealed it to be the species of two-toed sloth which possesses only six cervical vertebræ, and to which Professor PETERS has given the name *Cholœpus Hoffmanni.*‖

I lost no time in cutting into the abdominal cavity, and to my great gratification saw that the uterus was obviously in the gravid condition. It occupied not only the pelvis, but the adjacent part of the abdominal cavity; and it overlay the kidneys, and had pushed the coils of the intestine forwards and to

* Trans. Zoological Soc. vol. v. p. 303. 1863.

† Handbuch der Zoologie. Band I. 65. Leipzig, 1868.

‡ Natürliche Schöpfung's Geschichte. Berlin, 1868.

§ I wish to take this opportunity of thanking Dr RIDPATH not only for the above specimen, but for a number of other valuable zoological objects from Central America which he has from time to time presented to the Anatomical Museum of the University.

‖ Monats. Berichte Berlin Akad. 1858, p. 128, and 1864, p. 679; and Natural History Review, vol. v. p. 299. 1865.

the sides of the abdominal cavity. From each side of the uterus a well-defined broad ligament proceeded continuous with the peritoneum covering the adjacent kidney. With a sharp knife I cut through the broad ligaments, and removed the whole of the genital organs, together with the lower end of the rectum from the pelvis.

The uterus was a single organ, and exhibited no trace of a subdivision into cornua (fig. 1.) It was ovoid in form, with the broader, rounded end at the fundus. Its length was 7 inches, and the greatest circumference $10\frac{1}{2}$ inches. A slender cord-like round ligament was attached to the side of the uterus, a little below the Fallopian tube. The Fallopian tube was slender, about $1\frac{1}{2}$ inch long, and somewhat serpentine in its course. The trumpet-shaped mouth was situated on the free edge of a fold of peritoneum, which formed the anterior boundary of a deep pouch, in which the ovary was lodged. Through its mouth a bristle could be passed along the tube without difficulty. Its mesoarium was so short that the ovary was closely attached to the uterine wall. The left ovary, $\frac{4}{10}$ths-inch long by $\frac{3}{10}$ths-inch broad, was pale yellowish-white in colour; the right ovary, about one-third larger, was of a brighter yellow, and contained a well-defined corpus luteum.

The external genital orifice and the rectum, the former of which opened in front of the latter, were surrounded by a common fringe of hairs; and on each side of the genital orifice was a pouch-like depression of the integument. This orifice was surmounted by a clitoris, and was continued into a uro-genital chamber or vestibule. Half an inch from this orifice, the urethra opened into the vestibule immediately in front of the mouth of the genital passage: the urethra readily admitted a thick probe, and its canal, after a course of $\frac{3}{4}$ inch, imbedded in the inferior wall of the genital passage, dilated into a pyriform bladder. The genital passage, $1\frac{1}{4}$ inch long, opened directly and freely by a single orifice into the cavity of the uterus; and its vestibular orifice was encircled by a projecting lip, which gave to the orifice an appearance not unlike an os uteri externum. The passage contained yellowish mucus; the mucous membrane of its inferior wall was elevated into a feeble longitudinal mesial fold; and a slight circular fold marked where it became continuous with the uterine cavity. It is customary to regard this genital passage in the sloths as the vagina rather than the cervix uteri, though its characters in some respects more resemble the latter than the former.

Numerous tortuous arteries and veins were seen in the broad ligaments on their way to the ovaries and substance of the uterus; their ramifications beneath the serous membrane could be distinctly traced, and the veins from opposite broad ligaments freely anastomosed on the surfaces of the uterus. I introduced injecting pipes into a large artery and vein in each broad ligament, and an injection, consisting of carmine suspended in gelatine, was then gently passed into these vessels by my assistant, Mr STIRLING.

From the form of the uterus it was evident that only a single fœtus was contained within the cavity, and that the head presented at the orificium uteri. I then made a mesial longitudinal incision through the posterior wall of the vagina and adjacent portion of the uterus, carefully dividing the several coats. In the part of the uterine cavity which was opened, a translucent envelope, the chorion, was exposed. The outline of the head of the fœtus could now be distinctly seen and felt. I then made an incision through the membranes, over the head of the fœtus, with the view of extracting it, in order to pass a coloured injection into the fœtal part of the placenta along the umbilical vessels. No fluid escaped when the membranes were divided. The fœtus was now seen to be closely invested by a thin, semi-transparent membrane, which, as forming the immediate envelope of the embryo, I at first took to be the amnion. Further consideration, however, of its arrangement, which will afterwards be more fully described, satisfied me that it could not be that membrane, but a structure specially developed in connection with the fœtus itself.

A cut was now made into the umbilical cord, injecting pipes were introduced into the vein and into one of the arteries, and a prussian blue and gelatine injection gently passed along these vessels into the placenta. When the coloured gelatine had solidified, the incision previously made into the uterine cavity was prolonged upwards to the fundus, so as to give a full view of the interior of the cavity. As is not unusual when an organ which possesses complex vascular arrangements is injected with two colours, I observed that both colours had run freely into some parts of the placenta, into others only a single colour,—sometimes the blue, at others the red had passed; whilst a few limited areas had not received any injection at all. Opportunities were thus afforded of studying the vascularity of the placenta in every stage, from the perfectly injected to the uninjected state.

The placenta was large in proportion to the size of the uterus, and took up about three-fourths of the entire surface of the chorion (fig. 3).* It corresponded to the fundus, and to the greater part of the anterior, posterior, and lateral surfaces of the body of the uterus, reached to within two inches of the orificium uteri, and in its general form was dome-like, or bell-shaped. It concealed, therefore, the uterine orifices of the two Fallopian tubes. The posterior or non-placental part of the chorion which intervened between the lower edge of the placenta and the orificium was translucent, and had slender ramifications of the umbilical vessels, which were filled with the blue injection, distributed in it, and forming networks with polygonal meshes, though in some localities the vessels were arranged in greatly elongated and compact networks.

* To prevent any misunderstanding, I may state that in this and in my previous memoirs on the placenta, I use the term chorion, in the ordinary descriptive sense, to express the outer envelope of the fœtus, without committing myself to any theory of the mode of production of this membrane.

The amnion was closely adherent to the inner surface of the non-placental part of the chorion (fig. 3, *a m*). The two membranes could be separated by tearing through a thin layer of very delicate intermediate areolar tissue. The amnion was prolonged over the inner face of the placenta, and at the place of attachment of the funis was reflected upon the umbilical cord, which it invested, and by which it was conducted to the abdominal aspect of the fœtus. The inner free surface of the amnion was smooth and serous, although there was an absence of liquor amnii. This membrane was non-vascular, and contrasted, therefore, with the vascular chorion.

The placenta was subdivided into about thirty lobes or "cotyledons," but to prevent any possibility of misconception as to their nature, I shall in the course of my description speak of them not as "cotyledons" but as lobes. These lobes were not scattered over the surface of the chorion, as the cotyledons are in the Ruminants, but were more or less closely aggregated together. Not unfrequently they were in, or almost in, contact with each other by their margins; but sometimes narrow strips of chorion separated them from each other. A somewhat broader strip of non-placental chorion, continuous with that which lay adjacent to the os uteri, passed obliquely between the lobes forming the anterior part of the placenta, almost up to the fundus, so that the organ was imperfectly subdivided by it into right and left lateral halves (fig. 3, *ch'*). The umbilical cord, $5\frac{1}{4}$ inches long, joined the chorion about the centre of this strip, opposite the middle of the inferior wall of the uterus. Its vessels, consisting of a vein and two arteries, branched and radiated outwards, to ramify on the chorionic surface of the lobes prior to entering their substance. Owing to the separation between the lobes being much better marked than in the human placenta, so compact an organ was not formed as is met with in the human subject.

The lobes were irregularly discoidal in shape, and varied considerably in size. The largest had a long diameter of 2 inches, and a thickness of $\frac{4}{10}$ths inch, whilst the smallest were not more than half an inch in diameter. The greater number possessed distinct rounded margins, so that they had a well-defined individuality; but in a few instances some of the lobes were partially blended with each other by the fusion of their adjacent margins. The chorionic surface of the lobes was convex, and projected towards the interior of the ovum, after the manner described and figured by Carus in *B. tridactylus*. By the opposite surface the lobes were attached to the inner wall of the uterus, in a manner to be immediately described.

The non-placental area of the chorion lay free in the hinder part of the uterine cavity. It was in contact with, but not adherent to, the uterine mucous membrane. When raised from its position, its uterine surface was seen to be partially invested by a thin yellowish-brown layer, which could be easily peeled off, and which became continuous with the mucous membrane, covering the

non-placental part of the uterus at the junction of the placental and non-placental parts of the chorion. When examined microscopically, this layer was seen to consist of a very delicate connective tissue, in which ovoid and fusiform nucleated corpuscles, many of which were much larger than ordinary connective tissue corpuscles, and containing abundant granular matter, were imbedded. From its position, relations, and structure, this layer is undoubtedly a decidua reflexa, and its line of reflexion from uterus to chorion could be traced all around the line of junction of the placental with the non-placental part of the chorion. Growing from the non-placental part of the chorion into the decidua reflexa, more especially in proximity to the line of reflection, were elongated villi, which could not be regarded as aborted, because slender blue-injected blood-vessels, derived from the vascular network already described, were prolonged into them, and formed capillary networks in their interior. In those villi where the injection was complete, the capillaries were as abundant as in the placental villi themselves, so that there can I think be little doubt but they played a part, along with the cells of the decidua reflexa, in the nutrition of the fœtus. No utricular glands were recognised in the decidua reflexa.

I examined the surface of the non-placental part of the uterine mucous membrane both with the naked eye and a pocket-lens, and found it smooth, except where a longitudinal band, formed of slight folds, extending in the direction of the band itself, passed along the middle of the inferior wall. This band was directly continuous with the longitudinal band in the so-called vagina, already described. Neither with the aid of a simple lens nor with the compound microscope could I detect the mouths of any utricular glands on the free surface of the mucosa. I then examined numerous vertical sections through the mucous and submucous coats and adjacent parts of the muscular wall, but without seeing a trace of a gland. The blood-vessels were well injected; some, though not all, of the veins had a serpentine course; some, though not all, of the arteries were twisted in a cork-screw-like spiral, and a network of capillaries lay in the mucous membrane, parallel to the plane of its free surface.

I then proceeded carefully to peel some of the placental lobes from the placental area of the uterus. I first tore through the decidua reflexa along its line of reflection from the uterus to the placenta, and then found that, as I slowly stripped off the lobes, a delicate areolar tissue was torn through, some of which remained attached to the uterine wall, whilst a very thin layer adhered to the uterine face of the lobes themselves. The readiness with which the separation took place was in itself strong evidence that one had here the natural plane of separation between placenta and uterus as it takes place at the time of parturition, so that there can be no doubt that the layer which remained on the placental area of the uterus was the non-deciduous serotina,* whilst that which

* I adopt here the descriptive terms introduced by Professor Rolleston.

was retained on the placenta was the deciduous serotina properly so called. The tissue torn through resembled in its naked-eye appearance the decidual layer which connects the human placenta to the uterine wall; and the surfaces which had been in apposition were smooth and without either pits or villi, such as one sees in stripping off a diffused or cotyledonary placenta from the corresponding uterine area. When the non-deciduous serotina was removed, the thin muscular wall of the uterus was exposed. In microscopic structure, both the deciduous and non-deciduous serotina consisted of a very delicate connective tissue, in which the corpuscular element was large and distinct, though the cells did not assume either the size, or the proportion as regards numbers, of the colossal cells of the human serotina. In the former a very large number of plate-like crystals were seen after the placenta had been hardened in spirit.

The Serotina was not, however, the only structure exposed in the course of this dissection; for the arteries and veins proceeding to and from the placental lobes were distinctly recognised (fig. 2). Each lobe had at least one artery penetrating its uterine surface; sometimes the artery entered the centre of a lobe, at others nearer its margin. These arteries corresponded to the curling arteries of the human placenta, and possessed a very characteristic appearance as if twisted into a close spiral. Usually the curling artery and vein connected with a given lobe penetrated it at some distance from each other, but in one instance I saw them enter close together. I traced more than one of these arteries into their respective lobes, and found that the artery branched immediately on entering. In two cases it divided into three branches, and these again subdivided into smaller arteries. But it should be stated that the branches did not possess the twisted form of the trunk from which they proceeded; and owing to the brittleness of their coats, they readily tore through in the act of dissection.

One, and sometimes two, venous trunks, uniform in size with the curling arteries, left each placental lobe, usually near the margin. They differed from the arteries in being smooth and free from the peculiar twisted appearance, though in tracing them obliquely through the non-deciduous serotina and the muscular coat it was not uncommon to find them pursue a serpentine course. Those veins which proceeded from the lobes lying near the broad ligament passed obliquely through the muscular wall of the uterus into a venous plexus in that ligament. It is important to note that neither in the decidua serotina nor in the muscular wall of the uterus did these veins dilate into sinuses: in both localities they preserved the tubular, cylindrical form. Usually the veins lay for some distance partially imbedded in grooves on the uterine face of the lobes, and subdivided into two or more branches before they penetrated their substance. By a little careful dissection, I was able to follow these branches into the lobes, and to see that immediately after entering they again rapidly

subdivided. By snipping out specimens along with surrounding placental substance with a pair of fine scissors, and then teasing out the preparation with needles, on placing the object under a low power of the compound microscope, I could follow these branches for some distance. In one specimen I observed that the vein immediately after penetrating broke up at once into a penicillar cluster of branches (fig. 7). In another specimen the branches arose after the arborescent plan. When several of these branches were traced onwards into the placental substance, they were seen to lose their originally straight direction, and to become very convoluted (fig. 8). The coats of the veins were not so brittle as those of the arteries, so that these vessels could be followed for a longer distance with comparative ease. I may further mention that the lobes were well injected with the red gelatine in the immediate vicinity of the localities where they were penetrated by the maternal vessels.

It now became a matter of much importance to trace the maternal vessels further into the placental substance in order to ascertain their arrangements within the lobes, and their relations to the fœtal villi. As the intra-placental branches, both of the curling arteries and utero-placental veins, were too small to be followed out to their termination by the ordinary means of dissection, thin slices were then removed from some of the lobes, after hardening in spirit, with the aid of STIRLING'S section-cutter, and submitted to examination under both the lower and higher powers of the microscope.

In vertical sections made through the thickness of a lobe from the chorionic to the decidual surface, the intra-placental maternal vessels presented a very remarkable and characteristic appearance, which differs in various particulars from any that I have either seen or read of in other animals. These vessels were many times larger than capillaries, and possessed a transverse diameter varying from ·003 to ·008 inch (fig. 4). Their course was serpentine or even convoluted, and as they wound in and out between the villi, sometimes bending at an acute angle, at others possessing a more gentle curve, they had been repeatedly cut through by the razor in the plane of section; sometimes a maternal vessel was divided for a considerable distance, relatively speaking, parallel to its long axis, at others it was cut obliquely, at others transversely. Sometimes adjacent convoluted vessels anastomosed with each other, and here and there an appearance resembling a varicose dilatation of the wall of the vessel was recognised; but it is possible that these apparent varicosities in some instances may have been due to the cut having passed very obliquely through a main vessel at the origin of one of its branches. Under a low power, it not unfrequently seemed as if a convoluted vessel had extended so parallel to the plane of section, that a very considerable length of it had been exposed as a continuous undivided tube; but when a higher power was employed, and a more perfect definition of the

wall of the vessel therefore obtained, it was seen that the tube, owing to its tortuosity, had been cut across more than once, and that, instead of possessing an unbroken continuity, a series of oblique or transverse sections through its walls had been obtained.

It might have been supposed that the maternal vessels, in their course from the decidual to the chorionic aspect of the placenta, would have diminished in size, so that their diameter near the former surface would invariably have been greater than in the region of the latter; but this was not found to be the case. For although the main branches of the arteries and veins were necessarily nearer the decidual aspect, yet along with these were smaller branches of about the same calibre as those found close to the surface of the chorion.

I then proceeded to examine the intra-placental maternal vessels, with the view of ascertaining if they possessed definite walls capable of being isolated from the villi between which they lay; or if they were merely a freely anastomosing or cavernous system of blood spaces, which either did not possess definite walls at all, or whose walls were incorporated with the tissue of the villi. That the intra-placental branches, arising directly from the curling arteries and utero-placental veins, possessed distinct walls, there was no difficulty in at once deciding, as their coats could be traced in continuity with those of the trunks from which they arose, and they could be readily separated from the surrounding structures; but the serpentine and convoluted vessels continuous with these branches, on account of the mode in which they were intermingled with the villi, required to be more closely examined. I proceeded to study them under the microscope, both in sections through the lobes, where they were as far as possible *in situ*, and also in teased-out preparations, and from both methods of examination I obtained ample proof that these vessels possessed a definite wall. In thin sections, even when no displacement of the relative positions of the villi and convoluted maternal vessels had taken place, a limiting membrane, distinct from the tissue of the villus, could be traced around the vessels, whether they contained injection or not, more especially when they were transversely or obliquely divided; and the presence of an independent wall of the vessel was naturally more clearly seen when some displacement of a vessel from the villi between which it had originally been placed had occurred. In preparations teased out with needles, the structure of the wall of the tortuous vessels could be studied. It was of great delicacy, possessed no elastic tissue or muscular fibre-cells, and seemed to consist essentially of a nucleated protoplasm, in which faint traces of fibrillation were recognised. In one specimen I was so fortunate as to obtain, after the addition of acetic acid to an uninjected vessel, a demonstration of a delicate but perfectly distinct endothelial lining, the cells of which were ovoid in shape, with smooth and not jagged edges, and in some cases with nuclei in their interior (fig. 10).

I have already stated that differences in the degree of injection of the maternal vessels existed in different parts of the placenta. When the animal had died, its placental vessels were filled with blood, and when the injection was passed into the uterine arteries and veins, it had in many places entirely filled the lumen of the vessels, and in others it was intermingled with the blood corpuscles; but in various parts of the organ, where the injection had not penetrated, the serpentine and convoluted maternal vessels were occupied only by the blood-corpuscles. In some instances the corpuscles were so closely packed together as completely to fill the lumen of the vessel; in others they had shrunk into the centre of the vessel, so as to leave a space between them and the wall, which latter was in such instances clearly defined. And I may state that, as soon as the eye became familiar with the characters of these serpentine maternal vessels, their appearance and arrangement could be studied in those parts of the placenta, where they were filled only with blood-corpuscles, equally well as where they had been more or less perfectly distended with the coloured injection.

The characters presented by the red blood-corpuscles are worthy of consideration. They had the well-known circular form of the mammalian blood-disc, and the greater number were non-nucleated. But amongst them was a proportion of corpuscles in which the central part, differentiated from the peripheral portion of the corpuscle, was bounded by a sharp line, so as to give the appearance of a central nucleus. It should be stated that, before this observation was made, the placenta had been injected and hardened in spirit; but if the nucleated appearance had been occasioned artificially, it is probable that a greater proportion, or even the whole of the red corpuscles, would have been similarly affected. On the whole, then, I am disposed to think that the "nuclei" are normal and not artificial productions in a proportion of the red blood-discs of this animal. And here I may point out that previous observers have referred to the presence of nucleated red blood-corpuscles in other species of Tardigrada. KÜHNE remarks, that amongst the mammalia, only individuals (Camels, Tardigrades) possess nucleated blood-corpuscles.* Dr ROLLESTON states† that Mr MOSELEY called his attention to the appearance of nucleation in dried blood-corpuscles of *Cholœpus didactylus*. Further observations satisfied them that although a certain number of the dried blood-corpuscles of this sloth do contain one or more nuclei irregularly and eccentrically placed, the immense majority possess the usual mammalian non-nucleated character. Mr GULLIVER, in commenting‡ on these observations, says, that as the nuclei were not subjected to chemical examination, their real character is doubtful. But even if these nuclei are real structures, the small proportion of corpuscles

* Lehrbuch der physiologischen Chemie. Leipzig, 1868, p. 195.
† Quart. Journal of Microscopic Science, vii. p. 127.
‡ Journal of Anatomy and Physiology, ii. p. 1.

in which they exist shows that the nucleated character of the red blood-corpuscle in this mammal is an exceptional occurrence.

I have up to this time spoken of the serpentine and convoluted microscopic blood-tubes as "vessels," without stating whether I look upon them as arteries, as capillaries, or as veins. In the great delicacy of their coats, and in the absence both of muscular and elastic tissue, they approximate to capillaries, but they differ from them again in their large size and dilated character, so that they might be spoken of as forming a system of intra-placental maternal sinuses, continuous on the one hand with the curling arteries, on the other with the utero-placental veins, through which the maternal blood flows in order to be brought into relation with the capillaries of the fœtal villi. Of maternal vessels possessing the usual calibre of capillaries I saw not a trace, so that I am of opinion that these serpentine and convoluted vessels, or sinuses, are the only channels of communication within the placenta between the uterine arteries and veins. That these sinuses are neither artificially produced by the injection, nor a system of wall-less cavernous spaces, but a definite arrangement of blood-channels, is proved by their continuity with the uterine vessels, by the possession of a distinct wall capable of isolation, and by the enormous quantities of blood-corpuscles which they contained.

I carefully examined the decidua serotina in sections where the placenta was still *in situ*, as well as in fragments cut off after the lobes were peeled from the surface of the uterus, but could not detect any evidence of the presence of utricular glands; so that these structures appear to be absent both from the placental and non-placental areas of mucous membrane.

My attention was now directed to the arrangement and structure of the villi of the chorion. The stems of the villi arose at intervals from the surface of the chorion, and extended almost vertically through the thickness of the placental lobes close up to, or even as far as the decidual surface of the organ (fig. 4). In their course they diminished in thickness and gave off branches, which extended obliquely, and for a considerable distance, away from the main stem; these branches in their turn gave origin to smaller branches, from which again short bud-like sprouts proceeded, giving to the entire arrangement a characteristic tree-like character. Closely adapted to the sinuous outlines of the branching villi were the sinus-like maternal blood-vessels. Well-marked branches of the umbilical vessels coursed along the axes of the stems of the villi, and gave rise to smaller vessels, which extended down the branches of the villi, and ultimately joined a well-marked capillary plexus, which was distributed close to the surface, not only of the branches of the villi and their bud-like offshoots, but of the stem of the parent villus. In those parts of the placenta where the umbilical vessels were well injected these capillaries were filled with the blue gelatine, and were seen to lie in close relation to the tortuous maternal blood sinuses,

which were so arranged as to have the greater part, or sometimes the whole, of the circumference of their walls in contact with the highly vascular surfaces of adjacent villi (fig. 5 and 6). By this arrangement the fœtal and maternal blood-streams were brought so close to each other that the interchange of material which without doubt takes place between them could be readily effected. The transverse diameter of the terminal villi was, as a rule, equal to that of the maternal blood sinuses lying next them, so that the placental lobe consisted of fœtal and maternal structures in almost equal proportions. The average diameter of the fœtal capillaries was about ·0005 inch, or about $\frac{1}{16}$th of the diameter of the largest maternal blood sinuses.

The terminal branches of the villi reached as far as the decidual surface of the placenta, and gave off lateral offshots, which were interposed between the decidua and those intra-placental sinuses which lay nearest it; for fœtal capillaries filled with blue injection were seen in this region sometimes cut across transversely, at others with their long axis parallel and internal to those maternal blood sinuses that came close up to the uterine face of the placenta.

The basis substance of the villi consisted of delicate connective tissue in which the corpuscular element was strongly pronounced, not only because the corpuscles were numerous in a given area, but from their size. In shape they were by no means uniformly spindle-like, but many of the cells were irregular or broadly ovoid or even circular. They consisted of dimly granular protoplasm, and were distinctly nucleated (fig. 5 and 6). I examined a number of villi dissected out of the substance of the placenta with reference to the existence of an epithelial layer on the free surface outside the capillary network, but without obtaining any evidence of its presence. Neither could I detect any evidence of a layer of cells investing the villi similar to the well-known layer which forms a cap for the villi of the human chorion, and which has by some anatomists been referred to the decidua serotina. In dissected preparations, where the villi and maternal blood-vessels had been torn asunder, I sometimes saw flakes of delicate fibrillated tissue attached to the periphery of a villus, which I believe to be portions of the wall of the maternal vessel torn off during the dissection. I looked carefully for evidence of the prolongation into the placental lobes of processes or bars of the decidua serotina, but without seeing any, though, as has previously been stated, the uterine face of the placenta was invested by this membrane.

At the edges of those placental lobes, which were separated from each other by intermediate narrow strips of chorion, the uterine surface of that membrane gave origin to elongated and branched villi. The blue injection had entered the vascular trunks within the stems of the villi, but no capillaries were injected. The uterine mucosa was in contact with the free ends of the villi, and even in part gave off processes which passed between them, but no maternal blood

sinuses were developed in it. It is doubtful if these villi had possessed any functional activity.

The Fœtus and Epitrichium.

The fœtus, a male, was well developed, and had apparently reached nearly the full time. It measured $10\frac{1}{4}$ inches along the curve of the back, from the tip of the nose to the anal orifice; from the articulation of the shoulder to the tips of the anterior toes 5 inches; and an almost equal length from the hip-joint to the extremities of the hinder toes. The toes were flexed upon the soles. There was no external tail. The nipples were two in number, and pectoral in position. The umbilical cord was attached to the abdominal wall $1\frac{1}{2}$ inch in front of the anal orifice. The whole surface of the body, excepting the palms of the hands and soles of the feet, was thickly covered by well-developed hairs, which on the back and sides both of the body and limbs had a dark brown colour, but on the belly and the flexor aspect of the limbs were of a lighter yellowish-brown. The colour of the hair of the mother, again, was a tawny yellow, so that a complete change in the colour of the hairy coat takes place before adult life is reached. The length of some of the hairs on the side and back of the body was measured, and found to be $\frac{4}{10}$ inch. The fœtus was closely enveloped by a thin membranous bag which had remarkable and interesting relations, as was ascertained at the time when I removed the membrane from the young animal. This I accomplished by slitting up the membrane over the head, and then everting it as I extracted the fœtus from its interior. I found, however, that it was closely adherent to and continuous with the tender skin of the margins of the eyelids, of the nostrils, and of the margins of the lips, which adhesions had to be torn through before the head could be liberated. It was also prolonged down the external auditory meatus, and in the region of the muzzle was perforated by the tactile hairs, which projected for $\frac{4}{10}$ths inch through and beyond it. But, further, each limb of the fœtus was invested by an elongated tubular prolongation of this membrane, as closely as a tight-fitting stocking invests the leg and foot (fig. 3); and a distinct attachment existed between it and the soft cuticle around the roots of the claws, whilst the claws themselves were each invested by a prolongation of the membranous tube. After these prolongations were peeled off the limbs, an adhesion between the membrane and the tender skin surrounding the anus, and around the abdominal attachment of the umbilical cord, had to be torn through before the fœtus was finally liberated. A small quantity of a white caseous material was found in patches, more especially on the back of the animal, between the hairs and the enveloping membrane, but except in these localities the membrane was in close contact with the hairs themselves.

The membrane was translucent, and to the naked eye was not unlike the horny layer of the human cuticle. Examined microscopically after digestion in

solution of potash, it was seen to consist of a stratified squamous epithelium, the cells of which had the form of broad, flattened scales, with irregular jagged margins, and containing ovoid nuclei. The cells in a given layer were closely connected together by the interlocking of their adjacent serrated margins. The deep surface of the membrane in the region of the muzzle was irregularly pitted, and the pits obviously corresponded to hairs, which, in the progress of growth, had indented the deeper surface of the membrane, and even elevated its superficial aspect into papillary processes. In some cases these papillæ were perforated by the tactile hairs, but in others they were still imperforate. Delicate threads of a fatty sebaceous substance, obviously the secretion of the sebaceous follicles, were not unfrequently, but especially at the auricles and muzzle, continuous with the deep surface of the membrane.

It will now be advisable to express an opinion regarding the nature of this very remarkable fœtal envelope, and as a preliminary it may be well to ascertain if a similar arrangement, either in this or other groups of mammals, has been noticed by previous writers, and the views which they held regarding it. Von Baer (*op. cit.* p. 263) states he has seen in the embryo of the Tardigrada, that the epidermis, as in the pig, loosens itself as a complete sac, and appears like a second amnion within the amnion. Ibsen has observed,* closely investing the hair of an embryo sloth, a membrane, which he regarded as a continuation of the outermost covering of the umbilical cord, as indeed the amnion: in the embryo pig also he recognised an analogous membrane, which formed a peculiar outer-epidermic layer outside the hairs, which disappears in the later period of fœtal life. G. Simon has also described† this membrane in the embryo pig, and pointed out that it consists of squamous cells, such as the epidermis is itself composed of; but in a subsequent sentence he states that there is a circumstance opposed to the view of its epidermal nature, for beneath it another thin layer exists which corresponds to the epidermis. The nature of this membrane in the embryo sloth and pig has also been discussed by Bischoff, Kölliker, and Reissner.

The fullest description, however, of its structure, mode of arrangement, and distribution amongst mammals, has been given by Professor Hermann Welcker of Halle,‡ to whose Memoir I am indebted for the references to most of the authorities cited above. Welcker gives to this membrane the name of Epitrichium. He has seen it in two embryos of *Bradypus tridactylus*, in the fœtus of *Cholœpus didactylus*, of *Myrmecophaga didactyla* and *jubata*, of *Dicotyles*, the common pig, and probably also in the horse;

* See Eschricht's Essay, "Ueber die Richtung der Haare," in Müller's Archiv., 1837, p. 42.

† Müller's Archiv., 1841, p. 370.

‡ Ueber die Entwicklung und den Bau der Haut und der Haare bei Bradypus, in Abhand. der Naturforsch. Gesellschaft zu Halle. Bd. ix., 1864.

whilst it is absent, he says, in the embryos of *Dasypus*, *Cœlogenys*, *Dasyprocta*, *Hydro-chœrus*, *Cervus*, *Ovis*, *Bos*, *Didelphis*, *Ursus*, and *Felis*. But he describes it in *Cœlogenys paca*, *Cervus*, and the human embryo, as an epitrichoid layer, consisting of cells of smaller size than those which constitute the epitrichium proper in the animals which possess that membrane. This epitrichoid layer of cells probably takes a part in the formation of the vernix caseosa. He regards the epitrichium as no otherwise than the most superficial layer of the epidermoid-plate (*Epidermoidal-Blattes*) of the embryo. He believes that the outermost layer of the embryo (*Horn-Blatt*) divides into two parts, the deeper of which is the epidermis, properly so called, consisting both of its Malpighian and horny strata, whilst the superficial part is the epitrichium, which originally is blended with the epidermis, and only becomes elevated from it as a distinct membrane in consequence of the development of the hairs.

My examination of the structure and arrangement of the corresponding membrane in *Cholœpus Hoffmanni* substantiates, in every important particular, Welcker's description, and leads me to form the same conclusion of its morphology as that to which he had previously arrived. More especially I may refer to its continuity at the oral, anal, and nasal orifices with the mucous lining of those passages, to its union also with the epidermis at the roots of the nails, and to the fact that it is perforated by the stronger tactile hairs, as evidences of its development from the *Horn-Blatt*. Moreover, I have been able to point out, what Welcker and the previous observers were not, owing to the absence of the placenta in their specimens, in a position to ascertain, that a simple bag-like amnion exists in the sloth as in other mammals. The Epitrichium, therefore, is not the amnion, but is derived from the epidermoid layer of the embryo, the most superficial stratum of which is not perforated by the hairs (except the tactile bristles) in their development, but is elevated in the form of a distinct continuous membrane.*

Some observations by G. Simon on the development of the hairs in the pig (*op. cit.* p. 370), and of Welcker on their development in *Bradypus tridactylus* (*op. cit.* p. 14), would seem to throw some light on the mode of production of the epitrichium. In both these animals the hairs do not lie straight in their follicles and with their apices directed outwards, but the individual hairs are coiled on themselves, or bent in the form of a loop in their respective follicles, so that the apex of each hair is in proximity to the root. It is not improbable

* Through the kindness of my friend, Dr John Young, Professor of Natural History in the University of Glasgow, I was enabled, in the month of July, to examine a fœtus of *Bradypus tridactylus* in the Hunterian Museum in that University, though not one of Dr William Hunter's specimens. The fœtus was 9½ inches long from nose to tip of tail. It was covered with hair, and possessed a complete epitrichium similar to the larger of the two specimens figured by Welcker in his Memoir. The placenta, though greatly shrivelled and hardened by the prolonged action of spirit, was seen to be composed of a number of discoid lobes.

that these coiled or bent hairs, instead of in their growth outwards piercing the entire thickness of the epidermis, which they might have been expected to do if their apices had been directed to the surface, elevate its superficial stratum, and, as the hairs lie thickly together, the stratum is pushed before them as a distinct continuous membrane.

In the month of June 1870, my friend Dr DAVID CHRISTISON wrote me an account of a gentleman the hair of whose head had appeared in a very singular manner. He was born quite bald, and there was no sign of hair till he was fifteen months' old, when a crop of rounded swellings, about the size of a pea or less, appeared on the back of the head; the swellings speedily burst, and disclosed a little curl rolled up in each. Other similar swellings appeared in successive crops on the other parts of the head, which in their turn burst and liberated similar locks of curly hair, until the hairy covering of the scalp was complete; and now this gentleman at the age of thirty-three possesses a head of strong curly hair. I was not at the time able to give a satisfactory explanation of this singular mode of growth of the human hair, but the study of the epitrichium in the sloth appears to throw light on it. The hairs undoubtedly had, in this person, as their curly form shows, been bent on themselves in their respective follicles, and in their growth towards the surface had elevated the outermost stratum of the epidermis instead of pushing through it; but as the growth was not precisely cotemporaneous over the whole surface of the scalp, the whole of the superficial epidermal layer was not, as in the sloth, elevated at the same time, but only as much as lay superficial to any particular lock of hair. The epitrichium, therefore, was partial and not complete in its character.

A few words may now be said on the arrangement, within the abdominal cavity of the fœtus, of the umbilical vessels. The single umbilical vein passed to the liver, and the two umbilical arteries diverging from each other extended along the sides of the bladder, which was included between them. No trace of a urachus could be found. No remains either of the sac of the allantois or of the umbilical vesicle were seen either in the cord or at its placental and fœtal extremities. RUDOLPHI had previously seen* in the embryo of *B. tridactylus* a single umbilical vein and two arteries, but in these specimens the urachus arose from the anterior wall of the bladder nearer the neck than the fundus; and a similar mode of origin of the urachus was met with in *Myrmecophaga jubata* and *Manis pentadactyla*. VON BAER states (*op. cit.* p. 263) that he has also seen in the Tardigrada the urachus attached not to the summit but near the neck of the bladder, though in a fœtal *Dasypus* he found the occluded urachus arising from the tip of the bladder. In CARUS's specimen also the urachus is figured as attached to the anterior wall of the bladder. In

* Abhand. der Akad. der Wissensch. zu Berlin, 1828, pp. 40, 41.

the embryos of these animals, therefore, it would appear that the urachus persists longer than it does in *Cholœpus*.

*General Observations on the Placentation of the Edentata.**

From the description which I have now given of the form and structure of the placenta in this specimen of *Cholœpus*, and from the accurate interpretation which I am able to offer of Carus's figure of the placenta in *Bradypus*, it is evident that in the Sloths the placenta is not cotyledonary, *i.e.*,—if we use the term cotyledon, in the sense in which it is usually employed by zoologists, to express a particular form of non-deciduate placenta,—subdivided into distinct and scattered masses as in the Ruminants. It can only be called cotyledonary, if the term be employed, as is sometimes done in speaking of the deciduate human placenta, as equivalent to lobes. To avoid confusion in the use of terms, it may be well to speak of the sloth's placenta as a dome-like, multilobate, aggregate placenta, the lobes of which are discoidal. It is a deciduate placenta in the fullest sense of the word; for not only is a decidua reflexa shed along with the fœtal membranes, but if the plane at which I separated the placenta from the uterus be, as I think there can be no doubt it is, the natural plane of separation of these parts during parturition, then the fœtal membranes carry away with them the deciduous serotina, the curling arteries, the utero-placental veins, and the intra-placental maternal vessels. I have been able, therefore, to put on the basis of an actual demonstration the deciduate structure of the placenta in the Sloth, which Rolleston, Owen, and Milne Edwards from the study of Carus's drawing, had regarded as not improbable.

The character of the placentation in the Sloths having now been determined, it will be interesting in the next place to compare it with what has been recorded respecting the placentation of the other mammals included in the order Edentata. Unfortunately, however, we are not provided with such detailed information relative to the placental characters of these animals as to enable one to make so complete and exact a comparison as could be desired. Any remarks, therefore, which may be based on this comparison must be regarded as provisional merely, and to be subject to revision when more precise knowledge is obtained.

Of the placenta of the Armadillos nothing further has apparently been recorded than is contained in the brief statement made by Prof. Owen † that it is a single, thin, oblong disc, with which the maternal deciduous substance is interblended.

Our information as to the placenta of *Orycteropus* is also equally brief, and is limited to the observation recorded by Prof. Huxley in his "Introduction

* November, 1873.—This section has been re-written and added to since the paper was read.

† Anatomy of Vertebrates, iii., p. 731. 1868.

to Classification,"* based on the examination of a specimen in the stores of the Royal College of Surgeons, London, that in this genus the placenta is discoidal and deciduate.

Of the Hairy Ant-eaters, A. F. J. C. MAYER states† that in *Myrmecophaga* (*Cyclothurus*) *didactyla* a single orificium uteri exists, and in the gravid state the uterus and vagina are blended into a common cavity, and form a round membranous sac. The fœtus in the specimen he examined was well developed and strongly haired, with the head presenting. The placenta was a thick roundish cake (*Kuchen*), and lay to the right in a special pouch of the uterus. The chorion and amnion were distinct, but the erythrois and allantois could not be distinguished in the torn membranes. Prof. WELCKER, in his Memoir already quoted, incidentally mentions that in *M. didactyla* the amnion and chorion circumscribe in the usual way the border of a fungiform (*pilz-förmig*) placenta. The elder MILNE EDWARDS states‡ that he has found in *M. didactyla* a discoid placenta, but composed at its borders of small branched tufts: it did not appear to be united to the uterus by a decidua. His son, M. ALPHONSE MILNE EDWARDS, described only last year§ a placenta of *Tamandua tetradactyla*, which had been hardened in spirit for some years before it came into his possession. It occupied the larger part of the surface of the ovum, and was not composed of simple villosities like the placenta of the Pachydermata, but of vascular very compact vegetations. Its central part was thick and spongy; its borders well defined, with a smooth chorion beyond them. The vascular vegetations, he says, do not apparently resemble the reticulated folds and alveoli seen by Dr SHARPEY in *Manis*. Some débris of uterine tissue indicated the presence of a decidua, but he could not speak positively on the subject. The placenta is unilobed, dome-like in the mode in which it is set on the chorion, and is named by MILNE EDWARDS *placenta discoïdal envahissant*. The surface of the placenta next the embryo did not possess the projections seen by CARUS in *Bradypus*, or by myself in *Cholœpus*, but was smooth. No trace of an allantois was found.

From the above descriptions it is clear that in the hairy ant-eaters the placenta is not diffused over the whole surface of the ovum, but is localised in a particular area. From MILNE EDWARDS' figure the proportion of chorion occupied by the placenta is about equal to what I have seen in *Cholœpus*, but the organ is not, as in the latter animal and in *Bradypus*, subdivided into lobes. From the expressions *Kuchen* employed by MAYER, *pilz-förmig* by WELCKER, discoid and spongy by MILNE EDWARDS, *père et fils*, it is also clear that the

* P. 104. London, 1869.
† Analecten für Vergleich. Anatomie. Zweite Sammlung, p. 54. Bonn, 1839.
‡ Leçons sur la Physiologie, ix. part 2, p. 563, note. Paris, 1870.
§ Annales des Sciences Naturelles, xv. 1872.

organ possessed considerable thickness; and although the younger MILNE EDWARDS could not, from the condition of his specimen, speak positively of the presence of a decidua, the thick spongy character of the organ with the compact arrangement of the villi points rather to a deciduate than a non-deciduate structure.

In the Tardigrada, the Dasypodidæ, and the Orycteropodidæ, we have evidence then that the placenta is deciduate, and composed of one or more disc-shaped lobes. In the Myrmecophagidæ the evidence is not so complete, though I think it inclines in favour of the deciduate nature of the placenta.

But when we turn to the Scaly Ant-eaters we find a placenta of a very different character. Some years ago Dr SHARPEY examined the gravid uterus of a *Manis*, which, from the size of the fœtus, was presumably near the full time. He observed several most important features in the arrangement and structure of the placenta, which he communicated to Professor HUXLEY, who incorporated them in his "Elements of Comparative Anatomy."* "The surface of the chorion is covered with fine reticulating ridges, interrupted here and there by round bald spots, giving it an alveolar aspect, something like the inside of the human gall bladder, but finer. The inner surface of the uterus exhibits fine low ridges or villi not reticulating quite so much. The chorion presents a band free from villi, running longitudinally along its concavity, and there is a corresponding bald space on the surface of the uterus. The ridges of the chorion start from the margins of the bald stripe, and run round the ovum. The umbilical vesicle is fusiform."

In a letter to me, in reply to a request for further information about this specimen, Dr SHARPEY states that it had been in spirit before coming into his possession, and that the substance of the uterus and the tissues of the embryo were brown and fragile. An injection, both of the uterus and membranes, was attempted, but, from the condition of the parts, was unsuccessful. The elevations on the chorion corresponded to the finely-corrugated inner surface of the uterus. "I turned off the chorion like an everted stocking, and got the arrangement of the allantois, fusiform umbilical vesicle, omphalo-mesenteric and umbilical vessels. The ramifications of the umbilical vessels extended generally over the inner surface of the chorion, and were lifted off with it from the receptacular part of the allantois, which was very extensive, and passed into the diverticula of the chorion. The uterine glands were abundant and easily seen, but I could never distinctly trace their orifices; it seemed to me as if the ducts opened not abruptly, but gradually and funnel-like among the placental rugæ. I think you found this condition in the whale."

I have not only to express my thanks to Dr SHARPEY for this additional information, but to state that with great liberality he has allowed me to examine

* P. 112. London, 1864.

the original drawings of the chorion and uterine glands, and the specimen itself as it had been dissected by him, and to supplement his description by the following particulars. Although the preparation had now been for many years in spirit of wine, yet I had no difficulty in recognising the diffused arrangement of the fine villi of the chorion and the elongated band free from villi, such as Dr SHARPEY had described. In its general aspect the villous surface resembled the appearance which I had seen and described in *Balænoptera* and *Orca*,* though the ridges, folds, and villi of the chorion were finer than those seen in the Cetacea. The condition and age of the specimen were such as to render it impossible to make a satisfactory microscopic examination of the structure of the villi. The two extremities of the elongated chorion were unequal in capacity, and the non-villous band extended in closer proximity to the more dilated than to the narrower pole of the chorion. In the more dilated end, part of the fœtus had, in all likelihood, been lodged ; and it is probable that the poles of the chorion had been contained in two pouch-like recesses about the size of walnuts, one situated at each lateral extremity of the transversely elongated uterus. Each communicated freely with the general cavity of the uterus by an orifice somewhat less in diameter than that of the pouch itself, and was lined by a prolongation of the uterine mucosa. Into the outer end of each pouch the very fine orifice of the Fallopian tube opened.† I examined carefully the poles of the chorion to ascertain if a spot bare of villi, similar to what I had seen in *Orca* and in the mare, existed. At the narrower end the chorion was torn, so that the examination was not satisfactory, but the more dilated pole was entire, and in it no bare non-villous spot was recognised, so that if, as I suppose, the chorion enters the pouch-like recesses of the uterus, the villi investing it would have a relation to the mucosa lining each pouch as the villi covering the body of the chorion have to the mucosa lining the general cavity of the uterus. I saw no stellate bare spot on the chorion corresponding to the orificium uteri similar to what I have described in *Orca* and in the mare. Branched, cylindriform utricular glands, as figured in one of Dr SHARPEY's drawings, closely resembling those I have figured in *Orca*, and containing plenty of epithelial cells, the form of which was not very distinct, though apparently columnar, could be seen without difficulty in the uterine mucous membrane, but I could not precisely ascer-

* As described and figured in my memoirs in the Transactions of this Society, vol. xxvi. pp. 207 and 467, 1871.

† The presence of these uterine pouches in *Manis* is not without interest to the human anatomist, and may serve to throw some light upon the mode of production of some cases of so-called extra-uterine gestation, described by obstetrical writers, in the human female. It is not, I think, improbable that in some of the cases figured and described by BRESCHET (Répertoire Général, 1826), the ovum may have been lodged in a pre-existing uterine pouch, and not, as he supposed, in the thickness of the walls of the uterus, and the same explanation may perhaps be given to Dr BRAXTON HICK's case in Trans. Obstetrical Soc. of London, vol. ix. p. 57. Cases which have been described as tubo-uterine pregnancies may also have had a similar mode of origin.

tain the mode in which they opened on the surface of the mucosa, which was thickly studded with minute pits or fossæ; it is probable, however, that they opened obliquely into the bottom of these pits. The free surfaces, both of the chorion and uterine mucosa, presented, without doubt, the appearance which one recognises as characteristic of a diffused non-deciduate placenta.

The placenta, therefore, in *Manis*, differs in several most important particulars, both in arrangement and structure, from what I have described in *Cholœpus*. Not only is it diffused and non-deciduate, but both the allantois and umbilical vesicle remain as distinct sacs, and the utricular glands persist throughout the gravid uterine mucosa.

The demonstration, therefore, of the diffused, non-deciduate character of the placenta in *Manis* by Dr SHARPEY, and of the multilobate, discoid, deciduate placenta in *Cholœpus* by myself, will render it necessary for the systematic zoologist to reconsider either the value of the placenta as the basis of a system of classification, or the propriety of retaining the Sloths and the Scaly Ant-eaters in the same order. If the characters of the placenta are to be regarded as of more importance in classification than those furnished by any other organ or combination of organs, then it is clear that the non-deciduate *Manis* can no longer be placed in the same order as the deciduate Tardigrade. But if the characters of the other organic systems in the animals belonging to the order Edentata, as at present accepted, exhibit a series of affinities, which to the mind of the zoologist may seem to out-weigh the differences in placental structure, not only between *Manis* on the one hand, and *Cholœpus*, *Dasypus*, and *Orycteropus* on the other, but, if my inference as to the deciduate nature of the placenta in *Myrmecophaga* be correct, between *Manis* and *Myrmecophaga* also—affinities which would render it advisable that they should be retained in the same order,—then the placental system of classification is obviously not universally applicable, and will have to be abandoned. It would be out of place in this communication to enter into the consideration of the anatomy of the other organic systems in *Cholœpus*, and to discuss how far its various organs resemble, or differ from, those of the genera with which it is usually associated; but I hope, from the materials in my possession, to supplement this memoir, and draw up, in the course of the next few months, a detailed account of the structure of this animal, and to compare it, as far as the materials at my disposal will allow, with the other animals usually grouped with it in the order Edentata.

Before bringing this memoir to a conclusion, it may not be without interest briefly to compare the placentation of the Sloths with that of the other orders of Deciduate mammals.

It may help to make this comparison more complete if I introduce here some observations which I have recently made on the structure of the unimpregnated uterus of the Sloth. Through the kind permission of Dr SCLATER

I received from A. H. Garrod, Esq., Prosector to the Zoological Society, the fresh carcase of a female Hoffmann's Sloth, which had died in the Gardens early in the month of November. An injection of coloured gelatine was passed into the arterial system from the abdominal aorta, and the uterus was then removed from the abdomen. The uterus was $3\frac{1}{4}$ inches long by $\frac{8}{10}$ths inch broad. The Fallopian tubes were slender, the ovaries the size of peas, and lodged in peritoneal pouches. The cavity of the uterus was remarkably large for a non-gravid organ, and through its somewhat constricted orifice which opened into the vestibule, the finger could be introduced into the cavity, which showed no subdivision into cervix and body (fig. 11).

The mucous membrane both on the anterior and posterior walls was elevated into longitudinal ridges such as I have already described in the pregnant uterus, but these ridges terminated about $\frac{3}{4}$ths inch from the orificium uteri, leaving a smooth surface of mucosa. The mucous membrane was very vascular; the small arteries both in it and in the submucous coat presented a cork-screw like twist, the coils of which were close together, so that there can be no doubt that the curling arteries of the placenta pre-exist in the mucosa, and merely grow larger during pregnancy cotemporaneously with the development of the placenta. The veins were larger than the arteries, and serpentine in their course. The surface of the mucous membrane of the fundus uteri, and both horizontal and vertical sections through its thickness were examined with reference to the presence of tubular utricular glands. Distinct evidence of their existence was obtained, though they were much more difficult to see and much less numerous than in the pig, mare, cetacean or *Manis*; they were short tubes, and did not appear to give off more than one or two branches, which terminated in rounded closed ends. They did not lie perpendicular to the plane of the surface, for in vertical sections they were irregularly divided, and were arranged in groups, so that they were more numerous in some than in other portions of the fundus (fig. 12). Their orifices on the free surface of the mucosa, which were recognised with some difficulty, were nearly circular in form, and a somewhat elongated epithelium projected from the wall towards the centre, leaving, however, a small lumen; the polygonal ends of the epithelial cells were seen through the walls of the glands lying horizontally in the mucosa. Capillary loops surrounded the glands in the deeper part of the mucosa. In the smooth part of the mucosa, near the orificium, although a careful search was made, no glands were seen, and the arteries did not possess the corkscrew-like twist, so that from these structural differences this part of the mucosa did not present the same characters as that of the fundus uteri. The connective tissue of the mucous membrane contained numerous well-marked corpuscles, and its surface was covered by a layer of cells, only the ovoid nuclei of which could be defined with precision.

The placenta of the Carnivora is at once distinguished from that of the Sloth by several striking characters:—By its zonary form; by the presence, not only in the unimpregnated uterus, but in the non-placental area of the gravid uterine mucosa, and in the maternal part of the fully developed placenta of utricular glands; by the intra-placental maternal vessels retaining the form of a capillary net work;* by the brevity of the umbilical cord; and by the persistence both of the umbilical vesicle and allantois.

In the Insectivora, Rodents, and Cheiroptera again, the placenta, though with some slight modifications in its shape, forms a single "discoid" organ which in some cases at least shows a subdivision into lobes; the allantois and umbilical vesicle, the latter of which is of large size in many genera, persist as distinct sacs throughout intra-uterine life, and no evidence has been advanced that the intra-placental maternal vessels are dilated into sinuses. REICHERT has shown† that in these orders a decidua reflexa, more or less complete, exists. The condition of the mucous membrane, as regards the utricular glands, exhibits some variations in different genera. LEYDIG has observed them in the mole;‡ ERCOLANI in the hedge-hog;§ REICHERT in the guinea pig;‖ ERCOLANI in *Mus musculus*;¶ in which animal he says they are few in number, simple, and slightly sinuous. In the rabbit there is some difference of opinion as to the nature of the deep depressions which exist in its uterine mucosa, for though REICHERT compares the windings and folds of that membrane to the appearance exhibited by the convolutions of the brain, and speaks (MÜLLER's Archiv, 1848, p. 80) of the involutions of the mucous membrane as short and wide, yet he evidently regarded them as essentially the same as the tubular utricular glands in the pig, guinea pig, bitch, &c.** ERCOLANI again states that the rabbit, instead of possessing utricular glands, has numerous very short glandular follicles, which are only inflexions of the epithelial layer, and represent in this animal the uterine mucosa. These follicles, he says (p. 146), develope largely during pregnancy, are transformed into a glandular organ, and appear destined to replace, during pregnancy, the utricular glands which are wanting in these animals. I have had the opportunity of examining sections through not only the non-gravid uterine mucosa

* I am aware that ESCHRICHT, in his important memoir (De Organis, &c., p. 24), speaks of the vessels in the feline placenta as exhibiting dilatations, and that KÖLLIKER ("Entwicklungs Geschichte," p. 163), states that in the bitch the maternal blood-vessels are very strongly developed, and appear as very thin-walled capillaries $\frac{1}{6}$‴ in breadth, but I have not seen in the placenta of this animal vessels at all comparable with the sinuses in the sloth.

† Beiträge zur Entwicklungs Geschichte des Meerschweinchens in Abhand. der König. Akad. der Wissensch. zu Berlin, 1861.

‡ Lehrbuch der Histologie, p. 517. 1867.

§ Sur les Glandes Utriculaires de l'Uterus, p. 10. French translation. Algiers, 1869.

‖ Op. cit. plates i. ii. p. 117, and MÜLLER's Archiv. p. 80. 1848.

¶ Sulla le Glandole Otricolari dell'Utero. Mem. dell'Acad. delle Sc. di Bologna, p. 26. 1873.

** The naked-eye appearance of the folds of the mucous membrane of the rabbit's uterus has been carefully described and figured by M. H. HOLLARD, in Annales des Sciences Naturelles, p. 223. 1863.

of the rabbit, but through the non-placental portions of the mucosa in the gravid uterus, which have been carefully prepared by Mr STIRLING, and, from the appearances presented by these sections, there is, I think, good reason to regard these "glandular follicles" as only utricular glands somewhat modified in their shape. For though the part which lies next the uterine cavity has not the cylindrical-tubular form usually exhibited by the utricular glands, the deeper extremities of these follicles lying next the submucosa present, on transverse section, a circular or oval form, and on longitudinal section an elongated tubular form, such as the utricular glands themselves exhibit. Moreover, both the dilated and tubular portions of the so-called follicles are lined by a columnar epithelium, which projects into their cavity, but leaves a central lumen. It is clear, therefore, that these follicles or glands exist in the uterine mucosa prior to impregnation, and are not occasioned by the gravid condition.

The very important observations recently made by M. ALPHONSE MILNE-EDWARDS * on the placentation of the Lemurs, furnish some material for comparing this group of animals with the sloths. M. EDWARDS has examined specimens of the genera *Propithecus*, *Lepilemur*, *Hapalemur*, and *Chirogaleus*. He found the chorion almost entirely covered by dense, compact villosities constituting a sort of vascular cushion, the result of the confluence of a multitude of irregular cotyledons. The placenta had the appearance of a large sac which almost completely enclosed as in a hood the amnion. This form of placenta he calls bell-like (*placenta en cloche*), for the villi are most numerous at the upper and middle parts of the chorion, but almost entirely disappear towards the cephalic pole. The uterine mucosa corresponding to the villosities exhibited numerous irregular anfractuosities, and had developed a caducous layer. An enormous sac-like allantois was situated between the chorion and amnion. In its general mode of disposition on the chorion, the placenta of the lemur is not unlike that of the sloth, but in the latter animal the lobes or cotyledons are apparently less intimately blended than in the former, which has in addition a highly-developed allantois; but as no information is given on the arrangement of the utricular glands or maternal blood-vessels, I can make no comparison between the disposition of these important structures in the Lemurs and Sloths.†

* Annales des Sciences Naturelles, xv., 1871.

† Since this Memoir has been put in type my attention has been directed by Prof. R. O. CUNNINGHAM to a paper "On the Lemurs," by Mr ST GEORGE MIVART, in Proc. Zool. Soc. London, May 20th, 1873. Mr MIVART states that from a private communication made to him by M. ALPHONSE MILNE-EDWARDS, that naturalist is now of opinion that the Lemuroids have no decidua, and that the placenta is diffuse. It does not appear from Mr MIVART's paper whether M. MILNE-EDWARDS had received additional specimens since the publication of his Memoir quoted in the text, in which, when describing the uterine mucosa of *Propithecus*, he says, "Et la surface en est hypertrophiée de façon à former une couche caduque, très-analogue à celle qui, dans une très-faible étendue, adhère au placenta discoïde des Singes, des Insectivores et des Rongeurs." From which extract it is clear that when his Memoir was written, M. MILNE-EDWARDS had no doubt of the presence of a decidua.

In the new world Monkeys the placenta consists of a single disc-shaped organ, which is probably also the case in the anthropomorphous apes, though in the tailed apes of the old world, as HUNTER and BRESCHET's observations have sufficiently shown,* the placenta is subdivided into two large lobes by a greater or less interval. JOHN HUNTER has pointed out that each of these large lobes is made up of smaller ones, united closely at their edges—a feature which BRESCHET also has confirmed (*op. cit.* p. 445). The subdivision of the placenta in these monkeys into two parts is interesting in connection with the arrangement seen in the sloth's placenta, where a partial separation into a right and left lateral half was found, each of which in its turn consisted of smaller lobes. But though the form of the placenta, the existence of a decidua, the absence of the sac of the allantois, the absence, or at least the rudimentary condition, of the umbilical vesicle,† the arrangement of the amnion, and the comparative length of the umbilical cord, have all been determined in the quadrumana, and correspond in most particulars with what I have described in the sloth, yet there is, unfortunately, a want of precise information on the arrangement of the maternal blood-vessels, and the condition of the utricular glands in the former group of animals. JOHN HUNTER, indeed, says, veins or sinuses were placed in the fissures between the lobes, which received the blood laterally from the lobes, and that the substance of the placenta seemed to be "cellular," as in the human subject. Dr ROLLESTON, from the examination of a spirit-preserved placenta of *Macacus nemestrinus* (*op. cit.* p. 301), obviously inclines to the view that in it intraplacental maternal sinuses existed, though he points out that, from the age of the specimen, the examination was not so satisfactory as he could have desired. Prof. ERCOLANI states,‡ from an examination of a specimen preserved in spirit, of the placenta of *Cercopithecus sabaeus*, that in this monkey the placenta in its microscopic characters, as well external as internal, does not present any difference from that of woman; so much alike are they, that as he had just described the human placenta, he did not think it necessary to go into the details of structure in the ape. He does mention, however, that the intraplacental lacunæ in *Cercopithecus*, which contain the maternal blood, are smaller than in woman; and that on the uterine face of the placenta are manifest traces of serotina, which is continued on to the fœtal villi forming the external membrane, or the walls of ERCOLANI's glandular organ. As he had stated on p. 36 that he had never seen utricular glands in the human placenta, we must

* Figures of the placenta in the Quadrumana, or descriptions of its naked-eye characters, will be found in JOHN HUNTER's Collected Works, iv. 71; Plates xxxv., xxxvi., and fig. 2, xxxiv.; in RUDOLPHI's Memoir, "Ueber den Embryo der Affen," already quoted; in BRESCHET's important Essay in Mémoires de l'Institut, 1845, xix.; and in OWEN's Comparative Anatomy of Vertebrates, iii. 747, in which volume it is mentioned that the pregnant monkey, dissected by HUNTER, the name of which that anatomist had omitted to give, was a specimen of *Macacus rhesus*.

† Compare BRESCHET's description of *Simia Sabaea* with that of *Simia nasua*, pp. 444, 470.

‡ Mem. dell' Acad. di Bologna, 1870, p. 53.

assume from the similarity in structure that he had not observed them in this monkey.

As I could find no record of observations on the utricular glands, even in the unimpregnated uterus in the quadrumana, I examined the mucosa of a non-gravid spider-monkey, apparently the *Ateles gricescens*, preserved in spirit. I found numerous short glands, dilated into pouch-like recesses, and containing an abundance of epithelium, opening into the uterine cavity by constricted mouths. A few which possessed the form of short tubes were interspersed amidst these pouch-like glands. In this ape, as in the human female, the mucosa was in close relation to the subjacent muscular coat, and was not attached to it, as in the sloth and mammals generally, by a lax coat of submucous connective tissue.

I owe to Dr ROLLESTON the opportunity of examining a slice of the placenta of the *Macacus nemestrinus* in the Oxford University Museum. Both to the naked eye and with a simple microscope, a section through the organ showed a spongy character similar to that exhibited by the human placenta. The arborescent arrangement of the villi and the processes of decidua, prolonged from the serotina into the interior of the placenta, corresponded to Dr ROLLESTON's description. I examined the bud-like offshoots of the villi microscopically; the capillaries, filled with a yellow injection, were arranged, not in loops, but in networks. Between these vessels and the periphery of the villus, a relatively thick layer of tissue intervened, which seemed to consist of two strata; one next the capillaries, which consisted of cells such as ERCOLANI regards as the glandular organ, continuous with the serotina; the other, and more external, was apparently composed of flattened cells, which, on the supposition of an intra-placental circulation of maternal blood and an involution of the utero-placental vessels, probably represents the wall of the maternal blood-vessels reflected on to the villus. A microscopic examination of the serotina displayed numerous large fusiform cells with ovoid nuclei, mingled with which were circular or spherical cells possessing granular contents and nuclei often of a globular form. The processes of the decidua contained fusiform cells smaller than those just described, and a nucleated protoplasm in which a differentiation into definite cell forms was not clearly demonstrable.

The great attention which has been paid by various eminent anatomists to the structure of the human placenta, enables one to institute a closer comparison between it and that of the sloth than could be done with regard to the quadrumana. In man the lobes are more closely fused together into a single organ, though the original sub-division into separate lobes is not unfrequently shown by deep fissures extending from the uterine surface deeply into its substance; and in a case described by M. BRESCHET* the outer face of the chorion is said to

* Répertoire Général, p. 3, Paris, 1826.

have exhibited, not a regular placenta, but a number of distinct cotyledons. But, further, cases have been seen* in which the human placenta was subdivided into two not quite equal parts, a condition which is normal in the tailed monkeys of the old world, and an approximation to which, as I have already stated, is seen in the placenta of the sloth. Some recent observations by REICHERT† on a young human embryo at the 12th or 13th day, have shown that, at this very early period, distinct elevated islets or cotyledons had formed in the uterine mucosa, and a median cleft separated these islets into two halves, so as to give a bilaterally symmetrical arrangement. If, as is not improbable, these islets are the rudiments of the lobes of the future placenta, the human placenta approximates in its bilateral arrangement at this stage of development to what I have seen in the sloth.

Both in the human and sloth's placenta, curling arteries and utero-placental veins are present, though in the former their subdivision in the substance of the placenta into branches cannot be followed out as in the latter, in which animal, moreover, the dilatation of the veins into large sinuses within the muscular wall of the uterus and the serotina does not exist as in the human female. In both, intra-placental maternal sinuses, communicating on the one hand with the curling arteries, on the other with the utero-placental veins, are met with. In the sloth, however, these sinuses retain their individuality, their walls can be isolated from the adjacent villi, they have a tubular form, and their arrangement as an anastomosing network is preserved.‡ The mode in which they wind in and out between the villi bears a strong resemblance to the description and figures given§ by Dr PRIESTLEY of the arrangement of the maternal vessels in a human embryo at the second month; only in his case the vessels, though described as "capacious capillaries," were not dilated into large sinuses as in the sloth. In the fully-formed human placenta, on the other hand, though the walls of the curling arteries and utero-placental veins are distinct structures, yet the intra-placental maternal blood sinuses are not tubular, but consist of a system of irregularly-formed and freely-communicating spaces. Whether they possess delicate walls separating them from the tissue of the fœtal villi, or whether the villi float naked in the mother's blood, are questions which have been much debated amongst anatomists. Every one who has examined the villi of the human chorion is familiar with the layer of nucleated cells which invests the villi like a cap. It has been repeatedly figured, and is seen to lie immediately outside the single or double capillary loop which the bud-like offshoots of the human villi contain. Opinions are divided whether this layer of cells belongs to the

* See HECKER, as quoted by Dr J. MATTHEWS DUNCAN in "Edinburgh Medical Journal," Nov. 1873.

† REICHERT und DU BOIS REYMOND'S Archiv. p. 127. 1873.

‡ The arrangement in the sloth is, indeed, not unlike what E. H. WEBER conceived to be the arrangement in the human placenta.

§ Lectures on the Development of the Gravid Uterus, p. 62, figs. 19, 20. London, 1860.

villus, or is a layer of cells derived from the decidua serotina, which has become blended with the tissue of the villus; but whether we regard these cells as proper to the villus or as belonging to the serotina, no corresponding layer was seen in the sloth. In the course of observations made during the past two years on the minute structure of the human chorionic villi, I have more than once seen an appearance which led me to believe that outside this layer of well-defined cells—*i.e.*, nearer, or rather next to the maternal blood—was a layer of squamous cells, which would represent, therefore, the endothelium of the maternal blood-vessels, blended with the villus owing to the great expansion of these vessels into the irregularly-shaped, freely-communicating blood sinuses. In the sloth, again, as the intra-placental sinuses retain the individuality of their walls, their endothelium remains in its proper position as a layer of cells lining the maternal blood-tubes. In the sloth, therefore, the capillaries of the chorionic villi lie closer to the periphery of the villus than is the case in the human placenta; but, further, in the sloth these capillaries are arranged as a distinct network, whilst in man they form single or double loops. The branches which arise from the stems of the villi in the sloth are more elongated, and have more of a laminated arrangement than is the case in the human placenta.

The presence of a system of maternal sinuses within the placenta of the sloth, though, as I have just pointed out, it differs in several points from the corresponding arrangement in man, is of great anatomical interest; for it presents a transitional form between the simple maternal capillary plexus met with in a diffused cotyledonary or zonary placenta, and the irregularly-formed blood-spaces which exist in the placenta of the human subject. Several obstetrical writers have, from time to time, denied the existence of intra-placental maternal blood sinuses in the human female, and the objection has been advanced that the presence of such sinuses receives no support from comparative anatomy. In a communication read to this Society, May 20, 1872,* I adduced a number of facts, derived from the study of the human placenta, in support of the Hunterian doctrine of the intra-placental circulation of maternal blood, and I may now add to these facts the confirmatory evidence afforded by the structure of the placenta in the sloth.

The condition of the utricular glands in the mucosa of the human and sloth's uterus is also a feature of much interest. In both, in the quiescent, non-gravid state, the glands are short, comparatively simple in form, and are not demonstrable with the same readiness as in the uteri of animals, which, in the gravid state, possess a diffused or a zonary placenta. In the human uterus, at the commencement of gestation, the glands are well marked, but as pregnancy advances to its middle period they disappear, so that no traces can be seen of

* Abstract in Proceedings of that date, and more fully in Journal of Anatomy and Physiology, November 1872.

them, either in the decidua or in the fully-formed placenta.* Of the condition of the glands, in the early period of gestation in the sloth, we have no information, but that they are absent in the later period I have little doubt, for the careful examination to which I subjected both the placenta and the non-placental area of the mucosa would have disclosed them had they been present.

Both in the sloth and in the human subject the decidua reflexa is well marked. The serotina is not, however, so strongly developed in the sloth; more especially is there a deficiency in the granular colossal cells, which in the human placenta not only lie on its uterine aspect, but pass into its substance in the decidual dissepiments.

The great importance in fœtal nutrition, which has been ascribed by Ercolani in his several valuable and most instructive memoirs on placental structure to a system of gland follicles forming the maternal part of the placenta, which he believed to be new formed during pregnancy in all placental mammals, and in which the fœtal villi are lodged, naturally led me to examine the sloth's placenta with care, to ascertain if in it such gland follicles could be recognised. I failed, however, to see any indications of follicular structure, either in the defined form described by him to exist in the diffused cotyledonary or zonary placenta, or as a layer of cells, belonging to the decidua serotina, investing the fœtal villi as in the human female.

I have already, in my memoir on the Placentation of the Cetacea,† criticised and advanced some objections to the general applicability of Ercolani's theory, and from the study of the placenta in the sloth there appear to be additional reasons for doubting that anatomical unity in placental structure which he advocates. The presence of a gland secretion as an osmotic medium in fœtal nutrition, whether we regard it as produced by new-formed gland follicles, as Ercolani supposed, or by the utricular glands themselves, as Eschricht argued, does not, I believe, necessarily occur in all forms of placentæ. That a white fluid, subsequently termed uterine milk, which serves as aliment for the fœtus, is present in the cotyledons of the ruminants was known to Harvey and the older school of physiologists, and it is very probable that a similar fluid is produced in all placentæ where uterine glands or follicles continue to secrete during the whole period of placental formation. But in those placentæ, as the sloth, the apes, and the human female, where an unusual development of the maternal blood-vessels into large sinuses takes place, a modification in the anatomical structure is introduced which seems to render the presence of such a secretion unnecessary; the utricular glands seen in the non-gravid uterus

* Dr Priestley states (p. 27) that he could see them distinctly in the parietal decidua near the seat of the placenta in the 3d month, but they were undergoing granular degeneration, and, instead of being lined with epithelium, were filled with granules and molecules.

† Trans. Roy. Soc. Edinburgh, vol. xxvi. p. 467.

disappear, no new-formed follicles are produced, and the nutritive changes, in all probability, take place directly between the fœtal and maternal blood.

The amnion in the sloth is related to the chorion, placenta, and umbilical cord, as in the human female. The sac of the allantois and the urachus have disappeared, and I could see no trace of an umbilical vesicle. Further, I may state that the uterus is simple and uniparous, and that the mammæ are two in number and pectoral in position.

In classifying the Sloths and the other members of the Order Edentata it has been customary for zoologists to rank them with the lower orders of mammals. Professor Owen, for example,* directs attention to the supernumerary cervical vertebræ supporting false ribs, and the convolution of the windpipe in the thorax of *Bradypus*, as manifesting its affinity to the oviparous vertebrata, and to the unusual length of the dorsal and short lumbar spine in *Cholœpus* as recalling a lacertine structure; whilst the abdominal testes, single cloacal outlet, low cerebral development, absence of medullary canals in the long bones in the sloths, and long-enduring irritability of the muscular fibre in both the Sloths and Ant-eaters, show the same tendency to an inferior type. In his system of classification, based on the cerebral characters, he places them in the group Lissencephala, along with the Rodentia, Insectivora, and Cheiroptera. Professor H. Milne-Edwards, in his most recent defence of his placental system of classification,† whilst admitting the insufficiency of the information on the mode of development of the Edentata, considers that, from the structure of the teeth and the absence of incisors, these animals have affinities with the Cetacea more than with other mammals, though they appear to have some relations with the Monotremata, and he does not hesitate to form a separate phalanx for their reception. Professor Haeckel, whilst ranking them amongst the Indeciduata,‡ admits that the genealogy of the Edentata is very difficult. Perhaps, he says, they are nothing but a peculiarly-developed offshoot of the Ungulata, but perhaps their root may lie in a very different direction.

The comparison which I have just made between the placenta of the sloth and that of the other deciduate mammals reveals a correspondence in important features, both of arrangement and structure, between the placenta of the sloth, that of the human female, and of the monkeys, greater than exists between it and the same organ in any of the other orders of the Deciduata, so far as has yet been described. This correspondence in placental form and structure between mammals, which on general zoological grounds are so widely

* Reade Lecture "On the Classification of the Mammalia," p. 31, 1869.

† Considérations sur les affinités naturelles et la classification methodique des Mammifères, being the first chapter in the Rechérches pour servir a l'histoire naturelle des Mammifères, now in course of publication by himself and his son M. Alphonse. Preface, dated 27th April, 1868. Paris.

‡ Natürliche Schöpfungs-Geschichte. Berlin, 1868, p. 480.

separated, affords room for much speculation and thought, and throws a new light, not only on the position of the sloths in the order Edentata, but on their relations generally to the placental mammals.

Professor H. Milne-Edwards, in the Memoir above referred to, argues that similarity in the form of the placenta and in the arrangement of the membranes is associated with resemblances in other important structural characters, so that the classification of mammals founded upon the placenta rests on a natural basis. Thus Man, the Quadrumana, Cheiroptera, Insectivora, and Rodents, are grouped together by him in the Micrallantoid legion of the phalanx *Hématogénètes*, as they possess in common a discoid placenta, a small allantois, and a caduca uterina. But, further, they are all markedly unguiculated, their teeth are provided with a covering of enamel, and the dental series is continued around the front of the jaws.

As regards their placental characters, the Sloths would fall into this Micrallantoid legion, with which also they would be associated by their long claws; but in the structural characters of their teeth and the absence of incisors they are at once markedly distinguished from them, so that in these respects the correspondence between placental form and structure and these other well-pronounced natural characters breaks down.

Between *Choloepus* and *Homo* the divergence in most of their organic systems is so great that it is difficult to find evidence of any affinity except in their placental characters. With the Prosimii and Apes, however, affinities may be found. De Blainville had indeed many years ago* indicated correspondences in the skeleton of the Sloths and the Apes, more especially the Gibbons; I may here refer to the very remarkable vascular plexuses which exist in the limbs both of the Sloths and Lemurs; and now that I have called attention to the evidences of affinity with these higher mammals it is not improbable that other features of resemblance may in time be recognised. From the point of view of the descent hypothesis, it is possible that between the Sloths and the Lemurs genealogical relations may exist.

In conclusion, I may state that the study of the placenta in the Sloth has shown how difficult it is to predicate from the arrangement and structure of the other organic systems what the character of the placenta may be, and how necessary it is, before a proper estimate can be formed of the nature of the placentation, not only that the form of the organ and the arrangement of the membranes in the different orders of the mammals should be worked out, but the modifications in its minute structure should also be determined. Moreover, it would seem that affinities in placental form and structure may exist between mammals which in many other respects are widely separated, so that the placenta is not in itself sufficient to determine the position of an animal in

* Ostéographie des Mammifères. Paresseux, p. 1.

the mammalian series, and the use of this organ as a basis of classification, though in many instances it may be relied on, yet, from the complex cross relations which exist between the several organic systems in the placental mammals, is not universally applicable.



This woodcut is a diagrammatic representation of the advanced ovum of *Cholœpus Hoffmanni*. The shaded part (*Pl*) shows the proportion of chorion on which the placenta is developed. *Ch* is the non-placental area of the chorion. The dotted line (*am*) the amnion. *u*, the umbilical cord. *Ep*, the epitrichium investing the fœtus.

EXPLANATION OF PLATES.

Figures 1, 2, and 3 were drawn from nature, under my superintendence, by Mr John Reid; and figures 4, 5, 6, 10, and 11, by Mr C. Berjeau. Figures 7, 8, 9, are from sketches made with the camera lucida, by myself.

Plate III.

Figure 1. Posterior surface of the gravid uterus of Cholœpus Hoffmanni. The rectum is drawn to one side; the anus and external opening of the genital passage are surrounded by a common fringe of hairs. Reduced about $\frac{1}{3}$d.

Figure 2 represents a portion of the placenta dissected off the surface of the uterus, to show the curling arteries, utero-placental veins, deciduous and non-deciduous serotina. *Ut*, uterus; *Pl.*, placenta; *c c'*, curling artery, torn through in separating the placenta from the uterus; *v v*, utero-placental veins; *n s*, non-deciduous serotina; *d s*, deciduous serotina, thin flakes of which are shown passing between the uterine and placental surfaces. At *c'* and *v'* the non-deciduous serotina has been dissected off, and the muscular coat, with its nutrient blood-vessels, exposed. Magnified about $\frac{1}{3}$d.

Plate IV.

Figure 3. The interior of the cavity of the uterus, with the placenta *in situ*, the umbilical cord, fœtus, and epitrichium. The uterus has been opened into by a median longitudinal incision through its posterior wall. The inner surface of the placenta, displaying its lobes and the ramifi-

cations of the umbilical vessels, is towards the observer; at the left border of the placenta two of the lobes cut through in opening into the uterus are seen in section. *ch*, the non-placental area of the chorion, with branches of the umbilical vessels ramifying in it. *ch'*, strip of chorion subdividing the placenta imperfectly into a right and left lateral half. *am*, the amnion, partially dissected off the inner surface of the chorion and placenta. *V*, the so-called vagina, into which the chorion did not extend; at the lower end of the vagina is seen the opening of the urethra. *Ep*, the epitrichium, which has been drawn off the body of the fœtus, and at the same time everted. The two stocking-like prolongations next the fœtus had invested the hind limbs, the two next the uterus the fore limbs. Fragments of the epitrichium, torn through in order to remove it, are represented attached to the auricle, eyelid, lips, and nostril.

Plate V.

Figure 4. Vertical section through the entire thickness of a placental lobe, from the chorionic to the decidual surface, to show the arrangement of the convoluted intraplacental maternal blood-vessels, and their relations to the chorionic villi. *Ch*, chorion. *D s*, deciduous serotina, somewhat ragged at the free edge, where it has been torn away from the non-deciduous serotina. *V V*, stem of a villus, extending through the entire thickness of the lobe, and giving off branches, *V' V' V'*; those to the left are represented *in situ*, situated between and in contact with the convoluted maternal vessels. The chorionic fœtal vessels are coloured blue, and are shown, some in transverse, others in oblique sections; whilst the long axis of others may be seen. The capillary network was only partially injected in this preparation. The convoluted maternal blood-sinuses are coloured red. Magnified 43 diameters.

Figure 5. Section through a small portion of a placental lobe, magnified 225 diameters. The structure of the fœtal villi is displayed, with their numerous fusiform and irregularly-shaped connective-tissue corpuscles. The fœtal capillaries, coloured blue, for the most part transversely divided, lie close to the periphery of the villi. The maternal sinuses, coloured red, contain a proportion of red blood corpuscles, some of which seem to be nucleated. At *a* a slight separation in the act of making the section has taken place between a villus and the adjacent maternal blood sinus, so that the independence of the wall of the sinus can be readily recognised.

Figure 6. A similar section, with the fœtal capillary network, however, more distinct. The plane of section has so passed through the tissue of a villus as to shave off the peripheral plexus, between the meshes of which a subjacent maternal blood sinus may be seen.

Plate VI.

Figure 7. A utero-placental vein, dissected out of the substance of a placental lobe, to display the origin of its branches. Magnified 43 diameters.

Figure 8. Another specimen, showing the continuity of a branch, with a convoluted intra-placental vessel.

Figure 9. Anastomosing intra-placental maternal blood sinuses, dissected out of the substance of a placental lobe. Magnified 43 diameters.

Figure 10. A portion of the wall of a maternal blood-sinus, after the addition of acetic acid, displaying the endothelial cells. Magnified 223 diameters.

Figure 11. Non-gravid uterus of Cholœpus Hoffmanni, cut open to show the longitudinal folds of the mucous membrane, and the want of any subdivision into body and cervix. Natural size.

Figure 12. A portion of the mucous membrane of the non-gravid uterus. At the right side of the figure the surface of the membrane is shown with the ovoid nuclei of its epithelial layer, and at *a* the mouth of one of the utricular glands. More to the left this layer has been removed, and the short simple utricular glands, corkscrew-like arteries, serpentine veins, and capillary network are represented lying in the corpusculated connective tissue. Magnified 320 diameters.

V.—*On Orthogonal Isothermal Surfaces.* Part I. By Professor Tait.

[Read January 2d, 1866.—Revised and Improved, January 15th, 1872.]

The following pages contain, in a comparatively compact form, part of the substance of a voluminous paper read to the Society six years ago. Of that paper, which employed ordinary analysis alone, only a few pages had been put in type when I succeeded in overcoming a formidable difficulty which had presented itself in my quaternion treatment of the subject. I therefore withdrew the paper in order that it might have the benefit of the simplification which quaternions always give; but it is only of late that I have found time to complete part of the translation into the new language. From the circumstances under which the paper has thus been produced, i, j, k come forward with undue prominence, a thing to be regarded (in Hamilton's words) "as an inelegance and imperfection in this calculus, or rather in the state to which it has hitherto been unfolded." Immense as is the simplification already attained, it is clear that in many places still more is attainable. But I have not postponed my paper till it should receive this final polish, partly because the time I can devote to such inquiries is extremely limited, and partly because I think that several of the results obtained, and of the modes of obtaining them, are new and remarkable. Besides, a question of this order of difficulty is admirably adapted to show in what respects quaternion methods require improvement. There must be some simple mode of deducing (13) and (21) below from (7) without the explicit use of i, j, k, but I have not yet been fortunate enough to discover it.

I append to this introduction, for comparison, a few extracts from the paper in its original form.

a. Let $\left(\frac{d\xi}{dx}\right)^2+\left(\frac{d\xi}{dy}\right)^2+\left(\frac{d\xi}{dz}\right)^2$ be written $\square\xi$,

$$\frac{d\xi}{dx}\frac{d\eta}{dx}+\frac{d\xi}{dy}\frac{d\eta}{dy}+\frac{d\xi}{dz}\frac{d\eta}{dz} \quad . \quad . \quad \mathrm{D}(\xi,\eta),$$

and

$$\begin{vmatrix} \frac{d\xi}{dx}, & \frac{d\xi}{dy}, & \frac{d\xi}{dz} \\ \frac{d\eta}{dx}, & \frac{d\eta}{dy}, & \frac{d\eta}{dz} \\ \frac{d\zeta}{dx}, & \frac{d\zeta}{dy}, & \frac{d\zeta}{dz} \end{vmatrix} \quad . \quad . \quad . \quad \Delta(\xi,\eta,\zeta)$$

Then, if

$$\mathrm{D}(\xi, \eta) = \mathrm{D}(\eta, \zeta) = \mathrm{D}(\zeta, \xi) = 0\,,$$

we have

$$\frac{\frac{d\xi}{dx}}{\begin{vmatrix} \frac{d\eta}{dy}, & \frac{d\eta}{dz} \\ \frac{d\zeta}{dy}, & \frac{d\zeta}{dz} \end{vmatrix}} = \frac{\frac{d\xi}{dy}}{\begin{vmatrix} \frac{d\eta}{dz}, & \frac{d\eta}{dx} \\ \frac{d\zeta}{dz}, & \frac{d\zeta}{dx} \end{vmatrix}} = \frac{\frac{d\xi}{dz}}{\begin{vmatrix} \frac{d\eta}{dx}, & \frac{d\eta}{dy} \\ \frac{d\zeta}{dx}, & \frac{d\zeta}{dy} \end{vmatrix}},$$

or

$$\frac{\frac{d\xi}{dx}}{\frac{d\Delta}{d\left(\frac{d\xi}{dx}\right)}} = \frac{\frac{d\xi}{dy}}{\frac{d\Delta}{d\left(\frac{d\xi}{dy}\right)}} = \frac{\frac{d\xi}{dz}}{\frac{d\Delta}{d\left(\frac{d\xi}{dz}\right)}} = \frac{1}{u} \text{ suppose,}$$

with two similar sets in η and ζ.

b. Since

$$\Delta = \frac{d\xi}{dx}\frac{d\Delta}{d\left(\frac{d\xi}{dx}\right)} + \frac{d\xi}{dy}\frac{d\Delta}{d\left(\frac{d\xi}{dy}\right)} + \frac{d\xi}{dz}\frac{d\Delta}{d\left(\frac{d\xi}{dz}\right)},$$

we have at once

$$\Delta = u\Box\xi = v\Box\eta = w\Box\zeta\,.$$

Hence

$$u = \frac{\frac{d\Delta}{d\left(\frac{d\xi}{dx}\right)}}{\frac{d\xi}{dx}} = \frac{\Delta}{\Box\xi},$$

or

$$\frac{1}{\Delta}\,\frac{d\Delta}{d\left(\frac{d\xi}{dx}\right)}\,d\left(\frac{d\xi}{dx}\right) = \frac{\frac{d\xi}{dx}\,d\left(\frac{d\xi}{dx}\right)}{\Box\xi},$$

which gives by integration

$$\log.\ \Delta = \mathrm{C} + \tfrac{1}{2}\log.\ \Box\xi\,.$$

Thus, finally,

$$\Delta = \sqrt{\Box\xi\Box\eta\Box\zeta}\,.$$

c. Thus we have

$$u = \sqrt{\frac{\Box\eta\Box\zeta}{\Box\xi}},\ \&c.$$

But, identically,

$$\frac{d}{dx}\cdot\frac{d\Delta}{d\left(\frac{d\xi}{dx}\right)} + \frac{d}{dy}\cdot\frac{d\Delta}{d\left(\frac{d\xi}{dy}\right)} + \frac{d}{dz}\cdot\frac{d\Delta}{d\left(\frac{d\xi}{dz}\right)} = 0\,,$$

or

$$\frac{d}{dx}\left(u\frac{d\xi}{dx}\right) + \frac{d}{dy}\left(u\frac{d\xi}{dy}\right) + \frac{d}{dz}\left(u\frac{d\xi}{dz}\right) = 0\,,$$

or

$$u\nabla^2\xi - \mathrm{D}(u, \xi) = 0\,,$$

with similar equations in v, η and w, ζ.

d. Now if $\xi = c$ be one of a system of surfaces isothermal as well as orthogonal, we must have, by the above equation,

$$\mathrm{D}(u, \xi) = 0,$$

But the orthogonality gives

$$\mathrm{D}(\eta, \xi) = 0,$$
$$\mathrm{D}(\zeta, \xi) = 0,$$

and the elimination of ξ among these three equations gives

$$\Delta(u, \eta, \zeta) = 0,$$

i.e., by the property of functional determinants, u is a function of η and ζ alone. Thus we have

$$\sqrt{\frac{\square\eta\square\zeta}{\square\xi}} = f(\eta, \zeta),$$

a well-known relation, &c.

1. Consider the equation

$$\mathrm{T}.(\phi + f(h))^{-1}\sigma = 1,$$

or, as it may be written,

$$\mathrm{S}.\sigma(\phi + f(h))^{-1}\sigma = -1 \quad . \quad . \quad . \quad (1),$$

where ϕ is any self-conjugate linear and vector function, of which i, j, k are the principal vector directions. We assume that the roots of HAMILTON's equation

$$\mathrm{M}_\phi = 0$$

are finite and different from one another, so that cylinders, surfaces of revolution, &c., are excluded from (1).

For any assigned value of σ, (1) gives in general three values of $f(h)$ and therefore of h. Omitting for the present the consideration that each value of $f(h)$ may give more than one value of h, these values may be any assigned functions of the position of a point in space; because, when they and the function f are assigned, the squares of the constituents of σ (or, what comes to the same thing, the values of σ^2, $\mathrm{S}\sigma\phi\sigma$, $\mathrm{S}\sigma\phi^2\sigma$) can at once be found in terms of them, by a system of three *linear* equations.

In this first part I confine myself to cases in which each of these squares is positive, so as to avoid for the present the use of biquaternions.

2. For any assigned constant value of h, (1) represents in general a surface whose normal vector, ∇h, is given by

$$\mathrm{S}.\sigma\psi^{-2}\sigma f'(h)\nabla h = 2\Sigma\left(i\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\sigma\right) \quad . \quad (2),$$

ψ being written for convenience in place of $\phi + f(h)$.

Now if h_1, h_2 be the other two values of h given by (1) for a particular value of σ, the conditions of orthogonality of the surfaces h, h_1, h_2, are of the form

$$0 = \mathrm{S}.\nabla h\nabla h_1 = \Sigma.\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\sigma\,\mathrm{S}.\frac{d\sigma}{dx}\psi_1^{-1}\sigma \quad . \quad . \quad (3),$$

where

$$\psi_1 = \phi + f(h_1) .$$

3. The three equations (3) may be put in the new form

$$\mathrm{S}.\psi^{-1}\sigma\Sigma\left(\frac{d\sigma}{dx}\mathrm{S}.\frac{d\sigma}{dx}\psi_1^{-1}\sigma\right) = 0 , \text{ \&c.},$$

whence

$$\Sigma\left(\frac{d\sigma}{dx}\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\sigma\right) \quad \| \ \mathrm{V}.\psi_1^{-1}\sigma\psi_2^{-1}\sigma \quad . \quad . \quad (4).$$

4. But, by the nature of self-conjugate linear and vector functions,

$$\mathrm{S}.\psi^{-1}\sigma\psi_1^{-1}\sigma = \mathrm{S}.\sigma\psi^{-1}\psi_1^{-1}\sigma = \frac{1}{f(h_1)-f(h)}\mathrm{S}.\sigma(\psi^{-1}-\psi_1^{-1})\sigma = 0 \quad (5),$$

with two other equations of the same kind. These give (when the values of h are different) three equations of the form

$$\psi^{-1}\sigma \quad \| \ \mathrm{V}.\psi_1^{-1}\sigma\psi_2^{-1}\sigma \quad . \quad . \quad . \quad (6),$$

where, of course, we may dispense with the V.

5. By (4) and (6) we see that we have three equations of the form

$$\psi^{-1}\sigma \quad \| \ \Sigma\left(\frac{d\sigma}{dx}\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\sigma\right),$$

and these show at once that

$$\frac{d\sigma}{dx}, \quad \frac{d\sigma}{dy}, \quad \frac{d\sigma}{dz}$$

are rectangular vectors whose tensors are equal. For

$$\tau\mathrm{S}.\alpha\beta\gamma = \alpha\mathrm{S}.\beta\gamma\tau + \beta\mathrm{S}.\gamma\alpha\tau + \gamma\mathrm{S}.\alpha\beta\tau$$

is the *only* decomposition of τ parallel to α, β, γ respectively; and we have here the equation

$$\tau \| \alpha\mathrm{S}\alpha\tau + \beta\mathrm{S}\beta\tau + \gamma\mathrm{S}\gamma\tau,$$

holding good for the three non-coplanar vectors $\psi^{-1}\sigma$, $\psi_1^{-1}\sigma$, $\psi_2^{-1}\sigma$, and therefore true for all vectors. Hence we must have

$$\alpha \| \mathrm{V}\beta\gamma, \ \beta \| \mathrm{V}\gamma\alpha, \ \gamma \| \mathrm{V}\alpha\beta,$$

of which any two include the third as a necessary consequence, and in all three of which the coefficient of proportionality is evidently the same. The only exception to this is when

$$\frac{d\sigma}{dx} \| \psi^{-1}\sigma, \text{ \&c.}$$

But, in this case, by (2)

$$\nabla h \parallel i, \text{ \&c.},$$

and we have series of rectangular planes.

6. Hence there must exist a scalar function u, and a quaternion q (which may obviously be taken as a mere versor), such that

$$\frac{d\sigma}{dx} = uqiq^{-1}, \text{ \&c.},$$

or, in one expression,

$$d\sigma = uqd\rho q^{-1} \quad . \quad . \quad . \quad . \quad (7).$$

Thus it appears that, in order that (1) (with the limitations above imposed) may represent a triple series of orthogonal surfaces, σ must be such a function of ρ that, if the extremities of a set of values of ρ form the corners of an indefinitely small cube, those of the corresponding values of σ (drawn from a common origin) form the corners of another such cube; and that, therefore, the passage from ρ to σ is that from one mode of dividing space into indefinitely small cubes to another.

Whatever, therefore, may be thought of the logic of the investigation above, it is worth while to pursue the inquiry thus suggested, by developing the consequences of the equation (7) to which it has led us.

7. From the equations just written we see that if

$$\sigma = i\xi + j\eta + k\zeta, \quad . \quad . \quad . \quad . \quad (8),$$

the direction cosines of qiq^{-1} are

$$\frac{1}{u}\frac{d\xi}{dx}, \quad \frac{1}{u}\frac{d\eta}{dx}, \quad \frac{1}{u}\frac{d\zeta}{dx}.$$

From these, and other six of similar form, we see that the direction cosines of i referred to qiq^{-1}, qjq^{-1}, qkq^{-1} are

$$\frac{1}{u}\frac{d\xi}{dx}, \quad \frac{1}{u}\frac{d\xi}{dy}, \quad \frac{1}{u}\frac{d\xi}{dz},$$

and similarly for those of j and k.

Hence it follows that $\nabla\xi$, $\nabla\eta$, $\nabla\zeta$ form a set of mutually perpendicular vectors whose common tensor is u.

The same result may be obtained as follows:—

$$\begin{aligned}\nabla\xi &= -\left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right)\mathrm{S}i\sigma \\ &= -u(i\mathrm{S}.iqiq^{-1} + j\mathrm{S}.iqjq^{-1} + k\mathrm{S}.iqkq^{-1}) \\ &= -u(i\mathrm{S}.iq^{-1}iq + j\mathrm{S}.jq^{-1}iq + k\mathrm{S}.kq^{-1}iq) \\ &= uq^{-1}iq \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).\end{aligned}$$

Hence we have

$$-d\xi = u\mathrm{S}.q^{-1}iqd\rho\,,$$

of which the condition of integrability is

$$\mathrm{V}.\nabla.uq^{-1}iq = 0\,.$$

Thus u and q must be determined so as to satisfy the equation

$$\mathrm{V}.\nabla.uq^{-1}\alpha q = 0 \quad . \quad . \quad . \quad . \quad (10),$$

whatever constant vector be represented by α.

We may state our present conclusions in the following simple form. In order that (1) may represent a triple series of orthogonal surfaces, it is necessary and sufficient that the constituents of σ satisfy three equations of the form (9); *i.e.*, that, when severally equated to constants, they represent three series of surfaces which together cut space into cubes.

8. As a verification of (9) we see that it and the similar expressions for $\nabla\eta$ and $\nabla\zeta$ give

$$\begin{aligned} -d\sigma &= u(i\mathrm{S}q^{-1}iqd\rho + \quad . \quad . \quad . \quad .) \\ &= -uqd\rho q^{-1}\,, \end{aligned}$$

which is equation (7).

9. Performing the operations indicated in (10), it becomes

$$\mathrm{V}.\nabla.uq^{-1}\alpha q + u\mathrm{V}.\Sigma\Big(iq^{-1}\alpha\frac{dq}{dx} - iq^{-1}\frac{dq}{dx}q^{-1}\alpha q\Big) = 0\,,$$

or

$$\mathrm{V}.\frac{\nabla u}{u}q^{-1}\alpha q + 2\Sigma\mathrm{V}.i\mathrm{V}.q^{-1}\alpha qq^{-1}\frac{dq}{dx} = 0\,,$$

(this simplification being permitted because (§ 6) the tensor of q may be regarded as unity) or, finally,

$$\mathrm{V}.\frac{\nabla u}{u}q^{-1}\alpha q - 2q^{-1}\alpha q\Sigma\mathrm{S}.iq^{-1}\frac{dq}{dx} + 2\Sigma.(\mathrm{S}.q^{-1}\alpha qi)q^{-1}\frac{dq}{dx} = 0\,;$$

which may be written

$$\mathrm{V}.\frac{\nabla u}{u}q^{-1}\alpha q + 2q^{-1}\alpha q\mathrm{S}.\nabla q^{-1}q - 2\Sigma.(\mathrm{S}.q^{-1}\alpha qi)\frac{dq^{-1}}{dx}q = 0\,,$$

or

$$\mathrm{V}.\frac{\nabla u}{u}q^{-1}\alpha q + 2q^{-1}\alpha q\mathrm{S}.\nabla q^{-1}q - 2\mathrm{S}.(q^{-1}\alpha q\nabla)q^{-1}.q = 0\,.$$

Here α has the values i, j, k, so that if we write

$$q^{-1}\alpha q = i'\text{, \&c.,}$$

we have three equations of the type

$$-\mathrm{V}.i\frac{\nabla v}{u} + 2i'\mathrm{S}.\nabla q^{-1}q - 2\mathrm{S}(i'\nabla)q^{-1}.q = 0 \quad . \quad (11).$$

From these we have

$$-\Sigma.i\nabla.i\frac{\nabla u}{u} - 6S.\nabla q^{-1}q + 2\nabla q^{-1}.q = 0,$$

or

$$\frac{\nabla u}{u} - 3S.\nabla q^{-1}q + \nabla q^{-1}.q = 0.$$

Hence

$$S.\nabla q^{-1}q = 0$$

and

$$\frac{\nabla u}{u} = -V.\nabla q^{-1}q = -\nabla q^{-1}.q = -\frac{\nabla q^{-1}}{q^{-1}} \quad . \quad (12),$$

or, finally,

$$\nabla.uq^{-1} = 0 \quad . \quad . \quad . \quad . \quad . \quad (13).$$

10. But this is not the only relation between u and q. For by (12) we may write (11) in the form

$$-V.(\nabla q^{-1}qq^{-1}\alpha q)q^{-1} - 2S.(q^{-1}\alpha q\nabla)q^{-1} = 0 \quad . \quad (14).$$

It is obvious that, by adding the three equations of this form, each multiplied by a proper scalar, we may derive from them three equivalent ones of the form

$$V(\nabla q^{-1}qi)\,q^{-1} + 2S(i\nabla)q^{-1} = 0 \quad . \quad . \quad . \quad (15).$$

This may be written by the help of (12) in the form

$$2\frac{dq^{-1}}{dx}q = -V.i\nabla q^{-1}q = V.i\frac{\nabla u}{u} = V.i\nabla \log. u \quad . \quad (16),$$

and we thus see that the constancy of the tensor of q is recognised.

Differentiating again after multiplying into q^{-1}, we have

$$2\frac{d^2q^{-1}}{dx^2} = V(i\nabla \log. u)\frac{dq^{-1}}{dx} + V.i\frac{d}{dx}\nabla \log. u.q^{-1}$$

$$= \tfrac{1}{2}(V.i\nabla \log. u)^2q^{-1} + V.i\frac{d}{dx}\nabla \log. u.q^{-1}.$$

Adding the three equations of this form, we have

$$-2\nabla^2q^{-1} = (\nabla \log. u)^2\,q^{-1} \quad . \quad . \quad . \quad (17),$$

for obviously $\nabla^2 \log. u$ is a scalar.

But we have also

$$\nabla.uq^{-1} = 0 \quad . \quad . \quad . \quad . \quad (18),$$

which gives

$$\nabla \log. u.q^{-1} + \nabla q^{-1} = 0$$

and

$$\nabla^2 \log. u.q^{-1} + \Sigma.i\nabla \log. u\frac{dq^{-1}}{dx} + \nabla^2q^{-1} = 0,$$

or

$$\nabla^2 \log. u.q^{-1} + \tfrac{1}{2}\Sigma.i\nabla \log. u \, \nabla.i\nabla \log. u.q^{-1} + \nabla^2 q^{-1} = 0 ,$$

which may be simplified into

$$\nabla^2 \log. u.q^{-1} + (\nabla \log. u)^2 q^{-1} + \nabla^2 q^{-1} = 0 \quad . \quad (19).$$

Together, (18) and (19) give

$$\left.\begin{aligned} \nabla^2 \log. u.q^{-1} - \nabla^2 q^{-1} &= 0 \\ 2\nabla^2 \log. u + (\nabla \log. u)^2 &= 0 \end{aligned}\right\} \quad . \quad . \quad (20).$$

The latter of these equations may be written

$$\nabla(u^{\frac{1}{2}} \nabla \log. u) = 0 = \nabla\left(\frac{\nabla u}{u^{\frac{1}{2}}}\right),$$

or finally

$$\nabla^2(u^{\frac{1}{2}}) = 0 \quad . \quad . \quad . \quad . \quad . \quad (21).$$

11. Hence u is the square of the potential of some distribution of matter, none of which is contained in the space occupied by the surfaces.

Hence the only strict solution, *i.e.*, the only one which holds at every point of infinite space, is

$$u = \text{constant},$$

and, of course,

$$q = \text{constant}.$$

From this we have

$$\nabla\xi = uq^{-1}iq = u(ia_1 + jb_1 + kc_1)$$
$$\xi = c_1 + u(a_1x + b_1y + c_1z) .$$

Thus the constituents of σ, separately equated to constants, give the equations of three series of mutually perpendicular planes cutting space into cubes, for u is the same for all. When we turn the axes so as to be perpendicular to these planes respectively, and adopt a suitable origin, we have

$$\xi = ux , \ \eta = uy , \ \zeta = uz ,$$

whence

$$\sigma = u\rho ,$$

and thus equation (1) gives in this case the confocal surfaces of the second order.

12. We omit for the present, in consequence of the remark at the beginning of last section, other obvious solutions of (21), such as

$$u^{\frac{1}{2}} = \mathrm{S}\alpha\rho , \text{ or } u^{\frac{1}{2}} = \epsilon^{\mathrm{S}(\alpha + \sqrt{-1}\beta)\rho}, \text{ \&c.}$$

But if we admit that at *one* point of space there may be a particle of matter of mass m, we have, of course,

$$u = \frac{m^2}{T\rho^2},$$

so that

$$\nabla q^{-1} q = -\frac{2\rho}{T\rho^2},$$

which gives as a particular integral

$$q^{-1} = U\rho.$$

Hence, in this case,

$$d\sigma = uqd\rho q^{-1} = -\frac{m^2}{T\rho^3}(2U\rho\, SU\rho d\rho + d\rho)$$

$$= +\frac{m^2}{T\rho^4}(2\rho S\rho d\rho + d\rho\rho^2) = -m^2\left(\frac{2\rho dT\rho}{T\rho^3} - \frac{d\rho}{T\rho^2}\right),$$

or

$$\sigma = m^2\rho^{-1}.$$

The corresponding surfaces are the electric images of the confocal quadrics, taken from the common centre, and include Fresnel's surface of elasticity.

13. It follows, from what we have just proved, that the only orthogonal surfaces which divide all space into indefinitely small cubes are planes and their electric images, or images of images, &c. These are all, therefore, included in a triple series of spheres having a common point, and their centres in three rectangular axes passing through that point.

In fact, if in (7) we put for σ

$$\sigma' = (\sigma + \tau)^{-1},$$

we have

$$\frac{d\sigma'}{dx} = -\sigma'^2\frac{d\sigma}{dx}\sigma', \text{ \&c.},$$

whence

$$S\frac{d\sigma'}{dx}\frac{d\sigma'}{dy} = \sigma'^4 S\frac{d\sigma}{dx}\frac{d\sigma}{dy} = 0, \text{ \&c.},$$

and

$$T\frac{d\sigma'}{dx} = T\frac{d\sigma'}{dy} = T\frac{d\sigma'}{dz} = \frac{u}{T\sigma'^2}.$$

Hence the electric image of any orthogonal system is also orthogonal; and, if the system cut space into cubes, so does the image.

14. We are now prepared to introduce the conditions that the surfaces (1) shall each be isothermal. If h, h_1, h_2 represent their temperatures, these conditions are simply

$$\nabla^2 h = 0, \quad \nabla^2 h_1 = 0, \quad \nabla^2 h_2 = 0 \quad . \quad . \quad . \quad (22).$$

To express these in another form we must now differentiate equations (2).

15. By (2) we have

$$\mathrm{S}.\sigma\psi^{-2}\sigma f'(h)\frac{dh}{dx} = 2\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\sigma\,.$$

This gives

$$2\mathrm{S}.\frac{d\sigma}{dx}\psi^{-2}\sigma f'(h)\frac{dh}{dx} - 2\mathrm{S}.\sigma\psi^{-3}\sigma\left(f'(h)\right)^2\left(\frac{dh}{dx}\right)^2 + \mathrm{S}.\sigma\psi^{-2}\sigma f''(h)\left(\frac{dh}{dx}\right)^2$$

$$+ \mathrm{S}.\sigma\psi^{-2}\sigma f'(h)\frac{d^2h}{dx^2} = 2\mathrm{S}.\frac{d^2\sigma}{dx^2}\psi^{-1}\sigma + 2\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\frac{d\sigma}{dx} - 2\mathrm{S}.\frac{d\sigma}{dx}\psi^{-2}\sigma f'(h)\frac{dh}{dx}.$$

Eliminating $\frac{dh}{dx}$ from these equations, we have

$$\frac{4\mathrm{S}.\frac{d\sigma}{dx}\psi^{-2}\sigma\,\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\sigma}{\mathrm{S}.\sigma\psi^{-2}\sigma} - 8\mathrm{S}.\sigma\psi^{-3}\sigma\frac{\mathrm{S}^2\frac{d\sigma}{dx}\psi^{-1}\sigma}{\mathrm{S}^2\sigma\psi^{-2}\sigma} + 4\mathrm{S}.\sigma\psi^{-2}\sigma\frac{f''(h)}{(f'(h))^2}\frac{\mathrm{S}^2\frac{d\sigma}{dx}\psi^{-1}\sigma}{\mathrm{S}^2\sigma\psi^{-2}\sigma}$$

$$+ \mathrm{S}.\sigma\psi^{-2}\sigma f'(h)\frac{d^2h}{dx^2} = 2\mathrm{S}.\frac{d^2\sigma}{dx^2}\psi^{-1}\sigma + 2\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\frac{d\sigma}{dx} - 4\mathrm{S}.\frac{d\sigma}{dx}\psi^{-2}\sigma\frac{\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\sigma}{\mathrm{S}.\sigma\psi^{-2}\sigma}.$$

Now, whatever vector ϖ may be, we have by §§ (5), (6)

$$-u^2\varpi = \frac{d\sigma}{dx}\mathrm{S}.\frac{d\sigma}{dx}\varpi + \frac{d\sigma}{dy}\mathrm{S}.\frac{d\sigma}{dy}\varpi + \frac{d\sigma}{dz}\mathrm{S}.\frac{d\sigma}{dz}\varpi\,,$$

so that, if ω be any other vector,

$$-u^2\mathrm{S}.\omega\varpi = \Sigma\mathrm{S}.\frac{d\sigma}{dx}\omega\,\mathrm{S}.\frac{d\sigma}{dx}\varpi\,.$$

Adding, then, the three equations, of which that containing $\frac{d^2h}{dx^2}$ is given above, we find

$$-4u^2\frac{\mathrm{S}.\psi^{-2}\sigma\psi^{-1}\sigma}{\mathrm{S}.\sigma\psi^{-2}\sigma} + 8u^2\mathrm{S}.\sigma\psi^{-3}\sigma\frac{\mathrm{S}.\psi^{-1}\sigma\psi^{-1}\sigma}{\mathrm{S}^2.\sigma\psi^{-2}\sigma} - 4u^2\frac{f''(h)}{(f'(h))^2}\frac{\mathrm{S}.\psi^{-1}\sigma\psi^{-1}\sigma}{\mathrm{S}.\sigma\psi^{-2}\sigma}$$

$$= -2\mathrm{S}.\nabla^2\sigma\psi^{-1}\sigma + 2\Sigma\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\frac{d\sigma}{dx} + 4u^2\frac{\mathrm{S}.\psi^{-2}\sigma\psi^{-1}\sigma}{\mathrm{S}.\sigma\psi^{-2}\sigma},$$

where the term in $\nabla^2 h$ of course vanishes by (22). This is seen at a glance to be equivalent to

$$-4u^2\frac{f''(h)}{(f'(h))^2} = -2\mathrm{S}.\nabla^2\sigma\psi^{-1}\sigma + 2\Sigma\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\frac{d\sigma}{dx}.$$

The last term here is seen at once, by HAMILTON's beautiful theory of linear and vector functions, to be equivalent to

$$2u^2\Sigma\mathrm{S}.i\psi^{-1}i = -2u^2\left(\frac{1}{\mathrm{A}+f(h)} + \frac{1}{\mathrm{B}+f(h)} + \frac{1}{\mathrm{C}+f(h)}\right). \qquad (23),$$

if A, B, C be the constants of ϕ. Calling the expression in brackets for the present H, we have

$$-\mathrm{H} + 2\frac{f''(h)}{(f'(h))^2} = \frac{1}{u^2}\mathrm{S}.\nabla^2\sigma\psi^{-1}\sigma \quad . \quad . \quad (24).$$

The left hand member, if multiplied by $f'(h)\,dh$, is the differential of a function of h only. If, as in § 11 we have

$$\sigma = u\rho\,,\ u = \text{constant}\,,$$

the right hand side vanishes, and integration gives

$$\mathrm{C}'h = \int \frac{f'(h)dh}{\sqrt{(\mathrm{A}+f(h))\,(\mathrm{B}+f(h))\,(\mathrm{C}+f(h))}} = \int \frac{df}{\sqrt{\mathrm{M}_f}}.$$

If σ have the value given in § 12, equation (24) is obviously not satisfied. Thus confocal quadrics are the only isothermal orthogonal surfaces included in equation (1) with our present limitations.

16. It is interesting in itself, and will be useful for the second part of this paper, to eliminate σ from (24) by the help of our previous equations. For this purpose we may write (2) in the form

$$\begin{aligned} -\mathrm{T}^2\psi^{-1}\sigma f'(h)\nabla h &= 2\Sigma\left(i\mathrm{S}.\frac{d\sigma}{dx}\psi^{-1}\sigma\right) \\ &= 2u\Sigma(i\mathrm{S}.iq^{-1}\psi^{-1}\sigma q) \\ &= -2uq^{-1}\psi^{-1}\sigma q \quad . \quad . \quad (25), \end{aligned}$$

the tensor of which is

$$\mathrm{T}\psi^{-1}\sigma f'(h)\mathrm{T}\nabla h = 2u\,.$$

But it is shown in (29) below that

$$\nabla u = q^{-1}\nabla^2\sigma q\,,$$

so that (25) gives

$$\mathrm{T}^2\psi^{-1}\sigma f'(h)\mathrm{S}.\nabla u\nabla h = 2u\mathrm{S}.\nabla^2\sigma\psi^{-1}\sigma\,,$$

or, by (24),

$$= 2u^3\left(-\mathrm{H} + 2\frac{f''(h)}{(f'(h))^2}\right). \quad (26).$$

The three equations of this form give

$$\Sigma.\mathrm{T}^2\psi^{-1}\sigma f'(h)\nabla h\mathrm{S}.\nabla h\nabla u = 2u^3\Sigma.\nabla h\left(-\mathrm{H} + 2\frac{f''(h)}{(f'(h))^2}\right),$$

or, by (25) and (3),

$$-4u^2\nabla u = 2u^3\Sigma.f'(h)\nabla h\left(-\mathrm{H} + 2\frac{f''(h)}{(f'(h))^2}\right).$$

Operating by $\mathrm{S}.d\rho$, this gives

$$2\frac{du}{u} = -\Sigma f'(h)\,dh\left(-\mathrm{H} + 2\frac{f''(h)}{(f'(h))^2}\right),$$

of which the integral, by (23), is

$$\mathrm{C}' - 2\log. u = \Sigma\left[2\log. f'(h) - \log.(\mathrm{A}+f(h))(\mathrm{B}+f(h))(\mathrm{C}+f(h))\right],$$

or, if we write,

$$\frac{f'(h)}{\sqrt{(\mathrm{A}+f(h))(\mathrm{B}+f(h))(\mathrm{C}+f(h))}} = \mathrm{F}(h) \quad . \quad . \quad (27),$$

then

$$\frac{\mathrm{C}'}{u} = \mathrm{F}(h)\mathrm{F}(h_1)\mathrm{F}(h_2) \quad . \quad . \quad . \quad . \quad (28).$$

17. The following is the first quaternion method that occurred to me. I give it here, though it is considerably more prolix than the preceding, because it exhibits, incidentally, many curious properties of the system σ, u, q above defined.

Starting again with equation (7), we see that it gives

$$-\nabla^2\sigma = q\nabla uq^{-1} - 2uq\Sigma\left(\mathrm{V}.i\mathrm{V}q^{-1}\frac{dq}{dx}\right)q^{-1},$$

or,

$$= -q\nabla uq^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29),$$

to which value, as we shall see immediately, it may be reduced.

18. From (7) we obtain at once

$$\frac{1}{u}\frac{d^2\sigma}{dxdy} = \frac{dq}{dy}iq^{-1} - qiq^{-1}\frac{dq}{dy}q^{-1} + \frac{1}{u^2}\frac{du}{dy}\frac{d\sigma}{dx},$$

$$= \frac{dq}{dy}iq^{-1} - qiq^{-1}\frac{dq}{dy}q^{-1} + \frac{1}{u}qiq^{-1}\frac{du}{dy},$$

$$\frac{1}{u}\frac{d^2\sigma}{dydx} = \frac{dq}{dx}jq^{-1} - qjq^{-1}\frac{dq}{dx}q^{-1} + \frac{1}{u}qjq^{-1}\frac{du}{du}.$$

Comparing the last two values, we have

$$\frac{1}{u}q^{-1}\frac{d^2\sigma}{dxdy}q = -2\mathrm{V}.i\mathrm{V}q^{-1}\frac{dq}{dy} + \frac{i}{u}\frac{du}{dy} = -2\mathrm{V}.j\mathrm{V}q^{-1}\frac{dq}{dx} + \frac{j}{u}\frac{du}{dx} \quad (30).$$

Operating by $\mathrm{S}.k$, we have

$$-\mathrm{S}.jq^{-1}\frac{dq}{dy} = \mathrm{S}.iq^{-1}\frac{dq}{dx}.$$

From this, and other equations similar to it, it is obvious that

$$\mathrm{S}.iq^{-1}\frac{dq}{dx} = \mathrm{S}.jq^{-1}\frac{dq}{dy} = \mathrm{S}.kq^{-1}\frac{dq}{dz} = 0 \quad . \quad . \quad (31).$$

For we should find, as their common value, the expression of their *sum*

$$\mathrm{S}.iq^{-1}\frac{dq}{dx}+\mathrm{S}.jq^{-1}\frac{dq}{dy}+\mathrm{S}.kq^{-1}\frac{dq}{dz}.$$

19. Also, from (30) and the other similar equations, we have the following series of values—

$$\left.\begin{aligned}
\frac{1}{u}\frac{du}{dy}&=2\mathrm{S}.kq^{-1}\frac{dq}{dx}\\
\frac{1}{u}\frac{du}{dz}&=2\mathrm{S}.iq^{-1}\frac{dq}{dy}\\
\frac{1}{u}\frac{du}{dx}&=2\mathrm{S}.jq^{-1}\frac{dq}{dz}\\
-\frac{1}{u}\frac{du}{dx}&=2\mathrm{S}.kq^{-1}\frac{dq}{dy}\\
-\frac{1}{u}\frac{du}{dy}&=2\mathrm{S}.iq^{-1}\frac{dq}{dz}\\
-\frac{1}{u}\frac{du}{dz}&=2\mathrm{S}.jq^{-1}\frac{dq}{dx}
\end{aligned}\right\}\quad\ldots\quad(32).$$

These give three equations of the form

$$\mathrm{V}.q^{-1}\frac{dq}{dx}=\frac{1}{2u}\left(j\frac{du}{dz}-k\frac{du}{dy}\right),$$

which enable us to make the transformation assumed in § 17 above, and which may be all summed up in the following—in which the omission of the V is due to the remark in § 6 that the tensor of q may be assumed constant—

$$2\mathrm{V}.q^{-1}dq=2q^{-1}dq=-\mathrm{V}.(d\rho\nabla)\log.u\qquad(33).$$

This is the equation determining the quaternion which gives the position of a rigid body in terms of the vector-axis of instantaneous rotation. (*On the Rotation of a Rigid Body about a Fixed Point.* Trans. R.S.E. 1868–69.)

20. But by § 6, we have

$$\left.\begin{aligned}
\left(\frac{d\sigma}{dx}\right)^2=\left(\frac{d\sigma}{dy}\right)^2=\left(\frac{d\sigma}{dz}\right)^2&=-u^2,\\
\mathrm{S}\frac{d\sigma}{dx}\frac{d\sigma}{dy}=\mathrm{S}\frac{d\sigma}{dy}\frac{d\sigma}{dz}=\mathrm{S}\frac{d\sigma}{dz}\frac{d\sigma}{dx}&=0
\end{aligned}\right\}\quad\ldots\quad(34).$$

From these

$$\mathrm{S}\frac{d\sigma}{dx}\frac{d^2\sigma}{dx^2}=-u\frac{du}{dx},$$

$$\mathrm{S}\frac{d\sigma}{dy}\frac{d^2\sigma}{dx^2}=-\mathrm{S}\frac{d\sigma}{dx}\frac{d^2\sigma}{dxdy}=u\frac{du}{dy},$$

$$\mathrm{S}\frac{d\sigma}{dz}\frac{d^2\sigma}{dx^2} = -\mathrm{S}\frac{d\sigma}{dx}\frac{d^2\sigma}{dxdz} = u\frac{du}{dz},$$

which give

$$-u^2\frac{d^2\sigma}{dx^2} = u\left(-\frac{du}{dx}\frac{d\sigma}{dx} + \frac{du}{dy}\frac{d\sigma}{dy} + \frac{du}{dz}\frac{d\sigma}{dz}\right)$$

$$= -2u\frac{du}{dx}\frac{d\sigma}{dx} + u\left(\frac{du}{dx}\frac{d\sigma}{dx} + \frac{du}{dy}\frac{d\sigma}{dy} + \frac{du}{dz}\frac{d\sigma}{dz}\right).$$

From the three equations of this form we obtain, first,

$$u^2\nabla^2\sigma = u\left(\frac{du}{dx}\frac{d\sigma}{dx} + \frac{du}{dy}\frac{d\sigma}{dy} + \frac{du}{dz}\frac{d\sigma}{dz}\right). \quad . \quad . \quad (35);$$

and, secondly, three equations of the form

$$\frac{d}{dx}\left(\frac{1}{u^2}\frac{d\sigma}{dx}\right) = -\frac{1}{u^3}\left(\frac{du}{dx}\frac{d\sigma}{dx} + \frac{du}{dy}\frac{d\sigma}{dy} + \frac{du}{dz}\frac{d\sigma}{dz}\right).$$

These are summed up in

$$\frac{d}{dx}\left(\frac{1}{u^2}\frac{d\sigma}{dx}\right) = \frac{d}{dy}\left(\frac{1}{u^2}\frac{d\sigma}{dy}\right) = \frac{d}{dz}\left(\frac{1}{u^2}\frac{d\sigma}{dz}\right) = -\frac{1}{u^2}\nabla^2\sigma \quad . \quad (36),$$

which express some of the conditions of *orthogonality* of the three series of surfaces given by equation (1).

21. To obtain the others, remark that by (30) we have

$$\frac{1}{u}q^{-1}\frac{d^2\sigma}{dxdy}q = -2\mathrm{V}.i\nabla q^{-1}\frac{dq}{dy} + \frac{i}{u}\frac{du}{dy} = -2\mathrm{V}.j\nabla q^{-1}\frac{dq}{dx} + \frac{j}{u}\frac{du}{dx},$$

or

$$\frac{1}{u}q^{-1}\frac{d^2\sigma}{dxdy}q = \mathrm{V}.i\nabla j\nabla.\log.u + \frac{i}{u}\frac{du}{dy} = \mathrm{V}.j\nabla i\nabla.\log.u + \frac{j}{u}\frac{du}{dx},$$

and that each of the two latter expressions may be written

$$\frac{i}{u}\frac{du}{dy} + \frac{j}{u}\frac{du}{dx}.$$

Hence

$$\frac{d^2\sigma}{dxdy} = \frac{1}{u}\left(\frac{d\sigma}{dx}\frac{du}{dy} + \frac{d\sigma}{dy}\frac{du}{dx}\right) \quad . \quad . \quad . \quad (37),$$

and there are, of course, other two vector equations of a similar form.

From these we have nine equations of the form

$$\frac{d^2\xi}{dxdy} = \frac{1}{u}\left(\frac{d\xi}{dx}\frac{du}{dy} + \frac{d\xi}{dy}\frac{du}{dx}\right) \quad . \quad . \quad . \quad (38).$$

Now

$$\frac{d}{dy}\left(\frac{1}{u^2}\frac{d\xi}{dx}\right) = \left(\frac{1}{u^2}\frac{d^2\xi}{dxdy} - \frac{2}{u^3}\frac{du}{dy}\frac{d\xi}{dx}\right)$$

$$= \frac{1}{u^3}\left(\frac{d\xi}{dy}\frac{du}{dx} - \frac{d\xi}{dx}\frac{du}{dy}\right).$$

Symmetry shows from this that

$$\frac{d}{dy}\left(\frac{1}{u^2}\frac{d\xi}{dx}\right) = -\frac{d}{dx}\left(\frac{1}{u^2}\frac{d\xi}{dy}\right) \quad . \quad . \quad . \quad (39),$$

which is one of another set of nine equations, three each for ξ, η, ζ.

22. Now, by (36), it is obvious that we may write, ϖ_1 being a new variable,

$$\frac{1}{u^2}\frac{d\xi}{dx} = \frac{d^2\varpi_1}{dydz}, \quad \frac{1}{u^2}\frac{d\xi}{dy} = \frac{d^2\varpi_1}{dzdx}, \quad \frac{1}{u^2}\frac{d\xi}{dz} = \frac{d^2\varpi_1}{dxdy} \quad . \quad . \quad (40),$$

and thus (39) becomes

$$\frac{d^3\varpi_1}{dy^2dz} = -\frac{d^3\varpi_1}{dx^2dz} \quad . \quad . \quad . \quad . \quad (41),$$

with two others in ϖ_1 and three each in ϖ_2, ϖ_3.

Putting

$$\omega_1 = \frac{d^3\varpi_1}{dxdydz},$$

these give by differentiation

$$\frac{d^2\omega_1}{dy^2} = -\frac{d^2\omega_1}{dx^2}$$

$$\frac{d^2\omega_1}{dz^2} = -\frac{d^2\omega_1}{dy^2}$$

$$\frac{d^2\omega_1}{dx^2} = -\frac{d^2\omega_1}{dz^2},$$

so that all three quantities vanish. Hence we have

$$\omega_1 = \frac{d^3\varpi_1}{dxdydz} = 2hxyz + 2lyz + 2mzx + 2nxy + 2ax + 2by + 2cz + e,$$

where h, l, m, n, a, b, c, e are absolutely constant. From this, and (40), we have

$$\left.\begin{aligned}
\frac{1}{u^2}\frac{d\xi}{dx} &= hx^2yz + 2lxyz + mzx^2 + nx^2y + ax^2 + 2bxy + 2czx + ex + f_1(y,z),\\
\frac{1}{u^2}\frac{d\xi}{dy} &= hxy^2z + ly^2z + 2mxyz + nxy^2 + 2axy + by^2 + 2cyz + ey + f_2(z,x),\\
\frac{1}{u^2}\frac{d\xi}{dz} &= hxyz^2 + lyz^2 + mz^2x + 2nxyz + 2azx + 2byz + cz^2 + ez + f_3(x,y),
\end{aligned}\right\} \quad (42).$$

Applying (26) to these, we obtain

$$\left.\begin{aligned} hx^2z + 2lxz + nx^2 + 2bx + \frac{df_1}{dy} &= -hy^2z - 2myz - ny^2 - 2ay - \frac{df_2}{dx} \\ hxy^2 + 2mxy + ly^2 + 2cy + \frac{df_2}{dz} &= -hxz^2 - lz^2 - 2nzx - 2bz - \frac{df_3}{dy} \\ hyz^2 + 2nyz + mz^2 + 2az + \frac{df_3}{dx} &= -hx^2y - 2lxy - mx^2 - 2cx - \frac{df_1}{dz} \end{aligned}\right\} \quad (43).$$

The elimination of f_2 from the two first, by differentiation, gives

$$h(x^2 - y^2) + 2lx - my + \frac{d^2f_1}{dydz} = \frac{d^2f_3}{dxdy} + h(z^2 - y^2) - 2my + 2nz\,,$$

and the third gives

$$hz^2 + 2nz + \frac{d^2f_3}{dxdy} = -\frac{d^2f_1}{dydz} - hx^2 - 2lx\,,$$

so that we have

$$hx^2 + 2lx + \frac{d^2f_1}{dydz} = hy^2 + 2my + \frac{d^2f_2}{dzdx} = hz^2 + 2nz + \frac{d^2f_3}{dxdy} = 0\,. \quad (44),$$

which proves that h, l, m, n are separately zero, and that each of the f's is the sum of two separate functions, each containing one of the constituent variables only, *i.e.*,

$$\left.\begin{aligned} f_1(y, z) &= \mathrm{Y}_1 + \mathrm{Z}_1 \\ f_2(z, x) &= \mathrm{Z}_2 + \mathrm{X}_2 \\ f_3(x, y) &= \mathrm{X}_3 + \mathrm{Y}_3 \end{aligned}\right\} \quad . \quad . \quad . \quad (45).$$

But by (43) and (45), we have

$$2bx + \frac{d\mathrm{Y}_1}{dy} = -2ay - \frac{d\mathrm{X}_2}{dx}\,,\ \&c.$$

whence

$$\frac{d\mathrm{Y}_1}{dy} = -p''' - 2ay$$

$$\frac{d\mathrm{X}_2}{dx} = p''' - 2bx\,,\ \&c.,$$

giving

$$\mathrm{Y}_1 = -ay^2 + \mathrm{C} - p'''y$$

$$\mathrm{X}_2 = -bx^2 + \mathrm{C}' + p'''x\,,\ \&c.,$$

so that, finally,

$$\left.\begin{aligned} \frac{1}{u^2}\frac{d\xi}{dx} &= a(x^2 - y^2 - z^2) + 2bxy + 2czx + ex + g_1 - p'''y + p''z \\ \frac{1}{u^2}\frac{d\xi}{dy} &= 2axy + b(y^2 - x^2 - z^2) + 2cyz + ey + g_2 - p'z + p'''x \\ \frac{1}{u^2}\frac{d\xi}{dz} &= 2azx + 2byz + c(z^2 - x^2 - y^2) + ez + g_3 - p''x + p'y \end{aligned}\right\} \quad (46).$$

If, in these, we write

$$ia + jb + kc = \gamma$$
$$ig_1 + jg_2 + kg_3 = \gamma_1$$
$$ip' + jp'' + kp''' = \gamma_2,$$

we obtain, by multiplication by i, j, k respectively and addition,

$$\frac{1}{u^3}\nabla\xi = e\rho - \rho\gamma\rho + \gamma_1 + \mathrm{V}\gamma_2\rho \quad . \quad . \quad . \quad . \quad . \quad (46'),$$

which is equivalent to the three equations (46), and may be put in the form

$$\frac{1}{u^3}\nabla\xi = (e - 2\mathrm{S}\gamma\rho)\rho + \gamma\rho^2 + \gamma_1 + \mathrm{V}\gamma_2\rho, \quad . \quad . \quad (46'').$$

23. It was shown above, § (7), that

$$\nabla\xi, \nabla\eta, \nabla\zeta$$

form a rectangular system of vectors whose common tensor is u. Hence, by (46'') we have three equations of the form

$$-\frac{1}{u^2} = (e - 2\mathrm{S}\gamma\rho)^2\rho^2 + \gamma^2\rho^4 + \gamma_1^2 + \mathrm{S}^2\gamma_2\rho - \rho^2\gamma_2^2$$
$$+ 2\rho^2\mathrm{S}\gamma\rho(e - 2\mathrm{S}\gamma\rho) + 2\mathrm{S}\gamma_1\rho(e - 2\mathrm{S}\gamma\rho) + 2\rho^2\mathrm{S}\gamma\gamma_1 + 2\rho^2\mathrm{S}.\gamma\gamma_2\rho + 2\mathrm{S}.\gamma_1\gamma_2\rho,$$

expressing the equality of the tensors; and three others of the form

$$0 = (e - 2\mathrm{S}\gamma\rho)(e' - 2\mathrm{S}\gamma'\rho)\rho^2 + \rho^2(e' - 2\mathrm{S}\gamma'\rho)\mathrm{S}\gamma\rho + (e' - 2\mathrm{S}\gamma'\rho)\mathrm{S}\gamma_1\rho$$
$$+ (e - 2\mathrm{S}\gamma\rho)\rho^2\mathrm{S}\gamma'\rho + \rho^4\mathrm{S}\gamma\gamma' + \rho^2\mathrm{S}\gamma'\gamma_1 + \rho^2\mathrm{S}.\gamma'\gamma_2\rho$$
$$+ (e - 2\mathrm{S}\gamma\rho)\mathrm{S}\gamma_1'\rho + \rho^2\mathrm{S}\gamma_1'\gamma + \mathrm{S}\gamma_1'\gamma_1 + \mathrm{S}.\gamma_1'\gamma_2\rho$$
$$+ \rho^2\mathrm{S}.\gamma\gamma_2'\rho + \mathrm{S}.\gamma_1\gamma_2'\rho + \mathrm{S}\gamma_2'\rho\mathrm{S}\gamma_2\rho - \mathrm{S}\gamma_2'\gamma_2\rho^2.$$

Here the constants in $\nabla\eta$, $\nabla\zeta$, are expressed by the application of one, and two, dashes respectively to those of $\nabla\xi$.

In the first set of three, the terms in the various powers of $\mathrm{T}\rho$ must be equal. This gives the following sets

$$\gamma^2 = \gamma'^2 = \gamma''^2$$
$$\mathrm{S}(\mathrm{V}\gamma\gamma_2 - e\gamma)\rho = \mathrm{S}(\mathrm{V}\gamma'\gamma_2' - e'\gamma')\rho = \quad . \quad . \quad . \quad .$$
$$(e^2 - \gamma_2^2 + 2\mathrm{S}\gamma\gamma_1)\rho^2 + \mathrm{S}^2\gamma_2\rho - 4\mathrm{S}\gamma\rho\mathrm{S}\gamma_1\rho = \quad . \quad . \quad . \quad . \quad . \quad .$$
$$\mathrm{S}(\mathrm{V}\gamma_1\gamma_2 + e\gamma_1)\rho = \quad . \quad . \quad . \quad . \quad . \quad . \quad = \quad . \quad . \quad .$$
$$\gamma_1^2 = \gamma_1'^2 = \gamma_1''^2.$$

In the other set of three, we have by the same process

$$0 = S\gamma\gamma' = S\gamma'\gamma'' = S\gamma''\gamma$$

$$0 = S\rho\{V\gamma'\gamma_2 + V\gamma\gamma_2' - e'\gamma - e\gamma'\} = \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

$$0 = \rho^2\{ee' + S\gamma'\gamma_1 + S\gamma_1'\gamma - S\gamma_2\gamma_2'\} - 2S\gamma'\rho S\gamma_1\rho - 2S\gamma\rho S\gamma_1'\rho + S\gamma_2\rho S\gamma_2'\rho = \quad . \quad . \quad . \quad .$$

$$0 = S\rho\{V\gamma_1'\gamma_2 + V\gamma_1\gamma_2' + e'\gamma_1 + e\gamma_1'\} = \quad . \quad . \quad . \quad . \quad . \quad . \quad . = . \quad . \quad .$$

$$0 = S\gamma_1\gamma_1' = S\gamma_1'\gamma_1'' = S\gamma_1''\gamma_1 .$$

We might easily have obtained this last set of equations from that which preceded, by a species of differentiation, ρ being constant, and $d\gamma = \gamma'$, $d\gamma' = \gamma''$, &c.

24. From these we conclude that, if they exist at all, γ, γ', γ'', and γ_1, γ_1', γ_1'', form rectangular systems with equal tensors. In terms of them we obtain

$$\begin{aligned} -\gamma_2 &= \kappa\gamma_1 - e''\gamma_1' + e'\gamma_1'' = c\gamma + e''\gamma' - e'\gamma'' \\ -\gamma_2' &= e''\gamma_1 + \kappa\gamma_1' - e\gamma_1' = -e''\gamma + c\gamma' + e\gamma'' \\ -\gamma_2'' &= -e'\gamma_1 + e\gamma_1' + \kappa\gamma_1'' = e'\gamma - e\gamma' + c\gamma'' , \end{aligned}$$

where κ and c are scalar constants to be determined.

Expressing, from these, γ_1 in terms of γ, γ', γ'', we have

$$\gamma_1 = -\gamma + \frac{c+\kappa}{D}\{(\kappa^2 + e^2)\gamma + (ee' + \kappa e'')\gamma' + (ee'' - \kappa e')\gamma''\} ,$$

where

$$D = \begin{vmatrix} \kappa, & -e'', & e' \\ e'', & \kappa, & -e \\ -e', & e, & \kappa \end{vmatrix} = \kappa\{\kappa^2 + e^2 + e'^2 + e''^2\} .$$

Now, the above expressions for γ_2, &c., show that

$$T\gamma_1 = T\gamma, \text{ &c.},$$

hence by expanding and simplifying

$$0 = \frac{c+\kappa}{D}(\kappa^2 + e^2)\left(2 - \frac{c+\kappa}{\kappa}\right) .$$

This admits of no values but

$$\kappa = \mp c$$

and

$$\kappa = e = e' = e'' = 0 .$$

The first of these three values of κ gives

$$\gamma_1 = -\gamma, \text{ &c.},$$

and thence, by the equations at the end of § 23, leads to an impossibility, which

requires that all three sets of vectors γ, γ_1, γ_2 shall be null, and thus gives no solution. A similar nugatory result is obtained from the second.

The third thus shows that the only solution is

$$\frac{1}{u^2}\nabla\xi = \gamma_1, \text{ \&c.}$$

Hence

$$\nabla\frac{1}{u^2} = 0,$$

and, finally,

$$\sigma = -u^2\,(i\mathrm{S}\gamma_1\rho + j\mathrm{S}\gamma_1'\rho + \kappa\mathrm{S}\gamma_1''\rho),$$

where γ_1, γ_1', γ_1'' form a rectangular system with equal tensors, whose common value must obviously be the reciprocal of u. But we have seen that

$$\frac{d\sigma}{dx}, \frac{d\sigma}{dy}, \frac{d\sigma}{dz},$$

also form a rectangular set of vectors with equal tensors. Hence

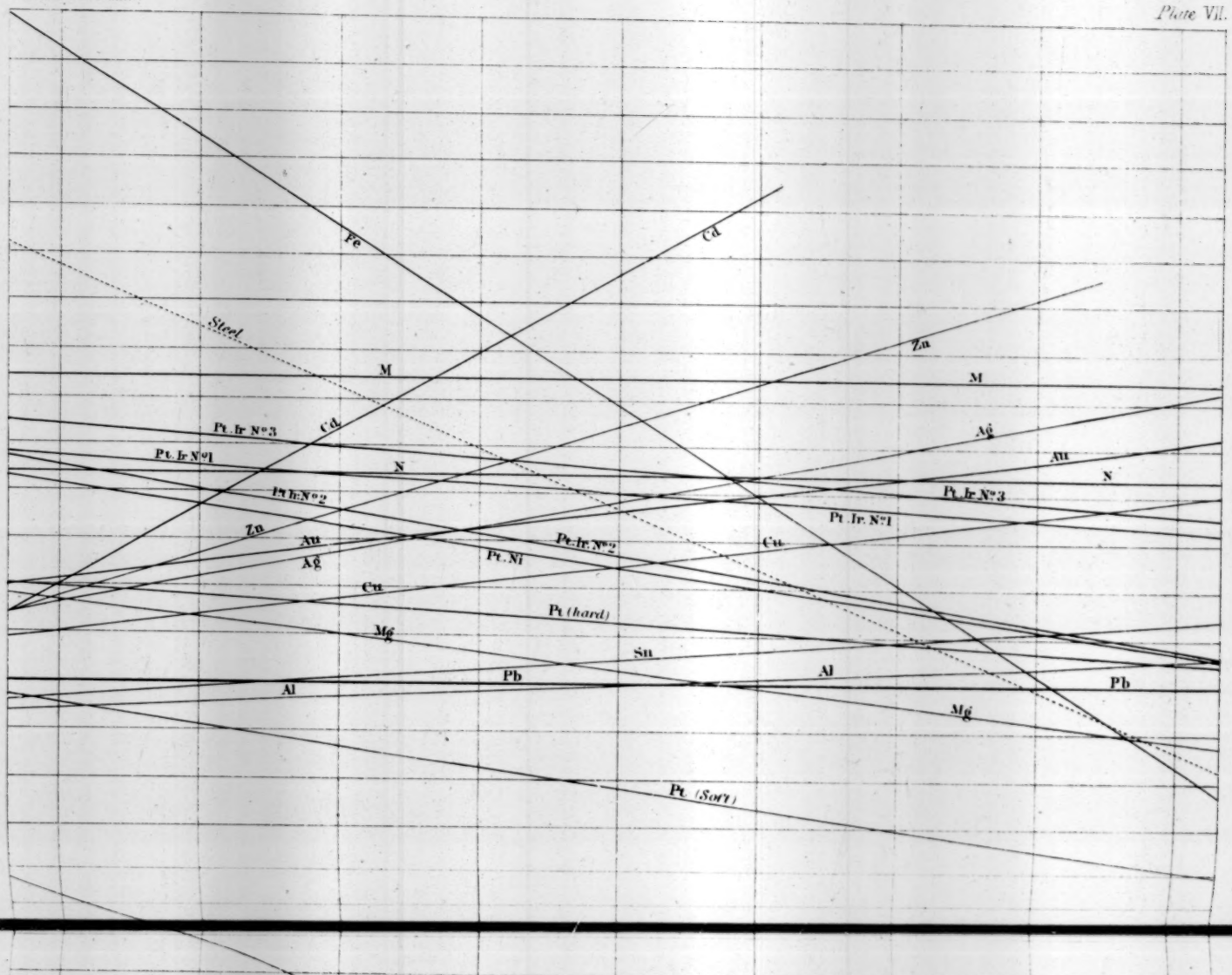
$$(\mathrm{S}i\gamma_1)^2 + (\mathrm{S}i\gamma_1')^2 + (\mathrm{S}i\gamma_1'')^2 = (\mathrm{S}j\gamma_1)^2 + (\mathrm{S}j\gamma_1')^2 + (\mathrm{S}j\gamma_1'')^2 = \ldots$$

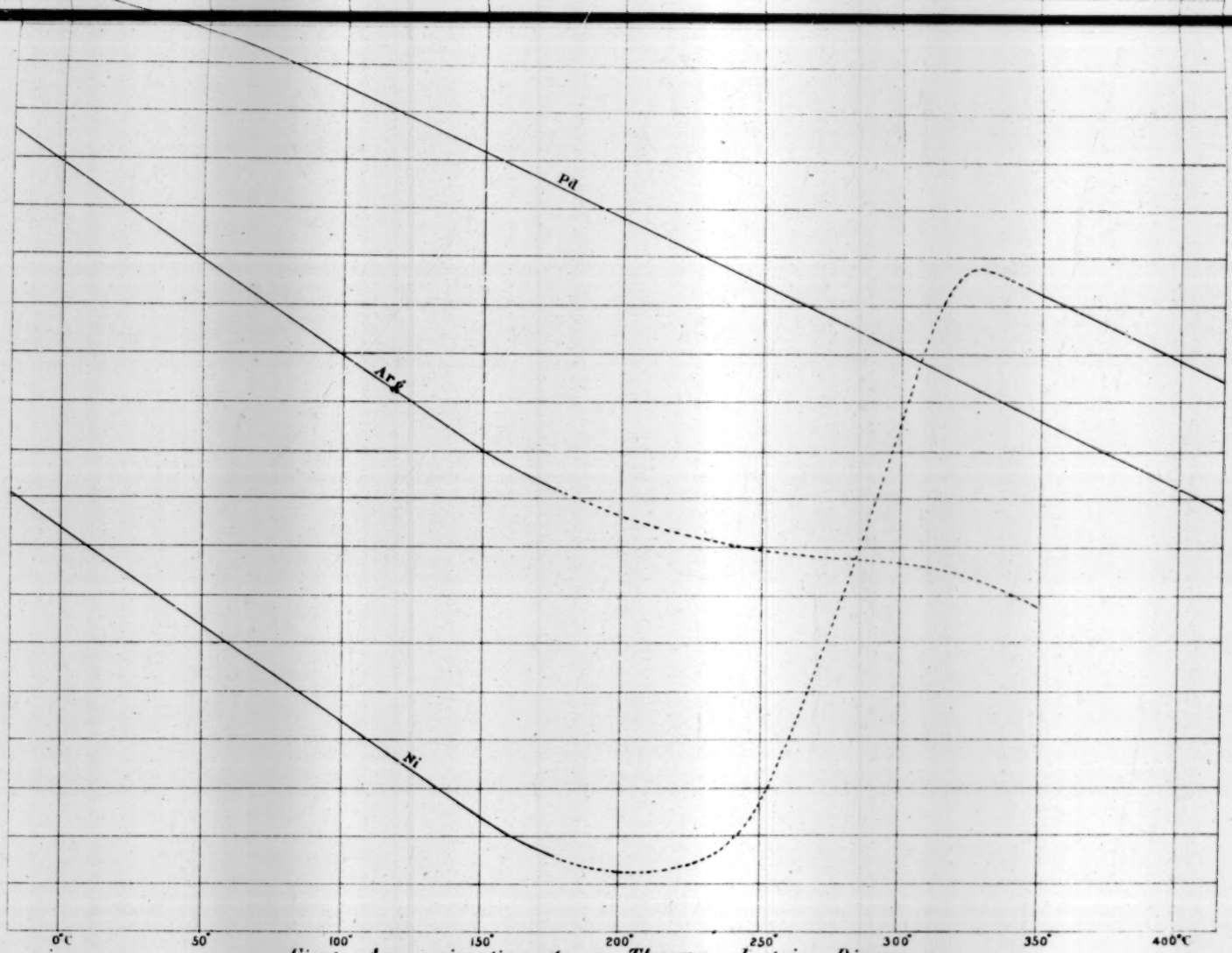
$$0 = \mathrm{S}i\gamma_1\mathrm{S}j\gamma_1 + \mathrm{S}i\gamma_1'\mathrm{S}j\gamma_1' + \mathrm{S}i\gamma_1''\,\mathrm{S}j\gamma_1'', \text{ \&c.}$$

These equations also are satisfied identically, and we therefore have, as before,

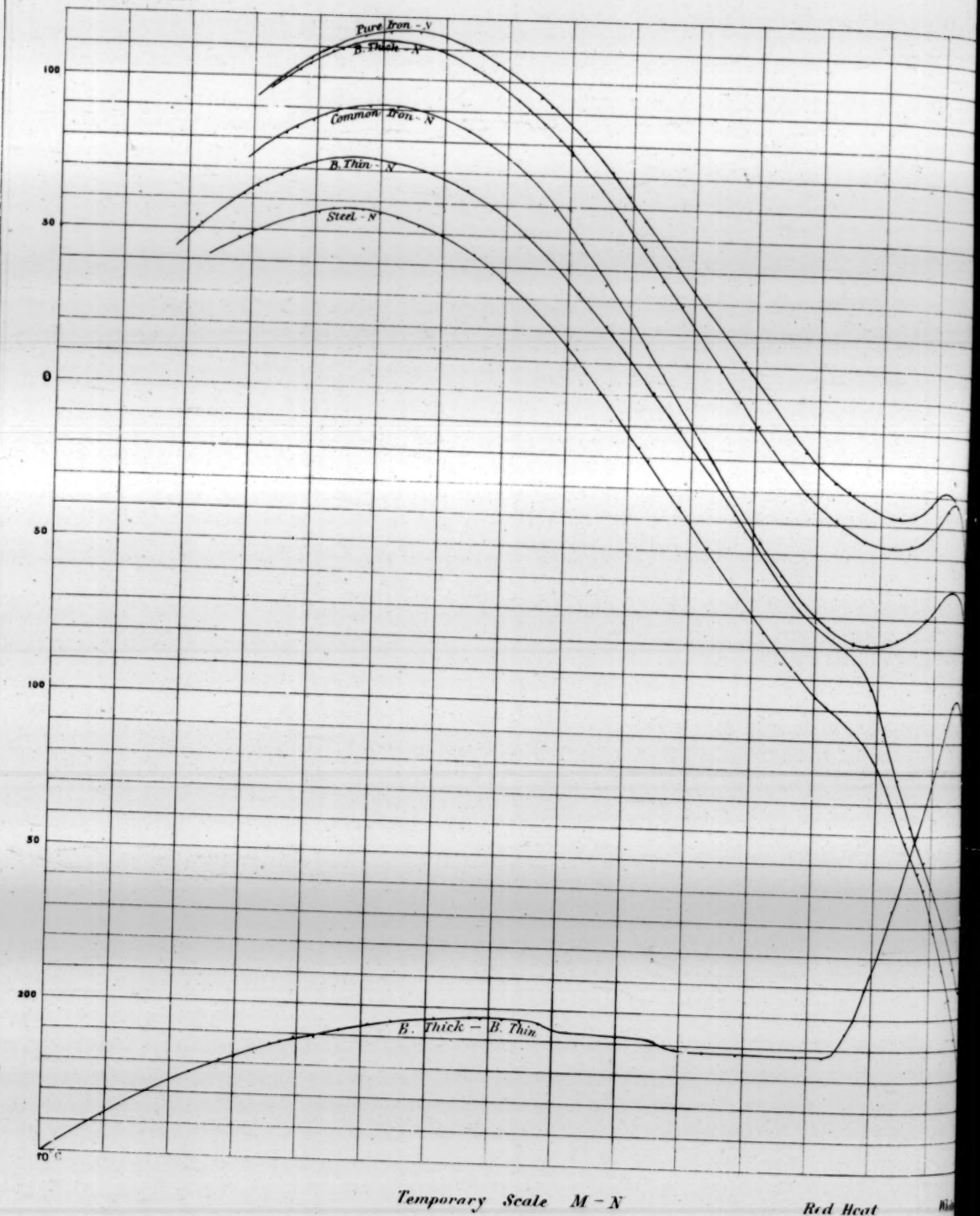
$$\sigma = uq\rho q^{-1}$$

where u and q are each constant.

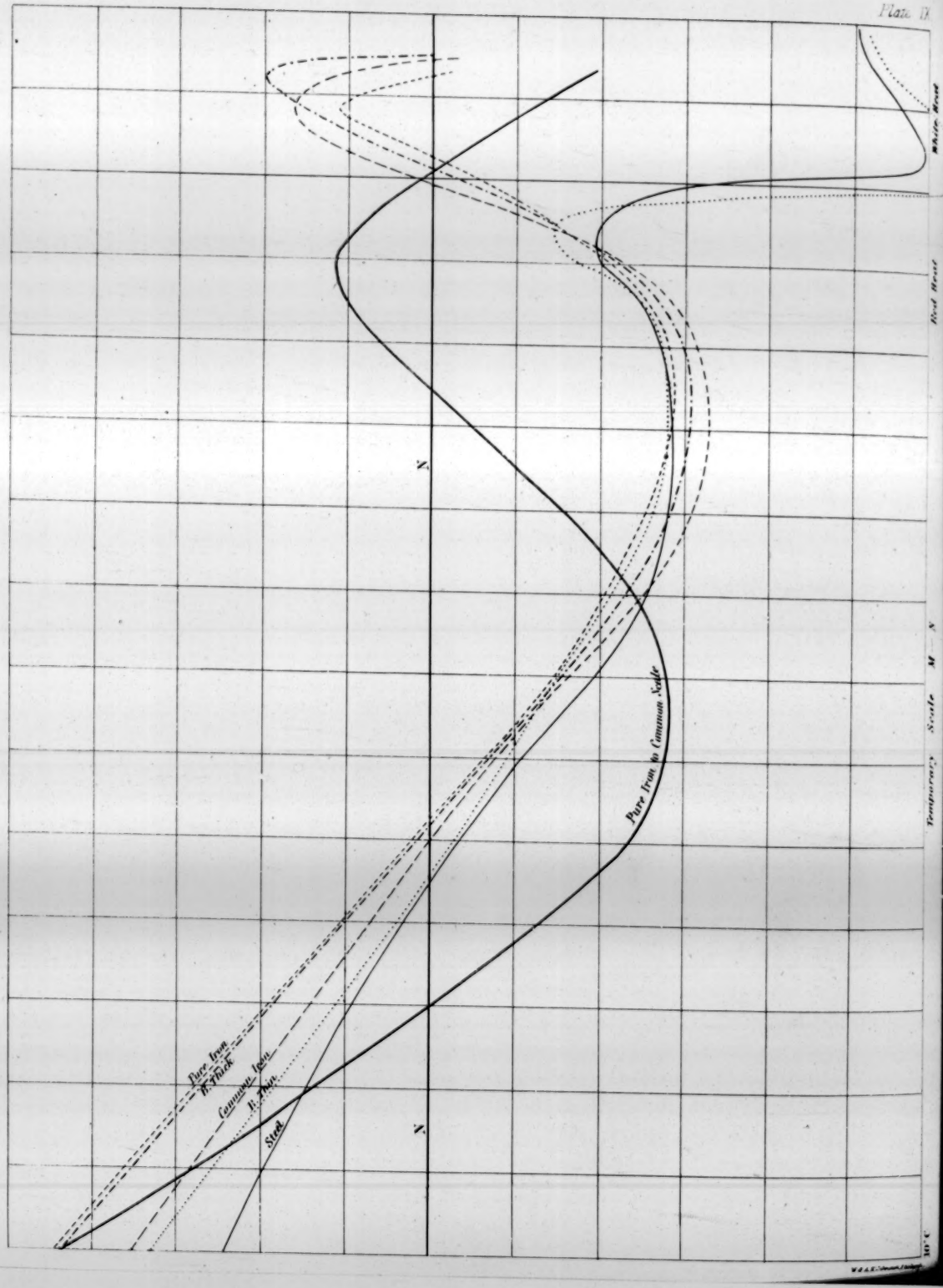

Fe
Cd
Steel
Zn
M
Pt. Ir. N°3
Cd.
Pt. Ir N°1
N
Pt. Ir. N°2
Zn
Au
Ag
Pt. Ni
Cu
Pt (hard)
Mg
Sn
Pb
Al
Pt (Soft)
Ag
Au
N
Pt. Ir. N°3
Pt. Ir. N°1
Pt. Ir. N°2
Cu
Al
Pb
Mg



First Approximation to a Thermo-electric Diagram.


Pure Iron - N
B. Thick - N
Common Iron - N
B. Thin - N
Steel - N
B. Thick — B. Thin
100
50
0
50
100
50
200
10°
Temporary Scale M - N
Red Heat

22 AP 71


White Heat
Red Heat
Temperature Scale M — N
M
N
N
Pure Iron to Common Scale
Pure Iron
½ Thick
Common Iron
½ Thick
Steel

VI.—*First Approximation to a Thermo-Electric Diagram.* By Professor TAIT.

(Plates VII., VIII., IX.)

(Read December 1, 1873.)

In the Session of 1867–8 I communicated to this Society a paper on the *Dissipation of Energy*, of which only a very brief abstract was published in the *Proceedings*. The main feature of that paper was the suggestion, as at least a valuable working hypothesis, that even in cases of the *steady* motion of heat, electricity, &c., the unexhausted energy is probably as small as possible, consistently with the conditions of each form of experiment.

Applied to the conduction of heat, this hypothesis was shown to lead to the result that thermal conductivity is inversely as the absolute temperature, a result closely agreeing with the experimental determinations of FORBES. A similar result follows (from the hypothesis) for electric conductivity, where it has long been known from experiment that the resistance is nearly proportional to the absolute temperature. As the latter experimental law, however, is subject to numerous exceptions, notably in the case of alloys, it was found necessary to introduce considerations of molecular change (such as alteration of specific heat, &c., with temperature); so that I determined to apply FORBES' methods as well as electric testing to other pure metals than iron, and also to an alloy such as German silver. The reduction of my observations is still far from complete, but I have already stated to the Society that the change of thermal conductivity by temperature in German silver is, like that of electric conductivity, at least much less than in iron.

These experimental determinations involved very great difficulties of various kinds, so that it was not till 1870 that I had an opportunity of testing experimentally the working hypothesis above mentioned in its application to the very curious phenomena of thermo-electricity. After a few experiments, however, I found that (at least within the limits where mercury thermometers can be employed) the so-called *Specific Heat of Electricity* is proportional to the absolute temperature, precisely the result indicated by the hypothesis. The following note is reprinted from the *Proceedings* of the Society for Dec. 19, 1870:—

"In a paper presented to the Society in 1867–8 I deduced from certain hypothetical considerations regarding Dissipation of Energy results connected with the thermal and electric conductivity of bodies, the electric convection of

heat, &c. As these were all of a confessedly somewhat speculative character, I printed at the time only that connected with thermal conductivity, which I had the means of comparing with experiment, and which seemed to accord fairly with FORBES' experimental results. But the assumption on which this was based was essentially involved in all the other portions of the paper.

"With a view to the testing of my hypothetical result as to electric convection of heat, several of my students, especially Messrs MAY and STRAKER, last summer made a careful determination of the electro-motive force in various thermo-electric circuits through wide ranges of temperature. Their results for a standard iron-wire, connected successively with two very different specimens of copper, when plotted, showed curves so closely resembling parabolas that I was led to look over my former investigations and determine what, on my hypothetical reasoning, the curves should be. This I had entirely omitted to do. I easily found that the parabola ought, on my hypothesis, to be the curve in every case, and I made last August a numerous and careful set of determinations with Kew standard mercurial thermometers as an additional verification.

"My hypothetical result was to the effect that what THOMSON (*Trans. R.S.E.* 1854, *Phil. Trans.* 1856) calls the specific heat of electricity, should be, like thermal and electric resistance, directly proportional in pure metals to the absolute temperature, the coefficient of proportionality being, for some substances, negative.

"Hence, using THOMSON'S notation as in *Trans. R.S.E.*, we have for any two metals

$$J\sigma_1 = k_1 t\ ,\ J\sigma_2 = k_2 t\ ,$$

where k_1 and k_2 are constants, whose sign as well as value depends on the properties of each metal, σ_1 , σ_2 are the specific heats of electricity, and J is JOULE'S Equivalent.

"Thus, introducing these values into THOMSON'S formulæ, we have

$$(k_1 - k_2)\, t = J\ (\sigma_1 - \sigma_2) = J\left(\frac{\Pi}{t} - \frac{d\Pi}{dt}\right),$$

where Π is the Peltier effect at a junction at absolute temperature t. Integrating, we have

$$C - (k_1 - k_2)\, t = J\frac{\Pi}{t},$$

or

$$J\frac{\Pi}{t} = (k_1 - k_2)\ (t_0 - t)\ ,$$

where t_0 is the constant of integration, obviously in this case the temperature at which the two metals are thermo-electrically neutral to one another. Hence the Peltier effect may be represented by the ordinates of a parabola of which

temperatures are the abscissæ; the ordinates being parallel to the axis of the curve.

"The electro-motive force in a circuit whose junctions are at absolute temperatures t and t' is then represented by

$$E = J\int_{t'}^{t} \frac{\Pi}{t} dt = \tfrac{1}{2}(k_1 - k_2)[2t_0(t - t') - (t^2 - t'^2)]$$

$$= (k_1 - k_2)(t - t')\left[t_0 - \frac{t + t'}{2}\right].$$

This, of course, is again the equation of a parabola. That $t - t'$ is a factor of E has long been known, and THOMSON has given the results of many experiments tending to show that $t_0 - \frac{t+t'}{2}$ is also a factor. But it was not till the experiments in my Laboratory had been carried on for some months that I was referred by THOMSON to a paper by AVENARIUS (*Pogg. Ann.* 119), in which it is experimentally proved (partly in contradiction of an assertion of BECQUEREL) that in a series of five different thermo-electric circuits the electro-motive force can be very accurately expressed by *two* terms of the assumed series

$$E = b(t_1 - t_2) + c(t_1^2 - t_2^2) + \quad . \quad . \quad .$$

where t_1 and t_2 are temperatures as shown by the ordinary mercurial thermometer. It follows from this that (neglecting the difference between absolute temperatures and those given by the mercurial thermometer) E has no other variable factor than those above given.

"Curiously enough, AVENARIUS, whose paper seems to have been written mainly for the purpose of attempting to explain (by the consideration merely of the effect of heat on electricity of contact of two metals) the production of thermo-electric currents, does not allude to the fact that the above equation represents a parabola. In fact he gives several figures, in all of which it is represented as a very accurately drawn *semicircle*. He makes no application of his empirical formula to the determination of the amount of the Peltier effect, nor does he seem to recognise the existence of what LE ROUX has called 'l'effet Thomson,' which is indispensable to the explanation of the observed phenomena.

"All the curves plotted by Messrs MAY and STRAKER, which were derived from iron, copper, and platinum alone, as well as my own, which included cadmium, zinc, tin, lead, brass, silver, and various other substances (sometimes arranged with a double arc of two different metals connecting the hot and cold junctions) were excellent parabolas. When the temperatures were very high, the parabola was slightly steeper on the hotter than on the colder side. This, however, was a deviation of very small amount, and quite within the limits of error introduced by the altered resistance of the circuit at the hotter parts, the deviations of the mercury thermometers from absolute temperature, and the non-

correction of the indication of the thermometers for the long column of mercury not immersed in the hot oil round the junction.

"To settle the question rigorously, I have been for some time experimenting with an arrangement sometimes of double metallic arcs, sometimes of two separate thermo-electric circuits acting on a differential galvanometer—a second object being to obtain, if it be possible, an arrangement capable of replacing with sufficient accuracy the air-thermometer in the measurement of very high temperatures, and where very exact results are not required.

"In fact, if the formula above be correct, we have for two circuits with their junctions immersed in the same vessels

$$\mathrm{E} = a(t - t_1)\left(t_0 - \tfrac{t+t_1}{2}\right), \quad \mathrm{E}' = a'(t - t_1)\left(t'_0 - \tfrac{t+t_1}{2}\right),$$

so that if the resistances in the circuits be made as a to a', their resultant effect on the differential galvanometer will be proportional to

$$(t_0 - t'_0)(t - t_1).$$

"It is obvious that so far as these factors are concerned, the most sensitive arrangements will be such as have their neutral points farthest apart. On a future occasion I hope to lay the results of my new experiments before the Society. They appear to promise to be of great use in furnishing an easily working and approximately accurate substitute for the air-thermometer in an inquiry on which I am engaged respecting specific heats and melting points of various igneous rocks, &c., while the comparison of the indications of two such arrangements at very high temperatures will give the means of determining whether the quantities called k above are really constants."*

A year later (Dec. 18th, 1871) the following communication, giving rough materials for the construction of a thermo-electric diagram, was made to the Society, and appeared in the *Proceedings*:—

"For some time back I have been endeavouring to prove, by experiment, through great ranges of temperature, the result announced by me in December last, viz., that the electro-motive force of a thermo-electric circuit is in general, unless the temperature be very high, a parabolic function of the absolute temperature of either junction, that of the other being maintained constant.

"For moderate ranges of temperature the experiment presents little difficulty; but when mercurial thermometers cannot be employed, a modification of the experimental method must be made. I have employed in succession several such modifications, of which the following are the chief:—

"The simplest of all is to dispense altogether with thermometers, and to

* In *Pogg. Ann.* 1873, Heft 7, which has just reached this country, there is another paper on this subject by AVENARIUS, in which he altogether deserts his earlier assumptions and line of reasoning, and comes to conclusions somewhat resembling those just quoted from my paper of 1870.

employ two thermo-electric circuits, whose hot and whose cold junctions are immersed in the same vessels; and to plot the curve whose abscissæ and ordinates are simultaneous readings of the electro-motive forces in the two circuits. In every case I have tried, the curve thus obtained is almost accurately a parabola, most of the few deviations yet observed being in the case of silver and other metals at temperatures not very much below their melting points—under circumstances, in fact, in which we should naturally expect that the law would no longer hold. There are, also, cases in which the whole electro-motive force is so small, even for very large differences of temperature, that very much more delicate apparatus would be required for their proper investigation. And there are cases in which the neutral point is so far off that for moderate ranges of temperature the curves obtained are sensibly straight lines. I intend to examine these cases with care—the former by using more delicate galvanometers, the latter by employing metals which are practically infusible. The difficulty of obtaining wires of such metals has been the chief one I have had to face.

"If we assume the experimental curve to be a parabola, then it is easily seen (*Proc.* May 29, 1871) that in each circuit the electro-motive force must be a parabolic function of some function of the absolute temperatures of the junctions. And, as in the iron-silver, iron-zinc, iron-copper, iron-cadmium, &c., circuits, this function has been proved to be simply the absolute temperature itself (at least, within the range of mercury thermometers), it is probable that such is the general law, at least for ranges of temperature short of those which materially alter the molecular structure of the metals employed.

"The second method consisted in employing two pairs of circuits, all four hot junctions being in the same heated substance, and all four cold junctions kept at a common temperature. The members of each pair acted on a differential galvanometer (as explained in *Proc.* Dec. 19, 1870) in such a way as to eliminate the term containing the square of the absolute temperature. In this case the readings of the galvanometers should be simply proportional to one another, and likewise to the differences of absolute temperature of the junctions. The method is exact in theory, but by no means easy in practice, especially with the very limited number of metals capable of resisting a high temperature which I could manage to obtain. That a very exact and useful thermometric arrangement can be made on this principle admits of no doubt, when we examine the results of the experiments.

"The third method consisted in assuming the parabolic law, and the following consequence of it which follows directly by the use of THOMSON's general formulæ. These may easily be reproduced as follows:—Suppose a sliding ring or clip to be passed round the wires, so as to press together points of the wires which are at the same temperature, t. Its effects are known by experiment to be nil, whatever be its material. Let it be slid along so that the temperature

of what is now effectively the hot junction becomes $t + \delta t$, then the two laws of thermodynamics give, respectively,

$$\delta E = J(\delta \Pi + (\sigma_1 - \sigma_2)\,\delta t),$$

and

$$0 = \delta \frac{\Pi}{t} + \frac{\sigma_1 - \sigma_2}{t}\delta t.$$

Here E is the electro-motive force, Π the Peltier effect at a junction at temperature t, and σ_1, σ_2, are the specific heats of electricity in the two metals.

Hence

$$\delta E = J\left(\delta \Pi - t\delta \frac{\Pi}{t}\right) = J\frac{\Pi}{t}\delta t.$$

Introducing the hypothesis, obtained from considerations of Dissipation of Energy (*Proc.* Dec. 19, 1870), that

$$\sigma_1 = k_a t,\ \sigma_2 = k_b t,$$

we have

$$J\frac{\Pi}{t} = \frac{dE}{dt} = (k_a - k_b)(T_{ab} - t),$$

where T_{ab} is the well-known 'neutral point.'

Also

$$E = (k_a - k_b)(t - t_1)\left(T_{ab} - \frac{t + t_1}{2}\right),$$

since it vanishes for $t = t_1$, the temperature of the cold junction. Now, if the neutral point be between such limits as 0° C. and 300° C., the exact determination of it is an easy matter; and this exact knowledge of it greatly facilitates the determination of $\frac{dE}{dt}$, which cannot be *very* accurately found by drawing a tangent to the plotted curve. For if one junction be at t, the other at T_{ab}, we have

$$E_\tau = \tfrac{1}{2}(k_a - k_b)(T_{ab} - t).^2$$

E_τ and $T_{ab} - t$ are easily measured on the experimental curve, and thus $k_a - k_b$ is found. The following values have thus been (roughly) calculated from observations. Where the neutral point was not reached, it is put in brackets. The unit for $k_a - k_b$ is 3 or 4 *per cent.* less than $\frac{2}{10^5}$ of the electro-motive force of a good Grove's cell.

	T	$k_a - k_b$		T	$k_a - k_b$
Fe – Cu (bad)	265 C.	– 0·00147	Fe – Al	(387) C.	– 0·00105
„ – Cu (good)	260	– ·00145	„ – Arg.	(1357)	– ·00045
„ – Cd	159	– ·00209	Cu (bad) – Cd	– (23)	– ·00081
„ – Zn	199	– ·00189	„ – Zn	– (146)	– ·00048
„ – Ag	235	– ·00151	„ – Ag	– (687)	– ·00006
„ – Pb	(357)	– ·00112	„ (good) – Pb	– (213)	+ ·00016
„ – Brass	(318)	– ·00127	Pb – Cd	– (74)	– ·00096
„ – Pt	(519)	– ·00063	„ – Pd	– (188)	+ ·00080
„ – Sn	(416)	– ·00094	„ – Zn	– (78)	– ·00060
„ – Pd	(1908)	– ·00029	„ – Ag	– (262)	– ·00026

"Now, it is an immediate consequence of the second law of thermodynamics, that as Peltier effects are reversible with the direction of the current, and are the *only* sensible thermal effects when a very feeble current passes through a thermo-electric circuit all of whose parts are at one temperature, we must have

$$\Sigma \frac{\Pi}{t} = 0 ,$$

or, assuming the parabolic law,

$$\Sigma . (k_a - k_b)(T_{ab} - t) = 0 .$$

This holds for any number of separate materials in the conductor. As t is the same throughout, the terms involving it evidently vanish identically; but there remains the equation

$$\Sigma . (k_a - k_b) T_{ab} = 0 ,$$

establishing a relation between the specific heats of electricity in a number of metals and the absolute temperatures of the neutral points of each junction of two of them. Other relations may be obtained by altering the order of the metals if there be more than three—but they are all virtually contained in the formula for three, which we write at full length,

$$(k_a - k_b) T_{ab} + (k_b - k_c) T_{bc} + (k_c - k_a) T_{ca} = 0 .$$

From the direct experiments of LE ROUX on "l'effet Thomson," as he calls it, it appears that k is null in lead.* At all events, since THOMSON showed that it has opposite signs in iron and copper, we may imagine a substance for which $k = 0$. We may now construct an improved "*Thermo-electric diagram*" to represent these relations numerically, employing the line for this substance as our axis of absolute temperatures; while the ordinates perpendicular to it give, for this substance employed with any other in a circuit of two metals, the values of $\frac{\Pi}{t}$, or $\frac{dE}{dt}$, or (what comes to the same thing) the electro-motive force of a circuit whose junctions are both very nearly at t, but have a small constant temperature difference. This quantity corresponds with what has been called the *thermo-electric power* of the circuit.

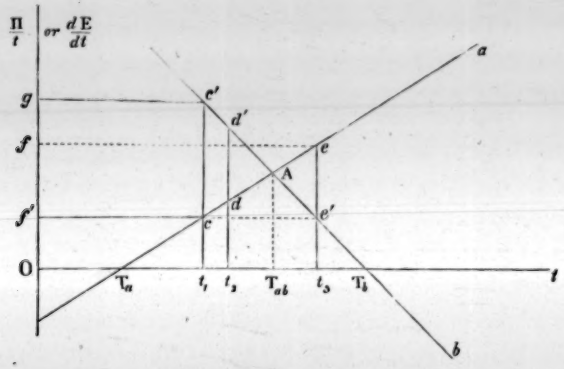
"The two oblique *straight* lines in the diagram belong to the metals a, b, respectively. The tangents of their inclination to the horizontal axis (the line of the supposed metal for which $k = 0$) are k_a, k_b—and they cut it at the points T_a, T_b, where they are neutral to it; cutting one another at a point A whose abscissa is their own neutral point T_{ab}. The only change which would be introduced, by taking as horizontal axis the line corresponding to a metal for which

* Annales de Chimie, 1867, vol. x. p. 277.

k does not vanish, would be a dislocation of the diagram, by a simple shear. This follows at once from the equation of one of the lines—

$$y = k_a (x - \mathrm{T}_a).$$

"The diagram gives the Peltier effect at the junction of a and b for any temperature t_1, by drawing the ordinate at t_1, and completing a rectangle $cc'gf''$ on the part intercepted, its opposite end being at absolute zero. The area of this



rectangle is to be taken positively or negatively according as the corner corresponding to a is nearer to, or further from, the horizontal axis than that corresponding to b, the current being supposed to pass from a to b.

"The electro-motive force in a circuit of the two metals a and b, with its junctions at t_1 and t_2 respectively, is found by drawing ordinates at these temperatures, so as to cut off triangular spaces Acc', Add', whose vertices are at the neutral point. The difference of the areas of these spaces, $cdd'c'$, is proportional to the electro-motive force. When the higher temperature t_3 is above the neutral point, the electro-motive force is the difference of the areas Acc', Aee'. The case above mentioned, in which, by a differential galvanometer, we get rid of the terms in t_2, is obviously a process for making the curves of two separate complex arrangements into parallel straight lines.

"In conclusion, I may give a few instances of the comparison of results of calculation of the neutral point of two metals from their observed neutral points, and differences of k, as regards iron, with calculation of the same neutral point from the portion of the curve (assumed to be a parabola) which expresses their electro-motive force within ranges of temperature where mercurial thermometers can be applied.

"Thus with Fe, Cd, Pb, we have from the iron circuits $0{\cdot}00112 - 0{\cdot}00209 = -0{\cdot}00097$, while the direct experiment with Cd, Pb gave $-0{\cdot}00096$.

"The neutral point, as calculated from the data for the iron circuits is $-69°$ C., while the calculation from direct experiment gives $-74°$ C.

"When the quantities to be found are very small, as for instance in the case Ag — Cu, we cannot expect to get a good approximation by introducing a third metal. In fact, introducing Fe we find indirectly 0·00147 — 0·00151 = — 0·00004, while the direct determination gives — 0·00006.

"Again with Zn and Cu, indirectly we get

— 0·00042 and — 144° C.

Directly — 0·00048 and — 146° C.

"Several of the other groups give results as closely agreeing with one another as these, others are considerably out.

"The numerical determinations above are founded entirely on a series of experiments made for me by Messrs J. MURRAY and R. M. MORRISON. Mr W. DURHAM is at present engaged in determining the electro-motive force of contact of wires of the same metal at different temperatures, with the view of inquiring into its relation to ordinary thermo-electric phenomena which appears to be suggested by some of the formulæ above given."

Mr DURHAM's results were published in the *Proc. R.S.E.* (June 17th 1872), and showed that in the case of platinum, the only metal he examined, the integral deflection of a somewhat massive galvanometer needle is independent of the absolute temperature of either wire, and proportional simply to the difference of their temperatures. This was the result I had expected from the formulæ given above (p. 130); for if

$$k_a = k_b ,$$

we have

$$T_{ab} = \infty ;$$

but consistently with these we may have

$$(k_a - k_b)\, T_{ab} = \tau ,$$

a finite quantity. Hence

$$E = J\tau\,(t - t_1) .$$

Various other communications on the subject were made by me to the Society, and published in the *Proceedings*; but of these I need quote only the following, of date June 3d, 1872, as it shows a novel difficulty which I met with, and which prevented me from publishing earlier an attempt at constructing a thermo-electric diagram:—

"Having lately obtained from Messrs JOHNSON and MATTHEY some wires of platinum, and of alloys of platinum and iridium, I formed them into circuits with iron wire of commerce; and noticed that with all, excepting what is called 'soft' platinum, there is more than one neutral point situated below the temperature of low white heat, and that at higher temperatures other neutral points occur. This observation is, in itself, highly interesting; but my first

impression was one of disappointment, as I imagined it depended on some peculiarity of the platinum metals, which I had hoped would furnish me with the means of accurately measuring high temperatures (by a process described in previous notes of this series). As this hope may possibly not be realised, I can as yet make only rough approximations to an estimation of the temperatures of these neutral points.

"So far as I am aware, the phenomenon discovered by CUMMING and analysed by THOMSON has hitherto been described thus: When the temperature of the cold junction is below the neutral point, the gradual raising of the temperature of the other produces a current which increases in intensity till the neutral point is reached, thenceforth diminishes; vanishes when one junction is about as much above the neutral point as the other is below it, and is *reversed* with gradual increasing intensity as the hot junction is farther heated. To discover how my recent observation affects this statement, I first simply heated one junction of a circuit of iron and (hard) platinum gradually to whiteness, by means of a blowpipe, and observed the indications of a galvanometer—both during the heating and during the subsequent cooling when the flame was withdrawn. The heating could obviously not be effected at all so uniformly as the cooling; but, making allowance for this, the effects occurred in the opposite order, and very nearly at the same points of the scale in the descent and in the ascent. [I have noticed a gradual displacement of the neutral points when the junction was heated and cooled several times in rapid succession; but as my galvanometer, though it comes very quickly to rest, is not quite a *dead-beat* instrument, I shall not farther advert to this point till I have made experiments with an instrument of this more perfect kind, which is now being constructed for me.] The observed effect of heating, then, was a rise from zero to 110 scale divisions when the higher temperature was that of the first neutral point, then descent to 95 at a second neutral point, then ascent to a third, descent to a fourth, neither of which could be at all accurately observed, and finally ascent until the junction was fused.

"With an alloy of 15 per cent. iridium and 85 per cent. platinum, the galvanometer rose to 53·5 at a neutral point, then fell to — 50 at a second, then rose to a third, at — 39·5, and thence fell, but I could not observe a possible fourth neutral point on account of the fusion of the iron. As shown on the plate, the first of these occurs at about 240° C. of a mercurial thermometer.

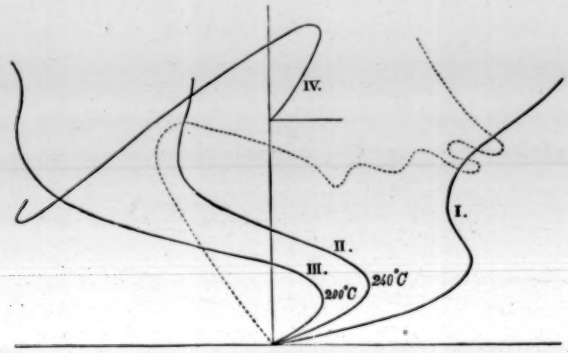
"With another alloy supposed to be of the same metals, but of which I do not yet know the composition, also made into a junction with iron, the behaviour was nearly the same, but the readings at the successive neutral points were 28, — 137, — 132. The temperature of the first is about 200° C. by mercurial thermometer.

"An iron-palladium circuit showed no neutral points within the great range

of temperatures mentioned above; though it showed a remarkable peculiarity which must be more closely studied, as it appears to point to the cause of the above effects in a property of iron. It was therefore employed to give (very roughly) an indication of the actual temperatures in these experiments. But as for this purpose it is necessary to measure the simultaneous indications of two circuits whose hot and whose cold junctions are respectively at the same temperatures, I was obliged to employ a steadier source of heat than the naked flame. I therefore immersed the hot junctions in an iron crucible containing borax glass, subsequently exchanged for a mixture of fused carbonate of soda and carbonate of potash; but, to my surprise, the former of these substances at a red heat disintegrated both the platinum and the alloy, and thus broke both circuits without sensibly acting on the iron, while the mixture (evidently by the powerful currents discovered by ANDREWS, *Phil. Mag.* 1837) interfered greatly with the indications of the thermo-electric circuit, as will be seen by the dotted curve in the wood-cut. [I may remark here that the deviations of this curve from its form when these currents are prevented are quite easily observed and plotted by the process next to be mentioned, so that the study of the Andrews effect may be carried out with great accuracy by my method.] Finally, determining to dispense altogether with fused salts, which conduct too well besides acting on the metals, I simply suspended a red-hot bombshell, vent downwards, in such a way that the hot junction was near its centre. This arrangement worked admirably, until a white heat was required, for this melted the shell. In its place a wrought-iron tube (an inch in bore, four inches long, half an inch thick, and closed at the upper end) has been substituted, and answers excellently. It does not cool too fast for accurate reading at the higher temperatures, and by elevating it by degrees from over the hot junction we can make the cooling fast enough at the lower ranges. In fact, I believe that if I do not succeed in getting a sufficient number of practically infusible metals to construct my proposed thermometric arrangement, I may be able to make a fair approximation to temperatures by simple time observations made with the hot tube, surrounded by some very bad conductor, such as sand, where the surface in contact with the air is always comparatively cool, and where therefore we can accurately calculate the rate of cooling.

"Curves I., II., III., in the wood-cut were drawn by means of this apparatus. The hot junction consisted of an iron wire, a palladium wire, and (for the several curves in order)—I. Hard platinum; II. Pt 85, Ir 15; III. The other alloy of Pt and Ir. The free ends of the palladium wire, and of the platinum or alloy, were joined to iron wires, and the junctions immersed in test-tubes filled with water resting side by side in a large vessel of cold water. The other ends of these three iron wires, and the wires of the galvanometer,

were led to a sort of switch, by means of which either circuit could be instantly made to include the galvanometer. Readings were taken of each circuit as fast after one another as possible (with the galvanometer I employed about 6·5



seconds was the *necessary* interval), and the mean of two successive readings of one circuit was taken as being at the same temperature as that of the intermediate reading of the other.

"The indications of these curves are very curious as regards the effect of even small impurities on the thermo-electric relations of some metals. It is probable, from analogy, that the curve for iron and *pure* platinum, in terms of temperature, would be (approximately, at least; even if it should be the iron, and not the platinum metal, which is represented by a broken or curved line) a parabola with a very distant vertex. And it appears probable that when the wire of curve III. is analysed it will be found to contain even a larger percentage of iridium (?) than that of curve II.

"I find by tracing these curves on ground glass, allowing for the difference between temperatures and the indications of an Fe-Pd circuit, and superposing them on a nest of parabolas with a common vertex and axis, that they can be closely represented by successive portions of different parabolas (with parallel axes) whose tangents coincide at the points of junction, though the *curvature* is necessarily not continuous from one to the other. Hence, as at least a fair approximation to the electro-motive force in terms of difference of temperature in the junctions, we may assume a parabolic function, which up to a certain temperature belongs to one parabola, then changes to another without discontinuity of direction, and so on.

"Hence either the iron, or the hard platinum and the platinum-iridium alloys, will be (approximately, at least) represented on my form of THOMSON'S thermo-electric diagram (*ante*, p. 132) by *broken* lines, of which the successive parts are straight. This, contrasted with the (at least nearly) straight lines for pure metals, seems to show that some bodies take successively different

states (*i.e.*, become *different substances*) at certain 'critical' temperatures, retaining their thermo-electric properties nearly unchanged from one of those critical points to another.

"The curve marked IV. in the woodcut was obtained by plotting against each other the simultaneous indications of the alloy of curve III. and iron, and of the alloy of curve II. and iron, so as to avoid any disturbance from possible peculiarities of palladium. Then, to obtain an idea of the share taken by iron in the results, it was found that the electro-motive force in a circuit formed by the two alloys, or by either with hard Pt, is (for a very great range of temperature) sensibly proportional to the temperature difference of the junctions.

"The same result is easily seen from the plate, if we notice that the difference of corresponding ordinates in any two of curves I., II., III., is nearly proportional to the corresponding abscissa. Now, it seems a less harsh supposition that the lines representing platinum and its alloys are nearly straight and parallel, while that of iron is a broken line, than that the latter should be straight and the former all broken at the same temperatures. On the other hand, this latter hypothesis would make k alternately negative and positive in iron, while the former would only require the platinum metals to have values of k alternately less and more negative than that of iron.

"I may add that none of the above-mentioned effects can be due to altered electric resistance of the heated junctions, because the galvanometer resistance was about 23 B. A. units, while that of the iron and platinum wires together was in each case not more than one such unit. The palladium-iron circuit was so much more powerful than the others that a resistance coil of about 146 B.A. units had to be inserted in its course. . . . . ."

To this paper was added during printing the following postscript:—"I have since made out that the lines of the diagram are approximately straight, and parallel to the lead line, for the platinum metals, that of hard platinum being below the lead line, while those of most of the other alloys are above it, and that the multiple neutral points depend upon the peculiar sinuosity of the line for iron. I have also obtained curious results of a somewhat similar kind with steel wire. The method I employed was to explore the part of the thermo-electric diagram included between the lines of gold and palladium, by making a multiple arc of these two metals, and varying the ratio of their separate resistances. But I reserve details until I have carefully examined the behaviour of nearly pure iron."

The peculiarity thus exhibited by iron I afterwards found to be also possessed by nickel, and with the farther advantage that the changes of sign of specific heat of electricity occur in that metal at temperatures within the range of mercury thermometers. (*Proc. R.S.E.*, May 1873). These results I developed in the *Rede Lecture* of 1873, a full abstract of which was printed in

Nature [May 29th and June 12th], and to this I refer the reader for some speculations as to the connection of these phenomena with known chemical and magnetic relations, as well as for a great deal of additional matter connected with Thermo-electricity, but not so directly connected with my present subject, the construction of a Thermo-electric Diagram.

I have given this *résumé* of a few of my former papers to show how I was led to attempt the construction of a thermo-electric diagram, by the result of experiments originally devised to test the truth of a hypothetical application of the Dissipation of Energy.

The following results were obtained mainly during the summer of the present year, the experiments being in great part made by Messrs GREIG and C. G. KNOTT in my laboratory. The extracts above show sufficiently the nature of the processes employed, so that but a very few remarks need be made about the thermo-electric diagram (Plate VII.), which is constructed from them, and embraces the greater part of the temperature-region in which mercury thermometers can be used. Metals like bismuth and antimony are quite beyond the capabilities of even a double plate on this scale.

1. A very small amount of impurity, or even of permanent strain, is capable of considerably altering the line of a metal in the diagram; so that I have given in general a sort of average position to each line, and have not attempted absolute exactness where it was obviously not requisite nor even desirable. N is the alloy of 15 Ir, 85 Pt described in the last extract above, M is the other alloy. Nos. 1, 2, 3 denote platinum-iridium alloys containing respectively 5, 10, 15 per cent. of the latter metal. These were prepared for me from pure metals by Messrs JOHNSON and MATTHEY, as I fancied from the behaviour of M and N that I might get a series of alloys whose lines should be parallel to that of lead. The result does not for the present appear encouraging.

2. I have not yet been able to arrive at any definite conclusion with regard to the form of the dotted portions in the lines of nickel and of German silver. In fact, had it not been that the palladium line intersects that of nickel near the middle of the most interesting region, I might have missed altogether the detection of the peculiarities of nickel, though I was led to seek for them near that region by induction from those of iron. It is obvious, in fact, from the diagram, that had copper, gold, iron, &c., been associated with nickel, the modification due to these peculiarities would have been only a very small fraction of the whole electro-motive force, and might easily have been attributed to errors of observation. As it is, my best specimen of pure nickel has been destroyed by exposure several times to a white heat, and I must wait for another before I can resume this part of the inquiry.

3. Having made no direct experiments on the electric convection of heat in lead, I have retained its line as the axis, on the authority of LE ROUX above

alluded to. As already stated, this is a question involving the *actual* specific heat of electricity in each metal; not the *difference* of the specific heats in any two metals, which is all that my experiments furnish.

Subject to these remarks we have the following table of the values of k, whose contents are represented graphically in Plate VII., and where the unit of electro-motive force employed is nearly 10^{-5} of a GROVE'S cell. The tangents of the inclinations of the lines in the plate may be reduced to the corresponding numerical values of k in terms of a GROVE'S cell by the factor 4×10^{-8}.

Fe	− ·00247	Cd	+ ·00218
Steel	− ·00171	Zn	+ ·00122
M	− ·00000	Ag	+ ·00076
Pt. Ir (No. 1)	− ·00028	Au	+ ·00052
Pt. Ir (No. 2)	− ·00068	Cu	+ ·00048
Pt. Ir (No. 3)	− ·00032	Pb	·00000
N	− ·00000	Sn	+ ·00028
Pt (soft)	− ·00056	Al	+ ·00020
Pt. Ni	− ·00056	Pd	− ·00182
Pt. (hard)	− ·00038	Ni (to 175° C.)	− ·00260
Mg	− ·00048	Ni (250° − 310° C.)	+ ·01225
Arg	− ·00260	Ni (from 340° C.)	− ·00260

Plate VIII. shows directly the galvanometric indications of circuits including various iron and steel wires; one of which is a ribbon of pure iron, prepared by Dr MATTHIESSEN, kindly put at my disposal by Dr RUSSELL. The other specimens of iron consist of two from the ordinary stock in my laboratory, and a third (probably, from its position so close to that of Dr MATTHIESSEN, very pure) which I owe to Sir R. CHRISTISON, who has used portions of it for chemical testing for more than thirty years. It was, therefore, prepared at a time when more care was employed to secure purity than in the present day. The circuits were completed by the platinum alloy called N above, whose line is nearly parallel to that of lead, but a little above it. The temperature scale is the temporary one given by the galvanometric indications of the two platinum alloys M and N. Their lines are drawn as almost exactly parallel in Plate VII., but they intersect at some temperature about a white heat; so that to reduce the diagram to something roughly corresponding to absolute temperatures, the whole must be extended parallel to the temperature axis, and in ratios continually increasing for higher ranges of temperature. The experimental work on which this diagram is based has been performed almost entirely by Messrs KNOTT and SMITH, and its general accuracy may be estimated by the smoothness of the curves obtained: particularly as all the observed points which do not lie *exactly* on the curves have been inserted in the diagram.

The points of contrary flexure in these curves correspond to the points of change of sign of specific heat of electricity in the specimens of iron and steel,

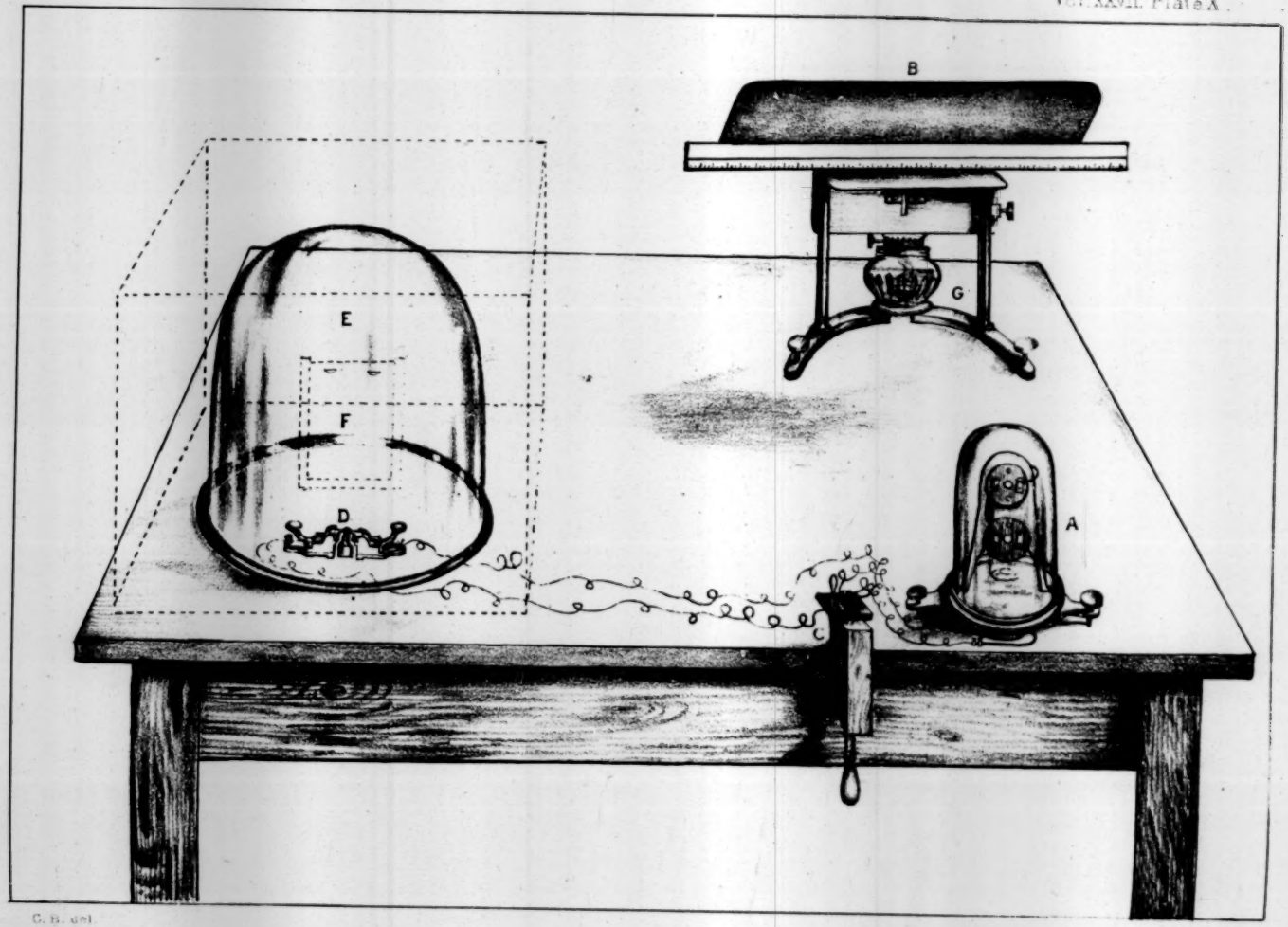
and it is obvious that it is a matter of great difficulty to estimate with precision where they lie. The wire called B Thin shows so remarkable a resemblance to steel in its thermo-electric properties—though it is certainly not steel—that, as a verification, I tried the electro-motive force of a circuit formed of it and of B Thick which so nearly coincides with pure iron. The result is given by the lower curve in Plate VIII., which is easily seen to be in entire agreement with the curves in the upper part of the plate, the ordinates of one being the differences of those of the other two.

In Plate IX. I have endeavoured, by drawing tangents to the curves of Plate VIII., to construct (to the same distorted temperature-scale) the thermo-electric diagram for N, and the various specimens of iron and steel. It will be seen that all of these specimens have at least two changes of sign of the specific heat of electricity. It is to be remarked, however, that as the heating of the junctions was effected by means of a white-hot iron cylinder (as described in one of the extracts above), the diagram belongs to specimens of iron and steel which have been raised to a white heat and are cooling. This process generally produces a marked change in the thermo-electric properties of steel, though a very slight one in those of iron.

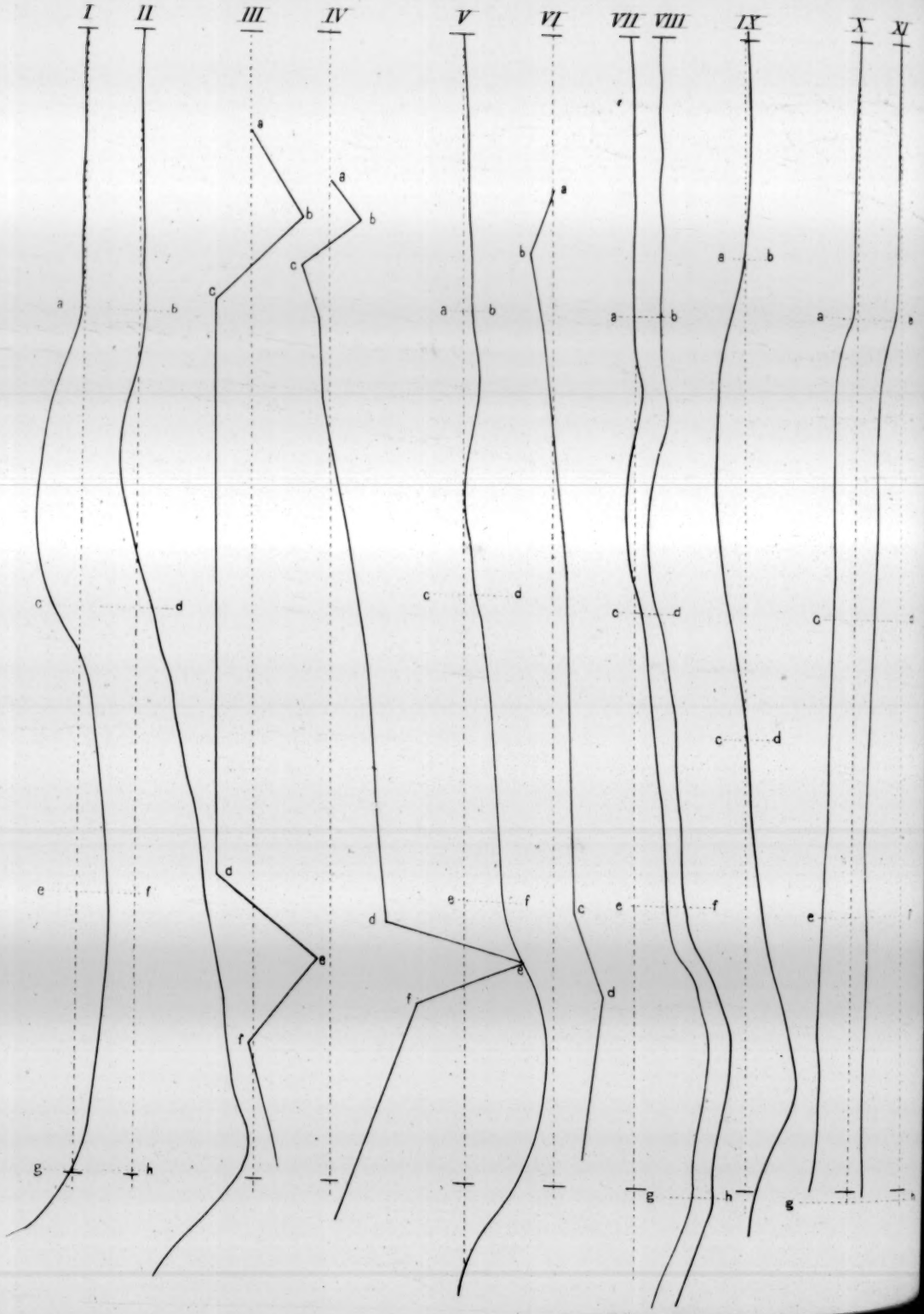
In the same Plate, IX., I have attempted (by means of the parabolic law, assumed for M, N) to approximate to the diagram for pure iron in terms of absolute temperature. The result is indicated by a double line, which may be compared with the line for nickel in Plate VII., to which it has more than a mere general resemblance. But this figure also shows one way of forming a thermo-electric circuit which shall give a current without any Peltier effect at either junction, and without electric convection of heat in one of the two metals concerned.

Note.—Since this paper was read to the Society I have seen in the *Phil. Mag.* for December 1873 a paper by Prof. BARRETT, in which he recalls attention to Mr GORE's singular observation regarding the sudden changes of length which take place in an iron wire at a low red heat, and adds his own very curious discovery of the sudden glow which occurs simultaneously with them. I have for some time been seeking for other physical changes, besides the well-known magnetic ones, and the above described thermo-electric ones, which may be expected to take place in iron about this temperature. A brief note on the change of electric resistance of iron appears in *Proc. R.S.E.* (Dec. 16, 1872) as a first instalment which I hope soon to be able considerably to extend.

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VII.—*On the Physiological Action of Light.* By James Dewar and John Gray M'Kendrick, M.D. (Part I. Plates X., XI.)

(Read 21st April, 5th May, and 2d June 1873.—Received, 6th February 1874.)

I. Introduction.

Sensory nerves are divided into two classes, those of general sensibility and those of special sense. The nerves of general sensibility are distributed to the skin, muscles, or viscera, and convey influences to the brain which give rise to sensations of touch, heat, &c., or to those vague sensations, not definitely localised, which we include under the name of the muscular sense. The nerves of special sense are endowed with special and individual physiological properties. When a nerve of this order is irritated in any way, either by mechanical, chemical, or electrical stimuli, an influence is conveyed to the brain which gives rise to the same kind of sensation as that produced by the normal stimulus on the terminal organ. For example, pressure on the eyeball, as shown by Newton and Young, electrical stimulation by a continuous current, as demonstrated by Pfaff,* Helmholtz,† Ritter,‡ Purkinje,§ Du Bois-Reymond,‖ and Schelske,¶ produce many of the phenomena of vision, including not only the perception of light, but the perception of various colours and tints. But while this is the case, it is equally certain that each terminal organ responds to its normal stimulus. Thus the retina, though capable of stimulation by pressure or electricity, is specially fitted by its histological structure for the reception of those minute vibrations of the ether which constitute light. But while the terminal organ is capable of receiving a most delicate action of the normal stimulus, the nerve in connection with it is not so affected. For example, although the retina is affected by light, the optic nerve is not so, as may be proved by Marriotte's** well-known experiment, by which it may be demonstrated that when the image of an external object falls on the entrance of the optic nerve, there is no corresponding sensation. The nerve is thus insensible to the normal stimulus of the sense organ, the retina. The current in the nerve cannot be excited by the direct

* Thierische Elektricität, 1795, p. 142.
† Physiologische optik, p. 203.
‡ Beiträge zur näheren kenntniss der Galvanismus und der Resultate seiner Untersuchung. Jena, 1805, p. 159, *et seq.*
§ Beobachtungen über Versuche zur Physiologie der Sinne. Band i.
‖ Thiersche Elektricität. Band i. p. 353.
¶ Zur Farben-empfindung. Graefes Archives ix. p. 49.
** Phil. Trans. 1668, p. 668; 1670, p. 1023.

application of the normal stimulus, but only through the intervention of a terminal apparatus. For each sense a special terminal organ is required, the function of which is to receive the normal stimulus. This stimulus affects a change in the terminal organ, and the result is conveyed by the nerve to the nerve centre.

Now arises the question of what is the specific effect of the external stimulus on the terminal organ. With regard to the ear, for example, there is the hypothesis that the hair-like processes of the cells situated on the delicate rods of Corti found in the *scala media* of the cochlea vibrate synchronously with waves of sound, and communicate these vibrations to the minute filaments of the auditory nerve which are in connection with them. With reference to vision, numerous theories have been put forward. NEWTON, MELLONI, and SEEBECK stated that the action of light on the retina consisted of a communication of mere vibrations; YOUNG conjectured that it was a minute intermittent motion of some portion of the optic nerve; DU BOIS-REYMOND attributed it to an electrical effect; DRAPER advanced the view that it depended on a heating effect of the choroid; and MOSIER compared it to the action of light on a sensitive photographic plate. Up to this date, however, our knowledge of the specific effect produced by light on the retina is without any experimental foundation.

Now, it is evident that, in accordance with the principle of the transference of energy, the action of light on the retina must produce an equivalent result, and this result may be expressed and measured as heat, chemical change, or electro-motive force. The change in the retina will likewise excite corresponding changes in the nerve, which will be conducted to the brain. What are the properties of a living nerve which can be affected by changes in the condition of the retina?

A nerve has three distinct properties: 1*st*, sensibility, or the property by which it is capable of receiving an impression and producing an influence; 2*d*, conductivity, or the property of conducting the influence to or from a nerve centre; and, 3*d*, electro-motive force, or the difference between the electrical potentials of two surfaces of the nerve, such as the longitudinal surface and transverse section. These three properties exist only in living nerve. When a nerve is removed from the body, and deprived of an adequate supply of healthy blood, it dies, and the rapidity of its death may be measured by the gradual loss of these properties. Now, we know that these properties are affected by a stimulus applied to the nerve. When a portion of a nerve is subjected to the influence of a continuous current of electricity, that portion passes into a condition termed the electro-tonic state, in which these three properties are altered, and altered differently in the neighbourhood of each pole. It is difficult, however, to ascertain with accuracy, by physical methods, the alteration in the properties of sensibility and conductivity; but it is comparatively a simple matter

to ascertain changes in the electro-motive force of the nerve. It has been shown by numerous physiologists that, in the case of a motor nerve, the electro-motive force suffers a negative variation; that is, its amount is diminished during an active condition of the nerve. This negative variation of the natural electro-motive force may be looked upon as the electrical expression of the physiological process taking place in the nerve during the passage of the normal nerve current. Analogy would lead one to expect a similar change in the electro-motive force of a sensory nerve during the transmission along its fibres of the influence produced by the action of the normal stimulus on the terminal organ with which the sensory nerve is connected. Thus, variations in the electro-motive force of the retina and optic nerve may be regarded as functions of the external exciting energy, which in this case is light.

Such considerations led to this investigation. It resolved itself into a matter of careful experiment in two directions: 1*st*, to determine the electro-motive force of the visual apparatus and optic nerve; and, 2*d*, to ascertain whether or not, and to what extent, this electro-motive force was affected by light.

The only experiment bearing on the first of these questions, namely, the electro-motive force of the optic nerve, was one made by Du Bois-Reymond, and thus described:* "Having prepared the optic nerve of a large tench in such a manner that one extremity was the artificial transverse section, and the other the globe of the eye freed from all adherent particles of muscle, found that every point of the external surface of the ocular globe was positive to the artificial transverse section of the optic nerve, but negative to the longitudinal section." From this experiment it therefore seemed to be quite possible, by using a delicate instrument, to obtain a measurement of the electro-motive force of the retina and optic nerve.

With regard to the second question, namely, the action of light on this electro-motive force, the problem seemed difficult to solve, but an experiment made by Mr, now Justice, Grove, held out the hope of success. It is detailed as follows:†—"A prepared daguerreotype plate is enclosed in a box filled with water, having a glass front, with a shutter over it. Between this glass and the plate is a gridiron of silver wire; the plate is connected with one extremity of a galvanometer coil, and the gridiron of wire with one extremity of a Breguet's helix —an elegant instrument, formed by a coil of two metals, the unequal expansion of which indicates slight changes in temperature; the other extremities of the galvanometer and helix are connected by a wire, and the needles brought to zero. As soon as a beam of either daylight or the oxy-hydrogen light is, by raising the shutter, permitted to impinge upon the plate, the needles are deflected. Thus, light being the indicating force, we get *chemical action* on the plate,

* Morgan's Electro-Physiology, p. 458.

† Grove. The Correlation of the Physical Forces, 5th edition, p. 153.

electricity circulating through the wires, *magnetism* in the coil, *heat* in the helix, and *motion* in the needles." A consideration of this remarkable experiment led to the hope that we might be able, by a somewhat similar method, to observe the action of light on the electro-motive force of the retina and optic nerve.

The apparatus adopted for obtaining the normal electro-motive force was that of DU BOIS-REYMOND, usually employed in determining the electro-motive force of muscle or nerve. This consists of DU BOIS-REYMOND'S well-known non-polarisable electrodes, which are shallow troughs of zinc, carefully amalgamated, containing a solution of neutral sulphate of zinc, and having inserted into them cushions of Swedish filter-paper, on which to rest the preparation, which, in our experiments, was the eyeball or optic nerve. To protect the latter from the irritant action of the sulphate of zinc, a film or pad of sculptor's clay, moistened with ·75 per cent. solution of common salt, and moulded, if necessary, by the fingers, to a point, is placed on each cushion. These electrodes are connected with a galvanometer, a key being interposed by which the current from the electrodes may be shut off at pleasure. The eyeball, carefully freed from adherent particles of muscle, is now placed on a glass support between the two clay points, and these are now adjusted to it so that one touches the cornea, and the other the transverse section of the optic nerve. On now opening the key, the galvanometer at once indicates a strong current passing from the longitudinal to the transverse section of the nerve. (The arrangement of the apparatus will be understood by referring to the plate. Plate, No. X.)

The first experiment we performed was on the eye of a rabbit. The galvanometer employed was a large multiplying instrument made by SAUERWALD of Berlin. The animal was killed by chloroform, and the eyeball removed as quickly as possible from the orbit. It was then placed between the troughs in the manner already described. We obtained a deflection of from 10 to 15 degrees. On allowing the beam of light from a magnesium lamp to impinge on the cornea there was no effect observable. This experiment was repeated many times with the same negative results. It then became evident that a much more sensitive instrument was required, because the amount of deflection was so small, and the variation, if any, of this deflection produced by the action of light would probably be so minute as to escape notice.

We then used a sensitive THOMSON'S galvanometer, which was kindly placed at our service by Professor TAIT. With this instrument we at once obtained a deflection of several hundred degrees on a millimetre scale placed at a distance of 26 inches, so that we had now an opportunity of observing whether light produced any variation in the electro-motive force.

After placing the galvanometer in position, secondary difficulties now presented themselves. It will be necessary to allude to these difficulties, and the methods by which we avoided or overcame them.

1. *The gradual death of the Retina and Nerve.*—When the eyeball is placed on the cushions in the manner above described, and the key interposed between the electrodes and the galvanometer is opened, there is at once a large deflection. But this deflection diminishes as the nervous apparatus dies. At first the nerve dies by fits and starts, afterwards more slowly, and with considerable uniformity. After a lapse of from 20 to 30 minutes, the rate of dying, as measured by diminution of electro-motive force, is almost arrested, and the vitality of the nervous apparatus remains nearly constant. These facts will be evident on considering the data of the following experiment :—

RABBIT'S EYE.—*Rate of Death as measured by diminution of Electro-motive Force.*
Zero, 164. *Polarity*, 156. *Total of first deflection*, 118.

Time P.M. H. M. S.	Readings on scale.	Difference.	Time P.M. H. M. S.	Readings on scale.	Difference.
1 43 0	274	0	1 55 0	250	4
1 44 0	282	8	1 56 0	246	4
1 45 0	288	6	1 57 0	242	4
1 46 0	285	3	1 58 0	238	4
1 47 0	282	3	1 59 0	234	4
1 47 30	Zero 164.		2 0 0	231	3
1 48 0	278	4	2 1 0	230	1
1 49 0	274	4	2 2 0	239	1
1 50 0	270	4	2 3 0	228	1
1 51 0	266	4	2 4 0	228	0
1 52 0	262	4	2 5 0	228	0
1 52 30	Zero 164.		2 6 0	228	0
1 53 0	258	4	2 7 0	228	0
1 54 0	254	4			

Time occupied in above experiment, 24 minutes.
Total fall in electro-motive force during that period 46°.
The polarity remained constant throughout the experiment at 156°.

2. *Change of the Zero point of the Galvanometer from Magnetic Alterations.*—This was greater at certain times than at others. On several occasions the variations were so sudden as to prevent any accurate experiments being made.

3. *Changes in the Polarity of the Electrodes.*—This was one of the greatest difficulties in the inquiry, but we have employed the most approved methods of overcoming it—such as by the use of neutral sulphate of zinc, by moistening the clay with a ·75 per cent. solution of pure chloride of sodium, and by connecting the electrodes together by a thick copper wire for several hours before using them. With a sensitive galvanometer it is almost impossible to maintain either a constant zero or a constant amount of polarity. This fact would be a very serious one in an inquiry such as this, if the variations to be observed in the deflections occurred slowly and through minute distances, because, in these circumstances, it would be manifestly impossible to discriminate between changes due to variations in polarity and those due to the action of light. But

we found that the changes produced by the action of light occurred suddenly, and lasted only for a brief period of time, so that there could be no possibility of mistaking them for changes in polarity. We have also eliminated as far as possible any error from variations in polarity by multiplying experiments and by performing them under as favourable conditions as could be devised. Frequently, during the course of a series of observations on an individual eye, the polarity was observed and any changes noted.

4. *Effects of Heat.*—The effects of radiant heat were eliminated by causing the light to pass through a spherical double shell of glass, having at least an inch of water between the walls. In our earlier experiments we found that heat produced immediately an increase in the electro-motive force. It was therefore absolutely necessary to eliminate radiant heat.

II.—Effects of Light on the Eye of the Rabbit removed from the Head immediately after Death.

Four series of experiments were made on one of the eyes of four individual rabbits, as follows:—

SERIES I.—Rabbit's Eye.—*Source of light, a magnesium lamp. Zero,* + 185; *polarity,* + 125; *total first deflection,* 130°.

Observations.	Degrees of Galvanometer scale.					Remarks.
	Immediately before the impact of light.	Immediately after the impact of light.	Immediately after removal of light.	Effect of the impact of light.	Effect of the removal of light.	
1	255	259	250	Rise of 4	Fall of 9	
2	250	256	250	Rise of 6	Fall of 6	
3	250	250	245	No rise	Fall of 5	
4	254	258	254	Rise of 4	Fall of 4	*Fresh transverse section of nerves now made.
5	255	258	254	Rise of 3	Fall of 4*	
6	185	186	179	Rise of 1	Fall of 7	
7	175	175	165	No rise	Fall of 10	
8	165	166	155	Rise of 1	Fall of 11	
9	155	157	147	Rise of 2	Fall of 10	
10	150	152	142	Rise of 2	Fall of 10	

The result of this experiment was, that *on the impact of light there was an average rise of nearly two degrees; the electro-motive force slowly diminished; and on the removal of light there was a rapid fall of from five to seven degrees.* The rise in this instance on the impact of light, we now know, was owing to the action of heat. At this early stage of the inquiry it was deemed necessary to focus by a lens the light of the magnesium lamp on the eyeball.

SERIES II.—RABBIT'S EYE.—*Source of light, a magnesium lamp. Neutral point,* + 195; *polarity,* + 175; *total first deflection,* 150.

Obs. 11. Before the impact of light the reading on the galvanometer scale was at 225, giving a total deflection of 150. On the impact of light it fell at once to 215—a fall of 10. The electro-motive force now diminished in the dark as follows, at intervals of 15 seconds :—Obs. 12 to 27—190, 185, 180, 175, 170, 165, 162, 160, 161, 159, 157, 156 155, 153, 152, 150, 149.

Obs. 29. A heated poker brought near it caused an immediate rise to 190. On the removal of the poker, it fell slowly to 153. The following four experiments were then made, the source of light being the flame of a magnesium lamp held at a distance of three feet, and allowed to fall directly on the eye :—

Observations.	Readings of Galvanometer scale before impact, at intervals of 15 seconds.	Effects of the impact of light.	Effects of the removal of light.	Remarks.
30	153, 153, 152, 152, 152, 151, 151, 150	143, or fall of 7	146, or rise of 3	The result of these four experiments is a fall of about 5 degrees on the impact of light, and a rise of 2 degrees on its removal.
31	145, 146, 146, 146, 145, 144, 144, 143	138, or fall of 5	140, or rise of 2	
32	140, 140, 140, 140, 140, 140	135, or fall of 5	137, or rise of 2	
33	135, 135, 135, 133, 133, 133	128, or fall of 5	130, or rise of 2	

After readjusting the eye on the cushions, the following experiments were made, in which the light was passed through a flat-sided glass vessel filled with water, one inch in thickness. The source of light, as before, was a magnesium lamp :—

Observations.	Before impact.	After impact.	After removal.	Effect of impact.	Effect of removal.	Remarks.
34	110	105	106	Fall of 5	Rise of 1	Result is a fall of about 3 degrees on impact of light, and a rise of 2 on removal of light.
35	104	103	105	Fall of 1	Rise of 2	
36	100	97	98	Fall of 3	Rise of 1	
37	94	91	92	Fall of 3	Rise of 1	
38	90	87	89	Fall of 3	Rise of 2	
39	86	83	85	Fall of 3	Rise of 2	
40	81	78	80	Fall of 3	Rise of 2	

SERIES III.—RABBIT'S EYE.—*Source of light, magnesium lamp. Zero,* − 60 ; *polarity,* − 80 ; *total first deflection,* 238.

Observations.	Before impact at intervals of 15 secs.	After impact.	After removal.	Effect of impact.	Effect of removal.	Remarks.
41	158, 158, 159, 159	155	156	Fall of 4	Rise of 1	Light fell in these experiments on the cornea.
42	155, 152, 152, 152	150	154	Fall of 2	Rise of 4	
43	155, 155, 155, 155	150	152	Fall of 5	Rise of 2	
44	152, 152, 152, 152	148	149	Fall of 4	Rise of 1	
45	150, 150, 150, 150	155	157	Rise of 5	Rise of 2	Light fell in these experiments on the back of the eye.
46	155, 154, 154, 155	160	161	Rise of 5	Rise of 1	
47	150, 150, 149, 148, 148, 148	152	153	Rise of 4	Rise of 1	
48	168, 168, 168, 168	166	167	Fall of 2	Rise of 1	The eye was readjusted on the electrodes, and light fell on cornea.
49	168	166	167	Fall of 2	Rise of 1	
50	168	165	168	Fall of 3	Rise of 3	
51	170	166	167	Fall of 4	Rise of 1	Readjusted.
52	180	176	178	Fall of 4	Rise of 2	
53	180	176	180	Fall of 4	Rise of 4	
54	180	178	180	Fall of 2	Rise of 2	

The general result of these observations is, that when light fell on the cornea, there was an average fall of over three degrees on the impact of light, and a rise of nearly two degrees on the removal of light ; while on the other hand, when the beam fell on the back of the eye, there was always a rise, which we believe to be attributable to heat.

Three observations were made on this eye with a lighted lucifer match as the source of illumination. It was found that if the match was held sufficiently near the eye so as to heat it, there was a sudden increase in the electro-motive force, to the extent of from ten to fifteen degrees; but the light from the match was insufficient to affect the retina. The effect of heat in increasing the electro-motive force was observed in many other experiments.

SERIES IV.—RABBIT'S EYE.—*Zero,* − 117 ; *polarity,* − 65 *to* − 80 ; *total first deflection,* 280.

Observations.	Before impact.	After impact.	After removal.	Effect of impact.	Effect of removal.	Remarks.
55	200	197	200	Fall of 3	Rise of 3	Source of light from a gas-flame. Passed through ½ in. of water.
56	196	195	No effect	Fall of 1	No rise	
57	195	194	No effect	Fall of 1	No rise	
58	196	194	195	Fall of 2	Rise of 1	
59	196	195	No effect	Fall of 1	No rise	
60	195	194	No effect	Fall of 1	No rise	

RABBIT'S EYE—(*Continued*).

Observations.	Before impact.	After impact.	After removal.	Effect of impact.	Effect of removal.	Remarks.
61	200	196	No effect	Fall of 4	No rise	Eye readjusted; light from magnesium lamp.
62	195	193	No effect	Fall of 2	No rise	
63	240	236	237	Fall of 4	Rise of 1	Eye readjusted; source of light was the gas-flame.
64	235	233	234	Fall of 2	Rise of 1	
65	244	242	243	Fall of 2	Rise of 1	
66	245	243	245	Fall of 2	Rise of 2	
67	245	243	244	Fall of 2	Rise of 1	Eye readjusted.
68	248	247	248	Fall of 1	Rise of 1	
69	244	240	244	Fall of 4	Rise of 4	Source of light from magnesium flame, passed through 1½ inch of water.
70	242	239	240	Fall of 3	Rise of 1	
71	240	238	238·5	Fall of 2	Rise of 5	
72	240	237	No effect	Fall of 3	No rise	
73	236	234	No effect	Fall of 2	No rise	
74	234	232	No effect	Fall of 2	No rise	

The result of the above twenty observations is that the impact of light produced a fall in the electro-motive force of about two degrees on the impact of light, while on the removal of light there was a rise of about one degree in most cases; but in a few there was no rise at all.

III.—EFFECTS OF LIGHT ON THE EYE OF THE FROG REMOVED FROM THE HEAD IMMEDIATELY AFTER DEATH.

At this stage of the inquiry it became evident that, owing to the rapid death of the nervous textures of warm blooded animals, it was difficult to obtain sufficiently great galvanometrical readings, the variations of which could be readily observed. We, therefore, began to employ the eye of the frog, the tissues of which, as is well known, retain vitality long after the systemic death of the animal. The eye was prepared by removing as far as possible all adherent tissues from the sclerotic, and was so placed between the electrodes that one electrode touched the cornea, while the other was in contact with the transverse section of the optic nerve. When the eye was placed in this position a large deflection was obtained, and frequently it was so great as to pass beyond the scale. As was the case with the eye of the rabbit, the electro-motive force slowly fell to a point at which it was on the whole tolerably permanent. Readings were not made until it reached this stable condition. To secure not only a large deflection, but also sensitiveness to light, accurate adjustment of the eye-ball on the cushions was necessary.

Ten series of experiments were made on ten frogs.

At this stage of the inquiry, it was found necessary to adopt arrangements for securing a sudden impact and a sudden removal of light without being obliged on each occasion to extinguish the flame which was the source of the light. For

that purpose we placed over the apparatus a rectangular box, blackened without and within, having a draw-shutter as seen in the diagram, Plate X. After placing the eye on the cushions the key was opened, and the amount of deflection observed. If not sufficiently great for experimental purposes, the eye was readjusted so as to secure accurate contact of the nerve with its corresponding cushion; if sufficiently great, the box was placed over it, and the eye was allowed for several minutes to remain in the dark. The impact of light took place on withdrawing the shutter. The change in the electro-motive force was noted at the instant of the impact. The subsequent change in the electromotive force produced by continuance of light was not noted till the spot on the galvanometer scale became tolerably steady. When this occurred the shutter was closed, and we noted the effect of the removal of light. In these experiments, therefore, time was not especially taken note of.

SERIES I.—Eye of Frog.*

The light from a magnesium lamp was passed through a glass vessel, containing a thickness of $1\frac{1}{4}$ inch of water, with the following result:—

Observations.	Before impact.	On impact.	During light.	After removal of light.	Effect of impact.	Effect of removal.
75	347	354	350	358,346	Rise of 7.	Rise of 8.
76	346	354	350	357,345	Rise of 8.	Rise of 7.
77	345	Went off the scale.	—	—	—	—

Two hours afterwards the following experiments were made with the same eye:—

Obs. 78. Galvanometer at 215. The eye nearly in the dark. On removing the paper from the galvanometer lamp, which was used as a shade, the electromotive force at once rose to 238. During the continuance of light the electromotive force fell to 225. On lighting the gas in the room it rose to 238, and then slowly fell during the continuance of light. On shutting off all the light it rose immediately to 230, and then fell to 205.

Obs. 79. Stationary at 205. Gas lit, rose to 224. In two minutes fell to 217, and in one minute more to 213. On turning off the gas, fell from 212 to 215, and then quickly to 197.

Two hours still later the following experiments were made with the same eye:—

Obs. 80. Stationary at 98. Gas lit in room, rose to 105, that is, a rise of seven degrees. During continuance of light, fell to 103. On removal of light, rose one degree.

* In the following series of experiments the zero, polarity, and total first deflection, are not given. These points, however, were always attended to, and observations were always made on a deflection amounting to, at least, 300 or 400 degrees.

Obs. 81. Stationary at 100. On impact of diffuse light, rose to 104, that is, a rise of four degrees. During continuance of light, fell to 102. On turning the light off, a slight and barely perceptible rise was observed.

SERIES II.—EYE OF FROG.

The eye having been adjusted on the clay cushions, the key was opened, and the spot of light reflected from the mirror of the galvanometer was at once deflected three inches off the galvanometer scale. Counting from the point of the polarity, this indicated a deflection of nearly 500 degrees. A small scale, divided into tenths of a millimetre, was attached to the galvanometer scale. The following observations were then made :—

1. A diffuse beam of light from a gas flame was passed through a globular glass jar, twelve inches in diameter, filled with a solution of ammoniacal sulphate of copper (blue light).

Observations.	Before impact.	On impact.	During light.	After removal of light.	Effect of impact.	Effect of removal.
82	57·2	59	56	58	Rise of 1·8	Rise of 2
83	56	58	55	59	Rise of 2	Rise of 4
84	55	56·5	54	55·5	Rise of 1·5	Rise of 1·5
85	55	56	55	58	Rise of 1	Rise of 3
86	58	59	56	57	Rise of 1	Rise of 1
87	52	53	52	54	Rise of 1	Rise of 2
88	51	52	50	53	Rise of 1	Rise of 3
89	51	51·5	49·5	50·2	Rise of ·5	Rise of ·7
90	45	46	47	48	Rise of 1	Rise of 1
91	46·5	47	46·2	47·5	Rise of ·5	Rise of 1·3

In all of the above experiments what we term the inductive effect, that is, the rise in the electro-motive force after removal of the light, was well marked. The impact produced a rise of about two degrees; there was a falling off during the continuance of light, and another rise on the removal of light.

2. In six experiments (Obs. 92 to 97) a lighted match, held up at a distance of two feet, produced a rise on impact of about one degree, a fall during continuance of light, and another rise, or inductive effect, of one degree on the withdrawal of light.

3. In all the previous experiments the glass of the galvanometer lamp was surrounded by a black shade, so as to prevent diffuse light from acting on the eye. It was now found (Obs. 98) that simply removing this protective shade caused at once a rise of 1·5 degrees; during the time the diffuse light acted on the eye there was a fall of about two degrees, and on again replacing the shade there was a rise of nearly two degrees.

4. The following experiments were then made by acting on the eye with

light after it had passed through a solution of bichromate of potash (red light) :—

Observations.	Before impact.	On impact.	During light.	After removal of light.	Effect of impact.	Effect of removal.
99	37	39	38	40	Rise of 2	Rise of 2
100	38	40	39	40	Rise of 2	Rise of 1
101	35·5	37·2	35·5	36·5	Rise of 1·7	Rise of 1
102	32	33·5	31	31·4	Rise of 1·5	Rise of ·4
103	36	38	34·5	35	Rise of 2	Rise of ·5
	At this stage the electro-motive force fell so as to permit readings to be made on the true galvanometer scale.					
104	340	360	350	355 and sudden fall	Rise of 20	Rise of 5
105	270	283	260	268 do.	Rise of 13	Rise of 8
106	235	240	235	238 do.	Rise of 5	Rise of 3
107	242	248	245	248 do.	Rise of 6	Rise of 3
108	235	240	238	239 do.	Rise of 5	Rise of 1
109	224	231	224	225 do.	Rise of 7	Rise of 1
110	217	224	217	219 do.	Rise of 7	Rise of 2
111	209	215	207	208·5 do.	Rise of 6	Rise of 1·5

These experiments show a rise on the impact of light, a fall during the continuance of light, and another rise, or inductive effect, on the withdrawal of light.

SERIES III.—Eye of Frog.

On placing the eye on the cushions, and opening the key, a deflection of nearly 500 degrees of the galvanometer scale was obtained. The readings were made on the attached scale just alluded to. The following observations were then made :—

Obs. 112. A platinum spatula was heated to a red surface in the flame of a Bunsen burner at a distance of $2\frac{1}{2}$ feet. Before it reached a red heat the galvanometer stood at 47. During the emission of red rays it slowly rose to 49. When spatula cooled, it fell to 48.

Obs. 113 and 114. A little carbonate of soda was placed on the spatula, and introduced into an almost colourless Bunsen flame. Rose from 48 to 51, and on withdrawal sank to 48. This experiment was repeated.

Obs. 115. A little nitrate of strontia was placed on spatula, and pushed into flame. Rose from 48 to 49·5. No inductive effect was observed.

Obs. 116. Experiment with nitrate of strontia was repeated. Rose from 50 to 53·5, and on removal sank to 50.

Obs. 117. Again repeated. Rose from 50 to 53. On removal rose from 52 to 54.

Experiments on the Eye of a Frog with a sudden Flash of Light.

A non-luminous Bunsen flame was held at a distance of $2\frac{1}{2}$ feet, and it was rendered instantaneously luminous by quickly placing the finger and thumb over the two air-holes at the base. The following are the results.—

Observations.	Before the flash.	After the flash.	Rise.
118	51	53·5	2·5
119	53	55	2
120	53·2	56	2·8
121	53	56	3
122	53	55·5	2·5
123	53	55	2

The Bunsen flame was removed to a distance of eight feet, and the experiment was repeated.

Observations.	Before the flash.	After the flash.	Rise.
124	53·2	55	1·8
125	53·2	54·8	1·6
126	53·2	54·6	1·4
127	53·2	54·4	1·2
128	53·8	54·4	·6
129	53·9	55	1·1

A series of observations of a similar nature to those just detailed were made on one of the eyes of other five frogs with exactly similar results; and it appears to us it would serve no useful purpose to give these in detail.

Experiments on the Action of Diffuse Daylight.

A number of observations were now made on the action of daylight on the eye of a frog. In these experiments diffuse daylight was admitted into the totally dark room by removing a piece of cardboard from one of the panes of the window. The following were the results:—

Observations.	Before impact.	On impact.	During light.	Removal of light.	Effect of impact.	Effect of removal.	Remarks.
130	85	100	100	105	Rise of 15	Rise of 5	
131	135	140	140	142	Rise of 5	Rise of 2*	
132	214	220	225	228	Rise of 6	Rise of 3	* Eye readjusted on pads.
133	230	240	240	250	Rise of 10	Rise of 10	
134	235	250	250	255	Rise of 15	Rise of 5	
135	274	285	285	290	Rise of 11	Rise of 5	Gradual rise of the electro-motive force during this experiment.
136	292	300	305	307	Rise of 8	Rise of 2	
137	310	326	326	328	Rise of 16	Rise of 2	
138	314	330	332	340	Rise of 16	Rise of 8	
139	325	338	338	342	Rise of 13	Rise of 4	

These experiments show that diffuse daylight produces results similar to those obtained by the action of rays from artificial luminous sources.

The action of light on the eye of the frog may be briefly stated as follows: When diffuse light is allowed to impinge on the eye of the frog, after it has

arrived at a tolerably stable condition, the natural electro-motive power is in the first place increased, then diminished; during the continuance of light it is still slowly diminished to a point where it remains tolerably constant; and, on the removal of light, there is a sudden increase of the electro-motive power nearly up to its original position.

IV.—Experiments as to the Action of the various Rays of the Spectrum.

With the view of observing the action of the various rays of the spectrum, we attempted, in the first instance, to place the eye in different parts of the spectrum produced by interposing a prism in the path of a beam from a lime light. We made numerous observations in this way. Another, and more satisfactory method was to pass light through a rectangular glass vessel, about one inch in thickness, containing a coloured liquid, the absorptive powers of which had been previously determined by the spectroscope. Both of these methods produced substantially the same results, which will be seen in the following record of observations made on the eye of a frog, which was in the condition giving a rise on the impact of light and a fall on its removal:—

Observations.	Beam passed through.	Absorptive effect of fluid on the Spectrum.	Effect of light on the Eye.				
			Before impact.	On impact.	Removal of light.	Effect of impact.	Effect of removal.
140	Water.	None.	200	220	200	Rise of 20	Fall of 20
141			200	220	200	Rise of 20	Fall of 20
142			210	225	220	Rise of 15	Fall of 5
143	Bichromate of potash.	Absorbs all beyond the low green—*red light.*	220	230	222	Rise of 10	Fall of 8
144			220	230	220	Rise of 10	Fall of 10
145			220	230	220	Rise of 10	Fall of 10
146	Chrome alum.	Absorbs all the yellow, orange, and a portion of the low green, leaving low red and violet—*rose tint.*	226	230	227	Rise of 4	Fall of 3
147			228	232	228	Rise of 4	Fall of 4
148			226	230	227	Rise of 4	Fall of 3
149	Neutral chromate of potash.	Absorbs all above the high green—*yellow light.*	230	238	225	Rise of 8	Fall of 13
150			220	231	221	Rise of 11	Fall of 10
151			221	232	221	Rise of 11	Fall of 11
152			221	232	220	Rise of 11	Fall of 12
153	Ammoniacal sulphate of copper.	Absorbs all below the high green—*blue light.*	235	239	234	Rise of 4	Fall of 5
154			237	241	236	Rise of 4	Fall of 5
155			240	244	239	Rise of 4	Fall of 5
156			241	246	241	Rise of 5	Fall of 5
157	Sulphate of nickel.	Low and high end of spectrum absorbed — *green light.*	246	250	246	Rise of 4	Fall of 4
158			244	250	246	Rise of 6	Fall of 4
159			249	260	250	Rise of 11	Fall of 10
160			250	261	251	Rise of 11	Fall of 10
161	Water, second time.	None.	256	270	257	Rise of 14	Fall of 13
162			255	270	255	Rise of 15	Fall of 15
163			255	269	255	Rise of 14	Fall of 14

These experiments show—(1), that the part of the spectrum which to our consciousness is the most luminous, namely, the yellow rays, produce the greatest effect in altering the electro-motive force of the retina; (2), that if we divide the spectrum into two parts, as, for example, into one portion above the high green, and another portion below the high green, the summation of the effects produced is about equal to that of white light; and (3), that certain of the colours of the spectrum may be arranged in the following order, in their greater relative power in altering the electro-motive force—yellow, green, red, blue.

V.—Crucial Experiment as to the Effects of Light on two Eyes placed on the Galvanometer Cushions in Reversed Positions.

Two eyes of the same frog were dissected out in the ordinary way. The first, which we will call No. 1, was so placed on the cushions as to give a deflection of nearly 400° to the right (the observer being supposed to be seated with his back to the galvanometer and his face to the scale). The other eye, No. 2, was placed in the reversed position. A current was thus obtained from No. 2, which passed through the galvanometer in the opposite direction to that of No. 1; but as the current of No. 1 was stronger than that of No. 2, the deflection of No. 1 was only considerably reduced. It is evident that we could now study the effect of light on No. 1 by placing a small black paper cap on No. 2, and *vice versâ*. The effects were what might have been anticipated. When one eye was thrown out of gear by not being subjected to the action of light, the deflection produced by the electro-motive force of the other eye was altered in the usual way. Suppose, for instance, that the impact of light in both cases produced (as in this instance it did) an increase in the electro-motive force, we would expect that when No. 2 was covered, the spot of light on the galvanometer scale would go to the right (as indicated by the rise of numbers in the following figures); whereas, if No. 1 was covered, the spot of light would go to the left (as indicated by the fall of numbers in the following figures). That is to say, when No. 2 was covered, and No. 1 acted, light caused an increase in the electro-motive force of No. 1 (to the right); whereas when No. 1 was covered, light caused an increase in the electro-motive force of No. 2 (to the left). Such was the case.

Paper cap on No. 1, the deflection of which is to the right. No. 2 was in action.

Obs. 164	Before light, 165	Impact, 163	Continued, 153	Removal of light, 150
„ 165	„ 175	„ 173	„ 165	„ 160
„ 166	„ 180	„ 177	„ 167	„ 165
„ 167	„ 175	„ 173	„ 163	„ 160
„ 168	„ 176	„ 174	„ 165	„ 160

Paper cap on No. 2, the deflection of which is to the left. No. 1 was in action. Eyes readjusted on the pads.

Obs. 169	Before light,	210	Impact,	212	Continued,	205	Removal of light,	205
„ 170	„	190	„	198	„	190	„	191
„ 171	„	178	„	185	„	178	„	182
„ 172	„	175	„	180	„	173	„	178
„ 173	„	168	„	174	„	168	„	173

VI.—Experiments with Varying Luminous Intensity.

For this purpose the eye was carefully guarded from extraneous light, as previously detailed, and the source of light was allowed to act on the eye at different distances. The following experiments were made on an eye of a frog that had been removed from the body two hours previously, and had been during that time lying on the clay pads of the electrodes :—

a. *Obs.* 174 to 178.—The gas flame was placed at a distance of *one* foot. Rose eight degrees ; fell eight degrees below original point during action of light ; and rose twelve degrees on removal of light. This was repeated five times with the same results.

b. *Obs.* 179 to 184.—The gas flame was now placed at a distance of *four* feet. The result was that on impact, rose five degrees, during continuance of the light fell five degrees below the original point, and there remained stationary. On removal of light rose to the original point. This happened in six experiments.

c. *Obs.* 185 to 188.—The gas flame was then placed at a distance of *eight* feet. Rose one degree ; fell during continued light five degrees below original point ; and rose five degrees on light being removed. This was repeated four times with same result.

From these observations it results that when the luminous intensity varies in the proportion of 64 to 1, the electrical variation is as 8 to 1, showing that it by no means varies directly as that of the luminous source, but much more slowly. At various times we have made observations on the diminution of electrical effect from a difference of luminous intensity in the proportion of 100 to 1, and as a mean of these observations we have found the electrical variation in different eyes to lie between proportions of 3 to 1 and 6 to 1. That is to say, if a luminous source, at a distance of ten feet, gives an initial impulse corresponding to one degree of our galvanometer scale, then at a distance of one foot the electrical effect would be from three to six degrees. In this comparison, we take the alteration produced in the galvanometer on the impact of light as the measure of the electrical variation, because the eye in this case is always compared under similar conditions, having been previously for a certain time in the dark, so as to allow it to recover from the effect of a previous experiment, whereas if we regarded the electrical effects produced on the withdrawal of light as comparable, we would have to deal with different degrees of

exhaustion of the retina and the action of light during different times. All such questions are left for examination in a subsequent paper on the effects of fatigue. Similar results to the above have been found for the compound eye. The natural query then arises, Are the physical effects we have described and measured really comparable in any way with our sensational differences in light perception when we eliminate all mental processes of association, &c., and leave only perception of difference of intensity? In other words, are these changes representative of what is conveyed to the sensorium? It would appear at first sight that this problem is altogether beyond experimental inquiry. There is, however, a way of arriving at very accurate measures of the variation of our sensational differences in the case of light, and this has been developed theoretically and experimentally by the justly renowned German physiologist FECHNER, and more recently extended and verified by DALBŒUF of the Belgian Academy. FECHNER's law as applied to light may be stated as follows:—If we call I and S respectively the luminous and sensational intensities, and a and k constant quantities, then the following equation is found to express the relation of the above quantities, viz.:—

$$S = a \log I + k \quad . \quad . \quad . \quad . \quad (1).$$

From this it follows, if we take any two new values, S_1 and I_1, we get the following relation—

$$S - S_1 = a \log \left(\frac{I}{I_1}\right) \quad . \quad . \quad . \quad . \quad (2).$$

That is, the difference of our sensations is directly proportional to the logarithm of the quotient of the luminous intensities. The formula (1) is, however, defective when considered with regard to small values of I, and gives an imaginary quantity when it becomes zero, viz., infinity. In order to remedy this defect, DALBŒUF has introduced a constant quantity along with the value of the luminous intensity, that may be regarded as representative of the normal condition of the retina in darkness, or the proper light of the eye. Not that there is a certain amount of light when the eye is kept in the dark, only that the receiving organ has a natural susceptibility that must be included in the equation.

The expression (1) becomes when we insert this constant quantity, C,

$$S = a \log (C + I) + k \quad . \quad . \quad . \quad . \quad (3).$$

And as S must be nothing when I is zero, we have—

$$S = a \log \left(\frac{C + I}{C}\right) \quad . \quad . \quad . \quad . \quad (4).$$

This formula has been verified experimentally by DALBŒUF, using the method of successive contrasts introduced by PLATEAU, and from it he has succeeded in deducing the value of the unknown constant C. The value of this constant is found in his experiments to vary at different times between the values 0·1 and

0·5. Introducing these values in equation (4), we can find what is the ratio of the sensational intensities for any assigned values of the luminous intensity. In this way, if the amount of light varies as 100 : 1, then the corresponding sensational values may differ as 3 : 1, or 5 : 1. Contrasting the above values with the electrical variations previously given for the same difference of luminous intensity, we observe a close agreement that can scarcely be regarded as accidental. It would thus appear that the psychophysical law of FECHNER is not dependent alone on perception in the brain, but in part on the structure of the eye itself. The effects which occur on, during, and after the action of light on the retina also take place after the eye has been removed from all connection with the brain. Thus the law of FECHNER is not a function of the brain alone, but is really a conjoined function of the brain and the terminal organ, the retina.

VII.—Experiment on the Effect of Moonlight.

Although in previous experiments we endeavoured to eliminate the effects of heat from those of light, we thought it advisable to note the effect of moonlight, which is well known to contain heat rays so few in number as to require the most sensitive arrangement for their detection. The experiment was accordingly made on a clear moonlight night. The troughs, with the eye placed on the cushions, were carried into the open air, and were connected by long wires with the galvanometer. It is sufficient to state that in these circumstances we found the eye as sensitive to moonlight as to the light of a candle placed at a distance of five or six feet.

VIII.—Effects of Certain Active Substances on the Sensibility of the Retina.

The subcutaneous injection into the frog of lethal doses of woorara, santonin, belladonna, morphia, and Calabar bean, does not destroy the sensibility of the retina to light. The change in the electro-motive force was not altered quantitatively to such an extent as to be appreciable. This is most remarkable in the case of santonin, which is well known to produce in man sensations of yellow light.

IX.—Examination of other Textures of the Eye.

It is manifest that in an investigation such as this, it is of great importance to examine the action of light on the various textures or tissues composing the globe of the eye and the adjacent skin, so as to determine whether or not these are affected by light.

1. *The Skin.*—As has already been shown by DU BOIS-REYMOND and many others, there is no difficulty in obtaining a strong deflection when the clay points are applied to the skin of the frog, one to the inner and the other to the outer surface. From the well-known experiments of LISTER on the pigment cells in the skin of the frog, in which he showed that the pigment molecules changed

position under the action of light, it might possibly have been expected that there would be a change in the electro-motive force. It has been found, however, in this research, that this current is not affected by light.

2. *Pigment Cells of the Choroid.*—A current is readily obtained from the pigment cells of the choroid, when a portion of this membrane is laid on the clay points. This current does not exhibit any sensitiveness to light.

3. *Sclerotic and Nerve without the Retina.*—These structures give a considerable current not affected by light.

4. *Anterior Segment of the Eyeball.*—The eye of a frog was bisected so as to include in the anterior segment the cornea, iris, and lens. This preparation was then placed on the pads, so that the corneal surface was in contact with the one pad while the posterior surface of the lens was in contact with the other. A strong deflection was obtained, which was, however, not affected by light.

5. *Posterior Segment of the Eyeball.*—On bisecting an eye of a frog, so as to remove completely the anterior portion, including cornea, aqueous humour, iris, ciliary muscle, and lens, and on bringing the retina into actual contact with one of the clay pads, we obtained a large deflection, which was as sensitive to light as when the whole eye was employed, thus eliminating any possibility of contractions of the iris under the stimulus of light, having to do with the results previously obtained.

6. *Distribution of the Electro-motive Force of the various Textures of the Eye.*—The distribution of the electro-motive force between the different portions of the eye and cross section of the nerve may be stated as follows:—The most positive structure is the cornea, then the sclerotic, then the longitudinal surface of the nerve; the cornea is also positive to the posterior surface of the crystalline lens, and the retina itself is positive to the transverse section of the nerve.

X.—Experiments on the Eyes of Living Birds and Mammals.

At this stage of the inquiry, we examined the action of light on the eyes of living animals. In our earlier experiments on the eyes of rabbits, cats, and dogs, we found that sensibility to light disappeared with great rapidity. This phenomenon is in conformity with the well-known fact, that the sensibility of the nervous system in warm-blooded animals quickly disappears after death from loss of arterial blood. It was important to ascertain, if possible, whether or not the variations in the electro-motive force, by the action of light, which we had noticed in the eyes of various animals removed from the head, also occurred in the eye while it remained in the head, and was nourished by blood. We, accordingly, instituted a series of experiments which were practically very troublesome. We examined the eye—(1), of the living cat; (2), of the living pigeon; and (3), of the living owl (*Strix flammea*, Lin.) In all cases the animals were deeply under the influence of chloroform during the experiments.

1. *The Cat.*—The animal was securely fixed in CZERMAK's rabbit-holder. The skin around the orbit was reflected. The zygomatic arch was snipped through by bone forceps, so as to expose as much as possible of the side of the orbit. The cellular tissue of the orbit was then pushed aside along the superior and the lateral aspect of the eyeball, so as to reach the optic nerve with as little disturbance as possible to the vascular arrangements of the eyeball. On exposing clearly the optic nerve, and staunching hæmorrhage, the nerve was cut through transversely with sharp scissors. When this was done, the globe could be pulled downwards, inwards, and forwards, so as to expose a clear transverse section of the nerve. With the head firmly fixed, one narrow clay point was now placed on the cornea, while the other was in contact with the transverse section of the nerve. On opening the key a strong deflection was obtained, in several instances so powerful as to drive the spot of light off the scale. In one experiment it was for a long time impossible to obtain sufficiently slight contact to procure a deflection small enough to lie on the galvanometer scale. In several experiments we succeeded in tracing the curve made by the movement of the spot of light on the surface of a horizontal cylinder, and it will be seen, on referring to this curve, as shown in the annexed plate (Plate XI.), that on the impact of light the electro-motive force diminished, during its continuance the electro-motive force gradually rose to a point where it became steady, and on the withdrawal of light the inductive effect, that is, a rise in the electro-motive force, was well marked.

2. *The Pigeon.*—The eye and optic nerve of the pigeon were more accessible than those of the cat, and the head of the bird was more easily held in a stable condition. The eye was found sensitive to light, and followed the same law as that of the cat.

3. *The Owl.*—As this bird is believed to be capable of seeing objects with a very feeble intensity of light, we considered it important to examine its eye in the same manner as we examined that of the pigeon. We found exactly the same results. We could not detect any quantitative differences between the effects of light on the eye of the pigeon and on the eye of the owl.

XI.—Experiments on the effect of Light, in which a portion of the Brain was included in the Circuit.

A number of experiments were made with the view of determining how far into the brain substance it was possible to observe any electrical variation produced by the action of light on the retina, the optic nerve and the eyeball remaining in their natural position.

1. *The Frog.*—On bisecting the head of a newly killed frog with a sharp pair of scissors, it is possible to obtain a longitudinal section of the brain, with

the various parts in their natural position, then to carefully cut away the anterior and posterior portions of the brain, leaving only the middle portion in contact with the optic nerve. On placing this preparation between the clay points, so that the one touches the surface of the cornea, while the other is in contact with the brain substance, a strong deflection is obtained which is sensitive to light, and follows the course observed in the case of the frog.

2. *The Pigeon.*—The effect was also traced into the optic lobes of a living pigeon (deeply under chloroform), the head of which was securely held between the clay points of the electrodes. The optic lobes in the pigeon are easily exposed. The following were the effects of this observation:—*a*, When one pole was applied to the left optic lobe, and the other to the cornea of the right eye, a deflection was obtained which was sensitive to light; *b*, when the pole was removed from the right eye, and applied to the cornea of the left, a smaller deflection was obtained, also sensitive to light; and, *c*, when light was allowed to impinge on both eyes, while the one pole was in contact with either eye, and the other with the left optic lobe, the result was nearly double that produced by the impact of light on one eye alone, either right or left. These effects may be explained by the decussation of the optic nerves in the optic commissure.

XII.—Experiments on the Eyes of Fishes.

The subject was examined in the case of three fishes—(1), the gold fish (*Cyprinus auratus*); (2), the rockling (*Motella vulgaris*); and (3), the stickleback (*Gasterosteus brachurus*).

1. *The Gold Fish.*—With the eye of this fish a deflection of about 600° was obtained with the smallest possible contact. On the impact of light it rose 50°, remained steady for a short time, and then slowly sank. On removal it fell about 8° or 10° below the starting point.

2. *The Rock Fish.*—Specimens of this fish were obtained from the rock-pools in the neighbourhood of Kinghorn in Fifeshire. The deflection was similar in amount to that of the gold fish, but the variations occurred in a much longer period of time.

3. *The Stickleback.*—The species examined is that frequently met with in brackish pools, a little beyond the level of high-water mark, at the sea-side. The eyes examined were very small. A deflection of 400° was obtained, which was remarkably sensitive to light, the variations occurring in very short periods of time.

On comparing these results, it will be seen that, in the active and alert stickleback, the variations occurred rapidly; in the more sedate goldfish, they occurred more slowly, while, in the case of the sluggish rockling, which hides beneath stones near low-water mark, the variations occurred very slowly.

XIII.—Experiments on the Eyes of Amphibia and Reptiles.

Observations have been made on—(1), the frog (*Rana temporaria*); (2), the toad (*Bufo vulgaris*); (3), the newt (*Triton aquaticus*); and (4), a snake (species unknown). Those made on the frog have been already fully described. The action on the eye of the toad was found to be exactly similar to that of the frog, only the variations were much more sluggish. In the case of the newt, the eye was found to be sensitive to light to the extent of about 12 per cent. of the electro-motive force. It was found also to be sensitive to a flash of light. These results were obtained in an eye having the following dimensions :—

Transverse diameter of eye, $\frac{1}{8}$th of an inch.
Diameter of the pupil, $\frac{1}{30}$th of an inch.
Diameter of the lens, $\frac{1}{20}$th of an inch.

The eye of the snake* resembled that of the frog.

XIV.—Experiments on the Action of Light on the Compound Eye.

We have examined the eyes of the following crustaceans, namely—(1), the common edible crab (*Cancer pagurus*); (2), the swimming crab (*Portunus puber*); (3), the spider crab (*Hyas coarctatus*; (4), the hermit crab (*Pagurus Bernhardus*); and (5), the lobster (*Homarus vulgaris*).

1. *The Common Edible Crab.*—In this case it was found that the nerve surface was positive to the other parts of the eye, not negative, as had been shown in the simple eye of various vertebrate animals. A deflection was obtained to the extent of about 300° of the scale of the galvanometer. The effect of light was to increase the current from 5° to 10° of the galvanometer scale, then to fall slightly. During the action of light the deflection remained very permanent, and on the withdrawal of light it fell.

2. *The Lobster.*—With this eye the following points were noted :—

(1.) The direction of the current was reversed, as in the case of the crab, that is to say, the corneal surface was negative, not positive, as was observed in the simple eye.

(2.) A deflection of about 600° of the scale were obtained with the smallest possible contact of the clay points with the cornea and nerve surface.

(3.) The action of light was to diminish the current to the extent of 10 per cent of the total amount of deflection, and the inductive effect, or effect of removal of light, was nearly equal to the depression produced by the impact of light.

* Kindly sent us by Mr Bartlett of the Zoological Gardens, Regent's Park. Genus and species not given.

(4.) It was found in this eye that at a distance of one foot, on the impact of light, there was a depression of 90° to 103°; at a distance of ten feet, there was a depression of from 15° to 20°. Here the depression did not at all correspond to the intensity. At a distance of ten feet, when the amount of light was $\frac{1}{100}$th part of what it was at one foot, the diminution in the alteration of electro-motive force by the action of light was not $\frac{1}{100}$th, but $\frac{1}{3}$th of the total amount.

3. *The Swimming Crab; and,* 4, *the Spider Crab.*—These animals, dredged from a depth of twenty fathoms from the Firth of Forth, had eyes which were found sensitive to light.

5. *The Hermit Crab.*—The length of the eye examined in this case was about one millimetre. Its breadth was about one-fourth of a millimetre. Still it was found very sensitive to light.

XV.—Experiments on the Effect of Fatigue on the Eye.

A few experiments, preliminary to a more detailed examination of the question, have been made as to the effect of fatigue on the eye, as estimated quantitatively by the amount of change in the electrical potential of the visual structures. We may, in this communication, shortly allude to these. Suppose we place the eye of a frog in connection with the clay pads of the troughs of the galvanometer. The box with the draw-shutter, already described, is placed over it, and one candle, giving a steady flame, is put a distance of twelve to sixteen inches from the box. The draw-shutter is now opened, and the amount of deflection on impact of, during the continuance of, and on the removal of, light is noted. If we then light another and exactly similar candle, so as to double the amount of light, the variation in the electro-motive condition of the eye is not much altered, certainly not to the extent of double the amount of that produced by one candle. If we then extinguish one candle, a slight alteration occurs; but when we extinguish the remaining candle, there is at once a great variation in the deflection of the galvanometer. That is to say, the eye becomes less sensitive to any increase in the intensity of light after it has become for some time subjected to the action of a light of certain intensity; or, in other words, it becomes fatigued. This experiment also shows that the eye is more sensitive to variations in light of weak intensity than to variations in light of great intensity.

XVI.—Modes of Graphic Registration in these Experiments.

During the course of our observations we have found it necessary to construct a true graphical representation of the variations of the electro-motive force occasioned by the impact, duration, and removal of light. It is clear that to register minute galvanometrical alterations from a Thomson's instrument, the only plan

that could be employed, yielding perfectly accurate results, would be to photograph on a sensitive surface, covering a cylinder rapidly revolving on a horizontal axis, the alteration of position of the spot of light reflected from the mirror, just as continuous magnetic observations are registered. As the apparatus required to execute these observations is very complicated, and will only become necessary in the second part of this memoir, where we must deal with the time occupied in the transmission of retinal impressions due to different coloured rays, we have, in the meantime, adopted two simple methods of registration. The first plan is to note the position of the galvanometer at equal intervals of time before, during, and after the impact of light on the eye. In these observations we have used a seconds pendulum giving a loud beat. One observer reads aloud the galvanometer, the other marks every interval of two seconds, registers the numbers obtained, and regulates the supply of light. A little practice in the method above described has enabled us to obtain very satisfactory results, agreeing very closely in different observations, and showing in a decided way the salient points of the variation curve (see curves, Plate XI.). The second plan of registration which may be of service in many physical and physiological researches, consists in placing at proper distances from the galvanometer, instead of the ordinary graduated scale, the surface of a cylinder covered with paper, and moving on a horizontal axis by clockwork. The spot of light reflected from the galvanometer mirror is rendered more precise by having the shade of the galvanometer lamp blackened over the entire surface, with the exception of a spot about three millimetres in breadth, in the centre of which a line or cross is made of soot. The image of this line or cross is, of course, reflected by the mirror upon the cylinder. When the cylinder is set in motion by the clockwork, the spot of light may be accurately followed by the hand of the observer after a little practice, with a fine brush moistened with ink. The cylinder we employed performed a complete revolution in eighty seconds. This time was divided into four equal parts, each representing twenty seconds, by four lines drawn transversely at equal intervals across the paper on the cylinder. The first space, between lines one and two, represented twenty seconds, in which the eye was in the dark, and in which the electro-motive force is represented by a straight line; the second space, between lines two and three, represented twenty seconds, during which the effect of the impact of light took place, and in which the variation of the electro-motive force is indicated, either by a curve to the right or to the left; the third space, between lines three and four, represented twenty seconds of continued action of light, during which the electro-motive force gradually rises; and lastly, the fourth space between lines four and one (the point of starting), represented twenty seconds, during which the electro-motive force at first rises on the withdrawal of light, and afterwards sinks rapidly. Illustrations of curves registered by

the two methods described above will be found in Plate XI. appended to this paper.

As this paper embodies only the experimental results obtained from a general survey of the subject, leaving many minute and intricate points of difference between the result of the natural stimulus applied to the nervous structure of the eye and the effect of stimulating a motor nerve (that is, what is called a negative variation of the normal nerve current), we do not enter into any theoretical explanations regarding the cause of these peculiarities, but leave all such matters for discussion and further elaboration in the second portion of this paper, on which we are at present engaged. In the meantime we may succinctly summarize the results of this part.

XVII.—Summary of Part I. on the Physiological Action of Light.

We have experimentally proved—(1), that the impact of light on the eyes of members of the following groups of animals, viz., Mammalia, Aves, Reptilia, Amphibia, Pisces, and Crustacea, produces a variation amounting to from three to ten per cent of the normal electro-motive force existing between the corneal surface and the transverse section of the nerve; (2), that this electrical alteration may be traced into the brain; (3), that those rays that we regard as most luminous produce the largest variation; (4), that the alteration of the electrical effect with varying luminous intensity seems to follow very closely ratios given by the psycho-physical law of Fechner; (5), that the electrical alteration is due to the action of light on the retinal structure itself, as it is independent of the anterior portion of the eye, eliminating, therefore, the natural supposition that the contraction of the iris might produce a similar result; (6), that it is possible, by experiment, to discover the physical expression of what is usually called in physiological language, fatigue; and (7), that the method employed in this research may be applied to the investigation of the special organs of the other senses.

[Appendix.

APPENDIX.

18*th February* 1874.

While this paper was passing through the press, we have been fortunate enough to have obtained information from a Swedish lady that a paper relating to the same kind of work had appeared in a Swedish Journal. We, accordingly, endeavoured to obtain this paper, which we find to be one by Professor HOLMGREN, a distinguished physiologist, published in the "Upsala Lakeref̈orenings F̈orhand lingar," vol. vi. 1870–71, No. 5, p. 419. It is with pleasure we at once acknowledge that he has the claim of priority in observing an electrical fluctuation by the action of light, and that his memoir is a valuable contribution to science. It is, however, almost needless to state that our work was done in entire ignorance of any previous observations on the subject, and that our methods of experiment, the delicacy of our instrument, and the distinct numerical data obtained, have enabled us to prosecute the matter in various new directions.

EXPLANATION OF THE PLATES.

PLATE X.

This Plate shows the arrangement of the apparatus.

A. Galvanometer.

B. Scale.

C. Key.

D. DU BOIS-REYMOND'S non-polarizable electrodes having the eye placed between the clay points.

E. Moist chamber, consisting of an annular glass vessel of at least two inches section, filled with water, and placed over the electrodes to keep the temperature uniform, and absorb radiant heat. Over this is placed a blackened box, the outlines of which are marked in dotted lines, having a moveable shutter in front, F.

G. Lamp.

PLATE XI.

This plate shows curves representing variations in the electro-motive force of the eye produced by the action of light; or they represent the path pursued at different times by the image reflected from the galvanometer mirror if the plate were supposed to move vertically upwards. The dotted lines represent the normal position of the galvanometer-image in the dark; and the complete dark lines represent the position of the image during and subsequent to the action of light.

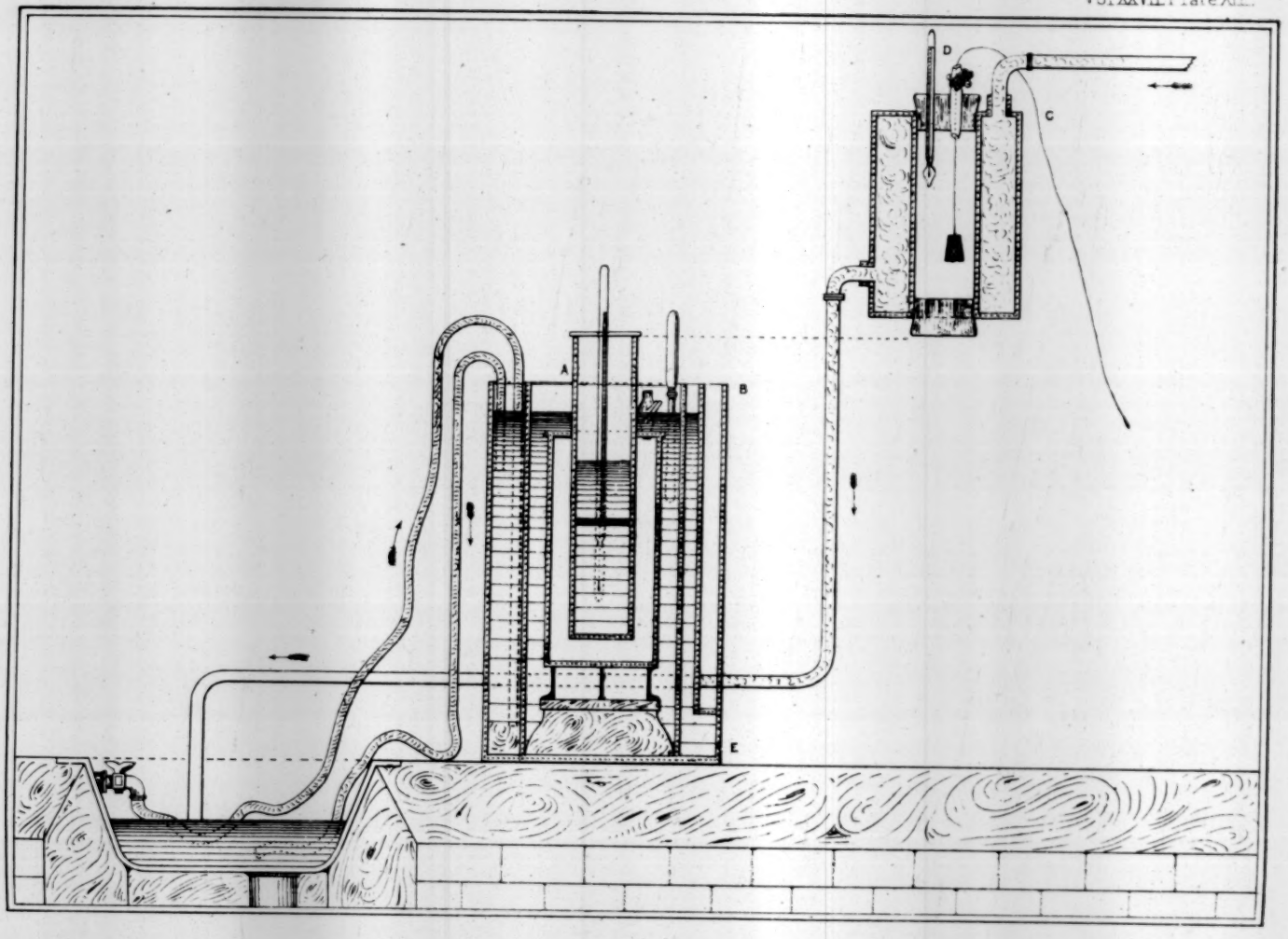
I. and II. Actual curves taken from the living eye of the cat on a cylinder moving horizontally.

III., IV., and VI. Graphical curves representing the general action taken by time observations. III. and VI. Frog. IV. Toad. V., VII., VIII., and IX. Actual curves taken on a cylinder from the eye of a frog.

X. and XI. Curves obtained from eye of cat removed as quickly as possible from dead body.

The four dotted horizontal lines divide the time of the experiment into four equal intervals, namely, 1*st*, in darkness; 2*d*, during action of light; 3*d*, during continued light; and, finally, on removal of light. A movement to the right of the dotted line indicates an increase, and to the left a diminution, in the electro-motive force.

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A
B
C
D
E

VIII.—*On the Physical Constants of Hydrogenium.* By JAMES DEWAR. (Plate XII.)

(Read January 20, 1873.)

In March 1869 I communicated to the Society a paper entitled "Motion of a Palladium Plate during the Formation of GRAHAM's Hydrogenium," which appears in the Proceedings for Session 1868–69. When engaged with this subject many points of interest regarding the behaviour of palladium containing occluded hydrogen, suggested themselves for investigation, and in concluding the paper I remarked "that careful determinations must be made of the electromotive force, latent heat, &c., of hydrogenium" before we could arrive at any conclusion regarding the condition of the absorbed hydrogen. Subsequently Professor TAIT made a series of determinations on the "Electrolytic Polarisation of Palladium Electrodes,"* devising a new and ingenious method for the purpose. Although at different times subsequent to my first communication, the problem of determining the physical constants of hydrogenium recurred, as my attention was in the meantime directed to the specific heat of carbon at high temperatures, no progress was made with the investigation until September 1872, when the results of my preliminary experiments were communicated to the "Philosophical Magazine," under the title, "Note on the Specific Heat of Hydrogenium." In that note it is stated that by means of a specially constructed calorimeter the specific heat of hydrogen in palladium is found to be 3·1 per atomic weight, nearly identical with that of gaseous hydrogen. The present paper deals with some of the physical constants of hydrogenium, more especially with the specific gravity, specific heat, and co-efficient of expansion.

GRAHAM, in his celebrated paper on hydrogenium, made many determinations of the specific gravity of the occluded hydrogen by observing the increase of length of palladium wire after being fully charged, thus finding the cubical expansion, and from it deducing the weight of unit volume of the absorbed hydrogen.

From experiments made in this way he found the specific gravity to be nearly 2. Afterwards he discovered the value was about three times what it ought to be from a contraction of length occurring when palladium wire is used. This he confirmed by the use of alloys of palladium that resist this contraction, and finally regarded the specific gravity as 0·733. No determinations were made when the palladium was partially saturated, and he rejected the ordinary

* Proceedings of the Royal Society of Edinburgh, Session 1868–9.

process of taking specific gravities because of the continual evolution of gas preventing exact weighings being taken in water.

In the experiments to be detailed a cubical mass of the metal was charged with hydrogen by electrolysis, taken at different times during the progress of the saturation, and weighed in air and in water. If the mass was allowed free exposure to the air for several hours little difficulty arose from the evolution of gas when immersed in water, and accurate results could be obtained.

From the experimental numbers the specific gravity of the absorbed hydrogen was calculated by the well-known formula—

$$\frac{w_1 + w_2}{\frac{w_1}{S_1} + \frac{w_2}{S_2}} = S\,;$$

when w_1 and w_2 are the weights of the substance, S_1 and S_2 the specific gravities, and S the mean specific gravity found by experiment. This formula assumes that no condensation takes place in the palladium.

TABLE I.—Specific Gravity of Bar Palladium at Different Times during Saturation with Hydrogen. Weight of Original Palladium, 31·802.

Experiments.	Weight of Palladium and Hydrogen in *air*.	Weight of Palladium and Hydrogen in *water*.	Specific gravity of substance.	Specific gravity of Hydrogen (calculated).	Weight of Hydrogen.
1	31·8748	29·1150	11·5488	0·6215	0·0728
2	31·9230	29·0860	11·2520	0·6230	0·1210
3	31·9425	29·0715	11·1259	0·6150	0·1405
4	31·9715	29·0615	10·9867	0·6081	0·1625
5	31·9860	29·0500	10·8944	0·6270	0·1840
6	31·9955	29·0455	10·8459	0·6299	0·1935
7	32·0040	29·0450	10·8158	0·6388	0·2020
8 (after some days)	31·9940	29·0325	10·8033	0·6024	0·1920

When the calculated values of the specific gravity of the hydrogen in the eight determinations of Table I. are examined it will be observed no material alteration can be said to take place in the specific gravity during the course of the saturation, regard being paid to the difficulty of getting accurate weighings immediately after removal from the electrolytic cell. All the above determinations were made as quickly as possible after the palladium was taken from the pole of the battery. The mean of the results gives a specific gravity of 0·620 for the hydrogen. The atomic volume of this condition of hydrogen is therefore

1·6, and the alteration of volume is equivalent to saying in the formation of this condition 7 litres of gas are condensed into 1 cubic centimetre. The maximum charge of hydrogen the palladium would take was given to No. 7 experiment, and this very nearly accords with the composition Pd_3H_2. In treating masses of palladium the above proportion was never exceeded.

As all the above experiments were made with bar palladium increasing to saturation, it was necessary to find similar values for a mass decreasing from saturation, the hydrogen being expelled by heating. For this purpose a piece of plate palladium was employed. When the plate was fully charged for the first time, and the specific gravity of the hydrogen found as in the previous experiment, the result was a density of 0·621, the plate in this case contained 0·21 grammes of hydrogen. When a portion of the hydrogen was expelled by heat, leaving 0·127 grammes in the plate, the density of the hydrogen was 0·633, and where only 0·0495 gramme was left, the value was 0·615. The mean specific gravity of the three different alloys of plate was 0·623, nearly the same as previously found for the bar. After palladium has been treated with hydrogen, and heated, the metal becomes porous, and often blisters, thus rendering specific gravity determinations very uncertain. The curious discovery of GRAHAM, that the strain on the particles of palladium produced by wire-drawing induces such a curious shrinking in the length after the hydrogen has been expelled, suggests the importance of investigating how a saturated piece of bar would resist the action of sustained pressure or tension. If the formation of the alloy is attended with an increase of volume in excess of the sum of the volumes of the constituents regarded as in the solid state, then partial decomposition ought to occur under great pressure. Experiments on this subject are left over for the present. After palladium has been used in the above experiments, and the last trace of hydrogen is removed by heating, the specific gravity is found to have diminished from 12·0359 to 11·9546. If this final value is taken in calculating the mean density, then the average result of the three hydrides of plate is 0·707 for the specific gravity of the condensed hydrogen. The mean of the first and second series of experiments is thus 0·664. A piece of palladium, weighing 31 grammes, was fused in a lime crucible with the oxyhydrogen flame, and the specific gravity was now found reduced to 10·5549. When this sphere of palladium was hydrogenised for a very long time (twenty-four hours) only 0·0684 grammes of hydrogen were absorbed, and by further treatment nothing was added. The specific gravity of the hydrogen in this case was 0·655, nearly identical with the mean of the values found from the plate experiments.

Specific Heat Observations.

The apparatus devised for this purpose is represented in the plate appended to this paper. The calorimeter used in the experiments had the form (A) shown

in Plate XII. It held conveniently 100 grammes of water, and was inserted in the middle of a stout brass envelope, thoroughly exhausted of air, the whole being placed in the middle of a large cylindrical tin vessel (E), having an outer annular compartment. The tin vessel was filled with water, and a constant current from the town supply was kept circulating by means of a syphon through the outer chamber. To the thermometer a ring of thin sheet india-rubber was attached as a stirrer, and when it was removed before the immersion of the palladium was placed in the little tin tube represented at (B). Immediately before an observation the whole apparatus was moved below the steam bath (C), and the palladium dropped in. This arrangement of the calorimeter is very convenient in a small chemical laboratory, where uniformity of temperature cannot be easily commanded. The constancy of the radiation in the calorimeter makes the correction for cooling very exact.

The same arrangement of the calorimeter is employed for registering very small amounts of heat by placing in (A) bisulphide of carbon or chloroform, packing the middle and outer compartments of the tin vessel full of pounded ice, and covering the exposed surface with sawdust. The thermometer is now provided with a thin sheet copper stirrer, instead of the india-rubber one formerly used. The rate of cooling is in this case determined once for all for a range of 5° above zero, and plotted in a curve. From this curve the correction for the radiation is determined for all subsequent experiments.

Two series of experiments were made. In the first series bar, and in the second plate, palladium was used, three different hydrides* of each. The experimental results are given in Tables II. and III.

In the case of the bar the specific heat of the occluded hydrogen increased as the charge diminished, the extreme values being 3·79 and 5·05. Similarly with the plate the values are greater, and have a wider range, viz., from 3·93 to 5·88. These results are calculated in each case for the excess of heat given to the calorimeter by the hydride in excess of that of the original palladium. But if a comparison is made between the different hydrides in both series of experiments, then the specific heat is found to exhibit no such regular increase as in the former series. In the bar the extreme values are 3·21 and 3·77, and the mean of the three results is 3·47. The plate, on the other hand, gives a range of from 2·7 to 3·94, the mean value being 3·31. The amount of variation in the plate is very great as compared with the bar, and is clearly due to some secondary action taking place.

The increase of specific heat for small charges of hydrogen, when comparison is made with pure palladium, is clearly due to some regular increase of the spe-

* In this paper the term hydride is not used in its strict chemical sense, but as a convenient abbreviation.

cific heat of the palladium, or of the hydrogen individually or conjointly. The mere fact that we have only a very small weight of hydrogen relatively to the amount of palladium does not explain the anomaly, because a small amount of hydrogen by the second mode of comparison does not yield such high results. The observational errors, although much greater when we are dealing with small quantities, cannot be expected to fall always in the same direction, and thus we are forced to admit there is some regular sequential change taking place in the relations of the hydrogen and palladium.

The palladium, after use, is not found to have increased in specific heat, but rather the reverse, so that the explanation can only rest on some altered condition of the bodies when united. So far as my experiments have led me, I am inclined to regard this alteration as a kind of molecular dissociation that increases with diminished charge of hydrogen. I am led to this conclusion from observing that the rate at which palladium loses hydrogen at constant temperature is dependent on the amount present increasing with diminished charge. In conducting these specific heat experiments, a greater variation in the results has always been observed when small charges were under experiment, especially in the first two or three determinations. This increase of dissociation may be explained in part from the formation of regular cracks or channels in the mass of palladium, together with the effect of pressure resulting from the contraction of the external layers of palladium from the loss of hydrogen. It is, however, premature to discuss the cause of this increase of dissociation until a more extended series of experiments is made, and the behaviour of the alloy to pressure is investigated.

TABLE II.—Specific Heat. Experiments with Bar Palladium. Three Hydrides.
Weight of Bar, 31·802 grammes. Water and Calorimeter, 107 grammes.

	I.	II.	III.	IV.
Weight of Hydrogen in Palladium,	0·1905	0·103	0·0415	Nothing.
Rise in Calorimeter for a fall of 100° C.,	2·426	2·178	1·944	1·740
	2·430	2·175	1·958	1·780
	2·429	2·175	1·958	1·760
	2·435	2·167	1·967	1·760
Mean rise,	2·436*	2·173	1·956	1·760
Rise due to Hydrogen,	0·676	0·413	0·196	Nothing.
Specific heat of Hydrogen,	3·79	4·29	5·05	

Weight of Hydrogen in—		Increase in Calorimeter due to difference of—		Specific Heat from difference of—		Mean.
I. in excess of II.	0·0875	I. and II.	0·263	I. and II.	3·21	
II. in excess of III.	0·0615	II. and III.	0·217	II. and III.	3·77	3·47
I. in excess of III.	0·1490	I. and III.	0·480	I. and III.	3·44	

* Mean of eight experiments.

TABLE III.—Specific Heat. Experiments with Palladium Plate. Three different Hydrides. Weight of Plate, 37 843 grammes. Equivalent of Water and Calorimeter, 107 grammes.

	I.	II.	III.	IV.
Weight of Hydrogen in Palladium,	0·210	0·129	0·0512	Nothing.
Rise in Calorimeter for a fall of 100° C.,	2·891	2·694	2·368	2·111
	2·870	2·657	2·362	2·100
	2·856	2·646	2·375	2·082
	2·858	2·659	2·402	2·086
Mean rise,	2·869	2·664	2·337	2·096*
Rise due to Hydrogen,	0·773	0·568	0·281	
Specific heat of Hydrogen,	3·930	4·70	5·88	

Weight of Hydrogen in—		Increase in Calorimeter due to difference of—		Specific Heat from difference of—		Mean.
I. in excess of II.	0·081	I. and II.	0·205	I. and II.	2·70	3·31
II. in excess of III.	0·0778	II. and III.	0·287	II. and III.	3·94	
I in excess of III.	0·1588	I. and III.	0·492	I. and III.	3·31	

Coefficient of Expansion.

As the observations on the coefficient of expansion had to be made with the same pieces of palladium used in the former experiments, the only method that could be employed was to weigh the hydride in distilled water of different temperatures, as Mathiessen did in his well-known paper on the expansion of metals and alloys ("Phil. Trans.," 1866), and to deduce the mean cubical expansion from the difference of weights and the known density of water. Experiments made in this way require great care in execution, and when every precaution is taken, are yet liable to considerable variation. Any difference of temperature in different portions of the water in which the alloy is weighed at the time of the observation, causing currents in the fluid, or the condensation of moisture on the fine platinum wire used to suspend the substance, renders the results useless. But in this case the difficulties are greatly increased from the minute bubbles of hydrogen that are apt to accumulate at any angular point of the mass, and must be removed by suddenly depressing or lifting the mass. After making a great many observations in this way, it became clear that the method would not yield results of very great accuracy when applied to the case of palladium containing hydrogen; in the meantime, the results obtained are provisionally stated. Generally a mass of palladium containing hydrogen nearly equivalent to

* Mean for six experiments.

the atomic proportion $Pd_3 H_2$, yields the following values for the mean coefficient of cubical expansion :—

Between 0° and 50°, . .	0·000058
Between 0° and 80°, . .	0·000066

The cubical expansion of palladium being 0·000033, we may say the alloy with hydrogen is just twice as great. If the expansion of GRAHAM'S alloy is assumed to be equal to the sum of the expansion of the respective volumes of the constituents, then the calculated result for the cubical expansion of hydrogen is 0·000246, a number about one and a half times the coefficient of expansion of mercury.

With a fused mass of palladium containing a small charge of hydrogen, the coefficient of expansion was found to be 0·00048, and the calculated value for the occluded hydrogen became then 0·00059.

Absorption of Hydrogen at a Red Heat.

GRAHAM'S theory of the rapid passage of hydrogen through palladium at high temperatures, assumes at first a direct absorption of the gas, and then a transmission of it by a kind of "cementation process." That an absorption of hydrogen takes place with its attendant increase of volume at high temperatures may be shown as follows :—

Take a strip of thin sheet palladium, four or five centimetres long, and about five millimetres in breadth, clamp it firmly by the end in a suitable support, so that the strip is free to vibrate, and insert it edgeways in the middle of a hydrogen flame, burning from a nozzle about a millimetre in diameter. If the palladium is now depressed into the inner dark cone it immediately begins to vibrate, producing a low musical note.

If the flame be extinguished by stopping the current of hydrogen for an instant, or allowing the gas to flow, the vibration commences again, and may be kept up without any actual flame.

The motion in this position in the flame is due to the absorption of hydrogen on the cool side next the inner cone, with its attendant increase of length, producing a bending of the sheet into the hot portion of the flame, where the hydrogen is instantly expelled from the palladium, which is forced to return to its original position from its natural elasticity.

The experiments detailed in this paper have been made with palladium very handsomely placed at my disposal by Messrs Johnson, Matthey, and Co. of Hatton Garden, London, and I gladly avail myself of this opportunity of thanking them for the means of conducting this research.

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IX.—*On the Establishment of the Elementary Principles of Quaternions on an Analytical Basis.* By GUSTAV PLARR, Docteur ès-sciences. Communicated by Prof. TAIT.

(Read March 2, 1874.)

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The extension which is required to be given to the meaning of algebraic addition and subtraction, for the purpose of representing a vector by a polynomial expression (by the algebraic sum, namely, of components differing in direction from one another), renders it necessary to investigate into the question of what the geometrical signification of the result of the other elementary operations of algebra will become when these operations are performed on those polynomial expressions?

The question, taken in its widest generality, is indeterminate; but we render it determinate by the introduction, step by step, of conditions and of limitations, and of rules, the leading condition being—that whatever be the result of the multiplication by the distributive law of two polynomial expressions of vectors into one another, that result shall be independent of the particular mode of composition of the vector factors. Those conditions and those propositions agreed to by definition, and the further condition, that the multiplication of two conjugate quaternions shall be made according to the distributive law applied to the two heterogeneous elements of the quaternions, give us as consequences the known relations between the unit vectors of a triple rectangular system, and the value of the square of a unit vector.

It may be remarked that the method followed in this paper is proceeding in a direction inverse to that which is commonly followed in the demonstration of the distributive property of vector multiplication. If there be an advantage in the present method, it may be the one that, by interverting the roles of the antecedent and the consequent of the proposition, it provides itself with a broader basis for deduction.

The associative property in multiplication is then easily established, and

without the help of any other auxiliary than the most elementary propositions of algebra or geometry; and then division can be treated of.

§ 1. *Definitions.*

We call *rector* the "compound" of, 1st, the mutual *distance* of two points, and of, 2d, the *direction* of the straight line drawn from one of the points, as origin of the vector, to the other point, as extremity of the vector.

A vector is usually represented by a Greek letter, ρ suppose, or by the letters designating its origin and extremity respectively, AA′ suppose, the first being always written to the left of the second, so that



$$\rho \text{ or } \overline{AA'}$$

are meant to represent the same.

We designate by $T\rho$, pronounced *tensor* of ρ, the absolute length of the vector; and by $U\rho$, pronounced *versor* of ρ, the symbol, *sui generis*, which represents the direction of ρ.

In order to express the vector ρ by means of $T\rho$ and $U\rho$, we agree by definition, (1), that

$$U\rho \times T\rho, \text{ or simply } U\rho\, T\rho,$$

expresses the *construction* of the length $T\rho$ into the direction belonging to ρ, and designated by $U\rho$, the extremities of $T\rho$ having to coincide respectively with the two points *given* as origin and extremity of ρ.

We may conceive a vector whose length is equal to the unit of length. In this respect

$$U\rho \times 1, \text{ or simply } U\rho,$$

is called also the *unit-vector* of ρ; it is the unit of length directed in the direction belonging to ρ.

We may conceive the *unit-vector* added to itself, end to end, on the same straight line a number of times expressed by the same number as $T\rho$; the vector comprised between the origin of the first of the unit-vectors and the extremity of the last (the last may be fractionary), will have for its expression

$$T\rho \times (U\rho \times 1), \text{ or simply } T\rho\, U\rho;$$

so that practically the expressions $U\rho\, T\rho$ and $T\rho\, U\rho$ are representing the same vector ρ.

We agree by definition that the symbol $U\rho$ is to be independent of position. The same value of $U\rho$ is to represent directions which are parallel, and drawn towards the same region of space. Such directions will be designated by the appellation of "*the same direction.*" We apply the appellation of "*opposed*

directions" to such which are parallel, but drawn towards opposed regions of space.

We agree to represent by $(-\mathrm{U}\rho)$ the versor in the opposed direction of that which $(+\mathrm{U}\rho)$ designates, or simply which $\mathrm{U}\rho$ designates.

A vector, considered in itself, may depend on position, but we agree that the *expression* of a vector is to be independent of position.

As a consequence of this, an expression like

$$w\mathrm{U}\rho,$$

where w is an ordinary quantity, will represent a vector parallel to ρ; and when w, reduced to a number, is positive, that vector $w\mathrm{U}\rho$ will be of the same direction as ρ. It will be of opposed direction when w reduces itself to a negative number.

The *sign of equality* made use of in connection with vectors is to express by agreement the *equality of the tensors* and the *sameness of direction*, so that

$$\rho' = \rho$$

expresses—1st, that $\mathrm{T}\rho' = \mathrm{T}\rho$; 2d, that $\mathrm{U}\rho' = \mathrm{U}\rho$, which equality expresses by agreement: *sameness of direction* of ρ' and ρ. The two equalities into which $\rho' = \rho$ decomposes itself are tacitly founded on the necessary hypothesis that

$$\mathrm{T}(\mathrm{U}\rho) = 1.$$

From what has been agreed to above, we have also $\mathrm{U}(-\rho) = -\mathrm{U}\rho$.

In opposition to a vector, we call *scalar* any ordinary quantity, which is not affected by a factor of the versor or vector kind.

We agree *by definition* that there is heterogeneity between scalars and vectors, so that an expression, composed of the aggregate of a scalar and a vector, cannot be equal to zero, unless both elements vanish separately.

Such an aggregate, the connection being established by the signs + or −, is called a *quaternion*.

Until we have explained the operation which can give rise to the connection between scalar and vector, we cannot give an analytical definition of the meaning of the signs + or −, applied to the linking together of the two elements. So long as this is not done, these signs may be looked upon *in this instance* as signifying "and" or "accompanied by."

When the *sign of equality* is used for establishing equality between two quaternions, its meaning is *by definition*, (5), that there is equality between the scalars separately, and between the vectors separately.

Designating a quaternion by q, and by $\mathrm{S}q$ and $\mathrm{V}q$ its *scalar* and *vector* respectively, we have

$$q = \mathrm{S}q + \mathrm{V}q.$$

We designate by Kq, pronounced conjugate of q, the quaternion

$$Sq - Vq = Kq.$$

We designate by Tq the tensor of q. As q cannot be zero unless both of its elements Sq and $T(Vq)$ are zero, we are led to *agree by definition* (6) to the formula

$$Tq = \sqrt[2l]{(Sq)^{2m} + (TVq)^{2n}},$$

where in anticipation we may state that m and n will have to be taken equal to units as well as l.

If we generally take for the definition of a versor the quaternion divided by its tensor, then we have for the versor of a quaternion

$$Uq = \frac{q}{Tq};$$

so that under another form we have

$$q = Tq\ Uq, \text{ and } T(Uq) = 1.$$

(By this a scalar may be said to have a versor, namely $+1$ or -1, according as the sign of the scalar is positive or negative in an absolute way. But we will never employ the expression "versor" in respect to a scalar.)

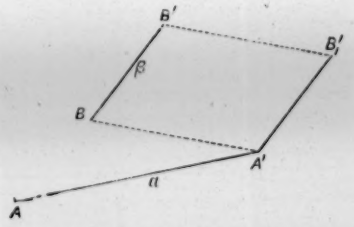
We shall refer directions in space to the directions of the axes of x, y, z, in a triply-rectangular system of axes, and we designate, according to use, by

$$i, j, k,$$

the versors in the direction of the positive halves of the axes x, y, z, respectively.

§ 2. *Addition and Subtraction of Vectors and Quaternions, and Expressions of Vectors founded thereon.*

Let α and β be vectors representing $\overline{AA'}$ and $\overline{BB'}$ respectively. Each considered separately has its position chosen at will.



In the expression

$$\alpha + \beta$$

we agree by *definition*, (7), *of addition* that the sign + has to represent the operation of *constructing* the term β following the sign, so as to put its origin into coincidence with the extremity of the term α preceding the sign +, and drawing β in its own direction.

We agree, further, to look upon the binomial expression

$$\alpha + \beta$$

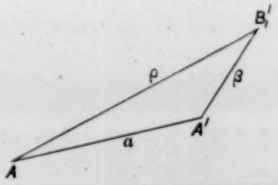
as a new vector, having its origin in common with that of α, and its extremity in common with that of β, in its position *constructed* by addition, namely in B_1'.

Let ρ be that vector; we have thus

$$\rho = \alpha + \beta$$

in the full meaning of a vector equation, namely

$$T\rho = T(\alpha + \beta) = AB_1' \text{ in length,}$$
$$U\rho = U(\alpha + \beta) \text{ in direction,}$$



parallel to AB_1', and in the *same* direction.

In like manner we might construct $\beta + \alpha$, and then it may be seen easily that $\beta + \alpha$ is equal to $\alpha + \beta$, namely, equal in length, and of the same direction; but what is different is the origin of $\beta + \alpha$, which is that of β arbitrarily given, whereas the origin of $\alpha + \beta$ is that of α; but position does not enter into the expression of a vector.

Subtraction may be *defined* in like manner, or may be reduced to addition, by the remark that, as for example,

$$\alpha - \beta' = \alpha + T\beta'\, U(-\beta').$$

We may generalise these constructions for *three* or more vectors; and form any polynomial vector - sum equal to a vector ρ.

In these definitions of vector - addition and subtraction the signs + and − have received an extended *meaning*, which returns to its original algebraic meaning when the vectors α and β are parallel.

The *addition* and *subtraction* of *quaternions* consist in adding scalar to scalar, vector to vector, each according to its appropriate rule.

Conversely, a given vector ρ may always be decomposed into the vector-sum of three components, parallel to three given directions.

Let $U\alpha$, $U\beta$, $U\gamma$, be the versors of the given directions. Then if a, b, c, designate the Cartesian co-ordinates of the extremity of vector ρ, in respect to its own origin, and parallel to axes of direction given by $U\alpha$, $U\beta$, $U\gamma$, we have

$$\rho = a\,U\alpha + b\,U\beta + c\,U\gamma.$$

In the case of the axes x, y, z, a vector ρ may be compounded of terms parallel to them, so that we have—

$$\rho = ia + jb + kc.$$

The tensor of ρ is evidently the length of the diagonal of the rectangular parallelepiped construed with ia, jb, kc. Therefore by *definition* (10), $T\rho$ *or*

$$T(ia + jb + kc) = \sqrt{a^2 + b^2 + c^2}.$$

This gives

$$\mathrm{U}\rho = i\cos\widehat{x\rho} + j\cos\widehat{y\rho} + k\cos\widehat{z\rho},$$

where, for shortness' sake, we designate by $\widehat{x\rho}$, $\widehat{y\rho}$, $\widehat{z\rho}$, under the sign cos, the angles which ρ forms with the three axes. We have also—

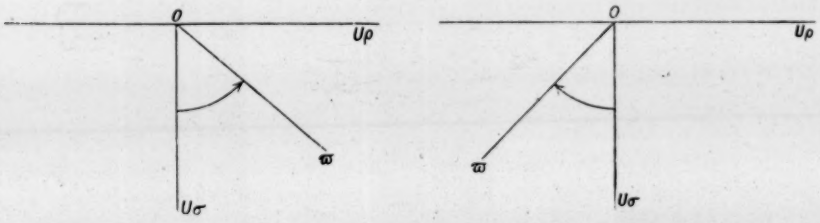
$$1 = \cos^2\widehat{x\rho} + \cos^2\widehat{y\rho} + \cos^2\widehat{z\rho}.$$

This expression of $\mathrm{U}\rho$ gives $\mathrm{U}(-\rho) = -\mathrm{U}\rho$.

We shall in the sequel, and concurrently with the system of the three rectangular directions i, j, k, make use of a triple rectangular system, whose vectors we designate by

$$\mathrm{U}\rho,\ \mathrm{U}\sigma,\ \mathrm{U}\tau.$$

Their directions are to be determined—1st, $\mathrm{U}\rho$, by the given direction of ρ; and 2d, $\mathrm{U}\sigma$, by being drawn through O as a common origin of ρ, and of another given vector ϖ, and in the plane determined by ρ and ϖ, and so as to form an



angle with ϖ, of a positive value, and not exceeding a right angle; 3rdly, $\mathrm{U}\tau$ will be drawn in respect to $\mathrm{U}\rho$ and $\mathrm{U}\sigma$, in the same relation of respective direction, as the direction of k is drawn in respect to the directions of i, j.

In this case we shall have $\cos\widehat{\sigma\varpi} = \sin\widehat{\rho\varpi}$, $\cos\widehat{\tau\varpi} = \cos 90^\circ = 0$; and the expressions for ϖ to be considered in the sequel, will be of the two forms—

$$\varpi = \mathrm{T}\varpi(\mathrm{U}\rho\cos\widehat{\rho\varpi} + \mathrm{U}\sigma\sin\widehat{\rho\varpi}),$$
$$\varpi = ix + jy + kz,$$
$$\mathrm{T}\varpi = \sqrt{x^2 + y^2 + z^2}.$$

§ 3. *Multiplication of Vectors one by another.*

According to the *principle of the influence of the order of the factors*, so happily introduced by the inventor of quaternions, we represent by $\rho\varpi$, the product of the *multiplicand* ϖ, by the *multiplier* ρ.

In this manner the two products

$$\rho\varpi, \text{ and } \varpi\rho,$$

are admitted to represent two different expressions, not equal to one another.

This principle does not apply to the product of a vector by a scalar, or *vice versâ*, and also it may be easily shown that if a scalar w is engaged as a factor, concurrently with vector factors, the place of the scalar may be shifted to any place in the product, namely, that there is equality between

$$w\alpha\beta = \alpha w\beta = \alpha\beta w .$$

We apply to $\rho\varpi$, and $\varpi\rho$ respectively, the principle of decomposition into two factors, a *tensor* and a *versor*. The versor is simply the quantity $\rho\varpi$ (or $\varpi\rho$) itself divided by its tensor (which is a scalar). That is, we put

$$\rho\varpi = \mathrm{T}(\rho\varpi) \times \mathrm{U}(\rho\varpi) .$$

The consequence is, that the absolute value of $\mathrm{U}(\rho\varpi)$ is unit—

$$\mathrm{TU}(\rho\varpi) = 1 .$$

As we have also

$$\rho\varpi = \mathrm{T}\rho\,\mathrm{U}\rho \times \mathrm{T}\varpi\,\mathrm{U}\varpi ,$$

it follows that

$$\mathrm{T}(\rho\varpi) = \mathrm{T}\rho \times \mathrm{T}\varpi ,$$

and

$$\mathrm{U}(\rho\varpi) = \mathrm{U}\rho\,\mathrm{U}\varpi , \ \mathrm{U}(\varpi\rho) = \mathrm{U}\varpi\,\mathrm{U}\rho .$$

The first equation is the expression of a *general principle*, which is applicable to any species of factors, namely, that the *tensor of the product* is *equal to the product of the tensors of the factors*. By an abbreviation we may call it the "*law of the tensors*" in a product.

The two other equations give for the tensor of the first member the value unit, because we have defined the tensor of a versor, or unit-vector, to be equal to unit, and by the application of the principle about the tensor of a product.

When $\mathrm{U}\varpi$, or both $\mathrm{U}\rho$ and $\mathrm{U}\varpi$, are replaced by expressions of a *polynomial form*, then we have *a priori* no indication of how the products are to be developed into a sum of partial products, because the partial factors differ in direction.

We shall be in need of a rule for effecting multiplication, whenever such expressions of a polynomial kind are introduced.

The choice of the rule stands at our free will, but in all cases we have to satisfy two distinct *conditions*, namely—1st, The expressions which will have to represent the products $\mathrm{U}\rho\,\mathrm{U}\varpi$ and $\mathrm{U}\varpi\,\mathrm{U}\rho$, and which will be based on different expressions for $\mathrm{U}\rho$, and for $\mathrm{U}\varpi$, [say in a first case on

$$\mathrm{U}\rho \text{ and on } (\mathrm{U}\rho \cos \widehat{\rho\varpi} + \mathrm{U}\sigma \sin \widehat{\rho\varpi}) \text{ for } \mathrm{U}\varpi ;$$

in a second case on

$$\frac{1}{\mathrm{T}\rho}(ia + jb + kc) \text{ for } \mathrm{U}\rho,$$

and on

$$\frac{1}{\mathrm{T}\varpi}(ix + jy + kz) \text{ for } \mathrm{U}\varpi;$$

in a third case on any other set of expressions for $\mathrm{U}\rho$ and $\mathrm{U}\varpi$ respectively] must be transformable or reducible to one and the same typical result. 2d, The tensor of the *expressions* which have to represent the products $\mathrm{U}\rho\,\mathrm{U}\varpi$, $\mathrm{U}\varpi\,\mathrm{U}\rho$, must be equal to unity.

These are two conditions the fulfilment of which will necessitate some developments.

As to the *choice of the rule of multiplication*, of course we make choice of the rule of the distributive law of algebraic multiplication, and of *the rule of signs* included therein, and we adopt, moreover, the *rule* concerning the *order of the vector-* (or versor-) *factors*, putting to the left place, in the partial products, those factors which belong originally to the multiplier.

As to the means of fulfilment of the conditions, we have at our disposal the, as yet, undetermined meaning of the products, *nine* in number, *ii*, *ij*, *ik*, *ji*, &c., and of the three products

$$\mathrm{U}\rho\,\mathrm{U}\rho,\ \mathrm{U}\rho\,\mathrm{U}\sigma,\ \mathrm{U}\sigma\,\mathrm{U}\rho.$$

Their values and meaning, and mutual relations, will, by these means, become determined.

Let us now treat of the first condition.

We apply the distributive law, and the other rules, to the expressions—

$$\rho = \mathrm{T}\rho\,\mathrm{U}\rho$$
$$\varpi = \mathrm{T}\varpi\,(\mathrm{U}\rho\cos\widehat{\rho\varpi} + \mathrm{U}\sigma\sin\widehat{\rho\varpi}),$$

and we provisionally put

$$\phi_1 = \mathrm{T}\rho\,\mathrm{T}\varpi\,[(\mathrm{U}\rho)^2\cos\widehat{\rho\varpi} + (\mathrm{U}\rho\,\mathrm{U}\sigma)\sin\widehat{\rho\varpi}]$$
$$\phi_2 = \mathrm{T}\rho\,\mathrm{T}\varpi\,[(\mathrm{U}\rho)^2\cos\widehat{\rho\varpi} + (\mathrm{U}\sigma\,\mathrm{U}\rho)\sin\widehat{\rho\varpi}].$$

Again, we apply the same rules to—

$$\rho = ia + jb + kc$$
$$\varpi = ix + jy + kz,$$

and put

$$\begin{aligned}\phi_3 &= a[i^2x + ijy + ikz]\\ &\quad + b[jix + j^2y + jkz]\\ &\quad + c[kix + kjy + k^2z]\\ \phi_4 &= x[i^2a + ijb + ikc]\\ &\quad + y[jia + j^2b + jkc]\\ &\quad + z[kia + kjb + k^2c].\end{aligned}$$

We have to equate φ_3 to φ_1 and φ_4 to φ_2, and then find the conditions of equality of the members of the equations.

We do not change the results of the deductions if we equate simultaneously

$$\varphi_1 - \varphi_2 \text{ to } \varphi_3 - \varphi_4$$
$$\varphi_1 + \varphi_2 \text{ to } \varphi_3 + \varphi_4 .$$

The first equation so formed will be (I.)—

$$\text{(I.)} \qquad \mathrm{T}\rho\,\mathrm{T}\varpi \sin \rho\varpi\,[\mathrm{U}\rho\,\mathrm{U}\sigma - \mathrm{U}\sigma\,\mathrm{U}\rho]$$
$$= (jk - kj)(bz - cy) + (ki - ik)(cx - az) + (ij - ji)(ay - bx) .$$

For the second equation let us put

$$\mathrm{U}\rho\,\mathrm{U}\sigma + \mathrm{U}\sigma\,\mathrm{U}\rho = 2\,\mathrm{F}$$
$$jk + kj = 2\,\mathrm{f}',\ ki + ik = 2\,\mathrm{f}'',\ ij + ji = 2\,\mathrm{f}''' .$$

This will give to it the following form (II.), after suppression of the factor 2—

$$\text{(II.)} \qquad \mathrm{T}\rho\,\mathrm{T}\varpi\,[(\mathrm{U}\rho)^2 \cos \widehat{\rho\varpi} + \mathrm{F} \sin \widehat{\rho\varpi}]$$
$$= (i^2 ax + j^2 by + k^2 cz)$$
$$+ (bz + cy)\,\mathrm{f}' + (cx + az)\,\mathrm{f}'' + (ay + bx)\,\mathrm{f}''' .$$

Let us examine equation (I.)

The coefficients $(bz - cy)$, $(cx - az)$, $(ay - bx)$ are proportional to the cosines of the angles which the direction of $\mathrm{U}\tau$, defined in the preceding paragraph, forms with the three axes, x, y, z. Namely, a perpendicular $\mathrm{U}\tau$ to the plane of the directions $\mathrm{U}\rho$, $\mathrm{U}\varpi$, drawn through a common origin, would be determined by the equations—

$$a \cos \widehat{x\tau} + b \cos \widehat{y\tau} + c \cos \widehat{z\tau} = 0$$
$$x \cos \widehat{x\tau} + y \cos \widehat{y\tau} + z \cos \widehat{z\tau} = 0 .$$

These give to the above coefficients the values comprised in

$$\frac{bz - cy}{\cos \widehat{x\tau}} = \frac{cx - az}{\cos \widehat{y\tau}} = \frac{ay - bx}{\cos \widehat{z\tau}} = \mathrm{T}\rho\,\mathrm{T}\varpi \sin \rho\varpi .$$

The sign of the last number has been taken so as to give the system of values (as a particular case) following—

$$\rho = ia ,\ \varpi = jy ,\ \cos \widehat{z\tau} = +1 .$$

The condition (I.) may therefore be transformed into

$$\mathrm{U}\rho\,\mathrm{U}\sigma - \mathrm{U}\sigma\,\mathrm{U}\rho = (jk - kj) \cos \widehat{x\tau} + (ki - ik) \cos \widehat{y\tau} + (ij - ji) \cos \widehat{z\tau} .$$

But we have also for $U\tau$ the expression—

$$U\tau = i \cos \widehat{x\tau} + j \cos \widehat{y\tau} + k \cos \widehat{z\tau}.$$

We see now that the condition (I.) will be identically fulfilled if we establish, *by definition*, (13), the following *four relations*:—

$$\begin{aligned} U\rho\, U\sigma - U\sigma\, U\rho &= 2\mathfrak{g}\, U\tau \\ jk - kj &= 2\mathfrak{g} i \\ ki - ik &= 2\mathfrak{g} j \\ ij - ji &= 2\mathfrak{g} k, \end{aligned}$$

where $\mathfrak{g}$ is the *same* in the four equations.

The most general expression for $\mathfrak{g}$ would be a quaternion. But if $\mathfrak{g}$ were to depend on the *particular* undetermined direction of the vector of the quarternion, then the solution of our problem would become indeterminate, and a practical solution might, in all probability, be despaired of.

We avoid this generality by introducing a condition which is analogous to that which we have already agreed to in respect of the *expression* for a single vector—the agreement, namely, that the expression of a vector shall be independent of the absolute position of the vector, a condition which is of practical use in the establishment of the analytical addition of vectors.

In the instance of multiplication we render its analytical operation practically possible by introducing the principle, that when similar systems of directions (like $U\rho$, $U\sigma$, $U\tau$) are to be bound by a mutual general relation, that relation must exist, and be expressed *independently of the absolute direction* of their system.

We therefore agree that $\mathfrak{g}$ does not depend on direction, and therefore must not contain a vector part, nor any angle in the expression of its scalar. There remains but the agreement that, (14),

$$\mathfrak{g} = \textit{a numerical quantity.}$$

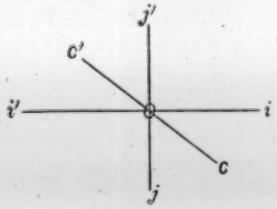
Let us consider equation (II.) In the case of the relations defining F, f′, &c., we cannot apply the principle laid down just now, because these quantities are *not bound*, like $\mathfrak{g}$, to enter into equations of definition ($jk + kj = \mathrm{f}'$, &c.) by one and the same value, and they can vary from one system to the other, *a priori*.

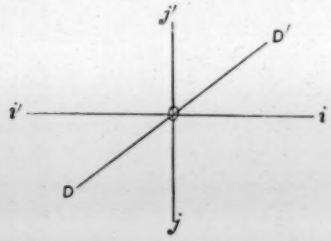
But we are able to establish, by reason of the symmetry of the expression of F, f′, &c., that these quantities cannot contain a vector part.

As for example, let $-i, -j$, be represented by i', j'; by the rule of signs agreed to, we have then

$$ij + ji = i'j' + j'i', \text{ and } i'j' + ji = ij + j'i'.$$

But the two first expressions are evidently symmetrical about both sides of—



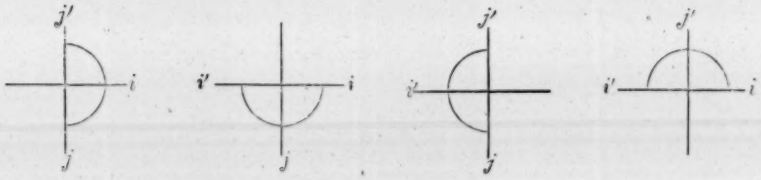


CC′, and the two last about DD′. Therefore, if $ij + ji = 2\mathrm{f}'''$ contains a vector, that vector must be directed in the direction either of $+k$, or of $-k$. But neither can take place, because the two first of the expressions are symmetrical in respect to the plane of the directions i, j, through a common origin. We must conclude that f‴ cannot contain any vector part.

The same mode of reasoning will apply to the expressions 2F, 2f′, &c., so that these four quantities must be *scalars*.

The reasoning founded on the symmetry of the expressions which define F, f′, &c., does not apply to the four expressions in which $\mathfrak{g}$ is concerned. We have, as for example—

$$2\mathfrak{g}k = ij + j'i = ij + ji'' = i''j'' + ji'' = i''j'' + j''i\,,$$



which shows that $2\mathfrak{g}k$ relates to two *consecutive* quadrants, and has therefore a rotatory character.

Further, we prove that

$$\mathrm{F} = \mathrm{f}' = \mathrm{f}'' = \mathrm{f}''' = \mathrm{f} \text{ suppose.}$$

Namely, by $ij - ji = \mathfrak{g}k$, and by $ij + ji = 2\mathrm{f}'''$ we have

$$ij = \mathrm{f}''' + \mathfrak{g}k\,.$$

The second member is a quaternion, of which we know *the* scalar f‴, and *the* vector $\mathfrak{g}k$; f‴, $\mathfrak{g}$, being admitted to be scalars. Therefore the tensor of ij may

be calculated by the general expression (defined in § 1) for the tensor of a quaternion. Thus—

$$T^{2l}(ij) = (f''')^{2m} + (g)^{2n}.$$

The first member is equal to unit by the "law of tensors," and the conditions $Ti = 1$, $Tj = 1$. Therefore we have

$$(f''')^{2m} = 1 - (g)^{2n}.$$

As the second member is also the value of $(f')^{2m}$, $(f'')^{2m}$, &c., we conclude to the equality of all the quantities F, f′, &c., to one value = f.

The equation (II.) becomes therefore

$$0 = T\rho\, T\varpi \cos \widehat{\rho\varpi}\, (U\rho)^2 - (i^2ax + j^2by + k^2cz) + [T\rho\, T\varpi \sin \widehat{\rho\varpi} - (bz + cy + cx + az + ay + bx)]\, f.$$

Remembering that

$$T\rho\, T\varpi \cos \widehat{\rho\varpi} = ax + by + cz,$$

the equation may receive the form

$$0 = [(U\rho)^2 - i^2]\, ax + [(U\rho)^2 - j^2]\, by + [(U\rho)^2 - k^2]\, cz + Df,$$

where we put

$$D = T\rho\, T\varpi\, (\cos \widehat{\rho\varpi} + \sin \widehat{\rho\varpi}) - (a + b + c)\, (x + y + z).$$

By the expression of $\rho = ia + jb + kc$, we see that $U\rho$ cannot be made to depend on x, y, z. The equation just written must, therefore, not establish any relation between $(U\rho)^2$ and x, y, z. This cannot be avoided unless we have the values zero, namely

$$f = 0,$$

and

$$(U\rho)^2 - i^2 = 0,\ (U\rho)^2 - j^2 = 0,\ (U\rho)^2 - k^2 = 0$$

for the coefficients in the equation.

This conclusion, avoiding any multiplication, may be arrived at thus: calling D_1, D_2, D_3, what D becomes respectively for $y = 0$, $z = 0$, and for $z = 0$, $x = 0$, and for $x = 0$, $y = 0$, we have, as the equation must hold good for any system of values of x, y, z,

$$[(U\rho)^2 - i^2]\, ax + D_1 f = 0,$$
$$[(U\rho)^2 - j^2]\, by + D_2 f = 0,$$
$$[(U\rho)^2 - k^2]\, cz + D_3 f = 0.$$

Calculating D_1, D_2, D_3, and substituting for $[(U\rho)^2 - i^2]\,ax$, &c., their values $-D_1 f$, &c., we get, after all reductions are made, the equation

$$0 = f \times \begin{bmatrix} T\rho\, T\varpi \sin \widehat{\rho\varpi} \\ -x\sqrt{b^2+c^2} - y\sqrt{c^2+a^2} - z\sqrt{a^2+b^2} \end{bmatrix}.$$

Now, $T\rho\, T\varpi \sin \widehat{\rho\varpi}$, when expressed by

$$\sqrt[2]{(T\rho)^2\,(x^2+y^2+z^2) - (ax+by+cz)^2},$$

is an irrational function of x, y, z; it cannot, therefore, be equal to a rational function, whatever be x, y, z. Therefore the coefficient of f cannot be equated to zero without giving rise to an equation between x, y, z, which must not be. Therefore we must have $f = 0$.

Let us designate by $\mathfrak{h}$ the common value of the square of a versor. Then

$$\mathfrak{h} = (U\rho)^2 = i^2 = j^2 = k^2.$$

This value can be but a numerical value, because it must represent the value of $(U\rho)^2$ in any direction of ρ. Therefore *we agree to* (15).

$$\mathfrak{h} = \text{numerical constant.}$$

This conclusion is an immediate consequence of the similar admission in respect to $\mathfrak{g}$, and it bridges over the gulf which we set *a priori* between a vector and a scalar, as the *square* of a versor (and therefore of a vector) is to be now *a scalar*.

We have now established identity between φ_1 and φ_3, and between φ_2 and φ_4. Namely, we have now

$$\varphi_1 = T\rho\, T\varpi\, [\mathfrak{h} \cos \widehat{\rho\varpi} + \mathfrak{g}\, U\tau \sin \widehat{\rho\varpi}]$$

$$\varphi_2 = T\rho\, T\varpi\, [\mathfrak{h} \cos \widehat{\rho\varpi} - \mathfrak{g}\, U\tau \sin \widehat{\rho\varpi}];$$

and introducing in these expressions the values

$$T\rho\, T\varpi \cos \widehat{\rho\varpi} = ax + by + cz,$$

$$\begin{aligned} (T\rho\, T\varpi \sin \widehat{\rho\varpi})\, U\tau = {} & i\,(bz - cy), \\ & + j\,(cx - az), \\ & + k\,(ay - bx), \end{aligned}$$

we transform by this the expressions of φ_1, φ_2 into φ_3, φ_4 respectively; and this identity is due to the relations agreed to, which give

$$(\mathrm{U}\rho)^2 = i^2 = j^2 = k^2 = \mathfrak{h},$$
$$\mathrm{U}\rho\,\mathrm{U}\sigma = -\,\mathrm{U}\sigma\,\mathrm{U}\rho = \mathfrak{g}\mathrm{U}\tau,$$
$$jk = -\,kj = \mathfrak{g}i,$$
$$ki = -\,ik = \mathfrak{g}j,$$
$$ij = -\,ji = \mathfrak{g}k,$$

as the consequence of $\mathrm{f} = 0$.

We now proceed to the treatment of the *second condition*, namely, that the tensor of the expressions which are to replace $\mathrm{U}(\rho\varpi)$ and $\mathrm{U}(\varpi\rho)$ must be equal to unity.

This condition applied to ϕ_1, ϕ_2, gives for one of them—

$$\frac{\mathrm{T}\phi_1}{\mathrm{T}\rho\,\mathrm{T}\varpi} = \mathrm{T}\left[\mathfrak{h}\cos\widehat{\rho\varpi} + \mathfrak{g}\mathrm{U}\tau\sin\widehat{\rho\varpi}\right] = 1.$$

As we have defined the tensor of a quaternion, the condition transforms itself into

$$(\mathfrak{h}\cos\widehat{\rho\varpi})^{2m} + (\mathfrak{g}\sin\widehat{\rho\varpi})^{2n} = 1$$

for both ϕ_1 and ϕ_2.

The absolute values of $\mathfrak{g}$ and $\mathfrak{h}$ are equal to unity, because the tensor of each must be equal to the products of the tensors of two versors, and these latter being equal to unity; therefore the squares

$$\mathfrak{g}^2 = +1,\ \mathfrak{h}^2 = +1.$$

The above condition will now be satisfied by the admission of

$$2m = 2n = 2;$$

and finally we *admit*

$$\begin{cases} \rho\varpi = \phi_1, \\ \varpi\rho = \phi_2. \end{cases}$$

The definition of the *tensor* of a quaternion has now been particularised to being, (16):—

$$(\mathrm{T}q)^2 = (\mathrm{S}q)^2 + (\mathrm{TV}q)^2.$$

§ 4. *Determination of* $\mathfrak{g}$ *and* $\mathfrak{h}$. *Discussion of Results as to the Rule of Multiplication.*

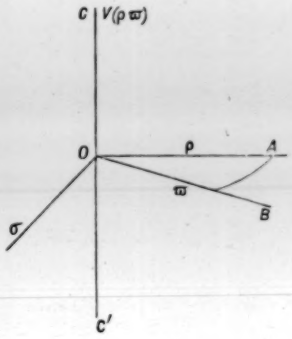
The values of $\mathfrak{g}$ and $\mathfrak{h}$ are both to be chosen between either $+1$ or -1.

As to $\mathfrak{g}$, we have to apply the rule given by Sir W. Rowan Hamilton regarding the sign to be attributed to the product of two versors (unit-vectors) perpendicular to each other.

That rule may be presented under a form in which it applies also to the vector $V(\rho\varpi)$ of the product of two vectors ρ, ϖ, generally not perpendicular to each other.

The vector of the product is

$$V(\rho\varpi) = (T\rho\, T\varpi \sin \widehat{\rho\varpi})\, \mathfrak{g} U\tau .$$



Through a common origin, O, we draw ρ and ϖ in their directions $\overline{OA}$, $\overline{OB}$. At O we erect a perpendicular C'OC to the plane AOB.

Then we take for the positive part of the direction $V(\rho\varpi)$ the direction OC, so situated that if an observer, standing in O, puts his eye into C, and directs it towards the opening of the angle AOB (not exceeding 180°, and not negative), he will *see* the *multiplier* ρ towards his *left*, and the *multiplicand* ϖ towards his *right*.

This verified, we will have to take

$$\mathfrak{g} = +1 \text{ or } \mathfrak{g} = -1 ,$$

according to the coinciding of $V(\rho\varpi)$ with either $+ U\tau$, or $(- U\tau)$; namely, $U\tau$ has been determined in such a way that its expression in function of a, b, c, x, y, z, makes it to coincide with the positive half of the axis, as for example, z, when $U\rho$ coincides with the axis $+x$, and $U\sigma$ with the axis $+y$; the angle between ϖ and $U\sigma$ being not greater than a right angle.

If, therefore, we originally did choose the positive halves of the axis in such a way that the direction of axis $+z$ is taken in respect with the axis $+x$, $+y$, by the same rule as that by which $V(\rho\varpi)$ is determined in its positive part in respect with $U\rho$ and $U\sigma$, *then* we have $U\tau$ coinciding with $V(\rho\varpi)$, and *vice versâ*, and

$$\mathfrak{g} = +1 .$$

We may, without loss of generality, admit this sole value, provided the axes are determined in their respective arrangements according to the above rule. The positive half of one of the axes, when they are considered in the cyclical order, x, y, z, x, y, &c., in respect with its two preceding ones, will be in the same relation for any set of three consecutive ones.

Determination of the Value of $\mathfrak{h}$.

We have to make a choice between the values $+1$ and -1. As long as not more than two vector factors are concerned in a product, the decision between $+1$ and -1 might be dispensed with; but the indetermination, when

there are more factors concerned, or in the case of the multiplication of quaternions, would lead to a useless complication.

So from the point of view of practical application we must determine the convenient value for $\mathfrak{h}$.

We shall determine $\mathfrak{h}$ so as to gain the applicability of the distributive law to the multiplication of two conjugate quaternions.

Let $w + \omega$, and $w - \omega$, be the two quaternions, w being a scalar, ω a vector.

Their product, indicated by $(w + \omega)(w - \omega)$, is to be expressed according to our *desideratum* by

$$\mathrm{R} = w + \omega w - w\omega - \omega^2 ,$$

namely, by

$$\mathrm{R} = w^2 - \omega^2 .$$

The sole condition to be satisfied is, that the tensor of the expression R be equal to the product of the tensors of the factors ; the versor of the expression may then become *what it may*.

Now we have

$$\mathrm{T}(w + \omega) = \mathrm{T}(w - \omega) = \sqrt[2]{w^2 + \mathrm{T}(\omega)^2} .$$

Therefore we have the condition

$$w^2 + (\mathrm{T}\omega)^2 = \mathrm{T}(w^2 - \omega^2) .$$

But

$$\omega = \mathrm{T}\omega \, \mathrm{U}\omega$$
$$\omega^2 = (\mathrm{T}\omega)^2(\mathrm{U}\omega)^2 = \mathfrak{h}(\mathrm{T}\omega)^2 .$$

Therefore we have

$$\mathrm{T}(w^2 - \omega^2) = \mathrm{T}[w^2 - \mathfrak{h}(\mathrm{T}\omega)^2] .$$

As the quantity of which the tensor is to be taken is a scalar ($\mathfrak{h}$ being a scalar), the tensor is the quantity itself taken with the appropriate sign. Thus the condition becomes

$$w^2 + (\mathrm{T}\omega)^2 = \pm [w^2 - \mathfrak{h}(\mathrm{T}\omega)^2] .$$

It is easily seen that the upper sign alone will make w^2 disappear, and then we have the condition

$$(1 + \mathfrak{h})(\mathrm{T}\omega)^2 = 0 .$$

Giving, for whatever values of w and $\mathrm{T}\omega$,

$$\mathfrak{h} = -1 .$$

In agreeing to this value of $\sqrt{-1}$, namely, taking

$$(\mathrm{U}\rho)^2 = i^2 = j^2 = k^2 = -1\,,$$

we gain the practical rule of multiplication of two conjugate quaternions, and, as we will show it in its place, of any two quaternions.

We add the remark, that in virtue of $\sqrt{-1} = -1$, we have for any vector

$$\rho^2 = -(\mathrm{T}\rho)^2, (\mathrm{V}q)^2 = -(\mathrm{TV}q)^2\,;$$

so that the tensor of a quaternion will be expressed now according to

$$(\mathrm{T}q)^2 = (\mathrm{S}q)^2 - (\mathrm{V}q)^2\,,$$

and as by the distributive rule the second member expresses the product of

$$(\mathrm{S}q - \mathrm{V}q) \times (\mathrm{S}q + \mathrm{V}q)\,,$$

namely, of

$$q\mathrm{K}q, \text{ or indifferently of } (\mathrm{K}q) \times q\,,$$

we have the results—

$$q\mathrm{K}q = (\mathrm{K}q) \times q = (\mathrm{T}q)^2\,.$$

We may also remark that the formula

$$(\mathrm{S}q + \mathrm{V}q)(\mathrm{S}q - \mathrm{V}q) = (\mathrm{S}q)^2 - (\mathrm{V}q)^2$$

defines the analytical character of the signs $+$, and $-$, used in connecting a scalar and a vector into a quaternion.

The vector of a product has now become

$$\mathrm{U}\rho\mathrm{U}\varpi = -\cos\widehat{\rho\varpi} + \mathrm{U}\tau\sin\widehat{\rho\varpi}\,.$$

It presents the anomaly of giving the value -1 for the product of two unit vectors of the same direction, and the value $+1$ for the product of two unit vectors opposed in direction, like in

$$\mathrm{U}\rho\,\mathrm{U}(-\rho) = +1\,.$$

This seems to be in opposition with what takes place for Cartesian co-ordinates, where, as for example,

$$x \times x' = +xx'$$
$$x \times (-x') = -xx'\,;$$

but the contradiction ceases when we reflect that these co-ordinates are *scalars* and *not vectors*, or scalars affected by versor factors.

As an application of therule of multiplication, let us determine $\mathrm{U}\sigma$, knowing that the vector $\mathrm{V}\rho\varpi$ of the product of two vectors ρ, ϖ, gives the perpendicular to both, its positive part being determined according to the rule above explained.

Let us take as factors $\mathrm{U}\tau$, $\mathrm{U}\rho$, in respect to which $\mathrm{U}\sigma$ is directed as $\mathrm{U}\tau$ was in respect to the factors $\mathrm{U}\rho$, $\mathrm{U}\sigma$, then we have $\mathrm{U}\sigma = \mathrm{V}(\mathrm{U}\tau\mathrm{U}\rho)$.

Now $\mathrm{U}\tau$ is determined by

$$\mathrm{V}(\rho\varpi) = \mathrm{T}\rho\,\mathrm{T}\varpi \sin\widehat{\rho\varpi}\,\mathrm{U}\tau$$
$$\mathrm{T}(\mathrm{V}\rho\varpi) = \mathrm{T}\rho\,\mathrm{T}\varpi \sin\widehat{\rho\varpi}\,,$$

and as $\mathrm{V}\rho\varpi$ is also

$$\mathrm{V}(\rho\varpi) = ia_2 + jb_2 + kc_2\,,$$

where we put

$$a_2 = bz - cy$$
$$b_2 = cx - az$$
$$c_2 = ay - bx\,,$$

we have

$$\mathrm{U}\tau = \frac{1}{\mathrm{TV}\rho\varpi}[ia_2 + jb_2 + kc_2]$$

$$\mathrm{U}\rho = \frac{1}{\mathrm{T}\rho}[ia + jb + kc]\,.$$

And then we must have

$$\mathrm{U}\sigma = \mathrm{V}\frac{(ia_2 + jb_2 + kc_2)(ia + jb + kc)}{\mathrm{TV}\rho\varpi\,\mathrm{T}\rho}\,.$$

Making the product, and calculating it by its scalar and its vector, we have for the scalar, in virtue of

$$i^2 = j^2 = k^2 = -1\,,$$

the expression whose numerator is

$$-(a_2a + b_2b + c_2c)\,,$$

and this is = zero by the very expressions for a_2, b_2, c_2, as it ought to be, $\mathrm{U}\sigma$ being a vector or unit vector.

The vector part of the effected product, by the application of

$$jk = -kj = i$$
$$ki = -ik = j$$
$$ij = -ji = k$$

becomes $\mathrm{U}\sigma$, namely,

$$\mathrm{V}(\mathrm{U}\tau\,\mathrm{U}\rho) = \frac{1}{\mathrm{TV}\rho\varpi\,\mathrm{T}\rho}[ia_1 + jb_1 + kc_1] = \mathrm{U}\sigma\,,$$

where we put

$$a_1 = b_2c - c_2b$$
$$b_1 = c_2a - a_2c$$
$$c_1 = a_2b - b_2a\,.$$

The general result we have arrived at is this. The products $\rho\varpi$ and $\varpi\rho$ are expressed by a quaternion, whose scalar is for both $\mathrm{S}\rho\varpi$, namely,

$$\mathrm{S}\rho\varpi = (-\mathrm{T}\rho\,\mathrm{T}\varpi\cos\widehat{\rho\varpi})$$
$$= -(ax + by + cz),$$

and whose vectors are $+\mathrm{V}(\rho\varpi)$ and $\mathrm{V}(\varpi\rho) = -\mathrm{V}(\rho\varpi)$ respectively.

Namely,

$$\mathrm{V}\rho\varpi = \mathrm{T}\rho\,\mathrm{T}\varpi\sin\widehat{\rho\varpi}\,\mathrm{U}\tau$$
$$= i(bz - cy) + j(cx - az) + k(ay - bx)\,.$$

It follows that

$$\rho\varpi = \mathrm{S}\rho\varpi + \mathrm{V}\rho\varpi$$
$$\varpi\rho = \mathrm{S}\rho\varpi - \mathrm{V}\rho\varpi\,.$$

Also, it follows that

$$\mathrm{U}(\rho\varpi) = -\cos\widehat{\rho\varpi} + \mathrm{U}\tau\sin\widehat{\rho\varpi}$$
$$\mathrm{U}(\varpi\rho) = -\cos\widehat{\rho\varpi} - \mathrm{U}\tau\sin\widehat{\rho\varpi}\,.$$

And these expressions are consequences of the relations

$$i^2 = j^2 = k^2 = -1$$
$$jk = -kj = i\,,\quad ki = -ik = j\,,\quad ij = -ji = k$$
$$(\mathrm{U}\rho)^2 = -1$$
$$\mathrm{U}\rho\,\mathrm{U}\sigma = -\mathrm{U}\sigma\,\mathrm{U}\rho = \mathrm{U}\tau\,.$$

The expression of the scalar shows that when the factors are at right angles, namely, when ϖ coincides in direction with that of $\mathrm{U}\sigma$, then the angle $\widehat{\rho\varpi}$ being $= 90°$, the scalar is $=$ zero, and the product $\rho\varpi$ or $\rho\sigma$ reduces itself to its vector.

The expression of the vector shows that it is of the same direction as the perpendicular to the plane of both factors drawn through a common origin, and having for its positive part the part which we have defined by the rule laid down at the instance of the determination of the value of $\mathfrak{g}$.

The condition that $\mathfrak{g} = +1$ being supposed to be fulfilled, the direction of

$\mathrm{U}\tau$, as to its positive part, is the same as that of $\mathrm{V}(\rho\varpi)$, so that the direction of

$$\mathrm{U}\rho\,,\ \mathrm{U}\sigma\,,\ \mathrm{U}(\mathrm{V}\rho\varpi) = \mathrm{U}\tau$$

form a triple rectanglar system arranged in respect to one another in the same manner as the directions of i, j, k respectively.

It follows that by the application of the rule of multiplication to the forming of the product

$$\mathrm{U}\tau\,\mathrm{U}\rho\,,$$

we get the result $\mathrm{U}\sigma$; because, *first*, the scalar of the product is zero, because the angle $\widehat{\tau\rho} = 90^\circ$; *second*, the vector of the product is perpendicular to the directions of both factors, and its positive part is directed in respect to $\mathrm{U}\tau$, $\mathrm{U}\rho$, in the same manner as $\mathrm{U}\tau$ is directed in respect with $\mathrm{U}\rho$, $\mathrm{U}\sigma$; *third*, and finally, the product is equal to its vector, the scalar being zero.

By actual multiplication, or by the generalisation of the result that

$$(\mathrm{U}\rho)^2 = \text{b} = -1\,,$$

in all possible directions we get also the results

$$(\mathrm{U}\sigma)^2 = -1\,,\ (\mathrm{U}\tau)^2 = -1\,.$$

It follows from all this that the three vectors

$$\mathrm{U}\rho\,,\ \mathrm{U}\sigma\,,\ \mathrm{U}\tau\,,$$

relating to any triply rectangular system of directions, arranged in the same mutual arrangement as those of

$$i\,,\ j\,,\ k\,,$$

satisfy the same relations as i, j, k, or, in other words, the relations between i, j, k are only a particular case of the relations—

$$\begin{aligned}
\rho'^2 = \sigma'^2 = \tau'^2 &= -1\\
\sigma'\tau' &= -\tau'\sigma' = \rho'\\
\tau'\rho' &= -\rho'\tau' = \sigma'\\
\rho'\sigma' &= -\sigma'\rho' = \tau'\,,
\end{aligned}$$

where ρ', σ', τ' represent respectively $\mathrm{U}\rho$, $\mathrm{U}\sigma$, $\mathrm{U}\tau$.

We remark the formulæ

$$\mathrm{U}\tau = \frac{\mathrm{V}(\rho\varpi)}{\mathrm{TV}(\rho\varpi)}\,;\ \mathrm{U}\sigma = \frac{\mathrm{V}(\rho\varpi)\times\rho}{\mathrm{T}(\mathrm{V}\rho\varpi)\times\mathrm{T}\rho}\,.$$

§ 5. *Generalisation of the Rule of Multiplication according to the Distributive Law, applied to Vectors and to Quaternions.*

We have to show that any expressions for ρ and ϖ, formed by vector-addition, may be treated by the distributive law, when multiplied one by the other.

We consider the result above

$$\rho\varpi = -(ax + by + cz) + i(bz - cy) + j(cx - az) + k(ay - bx).$$

We replace a, b, c, x, y, z respectively by $a' + a''$, $b' + b''$, $c' + c''$, $x' + x''$, $y' + y''$, $z + z''$ in the second member, and we effect the multiplication of

$$(a' + a'')(x' + x''), \text{ &c., &c.,}$$

by the distributive law applied to scalar multiplication.

If we designate by

$$\rho', \ \varpi', \ \rho'', \ \varpi'',$$

the vectors

$$ia' + jb' + kc', \ ix' + jy' + kz',$$

and two other similar expressions with two dashes ($''$), then the result of the substitution of $a' + a''$, &c., in the expression of $\rho\varpi$, will be composed of the aggregate of terms which represent respectively the products

$$\rho'\varpi', \ \rho'\varpi'', \ \rho''\varpi', \ \rho''\varpi''.$$

But in reality they are not in that order; the vectors, as for example, are in three groups or components parallel respectively to i, j, k. But neither addition, nor even vector addition, is changed in its result if we change the relative order of the terms to be added. We therefore add together, first the scalars belonging to $\rho'\varpi'$, and then the vectors belonging to it; then we group the terms belonging to $\rho'\varpi''$, and so on.

In the first member we have evidently also, for similar reasons,

$$\rho = i(a' + a'') + j(b' + b'') + k(c' + c'')$$
$$= (ia' + jb' + kc') + (ik'' + jb'' + kc'')$$
$$\rho = \rho' + \rho'',$$

and equally

$$\varpi = \varpi' + \varpi''.$$

Therefore we arrive at the result

$$(\rho' + \rho'')(\varpi' + \varpi') = \rho'\varpi' + \rho'\varpi'' + \rho''\varpi' + \rho\varpi',$$

which is the expression of the distributive law in its most general form as to vectors.

The formula is easily generalised to the case of three terms, $\rho = \rho' + \rho'' + \rho'''$, $\varpi = \varpi' + \varpi'' + \varpi'''$.

Let us apply this to the squaring of a vector, $\lambda + \mu + \nu$.

$$\begin{aligned}(\lambda + \mu + \nu)(\lambda + \mu + \nu) = \lambda^2 &+ \lambda\mu + \lambda\nu \\ &+ \mu\lambda + \mu^2 + \mu\nu \\ &+ \nu\lambda + \nu\mu + \nu^2 .\end{aligned}$$

Now we have

$$\lambda\mu + \mu\lambda = 2\mathrm{S}\lambda\mu ,$$

because

$$\mathrm{V}\mu\lambda = -\mathrm{V}\lambda\mu .$$

Therefore

$$\begin{aligned}(\lambda + \mu + \nu)^2 = \lambda^2 + \mu^2 + \nu^2 \\ + 2(\mathrm{S}\lambda\mu + \mathrm{S}\nu\lambda + \mathrm{S}\mu\nu) .\end{aligned}$$

Multiplication of Two Quaternions, not Conjugates necessarily, one by the other.

Let $p = a + \alpha$, $q = b + \beta$ be the quaternions, where

$$\begin{aligned}a &= \mathrm{S}p \quad , \quad b = \mathrm{S}q \\ \alpha &= \mathrm{V}p \quad , \quad \beta = \mathrm{V}q .\end{aligned}$$

Our demonstration will consist in the verification of the distributive rule applied to $(a + \alpha)(b + \beta)$, and the *test* will be the condition, that the tensor of the expression taken as product, will have to be equal to the product of the tensors of the factors.

Let us put

$$\begin{aligned}ab + \mathrm{S}\alpha\beta &= c \\ a\beta + b\alpha + \mathrm{V}\alpha\beta &= \gamma ,\end{aligned}$$

and let us verify if we have

$$\mathrm{T}(c + \gamma) = \mathrm{T}(a + \alpha) \times \mathrm{T}(b + \beta) ?$$

By what we have established in the instance of two conjugate quaternions, we have

$$\mathrm{T}(a + \alpha) = \sqrt[2]{a^2 - \alpha^2}, \text{ \&c.}$$

Therefore we have to verify if

$$c^2 - \gamma^2 = (a^2 - \alpha^2)(b^2 - \beta^2) ;$$

and as the second member is a scalar, we have to verify if

$$c^2 - \gamma^2 = a^2b^2 - \alpha^2b^2 - \beta^2a^2 + \alpha^2\beta^2 .$$

Now, we have by squaring—

$$c^2 = a^2b^2 + 2ab\mathrm{S}\alpha\beta + \mathrm{S}^2\alpha\beta$$
$$\gamma^2 = \alpha^2b^2 + a^2\beta^2 + \mathrm{V}^2\alpha\beta$$
$$+ 2[b\mathrm{S}.\alpha\mathrm{V}\alpha\beta + a\mathrm{S}.\beta\mathrm{V}\alpha\beta + ab\mathrm{S}\alpha\beta].$$

But $\mathrm{V}\alpha\beta$ is perpendicular to both α and β; therefore

$$\mathrm{S}.\alpha\mathrm{V}\alpha\beta = 0, \ \mathrm{S}.\beta\mathrm{V}\alpha\beta = 0 .$$

Therefore $c^2 - \gamma^2$ must be equal to

$$a^2b^2 - \alpha^2b^2 - a^2\beta^2 + \mathrm{S}^2\alpha\beta - \mathrm{V}^2\alpha\beta ;$$

and as we have found above

$$\mathrm{S}^2\alpha\beta - \mathrm{V}^2\alpha\beta = \mathrm{T}^2\alpha\beta = (\mathrm{T}\alpha)^2 \times (\mathrm{T}\beta^2 = \alpha^2\beta^2 ,$$

it follows that the expressions of c and γ are respectively the scalar and the vector of the product of $(a + \alpha)(b + \beta)$; and therefore *the distributive law* applied to the multiplication of *two quaternions gives us their product.*

The typical form of a quaternion being

$$q = \mathrm{T}q(\cos u + \mathrm{U}\theta \sin u) ,$$

where u is supposed to be between 0° and 180°, gives by multiplication, and comparison, for the irreducible, positive fraction $\frac{m}{n}$:

$$q^{\frac{m}{n}} = (\mathrm{T}q)^{\frac{m}{n}}[\cos v + \mathrm{U}\theta \sin v],$$

where

$$v = \left[\frac{m}{n}(u + 360^\circ \mathrm{N}) - 360^\circ \mathrm{M}\right],$$

N having to receive one of the n values

$$= \frac{n-1}{2}, \ldots -1, 0, +1, \ldots = \frac{n-1}{2} ;$$

and M taking *one* value for each value of N, in order to reduce the value of v, so as to be comprised between − 180° and + 180°.

The system of values N = 0, M = 0 in the case of

$$0 = \frac{m}{n} u = 180^\circ ,$$

may be noticed as being applied in spherical trigonometry.

§ 6. *Products of Three or more Factors. Associative Property in Multiplication.*

By definition, (17), the product of three factors, vectors or quaternions, suppose p, q, r, to be quaternions, is to be made so as to multiply the first multiplicand r, by its next multiplier q, so as to form the product qr; and taking this again for a multiplicand, multiply it by p. The product will then be :

$$p \times (qr) = pqr .$$

The question arises, if the product is the same when one multiplies r by the product pq? or, in formula, is

$$pq \times r, \text{ the same as } p \times qr ?$$

The affirmative to this question constitutes the *associative property* in multiplication ; namely, let be three quaternions—

$$p = a + \alpha , q = b + \beta , r = c + \gamma .$$

We get by the rule of multiplication of two quaternions—

$$\mathrm{S}qr = bc + \mathrm{S}\beta\gamma$$
$$\mathrm{V}qr = b\gamma + \beta c + \mathrm{V}\beta\gamma .$$

Then

$$p \times qr = (a + \alpha)(\mathrm{S}qr + \mathrm{V}qr)$$

can again be developed by the rule of multiplication of two ordinary quaternions.

If we develope the product we get ten terms, which, being grouped conveniently, reduce themselves to the eight terms which one would obtain in applying the distributive rule to the expression

$$(a + \alpha)(b + \beta)(c + \gamma) ,$$

and observing the rule of preserving the vector factors α, β, γ, in their order in the partial products, namely, α in the place to the left, then β, and, thirdly, γ in the place to the right.

Seven of these terms will contain not more than two vector factors; only one contains three, namely, the product

$$\alpha \times (\beta\gamma) .$$

If we effect the product

$$(\mathrm{S}pq + \mathrm{V}pq)(c + \gamma) ,$$

we will find again eight terms, seven of which will be at once declared identical with the seven corresponding ones of the product

$$(a + \alpha)(\mathrm{S}qr + \mathrm{V}qr) ,$$

because they contain not more than two vector factors, and they therefore are of the forms

$$\lambda\mu w, \text{ or } \lambda w\mu, \text{ or } w\lambda\mu ,$$

where w is a scalar, and λ, μ, vectors.

The eighth term will be

$$\alpha\beta \times \gamma ,$$

and the question is reduced to the demonstration that

$$\alpha\beta \times \gamma \text{ and } \alpha \times \beta\gamma$$

are identical.

The following demonstration may be, perhaps, the shortest.

We decompose β and γ into components parallel respectively to a triple rectangular system, of which the versors are ρ', σ', τ', and whose directions are the following—

ρ' parallel to α ;

σ' perpendicular to α, and in the plane comprising α and β, led through a common origin ;

τ' perpendicular to ρ' σ'.

Then we will have the expressions—

$$\begin{aligned} \alpha &= A\rho' \\ \beta &= B\rho' + B_1\sigma' \\ \gamma &= C\rho' + C_1\sigma' + C_2\tau'. \end{aligned}$$

The six scalar coefficients A, B, &c., have determinate values, but we need not effect their determination.

Then we have

$$\alpha \times \beta\gamma = A\left[\begin{array}{l} B(C\rho' \times \rho'^2 + C_1\rho' \times \rho'\sigma' + C_2\rho' \times \rho'\tau') \\ + B_1(C\rho' \times \sigma'\rho' + C_1\rho' \times \sigma'^2 + C_2\rho' \times \sigma'\tau') \end{array}\right]$$

$$\alpha\beta \times \gamma = A\left[\begin{array}{l} B(C\rho'^2 \times \rho' + C_1\rho'^2 \times \sigma' + C_2\rho'^2 \times \tau') \\ + B_1(C\rho'\sigma' \times \rho' + C_1\rho'\sigma' \times \sigma' + C_2\rho'\sigma' \times \tau') \end{array}\right].$$

Now ρ', σ', τ' satisfy the relations set down in § 4 analogous to those between i, j, k. Therefore we have

$$\begin{aligned} &\text{1st, } \rho' \times \rho'^2 = \rho' \times (-1) = -\rho' = \rho'^2 \times \rho' \\ &\text{2d, } \rho' \times \rho'\sigma' = \rho'\tau' = -\sigma' = \rho'^2 \times \sigma' \\ &\text{3d, } \rho' \times \rho'\tau' = \rho'(-\sigma') = -\tau' = \rho'^2 \times \tau' \\ &\text{4th, } \rho' \times \sigma'\rho' = \rho'(-\tau') = \sigma' = \tau'\rho' = \rho'\sigma' \times \rho' \\ &\text{5th, } \rho' \times \sigma'^2 = -\rho' = \tau'\sigma' = \rho'\sigma' \times \sigma' \\ &\text{6th, } \rho' \times \sigma'\tau' = \rho' \times \rho' = \tau' \times \tau' = \rho'\sigma' \times \tau' . \end{aligned}$$

The first members are the expressions entering into $\alpha \times \beta\gamma$, and the last members are those which enter into $\alpha\beta \times \gamma$.

The identity of $\alpha \times \beta\gamma$ and $\alpha\beta \times \gamma$ is therefore established. Therefore also the identity of $p \times qr$ and of $pq \times r$.

Applying the associative property to the product of four vector factors, $\alpha\beta\gamma\delta$, we liken α to p, β to q, and $\gamma\delta$ to r. Then we have

$$(\alpha\beta) \times (\gamma\delta) = \alpha \times [\beta \times (\gamma\delta)].$$

So that if two quaternions $\alpha\beta$, $\gamma\delta$ are the result of the products of two vectors each, their product may be formed as if the product of the four vectors had to be made according to the general definition of multiplication, namely, we have

$$(\alpha\beta) \times (\gamma\delta) = \alpha \times [\beta \times (\gamma\delta)] = \alpha\beta\gamma\delta.$$

General Remark.—The product of any number of vector, or quaternion factors, may be indicated irrespectively of the grouping together, or the indicating of the intermediate products.

As, for example, the product of n factors α_1, α_2 in α_n, may be looked upon as formed by two factors

$$(\alpha_1\alpha_2 \dots \alpha_h) \times (\alpha_{h+1}\alpha_{h+2} \dots \alpha_n),$$

for any value of h comprised in the enumeration $h = 1, 2, \dots n-1, n$; the last value giving to the second factor the value = *one.*

We may also remark the following *theorem*, founded on the associative property

$$\mathrm{K}q\,\mathrm{K}p \times pq = \mathrm{K}q \times (\mathrm{K}p \times p)q = (\mathrm{T}q)^2(\mathrm{T}p)^2,$$

because $\mathrm{K}p \times p$ is a scalar $= (\mathrm{T}p)^2$, and $\mathrm{K}(pq) \times (pq) = (\mathrm{T}pq)^2 = (\mathrm{T}p)^2 \times (\mathrm{T}q)^2$. Therefore

$$\mathrm{K}(pq) = \mathrm{K}q\,\mathrm{K}p.$$

§ 7. *Division, and the Definition of a Quaternion by that Operation.*

We *define*, (18), *division* by likening the *dividend* A to the *product* of the *divisor* B, as *multiplicand*, by the *quotient* C, as *multiplier*, A, B, C being quaternions generally.

This gives by definition

$$\mathrm{CB} = \mathrm{A}, \quad \mathrm{C} = \frac{\mathrm{A}}{\mathrm{B}}.$$

We multiply both members of the identity $\mathrm{KB} = \mathrm{KB}$ respectively by CB and by A. This gives

$$(\mathrm{CB}) \times \mathrm{KB} = \mathrm{AKB}.$$

Applying to the first member the principle of the *associative property*, we have for it

$$C \times (BKB) = C \times (TB)^2,$$

so that

$$C \times (TB)^2 = A \times KB.$$

We may now divide by the scalar factor $(TB)^2$, and replacing C by what it represents, $A \div B$, we have the result of *division*—

$$A \div B = \frac{1}{(TB)^2} \times A \times KB.$$

In the particular case of A being a scalar only, this formula gives:

$$\frac{1}{B} \text{ or } (B)^{-1} = \frac{KB}{(TB)^2},$$

We may now write the general result of division under the form

$$\frac{A}{B} = A \times \frac{1}{B} \text{ or } = A \times B^{-1}.$$

Thus *division* of a dividend by a divisor is effected by making the *product* of the inverse of the divisor (as multiplicand) by the dividend (as multiplier).

In other words, a fractional expression like $A \div B$, or $\frac{A}{B}$, may always be replaced by the *product* of the inverse of the *denominator*, multiplied by the *numerator*, namely, by

$$A \times \frac{1}{B} = \frac{AKB}{(TB)^2}.$$

This absolves us, to say once for all, from the consideration of quotients under the form of a fraction.

We may add the general remark, that *multiplication* of an expression by another is expressed by writing the multiplier to the left of the expression to be operated upon; and that *division* of an expression by another is expressed by writing the inverse of the divisor, as a multiplicand, to the *right* of the expression to be operated upon.

When the scalars of A and B are zero, namely, when both quaternions reduce themselves to their vectors, ϖ and ρ suppose, then we have $KB = -\rho$, and the quotient

$$\frac{\varpi}{\rho} = -\frac{\varpi\rho}{(T\rho)^2} = \frac{T\varpi}{T\rho}(\cos \rho\varpi + U\tau \sin \rho\varpi).$$

This is the typical expression which the inventor of quaternions took as the starting-point for their theory.

We may remark the formulæ—

$$\left.\begin{aligned} \varpi \div \rho &= \varpi \times \rho^{-1} \\ \mathrm{K}(\varpi \div \rho) &= \rho^{-1} \times \varpi . \end{aligned}\right\}$$

Supplementary Note.

With the rules on the four operations, addition, &c. . . . division, at our disposal, we are enabled to *reduce to a quaternion any algebraical function of vectors.*

The problem of this reduction, with its rules of abridgment for the separate formation of the scalar of the function, and of the vector of the function, constitutes a distinct chapter in the theory of quaternions, into the details of which we do not intend here to enter.

We confine ourselves to the indication of two results, which relate to the general question. Let α_1, α_2, &c., α_n, represent n vectors, generally different from one another, and let α_h represent one of them. We may conceive the product of $n - 1$ factors, for $h = 1, 2, \ldots n$,

$$p_h = \alpha_{h+1}\, \alpha_{h+2} \cdot \cdots \cdot \alpha_{h+n-1} ,$$

where the index exceeds n, but where it is to be reduced to be not greater than n, by the supposition

$$\alpha_g = \alpha_{g-n} = \alpha_{g-2n} ,$$

so that it may be positive, and not greater than n in each of the factors.

This conceived, let us form the sum of terms

$$\Sigma_1^n (-1)^{h+1} \alpha_h \mathrm{S} p_h ,$$

the sign Σ indicating by its limits that h is to receive the n values 1, 2, 3 . . . . $n - 1$, n, upper limit included.

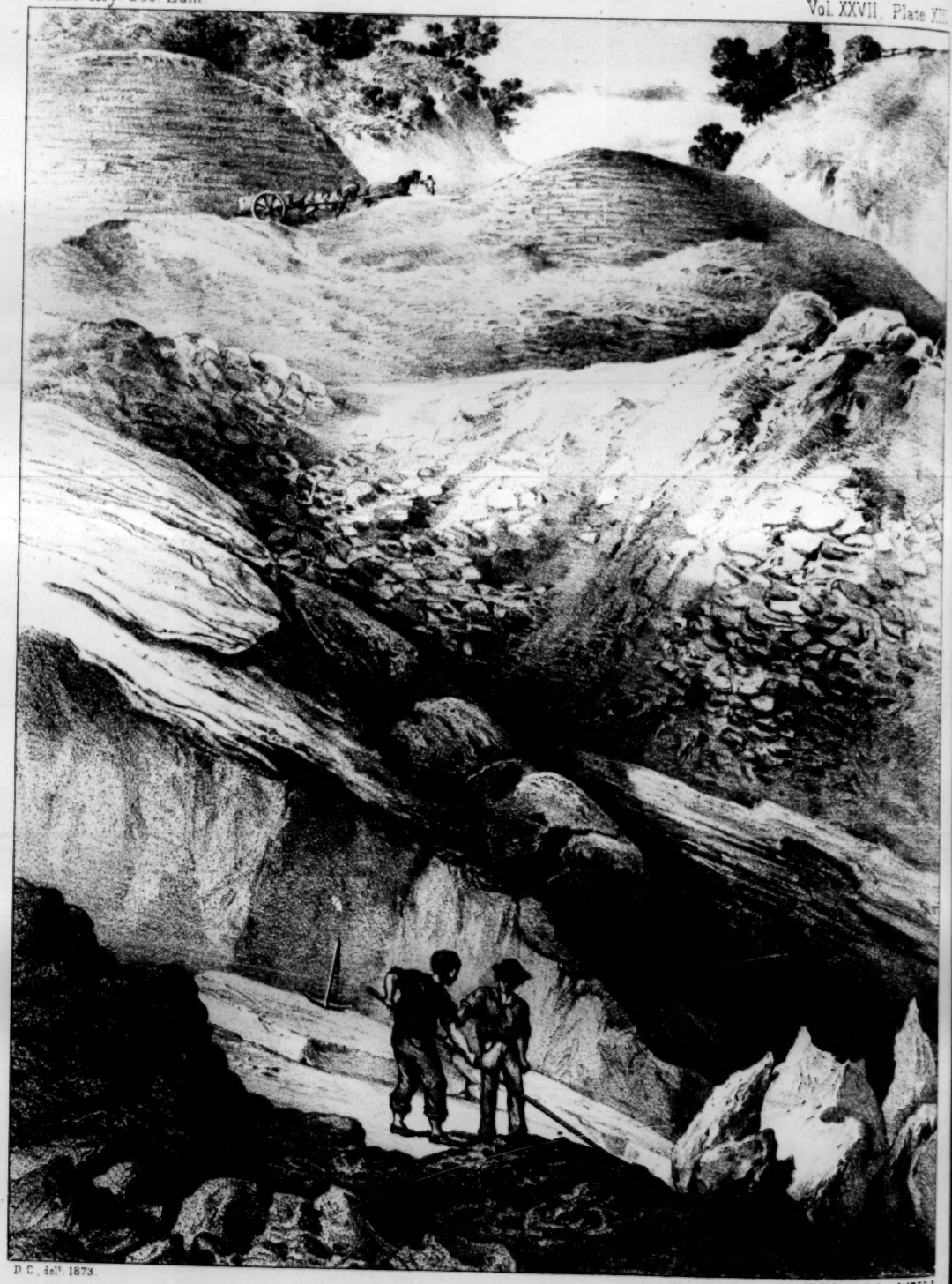
This sum expresses two different results, according as n is an even or an odd number.

When n is an *even number* the sum is equal to *zero.* When n is an *odd number*, the sum gives the expression of

$$\mathrm{V}(\alpha_1 \alpha_2 \ldots \alpha_n) ,$$

as a linear function of the single factors, respectively multiplied by a scalar.

It seems impossible to express in a similar form the vector of the product of an even number of vector factors. The expressions of such a vector by a linear function of the vectors of the $\frac{1}{2} n(n-1)$ combinations 2 by 2 of the factors is possible, but of little simplicity when n exceeds 4.



D. C., del.ᵗ 1873.

M.ᶜ Farlane & Erskine, Lith.ᴿ Edin.ᵍ

CRAIGLEITH FOSSIL TREE.

X.—*Notice of Fossil Trees recently Discovered in Craigleith Quarry, near Edinburgh.* By Sir Robert Christison, Bart., Honorary Vice-President, R.S.E. (Plate XIII.)

(Read May 5, 1873, and January 19, 1874.)

In February 1831 the late Mr Witham read to the Royal Society of Edinburgh, a paper of much interest on two fossil trees of great size which had been brought to light, the one in 1826, and the other in 1830, during the excavations carried on in Craigleith sandstone quarry, in the immediate neighbourhood of our city. In March of the same year this paper, with the addition of several chemical analyses of the fossils, was read also before the Natural History Society of Northumberland. In 1833 he included his observations on these fossils in a separate and more comprehensive treatise on "The Internal Structure of Fossil Vegetables found in the Carboniferous and Oolitic Deposits of Great Britain." The main purpose of Mr Witham's researches—in addition to an accurate description and delineation of natural objects previously little investigated—was to show that fossil vegetables in the oolitic and carboniferous formations were not, as had been generally supposed prior to his researches, always ferns, tree-ferns, lycopodiums, and other acrogenous plants, but on the contrary many of them woody exogenous trees, showing longitudinal ligneous bundles, transverse medullary rays, concentric annual layers, and other characters belonging to the intimate organisation of our existing forest trees. Mr Witham even went so far as to identify the structure of the Craigleith fossils with that of our modern pines, and to assign them to two separate species. His discoveries have been so far acknowledged by subsequent authorities, that the fossils are now generally known by the name of *Araucarioxylon Withami*. This name recognises their alliance with the now familiar Araucarias; but some inquirers are rather disposed to associate them with the yew tribe.

The recent disclosure in the same locality of two very perfect fossil trees, of still greater magnitude than those examined by Mr Witham, is an incident which seems to deserve being also recorded. I have therefore ventured to submit the following account of them to the Royal Society; and I shall take advantage of the opportunity to give a short historical notice of all the great vegetable fossils which can be traced as having been found in the remarkable quarry of Craigleith during the last fifty-five years.

These have been seven in number. 1. The first of which there is any record was uncovered in 1826. WITHAM describes it as a stem 36 feet in length, and 3 feet in diameter at its lower end. It has been only in part described by WITHAM, nor does he say that any portion of it was preserved. But there is reason to believe that it is represented by a fragment 3 feet 3 inches long, and 7 feet in girth at its widest, preserved at the Museum of Science and Art.—2. In 1830 was displayed the second, which was the chief object of WITHAM's researches. He describes it as 47 feet long; and he ascertained that there had been 12 feet more at the top before he saw it. The 12 lowest feet were in good preservation in the Royal Botanic Garden, and the next 18 feet in front of the Museum of Science and Art, to which this portion had been removed from the University, where it had been secured by Profesor JAMESON while keeper of the University Museum. By arrangement between Mr ARCHER and Dr BALFOUR, the whole remaining fragments of this fossil have been united in the Botanic Garden. It is 6 feet across where it is widest at the bottom; its length, accurately measured, is 30 feet 8 inches, and at its upper end the girth is 4 feet 4 inches.—3. The third fossil was uncovered about the year 1840, to the best of my recollection, and not far from where the two others had been found. No scientific account of it was ever published to my knowledge. But I have little doubt that, with the assistance of Mr CARRUTHERS of the British Museum, I have traced a large portion of it—and a fine specimen it is—as having been removed from Craigleith in 1854, by the late Mr Ross, donor of the fountain in Prince Street Garden, to his villa of Rockville, Murrayfield, where it sentinels in great blocks the avenue from the gate towards the house. The several segments, if united, would probably measure 24 feet. One of them is 7 feet high, and 10 feet in girth. A document in Mrs Ross's possession proves that they were conveyed to Rockville in 1854; and her gardener had seen the fossil in the quarry at least two years earlier.—4. The fourth, as I am informed by an old workman, after remaining several years in its bed in a different part of the quarry from the place where all the rest had been found, was removed not long before 1850 to his neighbouring mansion by the late Mr RAMSAY of Barnton, with the view of having it converted into polished slabs for furniture. It proved to be unfit for the purpose. But I have seen in good preservation behind Barnton House several segments of it, about 6 feet in girth, from 3 to 4 feet long, and sufficient to measure at least 12 feet if reunited. These segments exactly resemble Nos. 1, 2, and 3 in every main character.—5. The upper part of the fifth is believed to have been first brought into view in 1854. But after a few feet of it were torn out of its bed, it was covered over with rubbish of the quarry, until it was again recently displayed on excavation having been resumed in its vicinity. This is the largest of the fossils yet found in Craigleith. When I first saw it, as shown in the

drawing (Plate XIII.), 23 feet of it were exposed, firmly fixed in its sandstone bed; and its girth was estimated at 10 feet at least. As it was desirable that so magnificent a specimen should not be lost, I communicated, by permission of Mr HUNTER, lessee of the quarry, with Mr CARRUTHERS, botanical curator of the British Museum; and through his enterprise, and the liberality of the Museum trustees, not only has the then visible part of the fossil been conveyed to the National collection in London, but authority was also given to excavate, if possible, the remainder of it. This was not easy, for it descended at an angle of 70° through very tough sandstone, no longer within the operations of the quarry. It was most desirable, however, to see how the fossil terminated; and the opinion of the overseer was a great encouragement, who thought that the lowest sandstone bed at the spot would be penetrated at the depth of 6 feet. But an excavation was made, first of 14 feet, and then of 6 feet more, without any indication either of the fossil, or of the sandstone matrix, coming to an end. The lowest 6 feet too proved so brittle that it came away in small irregular fragments, which could not be again fitted together. Farther excavation was then abandoned, and a roadway now passes over the site. About 36 feet have been removed in large blocks to the British Museum, besides the comminuted fragments of 6 feet more; it is the intention of Mr CARRUTHERS to have the fossil set up, as far as possible, in imitation of its original attitude, and I have the assurance of the Museum authorities that it is "by far the most remarkable relic of Palæozoic vegetation known." Edinburgh geologists, however, will not regret its removal to so congenial a site as the great National Museum of the kingdom, considering that we have already in excellent preservation the fossil of 1830, which is nowise inferior except in size. The portion now in London varies in girth from 11 to 13 feet.—6. The sixth fossil came into view early in 1873, in the lowest workings of the quarry, at least 50 feet deeper than the last, and about 50 yards from it horizontally. There was a rumour, however, among the workmen of a higher part of it having been seen a few years since before the rock was excavated so low. For some time a cross section only was visible; for, being brittle, and the workmen's hammers lying conveniently near it when they left the quarry, it was easily broken up by visitors as it was gradually uncovered. Thus several feet were destroyed before I saw it. But it was then taken care of, and soon a round black fluted column, 9 feet in girth, stood up to the height of 6 feet from the white sandstone which had been removed from around it. Seven feet of it were conveyed to the Botanic Garden, and have supplied sections of great interest to the museum in the garden, the Museum of Science and Art, and the British Museum. In consequence of the deep workings of the quarry having been since given up, and to all appearance permanently, the spot has been flooded, and the remainder of this fossil has become inaccessible. This is to be regretted, because in several

respects it is the most interesting of all the Craigleith fossils.—7. To the foregoing list must be added a seventh, believed by the quarrymen to be a portion of a branch. It was found not far from the great stem, No. 5, but quite unconnected. It was originally 8 feet long; but when I first saw it in possession of the clerk of works, only 18 inches remained of its upper end. This is 5½ inches across where broadest, and 4½ inches where narrowest. As will be shown presently, it is not really a branch complete in its circumference, but a small, longitudinally-split section of a large branch, or possibly of a stem.

Mr WITHAM has alluded also, partly in his paper in the Royal Society's Transactions, and partly in that read to the Northumberland Natural History Society, to "a fragment of a third fossil stem with a branch," found after he had begun to write his paper, and to another portion having been got very near the last in August 1831. Of the former he has given no farther account than by describing and figuring a slice of a branch, which was a mere twig about an inch in diameter, and which showed concentric annual rings distinctly. Of the other he merely says that a slice 4 inches by 4 shows concentric zones ("Trans. Nat. Hist. Soc. of Northumberland," 1831, pp. 151, 152.) My attention has likewise been directed to several fragments of a large Craigleith fossil, preserved at the villa of Duncliffe, Murrayfield, by Mr BAILDON, who acquired them in 1866. If put together, these fragments would form a stem about 12 feet in length; and two of them, which are complete columnar sections, nearly round, measure 8 feet in girth. They are possibly the uppermost portions of No. 5, when it was temporarily uncovered a number of years ago. At least I have been unable to trace any other large fossil having been discovered in the quarry except the six whose history I have given above; and all but the top of No. 5 have been accounted for. Mr BAILDON never saw the fragments in his possession until they had been wrenched from their bed and scattered around it.

It is a matter of interest, not unconnected with the present inquiry, that during the time when the Craigleith fossils were brought to light, two others were discovered in the sandstone quarry of Granton, situated on the shore of our Firth two miles northward. This quarry, which was worked to a great extent, and to the depth of 80 feet, for the adjoining pier and harbour of Granton, produced a fine quality of sandstone, extremely like that of Craigleith. Much was added to this resemblance by the discovery of two large fossil trees, which presented all the characters of the Craigleith fossils. They were displayed in 1839, and were the subject of great interest to geologists at the second Edinburgh meeting of the British Association for Science in 1852. There is extant a lithographed sketch of them, taken from a camera-lucida drawing by the late Mr ROBERT ALLAN in that year; and I have obtained specimens of one of the fossils for examination through the kindness both of Dr

Balfour and of Mr Howkins, C.E., engineer in charge of Granton harbour. In October 1855, during the concurrence of a furious westerly storm and an unusually high tide, the sea threw down about 200 feet of the west sea-wall, and in a few minutes filled the quarry, which has remained drowned ever since, with many feet of water over the fossils even at low water of spring tides. One of the fossils was securely imbedded, high and inaccessible, on the inside face of the west sea-wall. The other, and much the larger of the two, lay on a lofty pedestal of sandstone, left in the middle of the quarry for its preservation. Both lay slightly inclined with their tops to the south, and their bottoms towards the north. Mr Howkins measured the larger one, and found it to be 75 feet long, 5 feet in diameter at the bottom, and 1 foot 8 inches at the top. Like No. 2 fossil at Craigleith, it was considerably flattened throughout its whole length.

Position and Form.—All the Craigleith fossil-trees save one have been found in the west end of the quarry, not far from one another. That removed to Barnton House alone lay at the east end. Nos. 2, 5, and 6 have alone been examined *in situ* by a competent observer. Mr Witham describes No. 2 as lying not many feet from the bottom of what he recognised as a basin in the stratification. The strata dip now from westerly to easterly, inclined at an angle varying from 19° to 38°. But Witham says they dipped from opposite directions towards his tree, and met horizontally where it lay. The basin thus formed must have been very limited, and has been dug out in the course of the much deeper excavations made since his time. But Witham's drawing remains to prove his accuracy; and the foreman of the works recollects the quarry having had, about twenty years ago, a limited horizonal bottom 50 feet or more above the present very deep workings. On every side now, however, the strata dip nearly in one direction, *i.e.*, more or less directly east.

No. 1 is stated by Mr Witham to have lain horizontally; but his account of it is so brief and imperfect, compared with that of No. 2, that probably he had not himself seen it in its bed. If the fragment mentioned above, as preserved at the Museum of Science and Art, belonged to No. 1, this fossil was substantially round in form.

No. 2 is described by Mr Witham as lying under about 100 perpendicular feet of sandstone rock, where the general dip of the beds is 12°, and the general direction of their dip towards N.N.E. The drawing made by Dr Greville, however, represents the fossil to lie where the stratification immediately adjoining is much contorted, and the general dip close to it only 5°. Both the direction and the dip of the fossil itself are different. As I read Mr Witham's description, the small upper end pointed 20° north of west. The dip was therefore in a direction 20° south of east; and by a line on the drawing, taken from

the small end or top, to the thick end or bottom, the angle of inclination was on an average 33°. But, as there are two bends in the trunk, in directions opposite to one another, the dip varied at different places, as Mr WITHAM ascertained, from 20° to 44½°.

The girth of this fossil increases very gradually from the small upper end towards the other extremity, and near the bottom it suddenly increases much more, like the lower end of most large forest trees of the present day. Along its whole length it is considerably flattened, so that a section is everywhere rudely elliptical. At the bottom, as it now lies in the Botanic Garden, it measures 6 feet across its widest diameter, which is a foot more than Mr WITHAM's measurement; and its narrowest diameter is about 2½ feet. At 12 feet above, the measurement is reduced to 27 inches by 17, and 73 in girth; and at 29 feet, the highest measurable part, it is 24 inches by 16, and in girth 52.

The portion preserved is divided into fourteen blocks by fractures generally transverse, but three of them oblique. The lower end is represented in the drawing as cut across abruptly without any vestige of root. As preserved in the Botanic Garden, too, there is no trace of any root, and the termination is rugged and shapeless. Neither is there in the whole of the fossil trunk any portion of a branch to be seen. At the upper end of the third block from the top there is a superficial cavity presenting very much the appearance of a branch having been torn out; and Mr WITHAM's drawing shows a small cylindrical fragment of what had been removed from the upper part of the fossil in its bed before the drawing was taken, bearing a similar and distinct mark of where a branch once had been. Moreover, at five different places, four of them situated in the middle of segments, and one at the seat of a fracture, there are more or less continuous transverse ridges, with several bumps on them more or less distinct, presenting altogether very much the appearance of the remains where whorls of small branches had been destroyed during the life of the plant. WITHAM mentions and figures a cross section of a small twig which was found not far from this fossil. But neither then nor since has there been discovered in any part of the quarry any remnant which can be positively called the portion of a branch.

The surface of this fossil is scored in some places with minute longitudinal grooves; in other places small crowded warty excrescences are seen; but nowhere does it show the fluted structure which abounds in some of the other fossils.

Of the position of Nos. 3 and 4 nothing can now be learned except that the former was found in the west end of the quarry not far from the last two, and No. 4 at a distance from these near the eastern boundary. Their form is known only from the segments preserved at Rockville Villa and Barnton House.

None of these are at all flattened. No. 3 approaches a correct cylindrical shape more than any other of the fossils. In several of the segments the surface is both finely grooved, and in some places coarsely fluted, in others warty. One segment, towards 4 feet long, being the half of the cylinder split longitudinally down the middle, shows, when the sunlight falls obliquely on it, the parallel lines of concentric annual layers, exactly as may be seen on a modern pine trunk when split in the same manner.

No. 5 lay near where No. 2 was found, and originally 110 feet of rock had covered its upper end. The strata around it dipped in a direction from 27° south of west to 27° north of east, and at an inclination of 28°. But the direction and inclination of the fossil, as in the case of No. 2, were different. It dipped in a direction from 5° south of west to 5° north of east. The 23 feet of it first seen by me in its bed were somewhat curved, the convexity being upwards; and the chord of the arch was inclined at an angle of 60°. Hence the fossil crossed the direction of the beds where it lay at an angle of 22°, and dipped through their parallel lines at an angle with them of 32°. The lowest 12 feet, however, of what was then seen had a dip of 70° to the horizon; and this dip continued the same 20 feet deeper, or as deep as the excavation was subsequently carried. The curvature was owing chiefly to two cracks, which were widest at the convexity, and were there not filled up in any way. At 31 feet from the lowest part excavated the trunk had apparently forked. There was a rough surface about 9 inches wide, which, had the fossil been erect, would have been not far from horizontal; and for 2½ feet above this mark the trunk had a longitudinal shallow hollow, such as would have been caused by a branch ascending for a few feet in contact with the trunk. At two other places less distinct marks might be seen of what might have been the attachment of small branches torn off. The surface of the fossil showed fine longitudinal furrows, but nowhere any fluting or warty excrescences. It came away from its bed chiefly in large blocks between 4 and 5 feet long, and 4 feet wide, not at all flattened. The fractured surfaces were most of them directly transverse, and some of them had the appearance of natural cleavages, more remarkable, however, in the next fossil.

No. 6 lay near the extreme west end, and in the deepest part of the quarry, at a place now flooded. It was covered by fully 180 feet of hard sandstone beds. The direction of the strata which it pierced was the same as in the case of the last fossil, namely, from W.S.W., dipping towards E.N.E.; and the angle of inclination was 36°. The direction in which the fossil lay was in this instance the same with that of the strata; but its angle of inclination was 61°, or 25° more than that of the beds which it traversed. Six feet of it only were ever seen at one time cleared of its matrix, and a most remarkable object it then was, standing up black and cylindrical from its white sandstone base. About

10 feet of it at most has been removed, and undoubtedly much more of it still remains in the floor of the quarry. The segments in the Botanic Garden show that they are not flattened, but rudely cylindrical, nearly 9 feet in girth, and more fluted on the exterior than any other of the Craigleith fossils.

No. 7, at first supposed to have been a branch, was not attached to any of the previous fossils, but was found some yards from No. 5. When entire it must have been very like a branch; but its internal structure, which has been preserved more exactly than in any other fossil, will presently be shown to be such as to prove that it is not a branch, but a longitudinal section, possibly of a trunk. The smaller end is roundly pointed and rugged, as if much worn by attrition.

I have nothing more to add here to what has been already briefly said of the position and form of the fossils of Granton quarry. Two fragments, all which now remain, except the inaccessible trunks in the flooded quarry, have been presented by Mr HOWKINS to the Botanic Garden collection. One of them shows that the fossil to which it belonged was, like No. 2 from Craigleith, considerably flattened.

STRUCTURE.—The Craigleith fossils are at first uniformly covered over their whole surface with a jet-black coal-like substance, like what is often observed on other fossils of the coal-formation. This is so brittle, and therefore so easily detached, that much of it has disappeared from WITHAM'S fossil of 1830, preserved in the Botanic Garden, as well as from those which were discovered not long afterwards. But in many places patches of it still remain. In No. 5, removed to the British Museum, the coal-like covering existed everywhere. On the upper 23 feet first displayed it was for the most part only $\frac{1}{4}$ th of an inch in thickness; in furrows and hollows $\frac{1}{10}$th. But lower down, at 6 feet from the lowest part excavated, it had increased to $\frac{1}{2}$ an inch in thickness, and at the lowest depth reached to $2\frac{1}{2}$ inches. Its adhesion to the surface beneath it was very slight; but it was impressed with striated longitudinal lines by the surface of the subjacent structure. It has exactly the conchoidal fracture, glassy lustre, and brittleness of caking-coal. Heat acts on the one exactly as on the other, that is, the coally crust froths up, cakes firmly, and gives off abundance of smoke, burning at the same time with a large dense white flame, becoming a coke like that from Newcastle coal, and leaving a very scanty ash when incinerated. It is, in short, a fine variety of highly bituminous coal, containing very little earthy matter. It does not present any appearance of structure when examined with the naked eye, or with the microscope by reflected light. Its great brittleness prevents it from being ground so thin as to be examined by transmitted light.

This crust, and the similar covering of numberless small fossil vegetables

found in the beds of the coal-formation, have been usually regarded as fossilised bark, and Mr WITHAM expressly endorses that opinion. But it will be seen afterwards that the coaly crust cannot have been the bark, at least in the case of the Craigleith fossils.

It had evidently covered every part of No. 7, the supposed branch,—its entire girth, and even the whole of its rugged worn point, although, for reasons to be given presently, the greater part of that fossil could never have had any bark upon it at all.

The black crust left here and there on WITHAM's fossil, No. 2, and on those at Rockville Villa and Barnton House, Nos. 3 and 4, has the same appearance as that of No. 5, but is less brittle, does not froth up or cake when heated, gives off less flame and smoke, and leaves more ash after incineration. It is still a very bituminous coal, but is much more earthy.

The structure of the fossils below this thin crust is totally different. All the investigations made on this subject agree exactly with the excellent description and drawings of Mr WITHAM's papers, and confirm his deductions by some facts which did not come under his observation.

The structure of No. 6, the lowest fossil in the quarry, and that of No. 7, the "branch," are the most distinct and interesting. No. 6 is much more brittle than the others; it is of a darker grey colour, and it leaves a fine brown dusty stain on the fingers when handled. The longitudinal fracture is in many places finely fibrous to the naked eye, and exactly like the fibrous appearance of wood; and, not unfrequently, such fractured surfaces present in great abundance to the naked eye the transverse parallel lines which denote transverse medullary rays. Cavities often occur, very ragged on their inner surfaces, filled with a fine charcoaly matter, and sometimes lined with crystals of pearl-spar and calc-spar. From such a cavity I have a specimen of transparent crystals of calc-spar as large as the finger nails. Thin translucent plates of the fossil show before the microscope, in a longitudinal section, crowded parallel fibrils; in a transverse section equally crowded hexagonal cells in regular lines, being the cross-cut cavities of the fibrils seen parallel to one another in longitudinal sections; and a lucky longitudinal slice shows also transverse medullary rays. The definition of all these objects is very clear from their blackness, which is owing apparently to very fine charcoal accumulated in the region of the cell-walls. In some places, however, these appearances of vegetable structure are displaced by little masses of roughly radiated crystallisation, obviously the fossilising material substituting its own crystalline arrangement.

The structure of the fossil being thus very distinctly shown in small sections for the microscope, a successful observation in the case of No. 7 led to a trial being made to obtain a demonstration of concentric annual layers on the large

scale by cutting across the entire trunk of No. 6, nearly 3 feet in diameter, and polishing the whole surfaces. The first trial failed to show concentric layers. A second section, of which one side is preserved in the Botanic Garden Museum, and the other in the Museum of Science and Art, shows in many places long stretches of rudely parallel concentric layers, rendered distinct by fine lines of white crystalline matter formed at the junction of the annual layers, while the boundaries of others are faintly marked by characters to be noticed in now describing the structure of No. 7.

This fragment, wherever it is stripped of its coaly crust, is finely fibrous on the surface, like that of wood. It is extremely tough. It has a dull grey colour, which, when it is polished, becomes shining black, like the finest black marble. A cross section, near its wider end, is of the shape of the letter D. On its polished surface may be seen in some lights nine faint marks, in somewhat parallel concentric curves, crossing from the straight line of the D to the opposite curved side, but interrupted by white calcareous veins. A thin transverse slice $2\frac{1}{8}$ inches long, and $1\frac{1}{8}$ inch wide, taken from the straight side where these marks were most distinct, and including five of them, shows by transmitted light to the naked eye, and still better before a common magnifier of three or four powers, that the marks are really the boundaries of annual rings, varying from $\frac{1}{10}$th of an inch to $\frac{1}{2}$ an inch in width. Rows of parallel cells and transverse medullary rays are seen crossing the surface, forming at each of the boundaries an open loop, always in one direction, and presenting a darker and a lighter shade on the opposite sides of the loop, so that the boundaries have much the appearance of a mountain chain as usually represented on a map. As the curvature of these boundary lines is very small, it is evident that they must have formed part of large circles. The fragment, therefore, is the longitudinally split segment either of a trunk or of a large branch. The only place where it is possible that bark had once existed is at the convexity, towards which alone the lines are convex. Nevertheless the whole circumference of the segment had been uniformly covered with the crust of bituminous coal. Hence this crust cannot have been bark.



Transverse Section of probably the portion of a branch of Fossilised Wood from Craigleith Quarry. Natural size. Showing the Annual Layers of Wood.*

The structure of Nos. 6 and 7, as now described, agrees in all respects with

* Mr Nichol, in his papers in the Edin. New Phil. Journal for 1834 on the "Structure of Vegetable Fossils," says he found that two Araucarias which now exist have no annual rings; and two others indistinct rings. Numerous sections of *A. imbricata* and *A. excelsa* in the Botanic Garden Museum show the annual rings most distinctly.

the excellent description by Mr WITHAM of the structure of Nos. 1 and 2. I am enabled, however, to make an interesting addition to his account of No. 2. When its segments were separated, I observed a cavity of the size of the fist at the end of one of them, and a corresponding cavity in the next segment, both containing a large quantity of charcoal. This was mostly in extremely fine powder. But in scraping the walls there came away small fragments of black, light, soft, fibrous matter, undistinguishable from common charcoal. By carefully breaking down these fragments in Canada balsam, Mr SADLER obtained several specimens which exhibit before the microscope, on the longitudinal surfaces, the quincuncial punctated structure of the pine tribe, as represented in the adjoining figure. I am not aware that this characteristic structure has been observed before in any of the Craigleith fossils since it was described by Mr Nichol in 1834.



Disc-bearing tissue from Craigleith fossil tree. The discs are arranged alternate as in Araucauria. The structure is magnified 200 diameters.

The structure of Nos. 4 and 5 has been ascertained to be also substantially the same as that of the preceding fossils. In the case of the Barnton House fossil, No. 4, a small transverse slice shows distinctly several little scattered portions of methodically-arranged hexangular or deformed cells, separated from one another by radiated crystalline masses of fossilising matter destitute of all organic form, or by broad black bands, consisting probably of charcoaly cell-walls agglomerated by pressure when the crystalliform structure was forming, and pushing aside the organic tissue.

CHEMICAL COMPOSITION.—Mr WITHAM, without giving his authority for the statement, says the fossil found in 1826 was composed in 100 parts, of 60 carbonate of lime, 18 oxide of iron, 10 alumina, 9 carbonaceous matter, and 3 loss, —that the tree found in 1830, and now in the Botanic Garden, contained 62 carbonate of lime, 33 carbonate of iron, and 5 carbonaceous matter; and that the "fragment and branch" found in 1831, consisted of 37·5 lime, 24·2 oxide of iron, and 36·1 coal. These analyses are certainly incorrect. In another analysis, supplied by my former University colleague, Dr WILLIAM GREGORY, the iron proved to be 21·8 per cent., apparently in the form of protoxide; the carbonaceous matter, partly coaly, 14·3 when all reduced to the form of charcoal, and the earthy matter largely composed of carbonate of lime, with mere traces of silica and alumina. These results, besides being incomplete, are in some respects erroneous. A subsequent analysis by Mr ROBERT WALKER, briefly noticed in Professor JAMESON'S "Philosophical Journal" for 1834, and referring apparently to WITHAM'S fossil of 1830 (xviii. 363), gives the composition as carbonate of lime 50·36, carbonate of magnesia 17·71, carbonate of iron 24·65, coal, silica, alumina, water 6·15, loss 1·13. This analysis appears to be sub-

stantially correct, and only in so far faulty as the carbonaceous matter of the substance of the fossil is called coal.

From many analyses, some quantitative, others qualitive only, I have come to the following conclusions:—1. That the thin black outer crust is caking coal, sometimes very pure, but in some united with the material which fossilises the interior; 2. That the whole vast interior is magnesian limestone, strongly charged with carbonate of protoxide of iron, and to a less amount with charcoal, or,—so to speak, charcoaly ferruginous dolomite; 3. That there is extremely little silica or alumina, and sometimes none, so that these ingredients are probably adventitious; 4. That the proportion of each ingredient varies in different parts of the same fossil, and consequently may seem to differ in the several fossils; but 5. That the composition of the interior of all the Craigleith araucarious fossils is substantially the same.

The coal-like crust of these fossils is everywhere in immediate contact with their woody structure, of the surface of which it bears the impression. The attachment is so slight that the crust is very easily removed. I have examined with greatest care the crust of No. 5, which is now in the British Museum. Its colour is jet-black, its lustre glassy, its fracture conchoidal, and its brittleness great. Its density is 1·225. Acids and alkalis do not act on it. When heated in small fragments or in powder in a platinum crucible, it gives out much black smoke or dense white flame, froths up and cakes, becomes a roundish mass of very light, vesicular, firm, glittering coke, and when then broken up may be burnt away with the exception of a scanty, greyish-red ash. The proportion of volatilisable bituminous matter, charcoal, and ash, varies a little. The very thin part of the fossil gave 66·8 charcoal, 29·7 bitumen, and 3·4 ash; but the ash sometimes amounted to 6·0. A mass, from the lowest part of the fossil, nearly as large as the fist, gave 74·0 charcoal, 24·2 volatilisable matter, and only 1·1 to 1·7 ash. Abstracting the ash, the proportion of the charcoal to the bituminous matter seems to vary very little. The ash yields no alumina to strong potash-solution aided by heat, and only a little lime, but neither magnesia nor iron, to diluted nitric or hydrochloric acid; in this respect differing entirely from the mineral matter of the great fossil which the coal-like covering encrusts. It is evident from this account that the crust presents all the properties of the finest caking splint coal.

The black crust is not in the case of all the fossils so fine a bituminous coal as that now described. What is still left attached to Mr WITHAM's fossil, No. 2, after forty-three years of exposure to the open air, froths up very little when heated, cakes but slightly, and leaves when incinerated a large amount of ash, apparently the same earthy matter which mineralises the interior of the fossils.

It may be here observed in passing, that the coal-like crust, which is well

known to envelope many fossils of the Coal-measures, has not received from geologists the attention which it deserves. Neither the circumstances in which it occurs, nor its composition, have been made the subject of inquiry. It covers many specimens of Sigillaria, Lepidodendron, and fossilised stems of other plants of loose cellular texture, found abundantly in the carboniferous strata of the Scottish lowlands; but it is not always present. A fine Sigillaria from Redhall quarry,—where, as will be presently mentioned, a large araucarious fossil has been lately found, covered with caking coal,—presents no coaly crust at all, either on its surface, or on that of the sandstone mould in which it had lain. On the other hand, a lepidodendron from Craigleith quarry, about an inch in diameter, is uniformly covered with a very brittle black shining coat, which is acted on by heat exactly as the crust of the araucarious fossil, No. 5, and is therefore a very pure caking bituminous coal. I found a similar specimen among seven in the Edinburgh Botanic Garden Museum; and another also among nine, which I owe to Dr Dickson, Professor of Botany in Glasgow University. Of the others all emitted much dense white flame, and several caked slightly, but some not at all. The frothing up, caking, and flaming seemed to be proportional to the scantiness of mineral matter in each specimen; but that point was not fixed numerically.

The composition of the great interior mass of these fossils proves to be entirely different from that of the thin black crust. Its fracture has a dull-grey earthy appearance, like the ordinary limestones of the neighbourhood. The density of No. 2 is 2·54, of No. 5, 2·64, of the "branch," No. 7, 2·88. Witham's fossil, No. 2, when subjected in fragments to the action of diluted hydrochloric or nitric acid, effervesces freely like marble; the fluid soon becomes black; and, with a sufficiency of acid, nothing is left undissolved but an impalpable carbonaceous matter, and a few siliceous particles. The silica seems adventitious, because the little particles are large enough to feel gritty and to be easily separated, and because they are not met with in others of the Craigleith araucarious fossils, such as No. 6. The black matter is charcoal, for it burns in a platinum spoon with a red glow, and no flame, or a transient, lambent blue flame only; and there remains 2·1 per cent. of loose greyish-red ash, with which acids cause scarcely any effervescence, and from which they remove only a trace of lime. It is ordinary charcoal, not graphite, because it is dissolved with the aid of powerful oxigenating fluids, such as sulphuric acid with chlorate of potash. Hence the carbonaceous matter, which partly composes the substance of the fossils, may be regarded as vegetable charcoal, with rather more than the usual proportion of earthy matters. It is not coal like the carbonaceous crust of the fossils; it contains no volatilisable bituminous matter.

The hydrochloric acid solution, if effected without access of atmospheric air,

is colourless. When neutralised, or nearly so, and filtered direct into solution of ferridcyanide of potassium, a dark prussian-blue precipitate is at once formed abundantly; but if the ferrocyanide be substituted, the precipitate is at first of the palest possible azure-blue, passing gradually to deep blue under exposure to the air. Farther, the hydrochloric solution, filtered into solution of carbonate of soda, yields a white precipitate; but if nitric acid be substituted for the hydrochloric acid to dissolve the fossil, the precipitate is of a dark ochre colour. Hence it follows that the fossil contains iron entirely in the state of protoxide. Its quantity was ascertained with sufficient approximation by throwing it down in the form of peroxide by adding ammonia to the nitric acid solution, acting on the washed precipitate with potash solution to remove a little alumina, and rendering the peroxide anhydrous at a full red-heat.

When the potash solution, obtained by the last part of the preceding process, contained alumina, which, however, was not always the case, the alumina was separated by heating the solution after the addition of hydrochlorate of ammonia. When the proportion of alumina was to be ascertained precisely, the alumina and peroxide of iron were thrown down from the nitric acid solution by precipitated carbonate of baryta instead of ammonia.

The nitric acid solution of the fossil, deprived of oxide of iron and alumina by ammonia, yields an abundant precipitate of white oxalate of lime on the addition of oxalate of ammonia. The lime was estimated in the usual way.

The filtered solution when treated with phosphate of soda, more ammonia being first added if judged necessary by the odour, yielded an abundant precipitate of crystallised phosphate of magnesia and ammonia, from which the amount of magnesia was estimated as usual.

All the arancarious fossils of Craigleith were found to present the same ingredients on being subjected to analysis. Some crude trials having led me to suspect that the proportion also of these ingredients might be the same in all of them, it was at first my intention to submit them all to careful numerical analysis. But, as subsequently several of the ingredients proved to vary considerably in proportion in the same fossil, the expectation of obtaining an exact ratio was necessarily given up. The *Charcoal* varied from 2·9 to 3·4, 4·9, and even 7·0 per cent. This variation depends upon whether the fragment used may happen to contain small cavities filled with charcoal, or, on the contrary, masses of fossilising matter in radiating crystals. The proportion 2·9 was obtained from a lump of No. 6 fossil, weighing almost a pound. At this rate the 36 feet of No. 5, in the British Museum, will contain rather more than a ton of charcoal. The iron estimated in the form of *Carbonate of Protoxide of Iron* has varied from 14·0 to 28·0 per cent. The circumstances regulating its proportion were not indicated in any of my analyses. The *Carbonate of Lime* has amounted on an average to 60 per cent., and the *Carbonate of Magnesia* to

17; and when they vary, their proportion remains nearly the same to one another, or as 3·5 to 1 nearly.

The composition of the larger of the two Granton fossils is precisely the same as that now stated. No chemist indeed, while analysing it, could know, unless told, that he was not examining an araucarious fossil from Craigleith quarry. The charcoal amounts to 3 per cent., and the three carbonates occur, each of them in large proportion. In one analysis I got 31·8 of carbonate of protoxide of iron.

The Sandstone Bed of the Fossils.—The sandstone of Craigleith quarry has been long celebrated for its whiteness, hardness, and durability. Until about thirty years ago scarcely any other stone was used in building the houses of the New Town of Edinburgh; but the railways have substituted a softer quality of stone, which, though inferior, is preferred by builders, because much more easily worked. The quarry is within a mile of the great trap outburst of Corstorphine Hill, to which it appears to owe the inclination of its beds eastwardly. The excavation of the quarry is now about 200 yards long, 150 yards wide, and at the deepest workings 185 feet deep from the sandstone surface. In most places nothing overlies the sandstone except some feet of gravelly sand, with also a few thin layers of intermingled sandstone and shale. The west cliff of the quarry is as it were split by a great wedge of coarse shale, entering from the north, between the sandstone beds, to a length of about 80 feet, with the point of the wedge directed and inclined southward. But the whole visible rock elsewhere is an unbroken cliff of sandstone. The sandstone is, for the most part, of a uniform very pale greyish-white colour. It has a finely granular dull surface, breaks with difficulty, and preserves its edges almost indefinitely under ordinary care. It resists the entrance of water into its texture with obstinacy, and retains its colour with little change under atmospheric exposure, except in the smoky air of a town. Its density is 2·45. When quite dry, by being kept some days in a warm room, its powder does not lose weight at all on being subjected to the action of diluted nitric acid; neither is there the slightest effervescence. Hence there is no earthy or iron carbonate or alumina to bind the particles together which compose the rock. It is a pure siliceous sandstone, formed by accretion of sand, not by agglutination. These are the characters of what the workmen call their "liver-rock," by which they mean the finest stone for building purposes.

But to a distance, occasionally of several feet, around the araucarious fossils, the rock has undergone a remarkable change, both in external characters and in chemical composition. It is tougher than the "liver-rock," scratches with the knife, though not easily, presents on the surface of a fresh fracture a decided

grey hue, and a splintery somewhat glistening appearance, breaks with very sharp edges, is reduced to powder with much greater difficulty than the pure sandstone, and has a density of 2·63. Moreover, its surface, after being exposed for some weeks to the open air, acquires an ochrey hue, which in a few months becomes a rather lively ochre-yellow. These alterations led me to suspect their cause; and analysis showed that the altered rock effervesces briskly in diluted nitric acid, loses weight in variable proportions, sometimes to the amount of 38 per cent., and yields to the acid a large quantity of lime, magnesia, and peroxide of iron. The iron, however, is dissolved as protoxide when hydrochloric acid is used, and atmospheric air excluded. In short, the sandstone must have been fossilised, so to speak, during consolidation by the same fluid which has fossilised the trees imbedded in it. When the quarrymen come upon this "whin," or "bastard whin," as they call it, they are apt to suspect the neighbourhood of a great fossil tree. But the same sort of altered rock is also met with, indeed too abundantly, in places far away from any fossil. It is bad stone for building purposes, as in time it becomes yellow, and in town black; and it is the cause of the black patches which deface some of the buildings of Edinburgh. It is easily distinguished, however, both by mineralogist and quarryman, so that the deformities referred to are inexcusable. The characters, whether external or chemical, are quite distinct when the proportion of of impurity is only 7 per cent.

The fossils are imbedded without in general any material intervening between their black coaly crust and the sandstone matrix. There is in some places a narrow vacuity between them, probably produced by the concussion caused by the blastings for removal of the fossils; possibly, too, arising from disturbances among the rocks at the time of their upheaval. At the concave part of the curve of No. 5 there was interposed, over a space of about a square foot, a narrow layer of slaty sandstone, containing black moulds of thin fern-like leaves and stems. A little farther up the same fossil, about a foot of the trunk had been crossed by a thicker band of rugged material, described to me as differing in appearance both from the coaly crust and from the matrix, and supposed by those who saw it to represent the remains of bark; but, unfortunately, it had been all removed before my visit to the quarry.

The sandstone of Craigleith does not abound in other fossils. Black very thin moulds of leaves and stems of acrogenous plants are indeed common in slaty beds, which here and there are thinly interposed betwixt the thick beds of hard pure rock. WITHAM, too, mentions several small casts of sandstone fossil stems which had been preserved for him by the quarrymen. They have been able to supply me with only two lepidendrons, one of them encrusted uniformly with caking coal, the other bare. Like all the specimens I have examined of fossilised plants from the coal-fields around Edinburgh, whose

internal structure had been loosely cellular, both of these lepidodendrons were casts merely, the whole interior consisting of the same material as the matrix, and without traces of vegetable tissue.

The sandstone of Granton, when worked for the harbour and adjacent houses, was considered to be of the same quality as that of Craigleith. That opinion is borne out by the properties of the large rectangular blocks, from two to three tons in weight, very many of which have lain unused, east of the quarry, since it was flooded in 1855. Most of them present the characters of the Craigleith "liver" rock; but many, too, are now yellow outside, possess all the other external properties of the "bastard whin," effervesce with acids, and yield to them largely lime, magnesia, and oxide of iron.

Conclusions.—The trees found in the fossilised condition in the quarries of Craigleith and Granton must have been water-borne, and by a tumultuous flood, for they have all been entirely stripped of branches, roots, and bark. They may have arrived at their present site either with the sand, which finally formed their hard sandstone bed, or subsequently, while it was a quicksand, through which they could easily sink, their heavier root-end foremost.

The sand in the course of time passed into the state of sandstone, for the most part by the simple accretion of particles through the long-continued influence of pressure and crystallising force. But in some places accretion has been aided by agglutination with carbonates conveyed among the particles by solution in water.

The trees, after arriving in their bed, had been fossilised by a peculiar process. It appears that acrogenous stems, lying in the same matrix, had been fossilised simply by the sandy particles invading their loose open cellular texture as it decayed, and subsequently accreting like the sand around them, so that interior structure is not preserved. But the compact, fine-grained wood of the araucarious trees could not be so penetrated. Fossilisation has been effected in them by a different, probably much slower method,—by long infiltration with water holding in solution carbonates of lime, magnesia, and protoxide of iron.

It is by no means easy to seize the whole details of this fossilising process. But the theory which occurs to me as accounting best for all the facts, is that of a process at ordinary temperatures of very slow destructive distillation, which bears, to the well-known process of destructive distillation of wood at a red heat, the same relation which Liebig's process of eremacausis, or slow natural oxidation in the atmosphere, bears to ordinary atmospheric combustion. In both cases of destructive distillation bituminous matter is discharged, and charcoal left behind. In both cases the woody structure is preserved. In the fossilising process the charcoal is squeezed up against the cell-walls by the

mineral mater crystallising in the cells, and thus defines the cell-walls sharply before the microscope. But in other places the crystallising force of the mineral matter is so great that the mixed carbonates assume their crystalline form on a much larger scale, pushing aside the organic textures altogether, and substituting masses of pure radiated crystallisation.

Addendum.

[*July* 1, 1874.—Soon after the Society's meetings were over for this season, a newspaper notice drew my attention to the discovery, in the sandstone quarry of Redhall, of a fossil tree of great size, which proves to be undistinguishable in any essential respect from the araucarious fossils of Craigleith and Granton. This is, on various accounts, too curious a fact not to deserve to go forth with those recorded in the preceding narrative; and I therefore add here a few notes from a paper on the subject which was read to the Edinburgh Botanical Society.

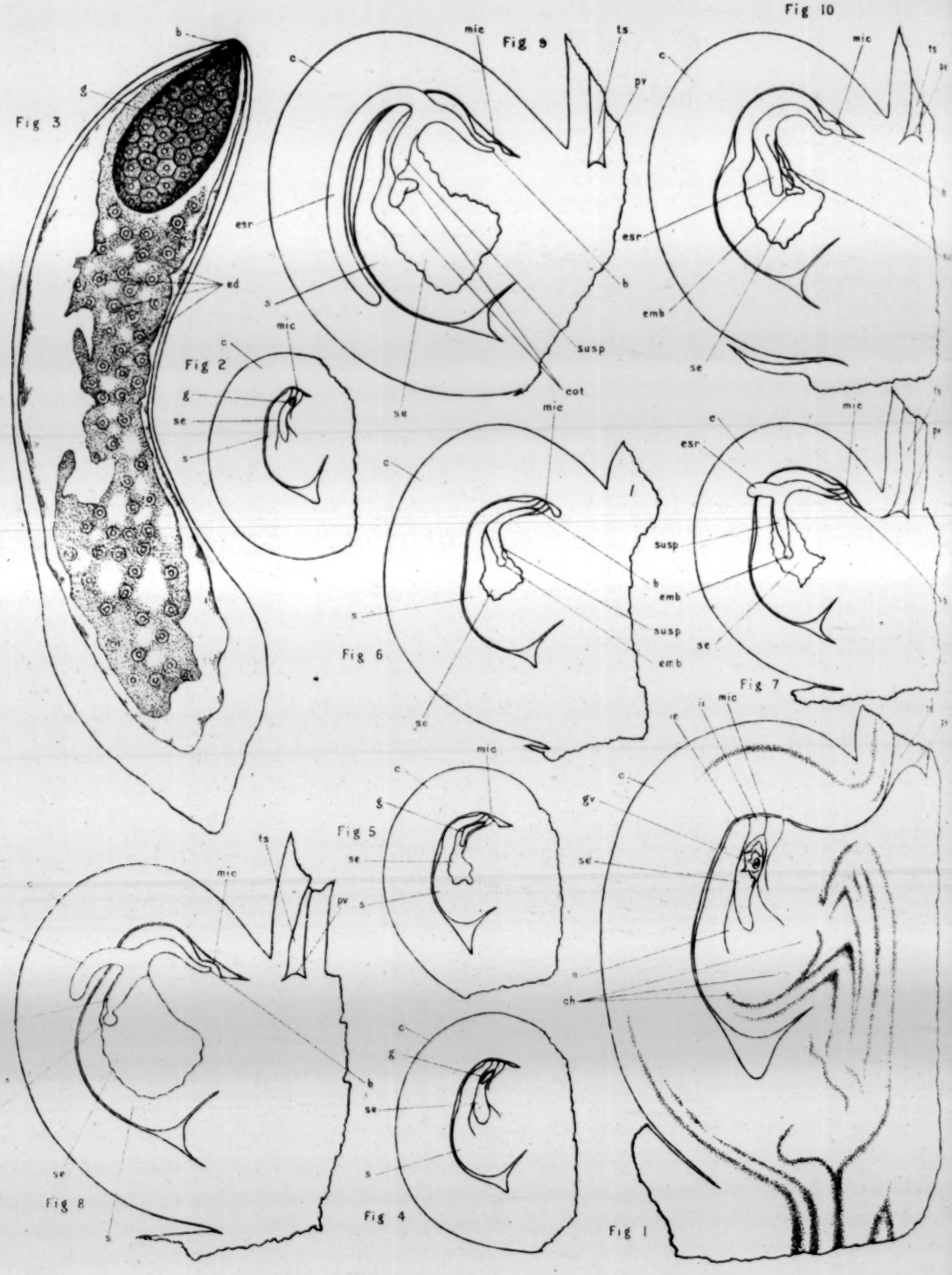
A new quarry, close to one which has been long worked, was lately opened on the property of Redhall, near Slateford, 2½ miles west from Edinburgh. It is 4 miles from Granton, and 2¼ from Craigleith; and the three quarries are nearly in a right line, in a direction from north to a little west of south. Under the soil is a thick bed of gravelly sand, from 8 to 20 feet in depth, containing very many boulders near the bottom, some of which are of great size. In the lower parts of this bed the sand has begun to consolidate into sandstone. Beneath it there is a succession of white sandstone beds about 15 feet thick, 2·426 in density, very like that of Craigleith, but less hard—more easily worked or pulverised. Lower beds succeed, of a pale yellowish hue, permanent, and not deepening under atmospheric exposure. Its density is 2·391. This, too, is brittle at first, but becomes tougher in time; and, being easily worked, is much in request by builders. Both the white and yellow stone have been formed by accretion, not by agglutination; for acids indicate only 1·5 per cent. of foreign matters, which are lime, magnesia, and oxide of iron,—the lime in the form of sulphate. But in various places the sandstone becomes ochrey under exposure, and then it presents the same external characters and the same chemical composition as the "bastard whin" of Craigleith. In particular, it effervesces strongly with diluted acids, and yields to them much lime, magnesia, and iron-oxide. There is a great resemblance, therefore, between the sandstone of the three quarries, and also the accidents which are apt to alter it.

A considerable variety of vegetable fossils has been met with in the new Redhall quarry. Of those shown to me, all but one seemed to belong to ferns and other acrogenous plants of coarsely cellular, loose texture, and consequently presented the appearance of casts, with the interior filled with sandstone destitute of traces of organic structure. The single exception proves to be a

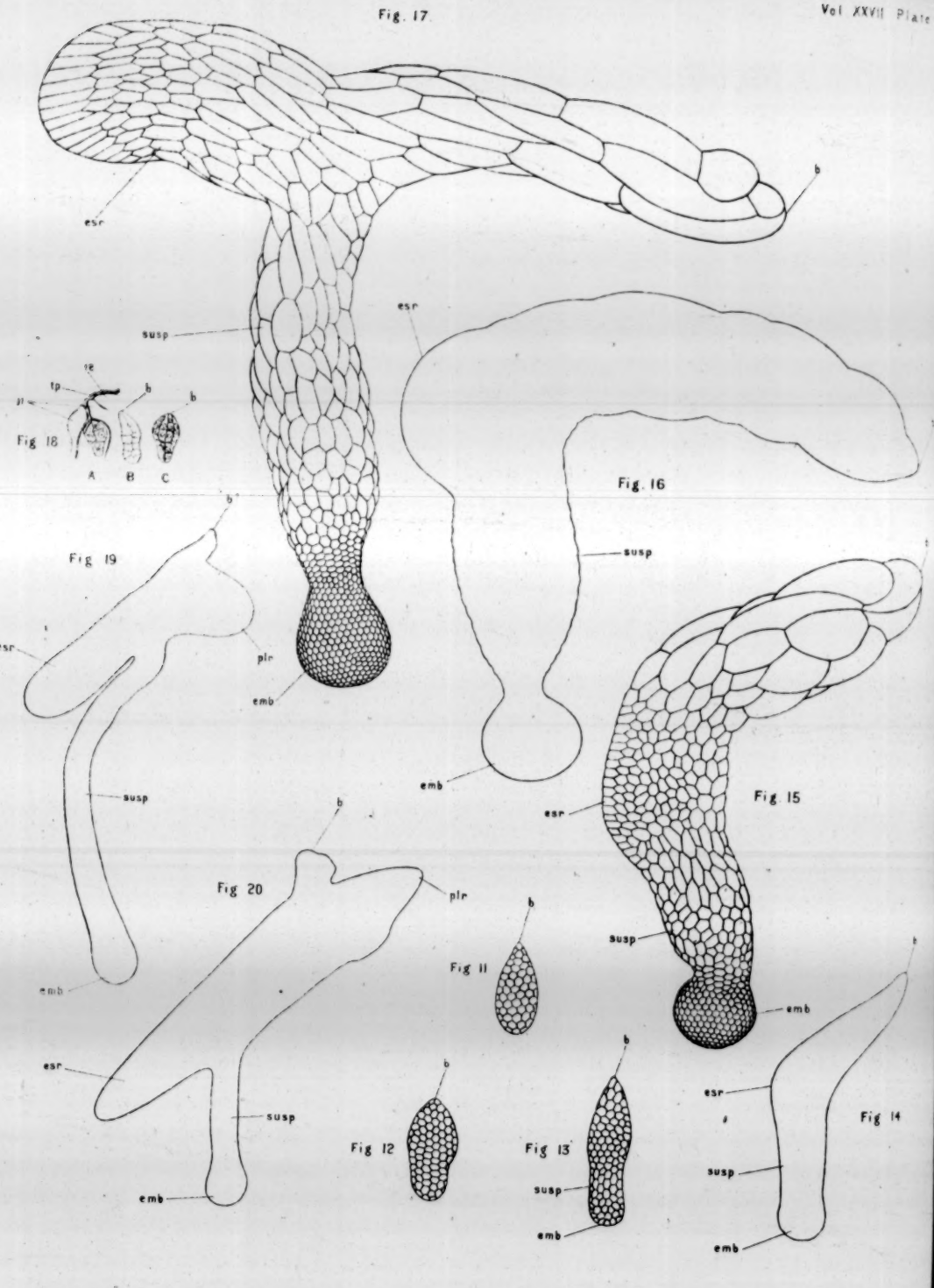
fossil tree of large size, undistinguishable by any essential character from the araucarious fossils of Craigleith and Granton. The beds where it was found dip towards 10° north of west at an inclination of about 15°. It was described to me by Mr GOWANS, lessee of the quarry, as lying horizontally in the lower part of the white sandstone rock under at least 15 feet of stone, and covered over its upper surface with 2 inches of shining, unattached, bituminous coal. Mr GOWANS thinks some of it had been carted away before his attention was drawn to it. What remains consists of four blocks, measuring together about 10 feet, neither roundish nor flattened, but quadrangular, somewhat rhombic, and with rounded edges. The girth varies from 6½ to 7½ feet. They had been uniformly covered with about $\frac{1}{10}$th of an inch of loosely attached, black, shining, very brittle coaly matter, much of which has been rubbed off in removing the blocks. These have been presented by Mr GOWANS to the collection of the Botanic Garden, along with several fragments of a fine Sigillaria 6 feet in length. The Sigillaria has nowhere any coaly crust on its surface.

The unattached and attached coaly matter of the large fossil tree proves to be a superior caking splint coal, the same in all its properties with the black crust of the Craigleith fossil, No. 5, now in the British Museum. The attached coal has a density of 1·274, and froths up in caking; that which was unattached has a density of 1·284, and cakes without frothing up. The fragments of the interior which I have examined have a density of 2·804, and a somewhat methodical nodulated fracture, and a grey-black, shining, spangled surface, like that of black mica-slate. Thin slices show before the microscope much radiated crystalline mineral matter, and small patches of deformed cells and fibrils. But Mr PEACH has shown me small portions which present, on longitudinal and transverse sections, the longitudinal fibrils crossed by medullary rays, and the formal transversely cut cells of the fibrils, which characterise the great fossils of the two other quarries.

The chemical examination of the substance of the Redhall fossil yielded precisely such results as would have led me to believe, had I not known otherwise, that I was analysing WITHAM's fossil from Craigleith, or that which has been sent to the British Museum. It effervesces with acids like a limestone. About 3·3 per cent. of charcoal remains undissolved; and lime, magnesia, and protoxide of iron are found largely in solution, the last in such proportion as to indicate the presence of 23 per cent. of carbonate of protoxide of iron in the fossil. The substance of the sigillaria, on the other hand, I found to be firmly accreted sand, like the sandstone matrix, and, like it, causing no effervescence with acids, but yielding to them 1·6 of foreign ingredients, which consist of lime, magnesia, and oxide of iron.]


Fig 10
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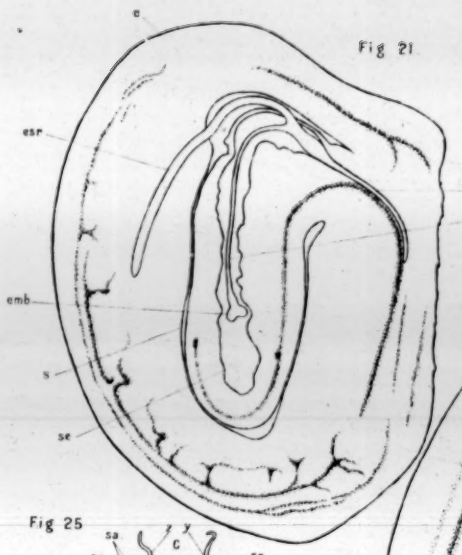
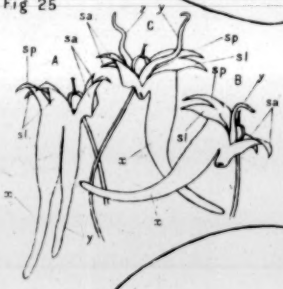


Fig 21



Fig 22

Fig 25



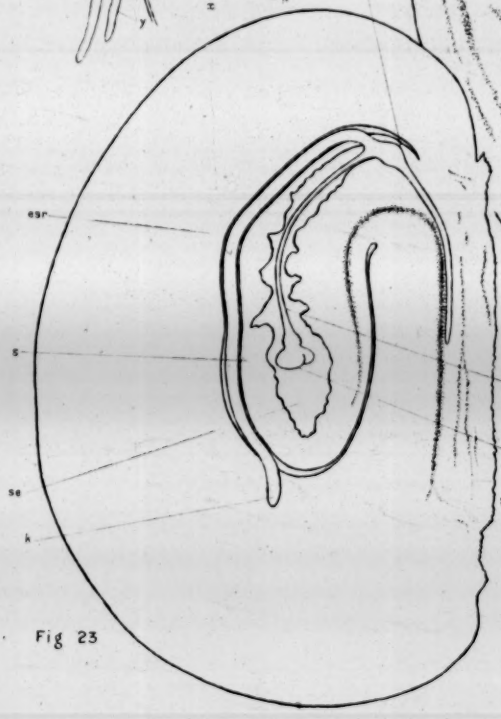
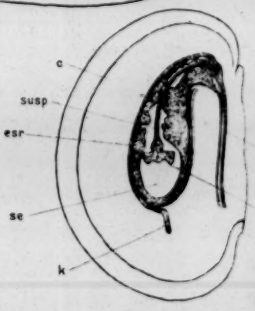


Fig 23

Fig 26



A Dickson, M.D. delt.

XI.—*On the Embryogeny of* Tropæolum peregrinum (*L.*) *and* T. speciosum (*Endl. and Poepp.*)

By Alexander Dickson, M.D., Edin. & Dublin. Regius Professor of Botany in the University of Glasgow. (Plates XIV.–XVI.)

(Read 19th January 1874. Given in for publication 14th May 1875.)

Introduction.

The extraordinary processes developed in connection with the base of the suspensor in the common Indian Cress (*Tropæolum majus*) have long excited the interest of vegetable embryologists, and have led to the publication of numerous observations on the embryogeny of that plant. As the following remarks are interesting chiefly in regard to the peculiar features in which the germs in two other species of the genus differ from that in the common Indian Cress, it will not be out of place for me briefly to recapitulate the principal facts connected with the development in that plant.

The repeated subdivision of the fertilised germinal vesicle in *T. majus* very soon results in the formation of a small flask-shaped structure, resembling in figure a soda-water bottle, having a short neck, ovoid body, and pointed base (Plate XV. fig. 18, A, after Hofmeister; B and C after Schacht). The pointed base fits into the correspondingly pointed apex of the embryo-sac. The neck of this structure becomes gradually elongated; and by an active cell-multiplication at its extremity, a somewhat globular head is produced, which ultimately becomes developed as the embryo proper; while the slenderer proximal portion constitutes the suspensor, which here is remarkable for consisting of several longitudinal rows of cells. At a very early period the outer side of the body of the bottle-shaped germ becomes the seat of active growth, resulting in the formation at first of a rounded projection, which afterwards increases and elongates into a thick cylindrical process, which pushes or bores its way through the coats of the seed a little to the outer side of the micropyle, and runs as a long root-like process free in the cavity of the seed-vessel. A little after the first appearance of this process a somewhat similar rounded projection appears on the inner or ventral aspect of the body of the germ, but a little nearer the basal point than the extra-seminal process. This new growth gradually elongates, becomes tapering and pointed, insinuating itself as a slender and delicate root-like structure through the tissue of the neck (*funiculus*) of the seed, and, reaching the placental vascular bundle just where it curves outwards to enter

the seed and form the raphe, runs downwards closely along the course of said bundle the whole length of the placenta. To the first mentioned thicker root-like process I have applied the term "extra-seminal root;" to the latter and slenderer one, that of "placental root." In applying the term "*root*" to these processes I do not mean to imply a *homology* with roots proper. They are wholly cellular in structure, and are destitute of the caps so distinctive of root-organs; but functionally they probably correspond to roots, which they certainly resemble in general appearance.

In 1863 I submitted to the Botanical Society of Edinburgh my "Observations on the Embryogeny of *Tropæolum majus*," in which I confirmed the statements of the late Mr Wilson, the distinguished author of the "Bryologia Britannica," as regards the course of the extra-seminal and placental processes, the latter of which he was the first to describe in 1843.* I also pointed out (as I then believed for the first time) that these processes are developed as lateral outgrowths from an originally bottle-shaped germ, in opposition to Schleiden, who had described the suspensor with embryo as originating as a lateral branch from an oblong body whose apex corresponded to the extremity of the extra-seminal process. At that time I was ignorant of the observations of Hofmeister (1849) and of Schacht (1855) on this subject. Both these authors had distinctly indicated the originally bottle-shaped germ with pointed base.† As to the placental process Hofmeister makes no mention of it, while Schacht falls into the most extraordinary error regarding it, describing it as passing out of the seed through the endostome, and making its way towards the canal of the style‡; the fact being (as originally stated by Wilson, and confirmed by myself) that this process makes its way at once through the tissue of the funiculus to the placental vascular bundle, along which it runs down in a course directly *away from* instead of *towards* the canal of the style.

In his paper Wilson described and figured an abnormality, where the extra-seminal process, instead of extending itself free in the cavity of the seed-vessel, penetrated by its extremity for a short distance into the tissue of the carpel, at a point corresponding in position to the chalazal or lower end of the seed. I have myself been fortunate enough to detect a precisely similar case, which I have represented in Plate XVI. fig. 23, along with a photo-lithographic reproduction of Wilson's figure for the purpose of comparison (Plate XVI. fig. 24). Such a deviation seems, at first sight, a trifling one; but, as the sequel of this paper will show, it is of great interest and significance, adding another to the

* W. Wilson, "On the Embryo of *Tropæolum majus*," London Journal of Botany, vol. ii. p. 623.

† Hofmeister, "Die Entstehung des Embryo," Leipzig, 1849; Schacht, "Ueber die Entstehung des Keimes von *Tropæolum majus*, Bot. Zeitung," 1855, p. 641 (translated in "Ann. des Sc. Nat." 4e sér. iv. p. 47). In Plate XV. fig. 18, I have given sketches, A after Hofmeister, B and C after Schacht, showing the earlier stages of this germ.

‡ Schacht, *l.c.* pp. 644–5.

numerous cases already observed, both in plants and animals, of an abnormal development in one species, shadowing forth, so to speak, the normal condition in another. In this *abnormal* penetration of the carpellary tissue by the extra-seminal process in *T. majus*, we have an imitation, though a feeble one, of the *normal* penetration which occurs in *T. peregrinum* and *T. speciosum*, to which I have now to direct attention.

Tropæolum peregrinum, L. (Canary Creeper).

The investigation of the embryogeny in this species is much more troublesome than that in either *T. majus* or *T. speciosum*. The delicate and slender character of the germ, together with the frequent difficulty in getting exactly mesial sections of the fruit-lobes from slight assymmetry, if these, as is often the case, are not all three equally developed, has forced me to be content with a comparatively imperfect result.

In Plate XV. fig. 20, I have given an outline figure of a young germ, the parts of which, though somewhat different in form, may at once be identified with the corresponding parts in the germ of *T. majus*, a similar outline of which, at about the same stage of development, I have also given for comparison (Plate XV. fig. 19). In *T. peregrinum* the basal projection (*b*) is very distinct, but is more knob-like and less pointed than in the common species. There is also considerably more marked obliquity of the body of the germ in *T. peregrinum* than in *T. majus;* the basal point (*b*) in *T. majus* being nearly in line with the suspensor, while this is far from being the case in *T. peregrinum*. At this stage the extra-seminal and placental root-processes are distinctly indicated. The latter process (*p/r*) is still rounded in the germ of *T. majus* figured, while it has become pointed, although not yet elongated, in that of *T. peregrinum*.

In Plate XVI. fig. 21 is represented a section of a somewhat advanced fruit-lobe, where I have succeeded in displaying all the parts of the germ *in situ*, and in tracing both root-processes to their extremities, so as to enable me to give a complete figure of the germ at that stage. Here the suspensor has become greatly elongated, and the embryo at its extremity exhibits rudiments of the two cotyledons. The placental process is now somewhat elongated, and has made its way to the placental vascular bundle, along the course of which it runs, all exactly as in *T. majus*. The extra-seminal process bores through the seed-coats, also exactly as in *T. majus*, making its way into the cavity of the seed-vessel. Unlike that in *T. majus*, however, this process, after running a short distance in the ovarian cavity, penetrates the carpel (say a little above its middle third), and pursues the remainder of its course embedded in the carpellary tissue. The direction of this process, after dipping into the carpel, is somewhat variable; sometimes it runs along just outside the inner surface of the carpel,

but more usually it extends itself obliquely outwards and downwards; sometimes very markedly so, as is seen in the more advanced stage represented in Plate XVI. fig. 22, where it extends quite out to the vascular bundle of the median rib or ridge of the fruit-lobe. I have not been able to trace how far the two root-processes in *T. peregrinum* ultimately extend themselves, but probably they proceed the whole length of the carpel and placenta respectively, as in *T. majus*. In fig. 22 the extra-seminal process is seen to extend for a considerable distance in the substance of the carpel; but its whole length is not seen in the section, the end having evidently been cut away in making the preparation.

T. speciosum, Endl. and Poeppig.

In this species (belonging to the *Chymocarpus* section of the genus, the fruit-lobes in which are destitute of ridges, and when ripe are somewhat drupaceous in character) the ovule departs widely from the marked anatropal form it presents in *T. majus* and *T. peregrinum*. The hilum (Plate XIV. fig. 1 *h*) is of great extent, reaching from the neighbourhood of the micropyle (*mic*) to that of the chalaza (*ch*). At the period of flowering the nucleus (*n*) is slightly curved, while its base or chalazal region is just sufficiently removed from the hilum to leave a small space traversed by vessels, which may be described as a very short raphe.* This peculiar form of ovule may be regarded as an intermediate one between the half-anatropal and the campylotropal. A somewhat analogous condition is that observed in such Leguminosæ as the common Bean or Pea, where, however, the chalaza is at some little distance from the hilum, and where, consequently, the raphe is more distinctly marked. The nucleus is broadest at its base, becoming rather rapidly narrowed towards its pointed apex. The embryo-sac (*se*) is elongated, nearly cylindrical, and pointed at its micropylar extremity or apex. At the time of flowering the embryo-sac appears to have produced complete absorption of the narrowed portion of the nucleus towards the micropyle, and thus comes to be in immediate contact with the integuments. The integuments are two in number, and the micropyle is formed by the endostome as in *T. majus*. The germinal vesicle (*gr*) (I can only speak to *one* with certainty, though probably there is a greater number) is a delicate ovoid nucleated cell with finely granular protoplasm, and occupies the apex of the embryo-sac. It is represented in the figure as entangled in a crumpled-up portion of the embryo-sac, and thus somewhat out of position. In this figure, and in some of the succeeding ones, the hemispherical, or slightly conical, extremity of the axile placenta (*pv*) is distinctly shown, representing the apex,

* Schleiden ("Nova Acta Acad. C. L. C. Nat. Cur.," tom xix. tab. viii. fig. 126) gives a figure of the ovule of *T.* (*Chymocarpus*) *pentaphyllum*, where it would appear that the anatropal character is much more distinctly present, the raphe being of considerable length.

or arrested *punctum vegetationis*, of the floral axis. A portion of the tube of the style (*ts*) is also shown, and this, it is to be noted, remains open, *i.e.*, does not become filled up with "conducting tissue;" just as has been observed by SCHACHT in *T. majus*.*

After fertilisation, I have observed the immediately succeeding stage where the germinal vesicle has undergone subdivision into two by a horizontal septum, like what has been noticed by SCHACHT in *T. majus*.

The stages immediately following this I have not seen, and the next observed is that represented in Plate XIV. figs. 2 and 3. Here the germ (*g*) is multicellular and pyriform with pointed base (*b*) fitting into the apex of the embryo-sac. On side view this pyriform germ exhibits about 50 cells. At this stage is very distinctly to be noted a single layer of nucleated particles, pretty uniformly distributed and embedded in the protoplasm which lines the inner surface of the embryo-sac. This layer, which I have attempted to indicate in Plate XIV. fig. 3 (*ed*), is interesting as undoubtedly representing a transitory endosperm which, as is known, is almost, if not altogether absent in *T. majus*.

In Plate XV. fig. 12, the germ is represented in the immediately succeeding stage, where it assumes the flask or soda-water bottle form, almost exactly like that observed in *T. majus* by HOFMEISTER, SCHACHT, and myself, the only important difference being in its consisting of a much larger number of cells than that in *T. majus*,—this standing in relation to the stouter and more massive general character of the germ of *T. speciosum*, as compared with that of the common species. In *T. majus* the flask form is assumed at a very early period, as may be seen in the figures after HOFMEISTER and SCHACHT (Plate XV. fig. 18, A, B and C); that at A giving a side view of only half a dozen cells.

In the next stage (Plate XIV. fig. 4 and Plate XV. fig. 13) the flask-shaped germ has become somewhat elongated, particularly in the neck and pointed base, in which parts the cells are becoming slightly enlarged. As yet the axis of the germ is straight. The chalazal extremity of the embryo-sac (Plate XIV. fig. 4) now exhibits signs of that enlargement which goes on progressively until maturation of the fruit, the embryo proper with its large amygdaloid cotyledons being lodged in this chalazal dilatation of the sac. Otherwise, the young seed at this stage hardly differs from that represented in Plate XIV. fig. 2, except in its somewhat greater size.

In Plate XIV. fig. 5, the micropylar extremity of the young seed and embryo-sac are seen to have become much more markedly curved, and coincidently with this (perhaps in consequence of it) the now still more elongated germ exhibits a distinct knee-like bend above its middle. The elongation of the neck and pointed base of the germ by enlargement of their constituent cells is now more distinctly marked, these regions appearing transparent by contrast

* SCHACHT, Bot. Zeit. 1855, pp. 641-2.

with the comparative density and opacity of the *apex* of the germ and of the *body* at the bend, in both of which situations cell-multiplication is actively going on. The apex or head of the germ now exhibits a slight rounded enlargement (*emb*), the rudiment of the *embryo* proper; the elongated neck represents the *suspensor* (*susp*); while the active cell-multiplication, giving opacity to the region of the knee-like bend, is the first indication of the formation of the *extra-seminal root-process*. I have given an enlarged outline figure of the germ at this stage in Plate XV. fig. 14.

In Plate XIV. fig. 6, the germ is a little further advanced. Its curvature is more strongly marked, the elongated and pointed base standing nearly at right angles to the suspensor. The terminal enlargement or embryonal globule (*emb*) is now very distinctly indicated as a globular head. A slight bulging or rounded projection of the outer aspect of the body of the germ at the knee-like bend is now to be seen (*esr*), the rudiment of the extra-seminal root-process. In Plate XV. fig. 15, I have given a much enlarged figure of the germ at a stage somewhat intermediate between the two last described, showing the cellular structure. The elongation and enlargement of the pointed base by dilatation of its constituent cells are very marked, as also is the elongation of the suspensor by a somewhat similar though not so excessive cell-dilatation. The regions where cell-multiplication is going on actively, viz., the "embryonal globule," and the body of the germ at the knee-like bend, are conspicuous by the comparatively small size of the cells.

In Plate XIV. fig. 7, a section of a fruit-lobe at a somewhat later stage is represented. The bulging on the outside of the knee-like bend is now developed into a short, thick, cylindrical process—the young extra-seminal root (*esr*). This has bored its way almost horizontally outwards through the seed-coats, at a point considerably distant from the micropyle, the space corresponding to the elongated base of the germ intervening between them. It is further to be noted, that this extra-seminal root-process has begun to force its way into the substance of the carpel (*c*) immediately on making its exit from the seed-coats. In Plate XV. fig. 17, an enlarged figure of the germ from this section is given, showing the cellular structure. The elongated base and extra-seminal process are seen in section; the suspensor and embryonal globule in surface view. The enlargement of the cells of the elongated base and of the suspensor is very distinct. The extra-seminal process exhibits very nearly the same characteristics (except in consisting of a much greater number of cells) as that of *T. majus* at a corresponding stage. Its rounded extremity is formed by a layer of more or less columnar cells of considerably greater length than those immediately behind them. The subsequent growth of this process is due in great measure to elongation of its constituent cells; but if cell-multiplication does go on during the elongation, it must, I think, be intercalary and not apical.

In Plate XIV. fig. 8, the germ is further advanced; the extra-seminal process having now run some little distance in the substance of the carpel, keeping just outside the inner surface of the cavity of the germen.

In the still later stage, represented in Plate XIV. fig. 9, the extra-seminal process is greatly elongated, continuing to run in the substance of the carpel just outside the inner surface of the cavity of the germen. The elongated base exhibits no change. The suspensor is somewhat longer and thicker, this being apparently due solely to enlargement of its cells. The young cotyledons are now distinctly developed, the "embryonal globule" stage being now passed.

The subsequent stages need not be minutely detailed here. They are characterised chiefly by the further elongation of the extra-seminal root process, which ultimately runs the whole length of the carpel; and by the enormous development of the fleshily thickened cotyledons, between which a well-developed plumule lies. The radicle, as in *T. majus*, is but slightly developed.

In Plate XIV. fig. 10, is represented an exceedingly interesting abnormality, which appears to illustrate, negatively, the function of the extra-seminal root. Here this process has attained a considerable length, but has failed to make its way out of the seed—is *intra-seminal*, in fact,—and, apparently in consequence of this, the suspensor and embryo are misthriven and shrivelled as if they had suffered from want of that nourishment which it is doubtless the function of the extra-seminal process to obtain from outside the seed.

One of the most striking peculiarities of the germ of this species, as compared with those of *T. majus* and *T. peregrinum*, is the total absence of the placental root-process, not the slightest trace of which is to be observed at any period of development. In looking at some of the more advanced stages in *T. speciosum* (*e.g.*, Plate XIV. fig. 9), one might at first sight imagine that the elongated, curved, and pointed base represented the placental root-process in the other two species; but in tracing it back through its developmental stages, it is demonstrably nothing more than an elongation of the pointed base of the originally flask-shaped germ—retaining throughout, moreover, its original position, impacted in the pointed apex or micropylar extremity of the embryo-sac.

Conclusion.

The phenomena above described suggest many interesting considerations. It cannot be doubted that the lateral branch or branches from the base of the suspensor in *Tropæolum* serve as organs of nutrition for the developing embryo —as *fœtal roots*, in fact. It seems probable, as has already been remarked by SCHACHT and myself, that their presence is physiologically conditioned by the absence or, at most, the very slight development of even transitory endosperm. In treating of this subject, SCHACHT has pointed out that those very different structures, the cæcal dilatations of the embryo-sac in many Labiatæ, Scrophu-

lariaceæ, &c.,* probably fulfil a similar function as organs for the absorption of nutritive material required by the developing embryo. One cannot reflect on such arrangements without being struck by a certain analogy which they present to the provisions for fœtal nutrition in the higher animals. In the fœtal root-process of *Tropæolum* we have an organ of nutrition which has its parallel in the mammalian *allantois*, which forms the root-process, so to speak, by which, when developed into the fœtal placenta, nutritive material is absorbed from the maternal system—the structure being in both cases a transitory portion of the germ itself. Again, as regards the cæcal dilatations of the embryo-sac in Labiatæ, &c., we have processes which may be regarded as villi for nutritive absorption. Now, inasmuch as the vegetable *embryo-sac* is the undoubted equivalent of the animal *ovum*, it is not too much to compare the processes therefrom with the villi of the chorion, which play so important a part in the earlier stages of the fœtal nutrition of the higher animals. It is, of course, impossible, as it would be absurd, to draw more than a very general parallel between organisms at such a distance from each other in the series; but the comparison, such as it is, is very interesting and instructive.

Not the least important fact brought out by the foregoing investigation is this, that in two species of *Tropæolum* we find normally a penetration of the carpellary tissue by the extra-seminal root-process—a phenomenon which seems to occur in *T. majus* only as a rare exception. In the three species, *T. majus*, *T. peregrinum*, and *T. speciosum*, we have a most striking series presented to us in the relations borne by the extra-seminal process to the carpel. In *T. majus* we have the extra-seminal process normally running its whole course free in the cavity of the seed-vessel. In the abnormalities observed by Wilson and myself, however, this process insinuates itself into the carpellary substance after it has run the length of the ovarian cavity; in *T. peregrinum* the penetration of the carpel is normal, but takes place at a much earlier period of development; while in *T. speciosum* the process plunges into the tissue of the carpel immediately on making its exit from the seed.

It would be easy here to indulge in that kind of speculation which is so common a feature of modern scientific thought. We might imagine that at one time there existed some old type, analogous, let us say, to the ordinary form of *Tropæolum majus*, where circumstances might chance to make a slight penetration of the carpel, at a comparatively late period, by the extra-seminal root, such as occurs in the abnormality above described, of *vital importance*, and hence capable of being made permanent by natural selection. Further, that in the course of time the penetration might show a tendency to take place at a somewhat earlier period in development, such variation becoming in like manner fixed

* Schacht, Bot. Zeitung, 1855, pp. 646–7. In Plate XVI. fig. 26, I have given a drawing carefully constructed after figures by Tulasne, showing these cæcal processes in a labiate plant, *Dracocephalum peltatum*.

by natural selection in a form more or less resembling *T. peregrinum*. Lastly, that circumstances might cause successively earlier penetrations to become fixed, until a form might be reached such as we have in *T. speciosum*, where the extra-seminal root penetrates the carpel as soon as it makes its exit through the seed-coats.

Such speculations are no doubt very attractive, but, in my opinion, are much to be deprecated. The Darwinian hypothesis is so essentially of the nature of a speculation which, at the best, can only have a *balance of probabilities* in its favour, that it must always be eminently hazardous to build, by speculation upon speculation, a structure which is liable at any time to fall to pieces from want of inductive foundation on fact.

With regard to the present case, the following objections occur to me as applicable to the supposed "descent-by-modification" of *T. speciosum*, which I have indicated above:—

1st, It seems highly improbable that such a variation as I have shown in *T. majus*, even although a favourable one, should ever have become of *vital importance*. It is far too slight to render it in the least probable that any change of conditions should have ever arisen to render its non-possession fatal.

2d, The impediments to self-fertilisation and the provisions for cross-fertilisation (dichogamy, conspicuous colour and shape of flowers, secretion of nectar, &c.) in these plants, would render extremely probable, if not almost certain, the obliteration of any such variation almost as soon as it appeared;* and thus *the chances of a concurrence of such variation and the change of conditions necessary to make it of vital importance would be almost infinitely small.*

It has been suggested to me by a critical friend, that the penetration of the carpel may possibly not have the importance attributed to it in the case as above given, and that the penetration may be merely a developmentally correlated adjunct to some more important modification which has been the immediate subject of natural selection. It appears to me, however, that the suggestion of such a possibility does not materially affect the question. If diverse species, either of plants or animals, have been produced by the fixture of modifications by natural selection—whether in the struggle with adverse conditions of life, in sexual competition, or otherwise—it may safely be assumed that the successive modifications so selected would always have been slight, inasmuch as any abrupt or violent divergence from the type almost necessarily partakes of the nature of a monstrosity, involving inherent weakness, and thus absence of the elements of permanence. My argument, therefore, may be regarded as capable of general application, whether it is the carpellary penetration or an unknown something, which may be supposed to have been the immediate subject of natural selection.

* As I have elsewhere stated, in a semi-popular lecture on "Consanguineous Marriages" (Glasgow Med. Journal, Feb. 1872), the use or function of sexual reproduction, as distinguished from gemmation, seems to be this very obliteration of individual variations, so as "to keep up an average tone or quality in the species, and, by dilution of individual peculiarities, to eliminate possible sources of evil on their appearance."

APPENDIX.

On the occurrence of supernumerary receptacular spurs in Tropæolum speciosum.

As may be expected, when a large number of specimens pass through one's hands a good many flowers have been met with more or less abnormal in character. I shall, however, confine myself to the consideration of those exhibiting supernumerary spurs.

In the year 1866 I published a note in the Transactions of the Botanical Society of Edinburgh, on four cases of this abnormality observed in *T. majus*. In these cases there was a supernumerary spur similar to the normal posterior one, but somewhat smaller, and placed in a line between the bases of a lateral and one of the anterior sepals, sometimes on one side of the flower, sometimes on the other. I commented upon the position of the supernumerary spur, as if it were there to show its want of connection with any sepal.*

In *T. speciosum* I have met with seven flowers exhibiting supernumerary spurs. Of these (Plate XVI. fig. 25) one (A) is essentially similar to those above described in *T. majus;* the additional spur having a general resemblance to the normal one, but being somewhat smaller, and occupying a position in line with the interval between the left anterior and lateral sepals. In the normal posterior spur (x) we have apparently six nerves; these, however, are in reality only three in number, which run *down* the inner aspect of the spur to its extremity, and then turn, running *up* its outer aspect to the base of the calyx. Of these three nerves one is in line with the posterior sepal; the other two with the intervals between the posteral sepal and the lateral ones on either side. In the supernumerary spur (y) we have but one nerve running *down* its inner and *up* its outer aspect to the base of the calyx, in line with the interval between the left anterior and lateral sepals.†

In the other six cases we have the very striking and *bizarre* peculiarity of the supernumerary spurs, after projecting externally to a variable but always slight extent, becoming invaginated or introverted like the foot of a stocking. The result of this introversion is that the spur makes its appearance turned inside out, as a more or less elongated, curved, horn-like process projecting from the interior of the flower *within* the corolla; these supernumerary spurs being, like the normal one, pouch-like dilatations of the receptacle between the insertion of the petals and that of the stamens. Of these six cases, *three* present a single introverted spur in line with the interval between the left anterior and lateral sepals, as in that figured in Plate XVI. fig. 25, C. In a *fourth* (Plate XVI. fig. 25, B), there are two supernumerary spurs similarly introverted, a right antero-

* Trans. Bot. Soc. Edin. vol. ix. p. 54, referred to in Masters' "Teratology," pp. 232–3.

† The same description applies to the arrangement of the vascular bundles in the above-mentioned abnormal flowers of *T. majus*.

lateral and an anterior one, *i.e.*, in line with the interval between the two anterior sepals. In a *fifth* there is a single supernumerary anterior spur, almost wholly turned inside out, and very small. The *sixth* case had one introverted spur, but unfortunately I lost the specimen before noting its position.

What the cause may be of the extraordinary and almost constant perversity of the supernumerary spurs in *T. speciosum* I am unable to say. It may be in consequence of the spur encountering some obstacle resisting its direct elongation; or it may be the result of some slight external injury to the point of the spur at an early period, leading to a reversal of the direction of the pouching. Anyhow, the phenomenon is a very remarkable one. In the three figures given I have omitted the partly withered petals and stamens present in the specimens as unnecessarily complicating the drawings, which, as they are, have sufficiently taxed my artistic powers.

I would here acknowledge the liberality with which I have, from time to time, been supplied by kind friends with materials for the foregoing investigation. My thanks are specially due to my former pupil, Wm. Macfarlane, Esq., Killin, Perthshire, in which county *T. speciosum* seems peculiarly at home; to Dr W. H. Lowe, Balgreen, Edinburgh; and to William Calder, Esq., Waterside, Dumfriesshire. I have to apologise to the Society for the long delay there has been in the publication of this paper, but I have been enabled thereby to make my observations much more complete than they would otherwise have been.

DESCRIPTION OF PLATES XIV., XV., AND XVI.*

Explanation of the lettering of the figures.

b.	Base of germ.	*mic.*	Micropyle, or region of same.
c.	Carpel.	*n.*	Nucleus of ovule.
ch.	Chalaza.	*plr.*	Placental root-process of germ.
ed.	Endosperm.	*pv.*	Extremity of axile placenta, or arrested *Punctum vegetationis* of the floral axis.
emb.	Embryo-proper.		
esr.	Extra-seminal root-process of germ.	*sa.*	Anterior sepal.
g.	Germ.	*se.*	Embryo-sac, or cavity of same.
gv.	Germinal vesicle.	*sl.*	Lateral sepal.
h.	Hilum.	*sp.*	Posterior sepal.
ie.	External integument of ovule.	*x.*	Normal posterior receptacular spur.
ii.	Internal do.	*y* & *z.*	Supernumerary receptacular spurs.

Plate XIV.

Tropæolum speciosum.

Figure 1. Longitudinal mesial section of ovarian lobe at the period of flowering, showing the heterotropo-campylotropal ovule, with germinal vesicle in the embryo sac. A small portion of the tube of the style is seen, at the bottom of which is the somewhat conical apex of the axile placenta—the arrested *punctum vegetationis* of the floral axis. The fibro-vascular bundles are indicated (somewhat conventionally) by dotted shading.

* The amplifications of the figures are, *approximately*, as follow :—Fig. 1, 45 times linear; Fig. 3, 250 times; Figs. 2, 4–10, and 21–23, 15 times; Figs. 11–17, 120 times.

Figure 2. Section of very young fruit-lobe. The germ is now pyriform, with pointed base fitting into the apex or micropylar extremity of the embryo-sac.

Figure 3. Embryo-sac and contents from preceding section, highly magnified. The cellular structure of the pyriform germ is seen. In the protoplasmic lining of the inner surface of the embryo-sac nucleated particles are embedded in a single layer, representing a transitory endosperm. The protoplasm has shrunk in the preparation, as represented in the figure; but I am not certain whether its well-defined outer boundary is the wall of the embryo-sac itself or merely the so-called "primordial utricle" of the sac.

Figure 4. Section of young fruit-lobe, further advanced. The germ is now flask-shaped (like a soda-water bottle), and has become somewhat elongated. Enlarged figure of this germ in Plate XV. fig. 13.

Figure 5. Section of young fruit-lobe, still further advanced. The campylotropal curvature of the micropylar extremity of the young seed is now much more pronounced. The flask-shaped germ is now more elongated, and exhibits a knee-like bend about its middle. Enlarged outline figure of this germ in Plate XV. fig. 14.

Figure 6. Section of young fruit-lobe, still further advanced. The pointed base of the germ is now much elongated. A slight bulging on the outer aspect of the body of the germ at the knee-like bend indicates the commencement of the extra-seminal root-process. The embryo ("embryonal globule") and suspensor are distinctly differentiated.

Figure 7. Section of young fruit-lobe, still further advanced. The extra-seminal process now appears as a short thick cylindrical structure which has bored through the seed-coat, and has forthwith begun to insinuate itself into the substance of the carpel. Enlarged figure of this germ in Plate XV. fig. 17.

Figure 8. The same, further advanced. The extra-seminal process has now run some little distance in the substance of the carpel.

Figure 9. The same, still further advanced. The extra-seminal process has now run for a considerable distance in the substance of the carpel, keeping just outside the inner surface of the cavity of the seed-vessel. The two cotyledons of the embryo have now distinctly appeared.

Figure 10. Abnormality. Section of young fruit-lobe apparently somewhat younger than the preceding. The "extra-seminal" root-process is *intra-seminal*, and the suspensor and embryo are shrunken and misthriven.

Plate XV.

Tropæolum speciösum.

Figure 11. Pyriform germ, with cellular structure indicated, the same as represented in Plate XIV. figs. 2 and 3.

Figure 12. Young germ, now shortly flask-shaped. Cellular structure indicated. Stage intermediate between that represented in Plate XIV. fig. 2, and that in fig. 4.

Figure 13. Flask-shaped germ, somewhat elongated, the same as represented in Plate XIV. fig. 4. Cellular structure indicated. Enlargement of the cells of the base (*b*) and of the neck of the flask (the future suspensor, *susp*), now commencing.

Figure 14. Outline figure of germ represented in Plate XIV. fig. 5. The embryo, suspensor, and (more faintly) the extra-seminal process, are now becoming differentiated. Body of germ with knee-like bend.

Figure 15. Germ somewhat further advanced. Cellular structure indicated.

Figure 16. Outline figure of germ at a stage a little further advanced than that in Plate XIV. fig. 6. The "embryonal globule" well marked, and the extra-seminal process visible as a very distinct rounded projection (*esr*).

Figure 17. Germ from section represented in Plate XIV. fig. 7. Cellular structure indicated.

Tropæolum majus.

Figure 18. Sketches, A after HOFMEISTER ("Die Entstehung des Embryo der Phanerogamen," 1849, tab. v. fig. 22); B and C, after SCHACHT (Bot. Zeit. 1855, tab. ix. figs. 8 and 9). Showing very young germs exhibiting the soda-water bottle form. In A, the pollen tube (*tp*) is seen passing through the micropyle.

Figure 19. Outline figure of considerably further advanced germ of same. The pointed base is distinctly seen, and the extra-seminal and placental root-processes, the suspensor and the "embryonal globule," are all distinctly differentiated.

Tropæolum peregrinum.

Figure 20. Similar outline figure of germ where the parts corresponding to those in the last are easily recognised, though the proportions are somewhat different.

PLATE XVI.

Tropæolum peregrinum.

Figure 21. Longitudinal mesial section of young fruit-lobe, showing embryo, suspensor, and the extra-seminal and placental root-processes. The extra-seminal root-process, after boring through the seed-coat, runs a short distance in the cavity of the seed-vessel, and then penetrates the substance of the carpel a little above its upper third, thereafter running an obliquely outward and downward course in its substance.

Figure 22. Similar section at a much more advanced stage. Extra-seminal process much elongated and pursuing a very obliquely outward course in the substance of the carpel. Cotyledons now of considerable size. Placental root-process and base of suspensor not seen in the section.

Tropæolum majus.

Figure 23. Abnormality. Section of young fruit-lobe, showing the extra-seminal process, instead of running its whole course free in the cavity of the seed-vessel, insinuating its extremity into the substance of the carpel at a point corresponding to the chalazal or lower end of the seed.

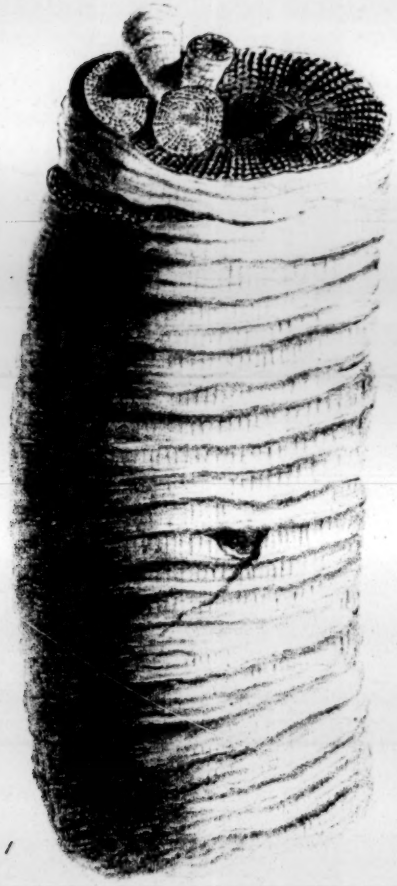
Figure 24. Photolithographic reproduction (somewhat reduced) of WILSON'S figure of a similar abnormality. Here the fruit-lobe is at a somewhat more advanced stage of development than in the last, but otherwise the cases are identical.

Tropæolum speciosum.

Figure 25. Abnormal flowers, showing supernumerary spurs, slightly over natural size. See Appendix.

Dracocephalum peltatum.

Figure 26. Constructed after figures by TULASNE ("Annales des Sc. Nat." 4e sér. t. iv. tab. iv. figs. 4, 5, and 9.) Section of young fruit-lobe, showing embryo-sac with cæcal dilatations (*cc*), which presumably have the function of absorbing nutritive material for behoof of the developing embryo. *a*, point of application of pollen-tube to embryo-sac, indicated by TULASNE (with a query) as the apex of the sac.

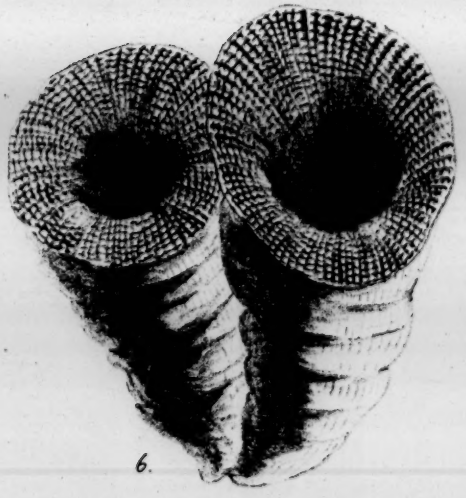

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XII.—*On the Mode of Growth and Increase amongst the Corals of the Palæozoic Period.*

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(Plate XVII.)

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The mode of growth and increase amongst the Coralligenous Actinozoa in general has been more or less fully treated of by various observers, including such distinguished naturalists as Milne-Edwards, and Haime, Dana, Martin Duncan, Fromentel, Agassiz, and others. I do not, therefore, in the present communication propose to pass the whole of this subject in review, but rather to consider the general and special peculiarities of growth and non-sexual reproduction exhibited by the corals of the Palæozoic Period alone. Many of these peculiarities are of great interest, both from the stand-point of the palæontologist, and also as concerns the systematic zoologist, and they have not yet met with all the attention they deserve. To carry out this inquiry, it will be necessary first to consider the general phenomena exhibited by the Palæozoic corals, as regards their mode of growth and increase. We may then examine the bearing of these phenomena upon various points connected with the classification of these ancient corals, and more especially upon their generic and specific affinities and differences. Finally, we may briefly consider the relations which exist between different parts of a compound corallum as regards their growth, and their influence upon the ultimate form of the colony.

I. General Modes of Growth in the Palæozoic Corals.

The Palæozoic corals belong, as is well known, almost exclusively to the groups of the *Rugosa* and *Tabulata*, and it is with these, therefore, that we are chiefly concerned. The *Tubulosa*, however, are wholly Palæozoic, and the *Aporosa* and *Perforata* are not altogether unrepresented. None of these three last-mentioned groups, as regards their Palæozoic representatives, exhibit anything in their mode of growth which is not shown by some member of the Rugose or Tabulate divisions, and they will, consequently, require nothing more than incidental mention. The general methods of growth and increase exhibited by the Palæozoic corals, may be considered under the following heads:—

A. *Simple Calicular Gemmation.*—This mode of increase has not been sufficiently distinguished as a separate form of growth; though it is not very

uncommon in certain of the Palæozoic corals, and gives rise to exceedingly well-marked results. In what is usually understood by calicular gemmation, the oral disc of the primitive polype produces two or more buds; these in turn repeat the process, and ultimately we have a mass of an inverted pyramidal shape, composed of numerous corallites diverging from the base. In the particular mode of growth to which I propose to apply the term of "*simple*" calicular gemmation, there is a well-marked modification of the above. The corallum, originally simple, after growing to a certain extent, sends up from its oral disc a *single* bud. The primitive calice may or may not be more or less completely obliterated by the gradual growth and extension of the epitheca over it; and the secondary bud may or may not produce a tertiary bud in the same manner as that in which it was itself produced. In any case, the mode of increase is by the production of *single* buds from the calicine disc, and, consequently, the resulting form of the corallum is in all cases altogether different to what is seen in the ordinary method of calicular gemmation.

I have, so far, only observed simple calicular gemmation in certain of the *Cyathophyllidæ* and *Cystiphyllidæ*, and it differs in different cases as to the extent to which it is carried. In *Cystiphyllum squamosum*, Nich., from the Devonian of Ohio, the primitive corallite seems never to produce more than one bud. This is developed from near the centre of the primitive calice, and has a direction more or less perpendicular to the plane of the old calice (or to the axis of the old corallite, with which the plane of the calice nearly coincides). In *Cystiphyllum Ohioense*, Nich., also from the Devonian of Ohio, most individuals of the species are simple. Others, however, throw up a single bud from the centre or one side of the primitive calice; but this bud, instead of being more or less perpendicular to the axis of the coral, as it is in *C. squamosum*, is usually continued in the direction of the original corallite. In *Cystiphyllum vesiculosum*, Goldfuss, again, the process of calicular gemmation is carried much further than in the two preceding forms. When the corallum in this species has attained a certain growth, it commonly, though not invariably, sends up a new bud from some point in the calice, generally directly above the old one; and this, too, continues to grow for a certain period. A third bud may then be produced in a like manner, and a fourth, fifth, or sixth may be similarly generated, until the aged corallum may consist of a series of short turbinate cups or inverted cones, superimposed one upon the other, the younger upon the older. In a general way, the successively produced corallites conform more or less in direction with the primitive one; but this is by no means universally the case, and the resulting form of the entire corallum is thus often very irregular and peculiar. The old cups are usually quite distinct; but they may be more or less completely obliterated by the gradual growth and extension of the epitheca over them. In these cases it becomes sometimes difficult to

distinguish the remains of the old cups from particularly strong and well-marked "accretion-ridges." There is, however, strong reason for believing that the production of accretion-ridges and the simple form of calicular gemmation just alluded to, are merely different degrees of the same physiological process.

Simple calicular gemmation is also particularly well exhibited in many examples of *Heliophyllum Halli*, Edw. and H.; though not as the only method of increase in this fine species. Sometimes the primitive corallite produces no more than one new cup, which it throws up from one side of the calice, and in such cases both the original and the secondary corallite usually attain considerable dimensions. In other examples several cups are produced successively, till the corallum may be composed of six or eight shortened corallites, arranged in a vertical series, and each springing from some portion of the calice of its predecessor. This mode of growth causes a considerable irregularity of form, old examples looking like a succession of inverted cones inserted into one another, the curvature of the whole mass being often rendered irregular by the bending of the successively produced corallites in different directions.

It seems extremely probable that the peculiar form of simple calicular gemmation, which I have just noticed as occurring in *Cystiphyllum vesiculosum* and *Heliophyllum Halli*, may really be due to some peculiarity in their surrounding conditions. Thus, examples of these species collected from the calcareous deposits of the Corniferous Limestone exhibit "acretion-ridges," but do not, so far as I have seen, produce a vertical series of cups. This latter mode of growth, on the other hand, is very common in specimens collected from the argillaceous beds of the Hamilton Group. It may therefore be suggested that the stimulus to the production of calicine buds in this peculiar fashion is, perhaps, to be found in the slow but regular deposition of fine clayey sediment, which ultimately buries the original polype to the lips, and thus threatens its existence.

B. *Compound Calicular Gemmation.*—In the ordinary method of calicular gemmation, to which I would apply the name of "compound," the primitive corallite throws up from its calicine surface two or more buds, which, after reaching a certain size, in most cases repeat the process. In the typical examples of this mode of growth, as exhibited in such a coral as *Cyathophyllum truncatum*, Linn., the primordial corallite attains no great size before commencing to bud; the buds produced are two or more in number, and they repeat the process. The necessary result of this is, that the aged corallum assumes the form of an inverted pyramidal mass, the base of which is formed by the primitive corallite. From the calice of this the secondary corallites diverge, and the surface of the entire mass is flattened or slightly convex. The calices of the secondary corallites, and the corallites themselves, may remain more or less completely separate, as is usually the case in *Cyathophyllum truncatum*. At other times the corallites are more intimately united by their walls, and the corallum

becomes truly astræiform, as is seen in such a species as *Cyathophyllum regium*, Phill., and in many other instances. It is to be remembered, however, that many of the astræiform corals, though often adduced as examples of calicular gemmation, really owe their form to a process which would be more appropriately termed "calicular fission."

In other cases, compound calicular gemmation gives rise to fasciculate corals, as is well seen in forms such as *Cyathophyllum articulatum*, Wahl., and *Eridophyllum strictum*, Edw. and Haime. This peculiarity is partly due to the fact, that the secondary corallites remain cylindrical, without expanding materially; that they rise side by side without uniting, and without greatly diverging; and that their vertical growth is continued for a considerable period before they in turn give rise to calicine buds. Another cause for their fasciculate form is, however, to be found in the fact, that in these cases true parietal gemmation is often combined with true calicinal budding. There need, therefore, be no surprise at the assumption by such coralla of the fasciculate form, which is so commonly found associated with parietal gemmation.

Finally, there is a peculiar modification of compound calicular gemmation, which I have observed to occur in certain examples of *Heliophyllum Halli*. In this modification, the calice of the primitive corallite throws up a number of buds; but the former does not seem to feel the stimulus to gemmation till it has reached a great age, and a corresponding size. The result of this is, that the secondary corallites remain more or less aborted, and do not appear to have sufficient energy to repeat the process of budding. Hence, in these cases, we have the fully-developed and often comparatively gigantic parent corallite, surmounted by a tuft of small undeveloped secondary corallites, springing from different points in its calicine surface.

C. *Basal Gemmation.*—This mode of increase, better known as "marginal" or "circumferential" gemmation, is effected by the extension from the base or margin of the original polype of a portion of its substance, which throws up a bud, and is ultimately converted into a new polype. Very different forms are produced in this way, according as the new buds are formed along definite lines or stolons, or are developed in a regular manner all round the circumference of the primitive mass. It is hardly requisite to add, that in the case of all compound coralla produced in this manner, the youngest corallites occupy the margin of the colony, whilst the oldest are placed in its centre.

So far as the Palæozoic corals are concerned, marginal gemmation is perhaps most perfectly and unmixedly exhibited by those forms which constitute thin crusts, parasitic upon foreign bodies, and composed of exceedingly short corallites. This is the case, for instance, with some of the encrusting species of *Chætetes*, such as *C. Ortoni*, Nich. (Lower Silurian), and *C. quadrangularis*, Nich. (Devonian). It is likewise the case with such a totally diverse type of

coral as the Lower Silurian *Protarea vetusta*. In forms such as the above we have basal gemmation in its purest form, constituting the only method of increase; and in all such the colony increases in dimensions solely by the addition of new corallites at the margins or circumference of the mass. There are, however, many cases in which increase is effected by a combination of basal gemmation with some other mode of growth. Thus, in all the typical species of the genus *Syringopora*, the colony extends itself laterally by marginal budding, whilst it is at the same time increasing vertically by means of parietal gemmation. The same is also true, though to a more limited extent, in the Tubulose genus *Aulopora*. In most, if not all, of the massive species of *Favosites*, again, in such massive forms of *Chætetes*, as *C. petropolitanus*, Pander, in the massive species of *Heliolites* and its allies, in the genus *Fistulipora*, and in many other Palæozoic corals, there is a combination of basal gemmation, with a peculiar form of lateral gemmation, or with fission. Basal gemmation, pure and simple, is therefore to be regarded as of comparatively rare occurrence amongst the ancient corals, and perhaps is only seen in those forms which constitute very thin crusts, the corallites of which are all of nearly equal height, and which do not tend to increase vertically.

D. *Parietal or Lateral Gemmation.*—Parietal or lateral gemmation is one of the commonest and best known of all the modes of increase of the coralligenous Actinozoa; and it consists in the production by the parent corallite of a bud at some point in its walls, between the lip of the calice and the base. Sometimes the primitive corallite may not produce more than one or two of these lateral buds. More commonly the secondary buds repeat the process of gemmation, and in a very great number of instances there is a combination of basal with lateral budding; so that whilst the corallum is increasing in height by virtue of the latter process, its lateral limits are simultaneously extended by the former. Excellent examples of parietal gemmation amongst the Palæozoic corals may be found in the genus *Diphyphyllum*, in some of the species of *Eridophyllum*, in *Lithostrotion* and *Syringopora*, and in certain species of *Cyathophyllum*, *Cystiphyllum*, *Pachyphyllum*, and *Heliophyllum*. The general form of corallum produced by means of parietal gemmation varies greatly in different cases. The typical form, so far as regards the Palæozoic corals, may be regarded as being the more or less loosely fasciculate corallum of *Diphyphyllum arundinaceum*, Billings, *D. Archiaci*, Billings, *Eridophyllum Verneuilanum*, Edw. and H., *Cystiphyllum fruticosum*, Nich., the ordinary form of *Cyathophyllum cæspitosum*, Goldfuss, *Lithostrotion irregulare*, Phill., and many other species which could be mentioned. In these cases the corallites are more or less elongated and cylindrical in form, and are not actually amalgamated with one another, being, on the contrary, usually separated by slight but conspicuous intervals; the buds are produced at compartively long intervals, and the parent corallites continue to

grow without being injuriously affected by the production of their buds; whilst all the stems preserve an approximate parallelism with one another, or only diverge slightly in proceeding from the base.

In other cases, as in *Heliophyllum proliferum*, Nich. and *H., subcæspitosum*, Nich, the principle of growth is the same as in the preceding, but the parent corallite does not produce more than one, two, or three lateral buds.

In other cases, as in *Lithostrotion basaltiforme*, the method of growth is the same, but all the corallites are in close contact; and in others they become actually amalgamated by their walls, giving rise to a regular astræiform corallum.

As has been before mentioned, parietal budding may be combined with other modes of growth, when its existence is sometimes difficult to demonstrate. Thus, it may be combined with calicular gemmation (*Eridophyllum strictum*, &c.), or with basal gemmation (most of the species of *Favosites*, *Chætetes*, and *Syringopora*). These complex methods of growth may give rise to simply fasciculate coralla, as in the species of *Eridophyllum* just alluded to; but they more commonly result in the production of a massive corallum. This latter result is well exhibited by forms like *Favosites Gothlandica*, Lam., *F. hemispherica*, Yand. and Shumard, and *Chætetes* (*Monticulipora*) *petropolitanus*, Pander. In these and similar cases, the corallum is usually of a more or less hemispheric or subspherical, or pyriform shape, and is composed of corallites, which diverge more or less from an imaginary axis, a basal point, or a basal plane. The corallites are closely contiguous, but are not amalgamated by their walls; and the increase of the corallum is effected partly by the production of buds round the margin of the mass, and partly by the interpolation of new corallites, produced by lateral gemmation from the sides of the old tubes. As will be noted however, subsequently, it is often extremely difficult to decide in certain of these cases whether the growth has been produced by fission, or by lateral gemmation.

E. *Increase by Fission.*—Fissiparous growth amongst the corals is effected by the cleavage or division of the calice, so that two corallites are produced out of one. This process is so well known, that it will not be necessary for me to enter into any details about it, further than to notice certain of its modifications, as exhibited amongst the Palæozoic corals. The chief practical point to be observed is, that there is great difficulty, in many instances, in distinguishing between corals formed by fission and those produced by parietal gemmation on the one hand, and calicular gemmation on the other hand. In many cases the distinction is, of course, an easy matter; but in others the results are extremely similar, though the processes by which they are brought about are very different. Some of these resemblances I shall discuss at a later stage; in the meanwhile, it is sufficient to point out the general reason of their occurrence.

It is obvious, namely, that a fasciculate corallum, similar, for example, to that of *Lithostrotion irregulare*, may very readily be produced by the repetition of the fissiparous process, provided only that the fission involve but a portion of the parent calice,—that the original corallite continues to grow, and that the new corallite extend itself in a direction approximately parallel to that by which it was produced. In the same way, fission is perfectly capable of producing a massive astræiform corallum, perfectly similar in its appearance to one which may owe its form to calicular gemmation. Without entering here into further details as to these perplexing resemblances, it need only be added, that the case is not uncommonly complicated by the coexistence of fission with gemmation in the same coral.

II. Classificatory Value of the Mode of Growth.

Under this head we have to consider what value is to be attached to the mode of growth of the corallum from a purely systematic point of view; and I hope to be able to show that its importance has been generally over-rated, so far at any rate as fossil corals are concerned. It has, namely, been generally assumed that difference in the mode of growth is to be regarded as being at least of generic value, and not a few genera are based essentially upon this alone. In the case of recent corals, where we are acquainted with the soft parts of the actinosoma, the mode of growth is doubtless a most important help to arriving at a correct classification. In the case, however, of fossil corals, where we must be guided solely by the nature of the skeleton, there appear to be good reasons for believing that too much stress has been laid upon the mere mode of growth.

a. In the first place, there is often extreme difficulty in the case of fossil corals in determining what the mode of growth actually may be. Two instances will be sufficient to exemplify this. Thus, the genera *Chætetes*, Fischer, and *Monticulipora*, D'Orbigny, include a number of corals which are in most respects precisely similar, being composed of closely aggregated polygonal, circular, or sub-circular corallites, traversed by well-developed tabulæ, with imperforate walls, and either without septa, or with these structures in a very rudimentary condition. The sole distinction by which the two genera are separated is, that the corallum of *Chætetes*, as redefined by Lonsdale and M'Coy, and as accepted by Milne-Edwards and Haime, is stated to increase by the subdivision or fission of the tubes; whereas the mode of growth in *Monticulipora* is stated to be by gemmation. This distinction, however, is one which it is extremely difficutl, if not impossible, to apply in practice. The truth of this remark is shown by the fact, that the msot eminent palæontologists have arrived at precisely opposite conclusions as to the mode of growth of the same species. Mr

LONSDALE, for example, regards the well known *Chætetes petropolitanus*, Pander, as increasing fissiparously, and consequently places it in the amended genus *Chætetes*. MILNE-EDWARDS and JULES HAIME, on the other hand, regard the same species as increasing by gemmation, and consequently refer it to the genus *Monticulipora*. If this discrepancy of opinion is possible about such a large, massive, and common species as *C. petropolitanus*, we must conclude that this distinction is quite valueless when we have to deal with many of the minute corals which belong to one or other of these groups. Upon the whole, therefore, even if we were to admit as a matter of theory that the mode of growth is of itself sufficient to constitute a generic distinction, we should be forced to conclude that the difficulties in the way of its application to many fossil corals are so great, that it becomes practically useless. It may be added in this connection, that in the corals which belong to *Chætetes* or *Monticulipora*, in which gemmation is clearly the chief or only mode of increase, the method in which gemmation is carried out, nevertheless, differs very widely in different species. Thus, the thin encrusting forms, like *C. Ortoni*, Nich., and *C. quadrangularis*, Nich., undoubtedly increase by basal or marginal gemmation alone. The ramose or dendroid species, like *C. pulchellus*, Edw. and H., *C. Fletcheri*, Edw. and H., and other similar forms, increase by lateral or interstitial gemmation. The more massive forms, like *C. petropolitanus*, Pander, appear to increase by a combination of lateral with basal gemmation. Finally, the strictly frondescent species, such as *C. mammulatus*, Edw. and H., and *C. clathratulus*, James, which consist of two layers of corallites directed in opposite directions from a median plane, appear to increase wholly by basal gemmation.

Again, considerable difficulty has been met with in the separation of the genera *Lithostrotion* and *Diphyphyllum*, in consequence of the fact that the mode of increase has been taken as one of the most important elements of the generic diagnosis. In the forms now usually referred to *Lithostrotion*, the mode of increase is clearly by lateral gemmation, the resulting corallum being sometimes compact and astræiform, sometimes fasciculate. The genus *Diphyphyllum* was founded by LONSDALE for forms which closely resemble the fasciculate species of *Lithostrotion*, but which this eminent palæontologist believed to increase fissiparously. It is now certain, however, that the mode of increase in *Lithostrotion* and *Diphyphyllum* is identical, both increasing by lateral gemmation. The real distinction between the two genera is to be found in the fact that a columella is present in *Lithostrotion*, whilst this structure, in spite of the doubts of MILNE-EDWARDS and HAIME, is certainly absent in *Diphyphyllum*. It seems wise, however, to conclude that a point upon which the most eminent authorities may differ should not be employed prominently in determining the generic position of fossil corals, the difficulties in the way of its practical application being so great as to preclude its use for this purpose.

b. In the second place, there are not a few instances in which the mode of increase differs widely in individuals even of the same *species*. Thus, in *Heliophyllum Halli*, Edw. and H., the ordinary individuals are truly simple and do not produce any buds at all; other individuals exhibit simple calicular gemmation, and form a vertical succession of cups produced singly and successively in the same longitudinal axis; other examples exhibit a modification of compound calicular gemmation, in which the aged corallum throws up a tuft of small and abortive corallites from its calicine surface, whilst a few individuals show the ordinary and typical form of compound calicular budding. In *Cyathophyllum cæspitosum*, Goldfuss, again, the mode of growth is typically by calicular and lateral gemmation combined with one another; but other examples of the species are stated by D'Orbigny, M'Coy, and Milne-Edwards, and Haime to increase by fission. Finally, there are numerous instances in which the form of the corallum, within the limits of the same species, becomes very mutable in consequence of variations in the mode of growth. Thus, the same species may be at one time fasciculate, and at another time astræiform, or at one time massive, and at another time ramose.

c. In the third place, the small classificatory value of the mode of growth of the corallum, comparatively speaking, is strongly shown by the marked and striking variations in this particular exhibited by closely allied species of the same genus. Very many examples of this could readily be brought forward, but a few will be sufficient. In the genus *Cystiphyllum*, for example, we find species such as *C. vesiculosum*, Goldfuss, *C. Americanum*, Edw. and H., *C. grande*, Billings, *C. mundulum*, Hall, *C. Ohioense*, Nich., *C. cylindricum*, Lonsdale, *C. Grayi*, Edw. and H., *C. Senecaense*, Billings, *C. Siluriense*, Lonsdale, and *C. squamosum*, Nich., in which the corallum is either invariably simple, or at most produces a few buds by a process of simple calicular gemmation. In *Cystiphyllum fruticosum*, Nich., however, we find a truly compound corallum, which has the internal structure peculiar to the genus, but possesses a fasciculate form, and increases by lateral gemmation; whilst in *C. aggregatum*, Billings, the corallum is also compound, and is produced either by lateral budding or by fission. In the genus *Eridophyllum*, one species (*E. strictum*, Edw. and H.), increases principally or solely by calicular gemmation; but others (such as *E. Verneuilanum*, Edw. and Haime, and *E. Simcoënse*, Billings), multiply by means of lateral budding. In the great genus *Cyathophyllum*, some species, such as *C. ceratites*, Goldfuss, *C. obtortum*, Edw. and H., *C. Damnoniense*, Phill., *C. Zenkeri*, Billings, *C. helianthoides*, Goldfuss, and the like are quite simple. Other species of the genus, such as *C. truncatum*, Linn., increase exclusively by means of calicular gemmation. In others, such as *C. cæspitosum*, Goldfuss, and *C. articulatum*, Wahl., we have a combination of calicular with lateral gemmation. In others, such as *C. æquiseptatum*, Edw. and H., *C. radicans*, Goldfuss,

and *C. Steiningeri*, Edw. and H., growth is by lateral gemmation; whilst dichotomous fission is stated to occur in some examples of *C. cæspitosum*, Gold. Examples very similar to the above could be drawn from other genera, and, indeed, the same fact is proved by the common co-existence in the same genus of simple and compound species. Some genera, such as *Zaphrentis*, *Aulacophyllum*, *Lophophyllum*, and the like, are so far only known as containing simple corals. In fact, the existence of a simple corallum is usually given as one of the characteristics of the entire family of the *Zaphrentinæ*. Even in this family, however, there are genera in which future researches will probably place certain compound forms. The genus *Amplexus*, for example, is usually stated to contain only simple coralla. There exists, however, in the Niagara and Guelph formations (Upper Silurian) of North America, and in the Corniferous Limestone (Devonian) of the same country, a coral which has all the structural peculiarities appertaining to *Amplexus*, but is truly compound, and grows in fasciculate masses. One course, in this and similar cases, would be to affirm that, being composite, it could not belong to the same generic group; but it appears to be more philosophical to alter the received definition of the genus, and to admit into it both compound and simple forms. Other instances in which the same genus contains both simple and compound species are so frequent as not to require special mention. It only need be added in this connection that there are not infrequent instances in which both simple and compound individuals are found in the limits of the same species. As examples of this may be mentioned *Heliophyllum Halli*, Edw. and H., *H. subcæspitosum*, Nich., and *Diphyphyllum Archiaci*, Billings.

d. Finally, it may be pointed out that precisely the same mode of growth, even when this is very peculiar, is exhibited by corals belonging to the most diverse groups. The best example of this that can be given is the existence of the same kind of calicular gemmation in such distinct and remote genera as *Palæocyclus* and *Cyathophyllum*, the one belonging to the *Aporose* division of *Zoantharia sclerodermata*, the other being one of the *Rugosa*.

e. The general conclusion from the above facts would seem to be that too much stress must not be laid upon the mode of increase of the corallum as a means of classification. When accompanied by other well-marked peculiarities it has undoubtedly a classificatory value; but not otherwise. Allied forms in the same genus often exhibit very different modes of growth; the same is true in some instances even of the individuals of a single species, whilst forms belonging to the most diverse groups may increase in the same way. Finally, there is often such great difficulty in determining what is the true mode of growth amongst the fossil corals, that this character, even if theoretically important, loses its value altogether when employed practically as a means of separating different genera.

III. Relations between the Growth of Different Parts of a Compound Corallum.

Amongst the various circumstances which conspire to regulate the growth of the corallum, one of the most important is to be found in the presence or absence of a common epitheca. As a general rule, the existence of a common epitheca more or less limits the growth of the colony, and the corallum assumes a compact and more or less definite form, as is well seen in the massive species of *Favosites*, in *Michelinia*, and in certain species of *Chætetes* and *Alveolites*. On the other hand, the absence of a common envelope is usually attended by a more or less lax or spreading mode of growth, as is well seen in the fasciculate coralla of *Diphyphyllum*, *Eridophyllum*, *Syringopora*, and the compound *Cyathophylla*, as well as in many other forms.

Even where a common epitheca is present, however, there are great differences as to its form and development. In some cases, as in *Favosites Gothlandica*, Lam., *Michelinia convexa*, D'Orb., and other species of the same genus, *Strombodes Murchisoni*. Edw. and H., *Acervularia luxurians*, Eichw., and other forms, the corallites radiate from a basal point, and the corallum has a more or less pyriform or sub-spherical shape, with a pointed base. The epitheca covers the under surface of the base, and envelopes the outer sides of the peripheral corallites; but the corallum increases mainly in height, and the accompanying increase in diameter is effected by the intercalation of new corallites between those already existing. In other cases, as in most examples of *Favosites hemispherica*, Yandell and Shumard, in *Fistulipora Canadensis*, Billings, *Alveolites Goldfussi*, Billings, *Chætetes petropolitanus*, Pander (in its typical form), and in many species of *Heliolites*, the epitheca forms an approximately flattened plate, to which the corallites are directed more or less at right angles, and upon which they rest by their bases, and not by their sides. In these cases, therefore, the corallum usually has the general form of a flattened or plano-convex expansion, which may increase laterally to almost any extent by means of basal or parietal gemmation. At the same time, the vertical growth of the corallum is often carried on by the interstitial addition of fresh corallites, or even by the superposition of additional layers of tubes.

In still another group of cases, namely, in *Favosites turbinata*, Billings, and in some examples of *F. Forbesi*, Edw. and Haime, the corallum has more or less of a cylindrical or turbinate shape, and the corallites radiate outwards from a central line, so as to open on the surface nearly or quite at right angles to the axis of the colony, except at its upper portion. The epitheca, which at first must be confined to the actual base of the colony, in these cases continues to grow, and gradually spreads over the calices of all the inferior corallites. Thus it comes to pass that in an adult example of *F. turbinata* we have an

inversely conical mass, the broad upper end of which is convex; and it is only at this upper end that the calices are actually open, or the corallites are truly alive. The whole of the lateral surfaces of the colony exhibits calices also, but these are covered by a well developed epitheca, through which they can be more or less dimly discerned.

Another circumstance which has a good deal to do with the growth and increase of the compound coralla, is the presence or absence of a cœnenchyma, and the extent to which it is developed when present. In none of the compound Palæozoic corals is the cœnenchyma a very conspicuous feature, and it is usually altogether wanting. Even when present, there may reasonably be entertained considerable doubts as to its true nature. In *Heliolites* and its Palæozoic allies it is possible that we have to deal with a true cœnenchyma, comparable to that of *Millepora*. In these cases the corallites are very markedly larger than the tubuli of the cœnenchyma, and are further distinguished by being septate. In *Fistulipora*, again, we have a transitional form, the corallites not being very greatly larger than the so-called cœnenchymal tubuli, and agreeing with them precisely in structure. The question arises, therefore, whether these tubuli are to be regarded as constituting a proper cœnenchyma, or whether they are not really of the nature of aborted or rudimentary corallites. Several facts would favour the latter view. Thus, in *Constellaria* (*Stellipora*), and in some species of *Callopora*, the difference between the larger and smaller tubes is not very striking, and the latter may well be nothing more than imperfect corallites. In many species of *Chætetes* (*Monticulipora*), further, the corallum is composed of three sizes of tubes, some unusually large, others of medium dimensions, and others very minute. The large tubes are comparatively few in number, are usually aggregated into definite groups, and are undoubtedly corallites. The medium-sized tubes form the bulk of the corallum, and are also undoubtedly corallites. The smallest tubes may be placed simply at the angles of the normal calices, when they would seem to be certainly young or rudimentary corallites; or they may be collected as well into groups occupying definite spaces. In this latter case it is uncertain whether they are to be regarded as being small corallites, or as belonging to the cœnenchyma, though I strongly incline to the former view. Additional support is given to this view by a consideration of species like *Favosites hemispherica*, Yand. and Shumard, and *F. dubia*, De Blainville. In the first of these we find, in many specimens, a most marked and extraordinary singularity in the size of the calices, but not at all as is seen in such a species as *F. Forbesi*, where the small calices are simply scattered irregularly amongst the larger ones. On the contrary, we have here the curious phenomenon that whilst the majority of the corallites have a certain average size, there are whole tracts, quite irregular in shape and

size, occupied by corallites which may be smaller by one half than the normal. In *F. dubia*, again, the corallites are normally of two sizes, small and large, the small nearly equalling the large in number, and being everywhere distributed amongst them.

The remaining causes which influence the form and mode of growth amongst the compound Palæozoic corals are of secondary importance, and need merely be alluded to. The relations of the different corallites of a compound corallum to one another vary greatly in different species, and sometimes even in different individuals of the same species, and the general form of the colony necessarily exhibits a corresponding variability. When the corallites are in close contiguity, with or without absolute fusion and amalgamation of their walls, then we get compact corals, as in *Columnaria*, *Favistella*, *Columnopora*, *Favosites*, *Chætetes*, *Michelinia*, &c. On the other hand, when the corallites are not closely contiguous, then we get fasiculate corals, as in *Syringopora*, *Diphyphyllum*, &c. Even where the corallites are contiguous, the form of the whole corallum is very variable, though any given portion of it is compact; the variations depending on laws of which we are at present very ignorant. Such coralla are typically massive; but they present themselves also in the form of dendroid or ramose growths, thin flattened horizontal expansions or vertical fronds, or sometimes as thin parasitic crusts; and all these variations may be found within the limits of a single genus. In certain instances, as in *Favosites*, the corallites though contiguous are not united by their walls; in other instances, as in *Favistella*, such union takes place. In other cases, as in *Phillipsastræa*, the walls of the corallites are wanting, and the different members of the colony are firmly united by the confluence of their septa. Finally, amongst the fasciculate coralla the chief differences of form are due to the comparative independence of the corallites (as in *Diphyphyllum*), or to the extent to which the corallites may be united by exothecal processes (*Eridophyllum*, *Syringopora*, *Haimeophyllum*, &c.)

EXPLANATION OF PLATE.

Figure 1. A large example of *Heliophyllum Halli*, Edw. and H., producing a small tuft of rudimentary corallites by calicular gemmation at the termination of its growth.

Figure 2. An example of the same, in which five calices have been produced in a vertical series by simple calicular gemmation.

Figure 3. Another example of the same, in which a single secondary corallite has been produced by simple calicular budding.

Figure 4. Another example of the same, in which ordinary compound calicular gemmation has occurred.

Figure 5. A simple individual of the same.

Figure 6. Another example of the same, composed of two closely amalgamated corallites, which have either been produced by fission, or have grown accidentally side by side.

Figure 7. An example of *Heliophyllum subcæspitosum*, Nich., showing lateral gemmation.

Figure 8. Another example of the same, showing the ordinary form of calicular gemmation.

Figure 9. *Cystiphyllum squamosum*, Nich., showing a secondary corallite, developed at right angles to the original one by simple calicular gemmation.

Figure 10. *Cystiphyllum Ohioense*, Nich., showing a secondary corallite developed in the axis of the original one by simple calicular gemmation.

(All the specimens figured are from the Devonian formation, and all are drawn of the natural size.)

XIII—*On the Elimination of* α, β, γ, *from the conditions of integrability of* $\mathrm{S}\,u\alpha d\rho$, $\mathrm{S}\,u\beta d\rho$, $\mathrm{S}\,u\gamma d\rho$. By G. Plarr, Docteur ès-sciences. Communicated by Prof. Tait.

(Read December 21, 1874.)

The proposed scalars represent the differentials of the three functions, X, Y, Z, of the co-ordinates x, y, z, put under the form

$$d\mathrm{X} = -\mathrm{S}\nabla\mathrm{X}d\rho\,,\quad d\mathrm{Y} = \text{etc.},$$

where ∇ represents the operator

$$i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\,,$$

and where

$$d\rho = idx + jdy + kdz\,.$$

These functions, equated respectively to constants, are intended to represent a system of orthogonal surfaces. The conditions to which X, Y, Z must for this end satisfy, are represented by

$$\nabla\mathrm{X} = u\alpha\,,\quad \nabla\mathrm{Y} = u\beta\,,\quad \nabla\mathrm{Z} = u\gamma\,,$$

where u designates the tensor which the three expressions must have in common, and where α, β, γ designate a system of treble rectangular unit-vectors; and u, as well as α, β, γ, being generally variable from one point of space to another, α, β, γ satisfying at every point to the nine relations:

$$\alpha^2 = \beta^2 = \gamma^2 = -1\,,\quad \alpha\beta = \gamma\,,\quad \text{etc. etc.}$$

The possibility of the expressions $u\alpha$, etc., for $\nabla\mathrm{X}$, etc., is expressed by the conditions of integrability of the proposed scalars, and as these conditions consist in three relations between u, α, β, γ, and as the vector relations between α, β, γ, give the required "fourth" equation, there is no impossibility *a priori* of conceiving the existence of an equation containing u alone after the elimination of α, β, γ.

In Professor Tait's paper "On Orthogonal and Isothermal Surfaces" (Trans. R.S.E. 1873–74), the problem of elimination is proposed and solved, but at the opening of the paper we read of the suggestion, that quaternion methods require improvements in their application to the proposed problem of elimination.

In the present paper we give an account of the attempts we have made to follow out the suggestion quoted. Our plan has been to define certain quaternion functions of α, β, γ, to which the question leads to, but which may present

themselves also in other questions, and then to make a study of the properties of these functions and of the relations which may exist between them. Having succeeded in establishing some of the relations which exist *a priori* between these functions, namely, independently of the conditions of integrability, we were enabled to carry out the elimination by taking into account concurrently (1st) the *a priori* relations, and (2d) the conditions of integrability, these latter ones giving, so to say, the special data of the problem.

Our definitions of those quaternion functions of α, β, γ, comprise expressions depending on two different operators, the one $\triangleleft$ being the common one, and the other $\triangleright$ being in certain respects conjugate.

Also we have made use of the property of both these operators, according to which the result, which they work out, remains unaffected in form, by any change in the direction of the axes of the Carthesian co-ordinates, and of the corresponding unit-vectors by which the operators are defined.

In a final paragraph we have reintroduced the representation of α, β, γ, by $q^{-1}iq$, $q^{-1}jq$, $q^{-1}kq$, and given the expressions of the above quaternion functions by the help of q.

§ 1. *Preliminaries.*

We establish by definition

$$(1) \qquad \begin{cases} \triangleleft r = i\frac{dr}{dx} + j\frac{dr}{dy} + k\frac{dr}{dz} \\ \triangleright r = \frac{dr}{dx}i + \frac{dr}{dy}j + \frac{dr}{dz}k. \end{cases}$$

The first of these operators is the one which has been designated by ∇. For altering this sign into $\triangleleft$ we invoke the authority of page 610 of HAMILTON's "Lectures," where the symbol $\triangleleft$ was introduced with its present meaning. The symbol $\triangleright$ for the second operator is then, so to say, a consequence of the first. If a denomination was wanted the first might be called the left handed, and the second the right handed operator.

Both operators give the same result when applied to a scalar, as for example w—

$$\triangleright w = \triangleleft w,$$

because the place of i, j, k is of no influence in respect to the scalars

$$\frac{dw}{dx}, \quad \frac{dw}{dy}, \quad \frac{dw}{dz}.$$

Applied to a vector ω, the operators give results satisfying to

$$(2) \qquad \triangleright\omega = \mathrm{K}\triangleleft\omega,$$

because the derivates $\frac{d\omega}{dx}$, etc., are vectors, like ω, and therefore

$$(3)\qquad \begin{cases} \mathrm{S}\triangleright\omega = \mathrm{S}\triangleleft\omega \\ \mathrm{V}\triangleright\omega = -\mathrm{V}\triangleleft\omega . \end{cases}$$

We may remark also—

$$(4)\qquad \begin{cases} \triangleleft\omega + \triangleright\omega = 2\mathrm{S}\triangleleft\omega \\ \triangleleft\omega - \triangleright\omega = 2\mathrm{V}\triangleleft\omega . \end{cases}$$

In the case of a quaternion $r = w + \omega$ and its conjugate $\mathrm{K}r = w - \omega$, we have, as $\triangleleft w$ is a vector,

$$\mathrm{S}\triangleleft r = \mathrm{S}\triangleleft\omega , \quad \mathrm{V}\triangleleft r = \triangleleft w + \mathrm{V}\triangleleft\omega$$
$$\mathrm{S}\triangleright r = \mathrm{S}\triangleleft\omega , \quad \mathrm{V}\triangleright r = \triangleleft w - \mathrm{V}\triangleleft\omega ;$$

and for $\mathrm{K}r = w - \omega$, we change the sign of ω in the preceding formula

$$\mathrm{S}\triangleleft\mathrm{K}r = -\mathrm{S}\triangleleft\omega , \quad \mathrm{V}\triangleleft\mathrm{K}r = \triangleleft w - \mathrm{V}\triangleleft\omega$$
$$\mathrm{S}\triangleright\mathrm{K}r = -\mathrm{S}\triangleleft\omega , \quad \mathrm{V}\triangleright\mathrm{K}r = \triangleleft w + \mathrm{V}\triangleleft\omega .$$

With these elements we may easily verify the two following formulas

$$(5)\qquad \begin{cases} \mathrm{K}\triangleleft r + \triangleright\mathrm{K}r = 0 \\ \mathrm{K}\triangleright r + \triangleleft\mathrm{K}r = 0 , \end{cases}$$

of which we will make an application in our § 4.

In the second order, both double operators $\triangleleft^2 r$, $\triangleright^2 r$, give the same result,

$$(6)\qquad = -\left(\frac{d^2r}{dx^2} + \frac{d^2r}{dy^2} + \frac{d^2r}{dz^2}\right).$$

The two operators $\triangleright(\triangleleft r)$ and $\triangleleft(\triangleright r)$ give also results the same for both. In the case of a scalar w the results are identical with $\triangleleft^2 w$ or $\triangleright^2 w$. But for a vector or quaternion generally they give the result—

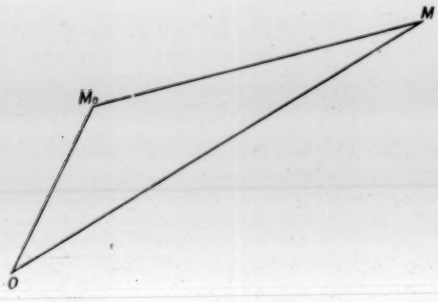
$$(7)\qquad \begin{cases} \triangleleft(\triangleright r) = \triangleright(\triangleleft r) = \\ = i\dfrac{d^2r}{dxdx} i + j\dfrac{d^2}{dydx} i + k\dfrac{d^2}{dzdx} i \\ + i\dfrac{d^2}{dxdy} j + j\dfrac{d^2}{dydy} j + k\dfrac{d^2}{dzdy} j \\ + i\dfrac{d^2}{dxdz} k + j\dfrac{d^2}{dydz} k + k\dfrac{d^2}{dzdz} k . \end{cases}$$

Let us introduce, into $\triangleleft$, variables related to x, y, z, by a linear relation, in the following manner :—Let O be the *fixed* origin of ρ, whose expression is

$$(8)\qquad \rho = ix + jy + kz ,$$

its extremity being in M.

Let us represent ρ by the vector sum (1st) of OM_0 represented by ρ_0, whose origin is O, and its extremity M_0, M_0 remaining invariable when M varies by differentiation, and (2d) of M_0M, represented by ω, so that when M varies ω varies also, but only by its extremity M and not by its origin M_0. We have thus



$$\rho = \rho_0 + \omega .$$

Through M_0 we conceive a system of treble rectangular axes, invariable in direction, and we designate by $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, the unit-vectors corresponding to their directions. Let a, b, c, be the Carthesian co-ordinates of M in respect to these axes. Then we have

$$\omega = \tilde{\alpha}a + \tilde{\beta}b + \tilde{\gamma}c .$$

For any displacement of M we have

$$d\rho = d\omega ,$$

namely

$$idx + jdy + kdz = \tilde{\alpha}da + \tilde{\beta}db + \tilde{\gamma}dc ,$$

but we consider three particular displacements corresponding to

1st, $dy = 0$, $dz = 0$, dx not zero,
2d, $dz = 0$, $dx = 0$, dy ,,
3d, $dx = 0$, $dy = 0$, dz ,,

This gives us severally

$$(9) \qquad \begin{cases} i = \tilde{\alpha}\dfrac{da}{dx} + \tilde{\beta}\dfrac{db}{dx} + \tilde{\gamma}\dfrac{dc}{dx} \\ j = \tilde{\alpha}\dfrac{da}{dy} + \tilde{\beta}\dfrac{db}{dy} + \tilde{\gamma}\dfrac{dc}{dy} \\ k = \tilde{\alpha}\dfrac{da}{dz} + \tilde{\beta}\dfrac{db}{dz} + \tilde{\gamma}\dfrac{dc}{dz} . \end{cases}$$

The nine quotients which correspond to as many *partial differentials* of a, b, c, are easily determined by the help of the properties of $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, as a treble rectangular system of unit vectors. Thus we get:

$$(10) \qquad \begin{cases} \dfrac{da}{dx} = -\,\mathrm{S}\tilde{\alpha}i , \dfrac{db}{dx} = -\,\mathrm{S}\tilde{\beta}i , \dfrac{dc}{dx} = -\,\mathrm{S}\tilde{\gamma}i \\ \dfrac{da}{dy} = -\,\mathrm{S}\tilde{\alpha}j , \dfrac{db}{dy} = -\,\mathrm{S}\tilde{\beta}j , \dfrac{dc}{dy} = -\,\mathrm{S}\tilde{\gamma}j \\ \dfrac{da}{dz} = -\,\mathrm{S}\tilde{\alpha}k , \dfrac{db}{dz} = -\,\mathrm{S}\beta k , \dfrac{dc}{dz} = -\,\mathrm{S}\tilde{\gamma}k . \end{cases}$$

We may express x, y, z in functions of a, b, c, and supposing we have a function of xyz, say r, we may express it by, or suppose it to be transformed into, a function of a, b, c; ρ_0 entering into the transformed function will not vary by differentiation.

Calculating, as for example, $\triangleleft r$, we put the partial differentiations under form:

$$\frac{dr}{dx} = \frac{da}{dx}\frac{dr}{da} + \frac{db}{dx}\frac{dr}{db} + \frac{dc}{dx}\frac{dr}{dc}$$

$$\frac{dr}{dy} = \frac{da}{dy}\frac{dr}{da} + \text{etc.}$$

$$\frac{dr}{dz} = \frac{da}{dz}\frac{dr}{da} + \text{etc.}$$

Now $\frac{da}{dx}$, $\frac{db}{dx}$, etc., expressing partial differential quotients represent precisely the same quantities as in equations (9), and are given by the values (10).

If now we multiply $\frac{dr}{dx}$, $\frac{dr}{dy}$, etc., respectively by i, j, k, and sum, the first members give $\triangleleft r$.

The second members give the sum

$$i\frac{da}{dx} + j\frac{da}{dy} + k\frac{da}{dz}$$

as the factor of $\frac{dr}{da}$. But by the substitution of the values (10) this becomes

$$-(i\mathrm{S}\tilde{a}i + j\mathrm{S}\tilde{a}j + k\mathrm{S}\tilde{a}k),$$

which is $= +\tilde{\alpha}$.

Likewise the factors of $\frac{dr}{db}$, $\frac{dr}{dc}$ are found to be $\tilde{\beta}$, $\tilde{\gamma}$ respectively.

Thus we have, for the expressions defined by (1), the transformations

$$(10\ bis) \qquad \begin{cases} \triangleleft r = \left(\tilde{\alpha}\frac{d}{da} + \tilde{\beta}\frac{d}{db} + \tilde{\gamma}\frac{d}{dc}\right)r \\ \triangleright r = \frac{dr}{da}\tilde{\alpha} + \frac{dr}{db}\tilde{\beta} + \frac{dr}{dc}\tilde{\gamma}, \end{cases}$$

the second of these formulas being formed in a similar way.

This transformation shows that the operators $\triangleleft$, $\triangleright$, work out results which are not dependent on the particular direction of the system of treble rectangular axes of co-ordinates to which the variables refer, provided that in the differentiation the axes themselves remain constant in direction.

If we differentiate the equations (9) partially in respect to any of the variables x, y, z, the differential quotients of a, b, c of the second order must *all* vanish.

To this circumstance the result is due that the operators $\triangleleft^2$, $\triangleright^2$, $\triangleleft(\triangleright)$, $\triangleright(\triangleleft)$, as given by (6) and (7) will, like $\triangleleft$ and $\triangleright$, preserve their primitive form, when the change of variables from x, y, z, to a, b, c, is introduced, so that we

may derive the transformed expressions from (6) and (7) in replacing in them x, y, z, i, j, k, respectively by $a, b, c, \tilde{a}, \tilde{\beta}, \tilde{\gamma}$. Thus, as for example,

$$(11) \qquad \triangleleft^2 r = \triangleright^2 r = -\left(\frac{d^2r}{da^2} + \frac{d^2r}{db^2} + \frac{d^2r}{dc^2}\right).$$

Generally the directions of $\tilde{a}, \tilde{\beta}, \tilde{\gamma}$ are quite arbitrary. We may form in respect to them expressions like

$$\triangleleft\alpha, \ \triangleleft\beta, \ \triangleleft\gamma, \ \triangleleft^2\alpha, \text{ etc. etc.}$$

And *when the differentiations have been effected,* assimilate $\tilde{a}, \tilde{\beta}, \tilde{\gamma}$ respectively with the directions of α, β, γ, whenever this assimilation be useful.

In the case of a formula in which the assimilation has been made, all further differentiation or integration would become the source of erroneous, or at least uncertain, results. But the formula, nevertheless, will express a quaternion relation which may be useful for ascertaining the relations which exist between α, β, γ (and any other element entering into the formula) when considered as fixed data, not variable. (These formulas, on all accounts, will not contain such elements as $\frac{d\tilde{a}}{da}, \frac{d\tilde{a}}{db}$, etc., because $\tilde{a}$, etc., cannot be differentiated.) The corresponding results may be called "*approximative,*" not as to actual value, but only as to their fitness, or better unfitness, for infinitesimal calculus.

We will adopt the notation

$$\alpha'_x = \frac{d\alpha}{dx}, \text{ etc.} \qquad \alpha''_{x,y} = \frac{d^2\alpha}{dxdy}, \text{ etc.}$$

In the use of the sign Σ, whenever confusion might arise otherwise, we inscribe immediately after the sign Σ, between bars, the letters to which permutation is to be applied, namely so:

$$\Sigma \mid \alpha, \beta, \gamma \mid \mathrm{F}\alpha = \mathrm{F}\alpha + \mathrm{F}\beta + \mathrm{F}\gamma,$$

$$\Sigma \left| \begin{matrix} \tilde{a} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{matrix} \right| f(\tilde{a}, a) = f(\tilde{a}, a) + f(\tilde{\beta}, b) + f(\tilde{\gamma}, c).$$

We shall make frequent applications of the following known formulas, which we record here, once for all; ρ being any vector, we have:

(A)	$\Sigma \mid \alpha\beta\gamma \mid$	$\alpha\mathrm{S}\alpha\rho$	$= -\rho$
(B)	$\Sigma \mid$	$\mid \alpha\mathrm{V}\alpha\rho$	$= -2\rho, \ \Sigma(\mathrm{V}\alpha\rho)\alpha = 2\rho$
(C)	$\Sigma \mid$	$\mid \alpha\rho\alpha$	$= +\rho$
(D)	$\Sigma \mid$	$\mid \mathrm{S}^2\rho\alpha$	$= -\rho^2$
(E)	$\Sigma \mid$	$\mid \mathrm{V}^2\rho\alpha$	$= +2\rho^2$
	$\Sigma \mid$	$\mid \mathrm{V}\alpha\rho\mathrm{S}\alpha\rho$	$= 0$
	$\Sigma \mid$	$\mid \mathrm{S}\alpha\mathrm{V}\beta\rho\mathrm{V}\gamma\rho$	$= +\omega^2$

α, β, γ being here any treble rectangular system of unit vectors.

§ 2. *The Quaternion Functions of* α, β, γ.

By definition we designate by I, II, III, IV, P, Q, R, Π', Π'', the following expressions—

$$(12)\qquad \begin{cases} \mathrm{I} = \Sigma\alpha\lhd\alpha\,, & \mathrm{III} = \Sigma\lhd\alpha.\alpha \\ \mathrm{II} = \Sigma\alpha\rhd\alpha\,, & \mathrm{IV} = \Sigma\rhd\alpha.\alpha \end{cases}$$

$$(13)\qquad \begin{cases} \mathrm{P} = \Sigma\mathrm{S}^2\lhd\alpha\,, \\ \mathrm{Q} = \Sigma\mathrm{V}^2\lhd\alpha\,, \end{cases} \qquad \mathrm{R} = \Sigma\mathrm{V}\lhd\alpha\mathrm{S}\lhd\alpha$$

$$(14)\qquad \Pi' = \Sigma\alpha\rhd^2\alpha\,, \qquad \Pi'' = \Sigma\alpha\rhd(\lhd\alpha)\,.$$

The four first of these sums are expressible by one of them. We choose II, which we will write here (also for other purposes) explicitly:

$$(15)\qquad \begin{cases} \mathrm{II} = \alpha(\alpha'_a\tilde{\alpha} + \alpha'_b\tilde{\beta} + \alpha'_c\tilde{\gamma}) \\ \quad + \beta(\beta'_a\tilde{\alpha} + \beta'_b\tilde{\beta} + \beta'_c\tilde{\gamma}) \\ \quad + \gamma(\gamma'_a\tilde{\alpha} + \gamma'_b\tilde{\beta} + \gamma'_c\tilde{\gamma}) \end{cases}$$

The terms which compose I are conjugates, with reversed sign, term by term, to the terms which compose IV. Example—In I we have $\alpha\tilde{\alpha}\alpha'_a$, the corresponding term in IV. is

$$\alpha'_a\tilde{\alpha}\alpha = -\,\mathrm{S}\alpha\tilde{\alpha}\alpha'_a + \mathrm{V}\alpha\tilde{\alpha}\alpha'_a\,,$$

and likewise for all the nine terms of I and IV. Therefore

$$\mathrm{IV} = -\,\mathrm{KI}\,.$$

The terms of III are conjugates of those of II, with reversed signs. Example—In III we have $\tilde{\alpha}\alpha'_a\alpha = -\,\mathrm{S}\alpha\alpha'_a\tilde{\alpha} + \mathrm{V}\alpha\alpha'_a\tilde{\alpha}$, and so on.
Therefore

$$\mathrm{III} = -\,\mathrm{KII}\,.$$

The relation between I. and II. may be derived from the remark that

$$\mathrm{S}\lhd\alpha = \mathrm{S}\rhd\alpha\,, \quad \mathrm{V}\lhd\alpha = -\,\mathrm{V}\rhd\alpha\,.$$

Thus we get:

$$\mathrm{I} = \Sigma\alpha\mathrm{S}\lhd\alpha + \Sigma\alpha\mathrm{V}\lhd\alpha$$
$$\mathrm{II} = \Sigma\alpha\mathrm{S}\lhd\alpha - \Sigma\alpha\mathrm{V}\lhd\alpha\,.$$

By addition and subtraction we deduce:

$$(16)\qquad \begin{cases} \mathrm{I} = 2\Sigma\alpha\mathrm{S}\lhd\alpha - \Pi \\ \mathrm{I} = \Pi + 2\Sigma\alpha\mathrm{V}\lhd\alpha\,. \end{cases}$$

Let us consider Π under its explicit form (15). We group the terms by columns, and put by definition:

$$\varpi_t = \alpha\alpha'_t + \beta\beta'_t + \gamma\gamma'_t \tag{17}$$

for any scalar variable t; in this present instance t represents a, b, c.

Thus we have:

$$\Pi = \Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} \varpi_a \tilde{\alpha}. \tag{18}$$

The terms which compose ϖ_t are vectors, because the differentiation of

$$\alpha^2 = -1, \beta^2 = -1, \gamma^2 = -1,$$

in respect to any scalar variable t, gives

$$S\alpha\alpha'_t = 0, S\beta\beta'_t = 0, S\gamma\gamma'_t = 0;$$

so that $\alpha\alpha'_t$, $\beta\beta'_t$, $\gamma\gamma'_t$, are their own vectors.

As we have $III = -K\,II$, we get

$$III = -S\Sigma\varpi_a\tilde{\alpha} + V\Sigma\varpi_a\tilde{\alpha};$$

But $V\varpi_a\tilde{\alpha} = -V\tilde{\alpha}\varpi_a$, because ϖ_a is a vector; thus:

$$III = -\Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} \tilde{\alpha}\varpi_a. \tag{19}$$

We remark, that we have generally

$$-\varpi'_t = +(\alpha'_t\alpha + \beta'_t\beta + \gamma'_t\gamma). \tag{20}$$

But one must not be tempted into taking ϖ_t as an exact differential quotient.

We express α'_t, β'_t, γ'_t by the help of ϖ_t, as follows:—

Multiplying, as for example, *by* α, both members of (17), and *into* α, both members of (20), and adding we get:

$$\begin{aligned} \alpha\varpi_t - \varpi_t\alpha = {} & \alpha^2\alpha' + \alpha\beta\beta' + \alpha\gamma\gamma' \\ & + \alpha'\alpha^2 + \beta'\beta\alpha + \gamma'\gamma\alpha. \end{aligned}$$

We group the terms of the second member into three groups, which give:

$$\begin{aligned} \alpha^2\alpha' + \alpha'\alpha^2 &= -2\alpha' \\ \alpha\beta\beta' + \gamma'\gamma\alpha = \gamma\beta' &+ \gamma'\beta = -\alpha' \\ \alpha\gamma\gamma' + \beta'\beta\alpha = -\beta\gamma' &- \beta'\gamma = -\alpha'. \end{aligned}$$

The first member is $2V\alpha\varpi_t$, and the second member becomes $-4\alpha_t$. Operating in the same way with β and with γ, we get the formulas:

$$(21) \qquad \begin{cases} \alpha_t' = -\frac{1}{2} V\alpha\varpi_t \\ \beta_t' = -\frac{1}{2} V\beta\varpi_t \\ \gamma_t' = -\frac{1}{2} V\gamma\varpi_t . \end{cases}$$

Let us with these values of α_t', etc., calculate the expression of VI, namely, of the vector of I.

We have by definition (as generally

$$\lhd\omega = \Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} \tilde{\alpha}\omega_a') ,$$

$$VI = \Sigma \left| \alpha\beta\gamma \right| \Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\gamma} & \tilde{\beta} \\ a & b & c \end{vmatrix} V\alpha\tilde{\alpha}\alpha_a' .$$

Developing $V\alpha\tilde{\alpha}\alpha_a'$, we get for it

$$\alpha S\tilde{\alpha}\alpha_a' + \alpha_a' S\alpha\tilde{\alpha} ,$$

the negative term being zero because of its factor $S\alpha\alpha_a' = 0$.

Substituting now (21) for α_a', etc., and inverting the order of summation:

$$VI = -\frac{1}{2}\Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} \Sigma \left| \alpha\beta\gamma \right| \{\alpha S(\tilde{\alpha}V\alpha\varpi_a) + V\alpha\varpi_a S\alpha\tilde{\alpha} .\}$$

In the first term we have:

$$S\tilde{\alpha}V\alpha\varpi_a = S\alpha\varpi_a\tilde{\alpha} = S\alpha V\varpi\tilde{\alpha}_a :$$

Thus by (A) § 1 we have

$$-\frac{1}{2}\Sigma \left| \alpha\beta\gamma \right| \alpha S(\alpha V\varpi_a\tilde{\alpha}) = +\frac{1}{2} V\varpi_a\tilde{\alpha} .$$

Likewise the second term under the form

$$-\frac{1}{2} V \left\{ \Sigma \left| \alpha\beta\gamma \right| \alpha S\alpha\tilde{\alpha} \right\} \varpi_a = +\frac{1}{2} V\tilde{\alpha}\varpi_a .$$

Therefore we have

$$(22) \qquad VI = \frac{1}{2}\Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} V(\varpi_a\tilde{\alpha} + \tilde{\alpha}\varpi_a) ,$$

a result which is identically zero, because $(\varpi_a\tilde{\alpha} + \tilde{\alpha}\varpi_a)$ is a scalar.

Thus we have

$$(22\ bis) \qquad V(I) = 0 .$$

Therefore $I = \Sigma\alpha\lhd\alpha$ is a *scalar*.

By (16) we have for this scalar

$$SI = -SII .$$

We recapitulate now these results of reduction. We have by the preceding:

$$(23)\qquad \begin{cases} \mathrm{I} = -\,\mathrm{S}\Pi \\ \mathrm{II} = \mathrm{S}\Pi + \mathrm{V}\Pi \\ \mathrm{III} = -\mathrm{S}\Pi + \mathrm{V}\Pi \\ \mathrm{IV} = +\,\mathrm{S}\Pi . \end{cases}$$

With the help of this we need occupy ourselves with the expression of II only.

Besides the formula of definition of $\mathrm{II} = \Sigma\alpha \rhd \alpha$, we have formula (18), and introducing $\mathrm{VI} = 0$ into (16) we get for VII, namely, the vector of II:

$$(24)\qquad \mathrm{V}\Pi = 2\Sigma\alpha\mathrm{S} \lhd \alpha .$$

$$(25)\qquad \mathrm{V}\Pi = -\,2\Sigma\mathrm{V}(\alpha\mathrm{V} \lhd \alpha) .$$

As we have expressed α'_a, α'_b, etc., by the help of ϖ_i, so we will now express $\rhd\alpha$, $\rhd\beta$, &c., by the help of ϖ_i and Π.

In treating (24) successively by S.α., S.β., S.γ. we get

$$(26)\qquad \begin{cases} \mathrm{S} \lhd \alpha = -\frac{1}{2}\mathrm{S}\alpha\Pi \\ \mathrm{S} \lhd \beta = -\frac{1}{2}\mathrm{S}\beta\Pi \\ \mathrm{S} \lhd \gamma = -\frac{1}{2}\mathrm{S}\gamma\Pi . \end{cases}$$

Then by (21) we get

$$\mathrm{V} \lhd \alpha = -\frac{1}{2}\Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} \mathrm{V}(\tilde{\alpha}\,\mathrm{V}\varepsilon\varpi_\iota)$$

$$= +\frac{1}{2}\Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} [\alpha\mathrm{S}\tilde{\varepsilon}\varpi_a - \varpi_\iota\mathrm{S}\tilde{\alpha}\alpha] .$$

By the expression (18) of Π this is

$$= \frac{1}{2}\alpha\,\mathrm{S}\Pi - \frac{1}{2}\Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} \varpi_\iota\mathrm{S}\tilde{\alpha}\alpha .$$

So that we have the three values:

$$(27)\qquad \begin{cases} \mathrm{V} \lhd \alpha = \alpha.\frac{1}{2}\mathrm{S}\Pi - \frac{1}{2}\Sigma \begin{vmatrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{vmatrix} \varpi_\iota\mathrm{S}\tilde{\alpha}\alpha \\ \mathrm{V} \lhd \beta = \beta.\frac{1}{2}\mathrm{S}\Pi - \frac{1}{2}\Sigma \begin{vmatrix} & & \\ & & \end{vmatrix} \varpi_\iota\mathrm{S}\tilde{\alpha}\beta \\ \mathrm{V} \lhd \gamma = \gamma.\frac{1}{2}\mathrm{S}\Pi - \frac{1}{2}\Sigma \begin{vmatrix} & & \\ & & \end{vmatrix} \varpi_\iota\mathrm{S}\tilde{\alpha}\gamma . \end{cases}$$

When these results are not to be used for differentiation, then we may identify in it $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, respectively with α, β, γ, so that *under the restriction* just expressed we have:

$$(28)\qquad \begin{cases} (\mathrm{V} \lhd \alpha) = \alpha . \frac{1}{2} \mathrm{S}\Pi + \frac{1}{2} \varpi_a \\ (\mathrm{V} \lhd \beta) = \beta . \frac{1}{2} \mathrm{S}\Pi + \frac{1}{2} \varpi_b \\ (\mathrm{V} \lhd \gamma) = \gamma . \frac{1}{2} \mathrm{S}\Pi + \frac{1}{2} \varpi_c . \end{cases}$$

According to our table of definitions we have now to calculate P and Q, defined in (13).

By squaring both members of (24), we have at once—

$$\mathrm{V}^2\Pi = -4\Sigma\, \mathrm{S}^2 \lhd \alpha = -4\mathrm{P},$$

Therefore

$$(29)\qquad \mathrm{P} = -\frac{1}{4}\mathrm{V}^2\Pi .$$

In squaring both members in the three results (28), we get *as to value:*

$$\Sigma(\mathrm{V}^2 \lhd \alpha) = \frac{1}{4}\{\mathrm{S}^2\Pi\Sigma\alpha^2 + 2\mathrm{S}\Pi\Sigma\mathrm{S}c\varpi_i + \Sigma\varpi_i^2\} .$$

By (18), in which we identify $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ with α, β, γ, the second term in the second member becomes $2\mathrm{S}\Pi \times \mathrm{S}\Pi$; the first term, owing to $\Sigma\alpha^2 = -3$, is $-3\mathrm{S}\Pi^2$. Thus:

$$(30)\qquad \mathrm{Q} = -\frac{1}{4}\mathrm{S}^2\Pi + \frac{1}{4}\Sigma\varpi_a^2 .$$

This result is the same with that which would have been arrived at if in expressions (27) we had not made the identification of $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, with α, β, γ.

The management of the formulas (27) will become easier when we pass the first terms of the second members into the first members, and then effect the squaring. The first members, after reduction, will then become

$$\Sigma\mathrm{V}^2 \lhd \alpha + \frac{1}{4}\mathrm{S}\Pi^2 .$$

The second members give then the sum

$$\frac{1}{4}\Sigma \left| \alpha\beta\gamma \right| \Sigma \left| \begin{matrix} \bar{\alpha} & \bar{\beta} & \bar{\gamma} \\ a & b & c \end{matrix} \right| \{\varpi_a^2 \mathrm{S}^2\bar{\alpha}\alpha + 2\mathrm{S}\varpi_a\varpi_b \mathrm{S}\bar{\alpha}a\mathrm{S}\beta\alpha\} .$$

Interverting the order of summation, and applying formulas (A), (D), § 1, and considering moreover that generally

$$\mathrm{S}\bar{\alpha}a\mathrm{S}\beta\alpha + \mathrm{S}\bar{\alpha}\beta\mathrm{S}\bar{\beta}\beta + \mathrm{S}\bar{\alpha}\gamma\mathrm{S}\bar{\beta}\gamma = \mathrm{S}\bar{\alpha}\beta ,$$

which is zero, the sum becomes

$$\frac{1}{4}\Sigma \left| abc \right| \varpi_a^2 ,$$

from which (30) is deduced free of restriction.

We come to the expressions II′ and II″ defined in (14). Such combinations, where a would follow $\triangleright^2 a$, or $\triangleright(\triangleleft a)$, or where $\triangleleft^2 a$ or $\triangleleft(\triangleright a)$ would enter, may be easily deduced from II′ and II″, so we consider only these two, and we confine ourselves to the valuation of their scalars.

We have, remembering $a''_{a,a} = \frac{d^2 a}{da\,da}$, etc.,

$$\triangleright^2 a = -(a''_{a,a} + a''_{b,b} + a''_{c,c}).$$

Therefore

$$S\Pi' = -\Sigma \left| \alpha\beta\gamma \right| \Sigma \left| \begin{matrix} \tilde{a} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{matrix} \right| S a a''_{a,a}.$$

If we differentiate $a^2 = -1$, $\beta^2 =$, etc., twice over in respect to a, or b, or c, we get

$$S a a'_a = 0, \text{ etc.}$$

and

$$S a a''_{a,a} + a'^2_a = 0, \text{ etc.}$$

Thus:

$$S\Pi' = +\Sigma \left| \alpha\beta\gamma \right| \Sigma \left| a\,b\,c \right| a'^2_a.$$

Interverting the order of summations, and replacing a'_a, etc., by their expressions (21), we get $S\Pi'$ to contain

$$\frac{1}{4}\Sigma \left| \alpha\beta\gamma \right| V^2 a\varpi_a = \frac{2}{4}\varpi_a^2,$$

because ϖ_a, etc., are vectors, and by (E), § 1.

Therefore we have

$$S\Pi' = \frac{1}{2}\Sigma \left| a, b, c, \right| \varpi_a^2. \tag{31}$$

For the evaluation of $S\Pi''$, we proceed from the formula

$$I = \Sigma a \triangleleft a,$$

and remark that I is a scalar $= -S\Pi$ (22 *bis*), and therefore $\triangleleft I$ will be a vector, and consequently $S\triangleleft I$ will be zero. Thus we get

$$0 = \Sigma \left| \alpha\beta\gamma \right| \left\{ S\triangleleft a \triangleleft a + \Sigma \left| \begin{matrix} \tilde{a} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{matrix} \right| S\tilde{a} a \frac{d\triangleleft a}{da} \right\}.$$

In the first term we have

$$S(\triangleleft a)^2 = S^2 \triangleleft a + V^2 \triangleleft a.$$

In the second term, being under the sign scalar, we shift $\tilde{a}$ to the third place. Thus we get

$$0 = \Sigma \left| \alpha\beta\gamma \right| \{S^2 \triangleleft a + V^2 \triangleleft a + Sa \triangleright (\triangleleft a)\},$$

and from this we deduce

$$S\Pi'' = -(P + Q). \tag{32}$$

We shall be in want of the expression of $S \lhd \Pi$. Having

$$\Pi = \Sigma a \rhd a ,$$

we get

$$\lhd \Pi = \Sigma \left| \alpha \beta \gamma \right| \left\{ \lhd a \rhd a + \Sigma \left| \begin{matrix} \tilde{a} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{matrix} \right| \tilde{a} a \frac{d \rhd a}{da} \right\} .$$

Taking the scalar, the first term is its own scalar

$$(S \lhd a + V \lhd a)(S \lhd a - V \lhd a) = S^2 \lhd a - V^2 \lhd a .$$

The second is, by change of place of $\tilde{a}$, etc.,

$$S a \rhd {}^2 a .$$

Thus we get:

$$S \lhd \Pi = P - Q + S\Pi' . \tag{33}$$

Now, by (29) and (30) we have:

$$P - Q = \tfrac{1}{4}(S^2\Pi - V^2\Pi) - \tfrac{1}{4}\Sigma \varpi_a^2 . \tag{34}$$

Therefore, and by (31),

$$S \lhd \Pi = \tfrac{1}{4}(S^2\Pi - V^2\Pi) + \tfrac{1}{4}\Sigma \varpi_a^2 . \tag{35}$$

But we have interest to express $S \lhd \Pi$ in P and Q alone, because these values will be immediate consequences of the data derived from the conditions of integrability. Therefore let us eliminate $\Sigma \varpi_a^2$. By (30) and (31) we get

$$S\Pi' - Q = \tfrac{1}{2} S^2\Pi + Q .$$

Therefore:

$$S \lhd \Pi = \tfrac{1}{2} S^2\Pi + P + Q . \tag{36}$$

This formula shows also that

$$S\Pi'' + S \lhd \Pi = \tfrac{1}{2} S^2\Pi . \tag{37}$$

We may also express $S \lhd \Pi$ in relation to $\Sigma S [a \lhd (S \lhd a)]$ in the following manner. Starting from (24), namely

$$V\Pi = 2\Sigma a S \lhd a ,$$

and applying the operator $S \lhd$, the first member becomes $S \lhd \Pi$, because $\lhd (S\Pi)$ being a vector, the scalar of it will disappear. Thus

$$S \lhd \Pi = 2\Sigma S \left[\begin{matrix} \lhd a S \lhd a \\ + a \lhd (S \lhd a) \end{matrix} \right] .$$

The first term in the second member is $2\Sigma(S \lhd a)^2 = 2P$. Therefore

$$\Sigma S[\alpha \lhd (S \lhd \alpha)] = \frac{1}{2} S \lhd \Pi - P. \tag{38}$$

The expression in the first member gives an example of how far one might dare in drawing conclusions from the identifying of $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$, with α, β, γ. Namely:

If we put in it $S\alpha\tilde{\beta} = 0$, $S\beta\tilde{\alpha} = 0$, etc., $S\tilde{\alpha}\alpha = S\tilde{\beta}\beta = S\tilde{\gamma}\gamma = -1$, and consider the remaining terms, they will be expressed correctly by $-S \lhd \Theta$, where $\Theta = \alpha'_a + \beta'_b + \gamma'_c$ not alone for its value, but also for $\lhd\Theta$. If we take for $\alpha'_a, \beta'_b, \gamma'_c$ their expressions (21), we get

$$\Theta = -\frac{1}{2} V(\alpha\varpi_a + \beta\varpi_b + \gamma\varpi_c),$$

and now, for differentiations, one must not be tempted into equating the second member to $\frac{1}{2} V\Pi$, which value would, according to (18), take place if $\alpha\varpi_a$, β etc., were $\tilde{\alpha}\varpi_a$, $\tilde{\beta}$ etc. Accordingly if $-S \lhd \Theta$ was becoming $\frac{1}{2} S \lhd V\Pi$, then equation (38) would not be satisfied, because of the term $-P$ in the second member, which shows that the operation would lead into error.

It would seem that between P and Q, in other words, between Π and $\Sigma\varpi'^2_a$, there exists no relation expressible in simple terms, such as a mere numerical relation *a priori*, that is, independently of the conditions of integrability. It is through those conditions that we shall find the ratio between the values of P and Q to be satisfying to $P + 2Q = 0$.

§ 3. *The Elimination.*

The conditions of integrability of the proposed scalars are expressed, either under the form:

$$V \lhd u\alpha = 0, \quad V \lhd u\beta = 0, \quad V \lhd u\gamma = 0, \tag{39}$$

or under the form

$$V \rhd u\alpha = 0, \text{ etc.} \tag{40}$$

Both sets of equations are comprised in

$$(\lhd u\alpha - \rhd u\alpha) = 0, \quad (\lhd u\beta - \rhd u\beta) = 0 \quad (\lhd u\gamma - \rhd u\gamma) = 0. \tag{41}$$

Developing and putting

$$\frac{du}{u} = dv \tag{42}$$

we get the equations:

$$\left\{ \begin{array}{l} \lhd v.\alpha - \alpha \rhd v + \lhd\alpha - \rhd\alpha = 0 \\ \lhd v.\beta - \beta \rhd v + \lhd\beta - \rhd\beta = 0 \\ \lhd v.\gamma - \gamma \rhd v + \lhd\gamma - \rhd\gamma = 0. \end{array} \right. \tag{43}$$

Considering that $\triangleright v$ is the same as $\triangleleft v$, because v is a scalar, the conditions take also the form:

$$(44) \qquad \begin{cases} V\triangleleft\alpha = V(\alpha\triangleleft v) \\ V\triangleleft\beta = V(\beta\triangleleft v) \\ V\triangleleft\gamma = V(\gamma\triangleleft v). \end{cases}$$

Let us consider the conditions first under their form (43.)

We multiply the equations (43) respectively by α, β, γ, and sum up the results; and then we multiply them *into* α, β, γ respectively, and sum up. Then remembering the definitions (12) of I, II, etc., we get the two results:

$$\Sigma\alpha\triangleleft v.\alpha - \triangleright v\Sigma\alpha^2 + I - II = 0$$
$$\triangleleft v\Sigma\alpha^2 - \Sigma\alpha\triangleright v.\alpha + III - IV = 0.$$

By formula (C) § 1, we have

$$\Sigma\alpha\triangleleft v.\alpha = \triangleleft v, \text{ and as } \Sigma\alpha^2 = -3,$$

and as $\triangleright v$ is identical with $\triangleleft v$, these equations become:

$$4\triangleleft v + I - II = 0$$
$$-4\triangleleft v + III - IV = 0.$$

Introducing the expressions (23) of I, etc., in function of II, the result will be

$$4\triangleleft v - 2S\Pi - V\Pi = 0$$
$$-4\triangleleft v - 2S\Pi + V\Pi = 0.$$

The two consequences to be drawn from these equations are therefore:

$$(45) \qquad \begin{cases} S\Pi = 0 \\ V\Pi = 4\triangleleft v. \end{cases}$$

In operating by $S\triangleleft$ on the second of these equations, we get for the first member

$$S\triangleleft\Pi,$$

because, whatever be the value of $S\Pi$, the result $\triangleleft S\Pi$ will be a vector, and therefore $S[\triangleleft(S\Pi)] = 0$.

Thus

$$S\triangleleft V\Pi = S\triangleleft\Pi.$$

For the second member we get $4\triangleleft^2 v$, which is a scalar by itself.

The result of the operation $S\triangleleft$ is therefore the condition:

$$(46) \qquad S\triangleleft\Pi = 4\triangleleft^2 v.$$

The second equation (45) and the conditions under their form (44), will enable us to express $S\triangleleft\Pi$ in function of $\triangleleft v$ alone. For this we take the expression (36) formed *a priori* for $S\triangleleft\Pi$, which for $S\Pi = 0$ becomes

$$S\triangleleft\Pi = P + Q.$$

We have by (45)

$$\text{(47)} \qquad P = -\frac{1}{4}V^2\Pi = -4(\triangleleft v)^2 .$$

Then by (44) being substituted for $V\triangleleft a$, etc. in Q, we have

$$\Sigma V^2 \triangleleft a, \text{ namely } Q = \Sigma\,|\,a\beta\gamma\,|\,V^2(a\triangleleft v),$$

which by E, § 1, gives

$$\text{(47 } bis\text{)} \qquad Q = 2(\triangleleft v)^2 .$$

Thus we get

$$\text{(48)} \qquad S\triangleleft\Pi = -2(\triangleleft v)^2 .$$

Equating this value to that of (46) we have

$$\text{(48 } bis\text{)} \qquad 4\triangleleft^2 v + 2(\triangleleft v)^2 = 0, \text{ or } \triangleleft^2(u^{\frac{1}{2}}) = 0 .$$

In this equation (48 *bis*) we may reintroduce the original independent variables, x, y, z, and the corresponding unit vectors, i, j, k; the change of variables will, according to what has been established in § 1, require no other transformation than putting $\rho_0 = 0$, and substituting $x, y, z,\ i, j, k$, respectively to $a, b, c,\ \tilde{a}, \tilde{\beta}, \tilde{\gamma}$.

The remark may be made that the function $I, = \Sigma a\triangleleft a$, whose vector is zero *a priori* has also its scalar, $= -S\Pi$, annulled by the conditions of integrability, so that I, as well as IV, have to vanish altogether.

We may also notice that it is through the conditions of integrability being fulfilled that a connection between $P = \Sigma S^2\triangleleft a$, and $Q = \Sigma V^2\triangleleft a$ is established, the former being *a priori* $= -\frac{1}{4}V^2\Pi$, and the latter becoming by the conditions $= 2(\triangleleft v)^2$, which by the consequence of (44), namely, by (45), becomes $= \frac{1}{8}V^2\Pi$. So that P and Q satisfy *a posteriori* to the relation $P + 2Q = 0$.

When the expression of u has been derived from (48 *bis*), and consequently, by (42), v and $\triangleleft v$ are known expressions, then the differential equations of α, β, γ, may be formed in the following manner.

Supposing

$$d\alpha = \alpha'_a da + \alpha'_b db + \alpha'_c dc ,$$

which expression may be replaced in the result by

$$\frac{d\alpha}{dx}dx + \frac{d\alpha}{dy}dy + \frac{d\alpha}{dz}dz ,$$

we have the expressions (21) of α'_a, etc., which may be comprised in the formula

$$\theta'_t = -\frac{1}{2}\,\mathrm{V}\theta\varpi_t\,, \tag{49}$$

where the letter θ represents successively α, β, γ; and t independently of it represents a, b, c.

Then, by the conditions of integrability under the form (44), the equations (27) which determine ϖ_a, ϖ_b, ϖ_c, in function of $\mathrm{V}\triangleleft\alpha$, etc., will become, for $\mathrm{S}\Pi = \text{zero}$:

$$\begin{cases} -2\mathrm{V}(\alpha\triangleleft\upsilon) = \Sigma \left|\begin{matrix} \tilde{\alpha} & \tilde{\beta} & \tilde{\gamma} \\ a & b & c \end{matrix}\right| \varpi_a \mathrm{S}\tilde{a}\alpha \\ -2\mathrm{V}(\beta\triangleleft\upsilon) = \Sigma \left|\begin{matrix} & & \\ a & b & c \end{matrix}\right| \varpi_a \mathrm{S}\tilde{a}\beta \\ -2\mathrm{V}(\gamma\triangleleft\upsilon) = \Sigma \left|\begin{matrix} & & \\ & & \\ a & b & c \end{matrix}\right| \varpi_a \mathrm{S}\tilde{a}\gamma\,. \end{cases} \tag{50}$$

Multiplying respectively by the three sets of multiplicators,

$$\begin{aligned} &\mathrm{S}\tilde{a}\alpha\,,\ \mathrm{S}\tilde{a}\beta\,,\ \mathrm{S}\tilde{a}\gamma\,, && \text{as first set,} \\ &\mathrm{S}\tilde{\beta}\alpha\,,\ \mathrm{S}\tilde{\beta}\beta\,, && \text{, as second set,} \\ &\mathrm{S}\tilde{\gamma}\alpha\,,\ \text{etc.}, && \text{, as third set,} \end{aligned}$$

and summing severally, we get for *the first member*, with the first set:

$$2\mathrm{V}[-\Sigma\,|\,a\beta\gamma\,|\,(a\mathrm{S}\tilde{a}\alpha)\triangleleft\upsilon] = 2\mathrm{V}(\tilde{\alpha}\triangleleft\upsilon)\,;$$

for the second set the result is

$$2\mathrm{V}(\tilde{\beta}\triangleleft\upsilon)\,;$$

and for the third it is

$$2\mathrm{V}(\tilde{\gamma}\triangleright\upsilon)\,.$$

The *second members* will be, for the first set, $=\varpi_a$; because its factor will be

$$\Sigma\,|\,a\beta\gamma\,|\,\mathrm{S}^2\tilde{a}\alpha = -\tilde{\alpha}^2 = +1\,;$$

and the factors of ϖ_b, ϖ_c, will be zero, that of ϖ_b being

$$\mathrm{S}\tilde{a}\alpha\,\mathrm{S}\tilde{\beta}\alpha + \mathrm{S}\tilde{a}\beta\,\mathrm{S}\tilde{\beta}\beta + \mathrm{S}\tilde{a}\gamma\,\mathrm{S}\tilde{\beta}\gamma = \mathrm{S}\tilde{a}\tilde{\beta}\,,$$

which is zero, and so likewise for the factor of ϖ_c.

Similar results are deduced from the second set and the third set of multiplicators. Thus we have the equations (27) resolved into

$$\begin{cases} \varpi_a = 2\mathrm{V}(\tilde{\alpha}\triangleleft\upsilon) \\ \varpi_b = 2\mathrm{V}(\tilde{\beta}\triangleleft\upsilon) \\ \varpi_c = 2\mathrm{V}(\tilde{\gamma}\triangleleft\upsilon)\,; \end{cases} \tag{51}$$

and it must be remembered that these expressions are formed *without* any identification having been made between $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$ and α, β, γ.

Now we substitute these values of ϖ_a etc. into the expression θ'_t. We may represent ϖ_t by

$$\varpi_t = (2V\bar{\gamma}\triangleright v),$$

where $\bar{\gamma}$ is to represent $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, when t represents a, b, c, respectively. Thus we have (nine expressions in one) by

$$\theta'_t = V[\theta V(\triangleleft v\,\bar{\gamma})].$$

Writing this for $t = a, b, c$, and multiplying respectively by da, db, dc, and remembering

$$d\rho = \bar{\alpha}da + \bar{\beta}db + \bar{\gamma}dc,$$

we get the results—

$$(52) \qquad \begin{cases} d\alpha = V[\alpha V(\triangleleft v d\rho)] \\ d\beta = V[\beta V(\triangleleft v d\rho)] \\ d\gamma = V[\gamma(V(\triangleleft v d\rho)]. \end{cases}$$

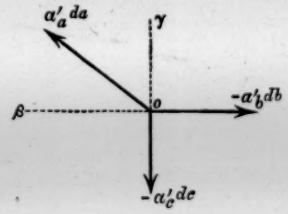
These are the differential equations which connect α, β, γ and their differentials with u, in virtue of the conditions of integrability.

The consequence which follows from these equations, at first sight, is that when $V(\triangleleft v d\rho) = 0$, the increments $d\alpha$, $d\beta$, $d\gamma$ are also zero.

Now we have $V(\triangleleft v d\rho) = 0$, when the displacement $d\rho$ is, in its direction, normal to the surface whose equation is $u =$ constant; and therefore the values of α, β, γ remain unchanged along the direction of that normal, when the displacement is infinitely small of the first order.

It will be interesting to see in what way the formulas (52) work out this result. For this end we assimilate in them $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, with α, β, γ; then for the three components of $d\rho$, namely, the three partial displacements $\bar{\alpha}da$, $\bar{\beta}db$, $\bar{\gamma}dc$, which compose the total displacement $d\rho$, the corresponding variations of, as, for example, α, namely, $\alpha'_a da$, $\alpha'_b db$, $\alpha'_c dc$, which compose $d\alpha$, will be

$$(52\ bis) \qquad \left\{ \begin{array}{rcl} \alpha'_a da &=& (\beta v'_b + \gamma v'_c)da \\ \alpha'_b db &=& -\beta v'_a db \\ \alpha'_c dc &=& -\gamma v'_a dc \end{array} \right\}$$



namely, all three perpendicular to α, as a matter of course, but the two latter ones will be of a similar direction to that of the components of $d\rho$ to which they correspond; whereas the first, corresponding to the component of $d\rho$ in the direction of α itself, will be generally oblique to the two other variations. By their sum

$$d\alpha = \alpha'_a da + \alpha'_b db + \alpha'_c dc,$$

the terms being grouped into

$$d\alpha = \beta(v_b' da - v_a' db) + \gamma(v_c' da - v_a' dc),$$

we see that indeed $d\alpha = 0$, when

$$\frac{da}{v_a'} = \frac{db}{v_b'} = \frac{dc}{v_c'},$$

which corresponds to $d\rho$ being parallel to $\triangleleft v$, and $\mathrm{V}(\triangleleft v d\rho) = 0$.

Similar remarks relate to $d\beta$, $d\gamma$, with relations similar to (52 *bis*). It is precisely these particular modes of variation of α, β, γ which work out the fulfilment of the conditions of integrability. But we must remember that (52 *bis*) is unfit for integration, and gives only actual values.

Having formed the expressions (51) of ϖ_a, etc., involving the conditions (39), we calculate the sum

$$\Sigma\varpi_a^2 = 4\Sigma\mathrm{V}^2(\breve{a}\triangleleft v) = 8(\triangleleft v)^2.$$

Having also found by (45)

$$\mathrm{V}\Pi = 4\triangleleft v,$$

the expression (35) of $\mathrm{S}\triangleleft\Pi$ may now be used, because its two terms

$$\mathrm{S}\triangleleft\Pi = -\tfrac{1}{4}\mathrm{V}^2\Pi + \tfrac{1}{4}\Sigma'\varpi_a^2$$

are expressible now both in function of the same quantity $(\triangleleft v)^2$, as

$$\Sigma'\varpi_a^2 = \tfrac{1}{2}\mathrm{V}^2\Pi = 8(\triangleleft v)^2. \tag{53}$$

§ 4. *Connections between the preceding method and the method founded on the representation of* α, β, γ, *by rotations of the directions of* i, j, k, *round the axis of a quaternion* $\mathrm{q}^{-1} = \mathrm{p}$.

The expressions of α, β, γ, will be

$$\alpha = piq, \quad \beta = pjq, \quad \gamma = pkq, \tag{54}$$

where we suppose

$$pq = \mathrm{T}q^2 = 1. \tag{55}$$

As we have $q = \mathrm{K}p$, or $p = \mathrm{K}q$, we may apply formula (5) of § 1, which gives

$$\triangleright q + \mathrm{K}\triangleleft p = 0.$$

Multiplying the first term by p, and the second by its equal $\mathrm{K}q$, and considering that by the known relation we have

$$\mathrm{K}(q) \times \mathrm{K}(\triangleleft p) = \mathrm{K}(\triangleleft p.q),$$

we get

$$p\triangleright q + \mathrm{K}(\triangleleft p.\ q = 0 \tag{56}$$

Applying (5) to $(\triangleleft p.q)$ we get

$$\triangleright\mathrm{K}(\triangleleft p.q) + \mathrm{K}\triangleleft(\triangleleft p.q) = 0.$$

By the preceding formula (56) the first term becomes $-\triangleright(p\triangleright q)$, thus we have

$$\triangleright(p\triangleright q) = \mathrm{K}[\triangleleft(\triangleleft p.q)].$$

If we limit the application of this to taking its scalar, we get

$$\mathrm{S}(p.\triangleright^2 q) = \mathrm{S}(\triangleleft^2 p.q), \tag{57}$$

because the terms

$$\mathrm{S}\Sigma\begin{vmatrix} x & y & z \\ i & j & k \end{vmatrix}\frac{dp}{dx}\triangleright q.i = \mathrm{S}\triangleleft p\triangleright q$$

$$\mathrm{S}\Sigma\begin{vmatrix} x & y & z \\ i & j & k \end{vmatrix} i\triangleleft p\frac{dq}{dx} = \mathrm{S}\triangleleft p\triangleright q$$

disappear, being common to both members. The result (57), we own it, may be easily guessed, in representing p and q respectively by $w+\omega$, $w-\omega$, etc.

However, we remark that the two results (56) and (57) take place *independently* of the *assumption* that the tensor of the quaternions $\mathrm{T}p$ or $\mathrm{T}q$ is *constant.*

But now we establish this condition, by definition, and differentiate $pq = 1$, and $\mathrm{T}p = 1$. This gives severally

$$p'_i q + pq'_i = 0, \quad \mathrm{S}p'_i q = 0. \tag{58}$$

Differentiating a second time in respect to the same variable t, we get

$$p''_{i,i} q + 2p'_i q'_i + pq''_{i,i} = 0, \tag{59}$$

where we remark that the condition $\mathrm{T}q =$ constant renders the second members equal to zero in (58) as well as in (59).

Writing (59) successively for $t = a$, $= b$, $= c$, and summing, we get the result

$$-\triangleleft^2 p.q + 2\Sigma\,|\,a\,b\,c\,|\,p'_a q'_a - p\triangleright^2 q = 0.$$

As the scalars of the first and third terms are equal, we draw from this and from (57) the result

$$\mathrm{S}\Sigma\begin{vmatrix} a & b & c \end{vmatrix}\frac{dp}{da}\frac{dq}{da} = \mathrm{S}\triangleleft^2 p.q = \mathrm{S}p\triangleright^2 q. \tag{60}$$

The first term may be expressed by the derivatives in respect to a, b, c, or by those in respect to x, y, z; in all cases we have

$$\Sigma\begin{vmatrix} x & y & z \end{vmatrix}\frac{dp}{dx}\frac{dq}{dx} = \Sigma\begin{vmatrix} a & b & c \end{vmatrix}\frac{dp}{da}\frac{dq}{da}.$$

The demonstration of this may be conducted in the same way as the transformation in § 1 of formulas (1) into formulas (10 *bis*).

We now have recourse to the intermediary of α'_i, etc., in order to form the relation between $p'_i q$, pq'_i, and ϖ_i.

We differentiate $\alpha = piq$, which gives

$$\alpha'_t = p'_t iq + piq'_t .$$

But $\alpha = piq$ gives $iq = q\alpha$, and $pi = \alpha p$; and as $pq'_t = -p'_t q$ we get

$$\alpha'_t = p'_t q.\alpha - \alpha p'_t q = -2\mathrm{V}\alpha p'_t q ,$$

because $p'_t q$ is its own vector, $\mathrm{S}p'_t q$ being zero, owing to $\mathrm{T}q = \text{constant}$.

Comparing this with (21), $\alpha'_t = -\frac{1}{2}\mathrm{V}\alpha\varpi_t$, we see that

$$(61) \qquad \begin{cases} p'_t q = \frac{1}{4}\varpi_t \\ pq'_t = -\frac{1}{4}\varpi_t . \end{cases}$$

By forming with the second the expression of $p \triangleright q$, and with the first that of $\triangleleft p.q$, we see, by expressions (18) and (19), that we have

$$(62) \qquad \begin{cases} p \triangleright q = -\frac{1}{4}\mathrm{II} \\ \triangleleft p.q = -\frac{1}{4}\mathrm{III} = \frac{1}{4}\mathrm{K}\Pi . \end{cases}$$

The first relation (16) giving I in function II, cannot by simple means be expressed with the help of p, q. However, the equation, which shows that $\mathrm{VI} = 0$, is comprised in (56), giving

$$\mathrm{V}(p \triangleright q) - \mathrm{V}(\triangleleft p.q) = 0 .$$

The expression of P, which is $-\frac{1}{4}\mathrm{V}^2\Pi$, becomes now

$$(63) \qquad \mathrm{P} = -4\mathrm{V}^2(p \triangleright q) = -4\mathrm{V}^2(\triangleleft p.q) .$$

That of Q (30), may be put under the form

$$\mathrm{Q} = -\frac{1}{4}\mathrm{S}^2\Pi - \frac{1}{4}\Sigma(\varpi_a) \times (-\varpi_a) .$$

For ϖ_a we take $4p'_a q$, and for $(-\varpi_a)$ we take $4pq'_a$, according to (61). Their product is, owing to $pq = 1$:

$$16p'_a q \times pq'_a = 16p'_a q'_a .$$

Thus

$$\mathrm{Q} = -4\mathrm{S}^2 p \triangleright q - 4\Sigma p'_a q'_a ;$$

or also, introducing the variables x, y, z,

$$(64) \qquad \mathrm{Q} = -4\mathrm{S}^2 p \triangleright q - 4\Sigma p'_x q'_x ,$$

We have in consequence

$$\mathrm{P} - \mathrm{Q} = 4(\mathrm{S}^2 p \triangleright q - \mathrm{V}^2 p \triangleright q) + 4\Sigma p'_x q'_x .$$

But

$$\mathrm{S}^2 p \triangleright q - \mathrm{V}^2 p \triangleright q = (\mathrm{K}p \triangleright q) \times (p \triangleright q)$$

In virtue of (56), or better, of its conjugate

$$K(p \triangleright q) + \triangleleft p \,.\, q = 0\,,$$

we have

$$K(p \triangleright q) \times p \triangleright q = - \triangleleft p \,.\, q \,.\, p \,.\, \triangleright q\,,$$

and thus, owing to $qp = 1$,

$$S^2(p \triangleright q) - V^2(p \triangleright q) = - \triangleleft p \triangleright q\,.$$

Therefore

$$\Sigma \mid \alpha\beta\gamma \mid \triangleleft \alpha \triangleright \alpha\,,$$

namely,

$$P - Q = - 4 \triangleleft p \triangleright q + 4\Sigma p'_x q'_x\,. \tag{65}$$

Comparing this to the expression (34), we have

$$\frac{1}{4} T^2\Pi = - 4 \triangleleft p \triangleright q\,, \tag{66}$$

as

$$\frac{1}{4} \Sigma \varpi^2{}_a = - 4\Sigma p'_x q'_x\,. \tag{67}$$

Of course the second members are scalars like the first, and we have :

$$S \triangleleft \Pi = - 4 \triangleleft p \triangleright q - 4\Sigma p'_x q'_x\,. \tag{68}$$

Thus far we can form the expressions independently of the conditions of integrability.

Introducing now these conditions, we replace ϖ_i by the expressions (51), which give

$$\left\{ \begin{array}{l} p'_a q = - p q'_a = \frac{1}{2} V(\tilde{\alpha} \triangleleft v) \\ p'_b q = - p q'_b = \frac{1}{2} V(\tilde{\beta} \triangleleft v) \\ p'_c q = - p q'_c = \frac{1}{2} V(\tilde{\gamma} \triangleleft v)\,. \end{array} \right. \tag{69}$$

These expressions take place also, in all rigour, when $a, b, c, \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$, are replaced respectively by x, y, z, i, j, etc.

We have thus, taking $p'_x q \times p q'_x = p'_x q'_x$,

$$\Sigma \left| x\, y\, z \right| p'_x q'_x = - \frac{1}{4} \Sigma \left| i\, j\, k \right| V^2(i \triangleleft v)\,,$$

which by (E) § 1, becomes

$$- \frac{1}{2} (\triangleleft v)^2,$$

Thus:

$$\Sigma p'_x q'_x = - \frac{1}{2} (\triangleleft v)^2 = - \frac{1}{16} \Sigma \varpi_a\,. \tag{70}$$

Then we draw from first and third members of (69),

$$\triangleleft p \,.\, q = -\frac{1}{2}\Sigma \left| \begin{matrix} \tilde{\alpha} \\ \tilde{\beta} \\ \tilde{\gamma} \end{matrix} \right| \left\{ \begin{matrix} \\ \\ \end{matrix} \tilde{a}\mathrm{V}(\tilde{a} \triangleleft v) \right\},$$

which by (B), § 1, becomes $= -\triangleleft v$. Likewise by second and third members

$$p \triangleright q = -\frac{1}{2}\Sigma(\mathrm{V}\tilde{a}\triangleleft v)\tilde{a} = -\triangleleft v\,.$$

We have thus:

$$(71) \qquad \begin{cases} \triangleleft p.q = -\triangleleft v = \frac{1}{4}\mathrm{K}\Pi \\ p \triangleright q = -\triangleleft v = -\frac{1}{4}\Pi\,. \end{cases}$$

The second member being a vector, the scalars of $\triangleleft p.q$ and of $p \triangleright q$ are zero. This consequence would also follow from a comparison of the scalar of (56):

$$\mathrm{S}p \triangleright q + \mathrm{S}\triangleleft p.q = 0\,,$$

with

$$p \triangleright q = \triangleleft p.q\,,$$

giving, as corresponding to $\mathrm{S}\Pi = 0$, the conditions

$$(72) \qquad \mathrm{S}p \triangleright q = 0\,, \quad \mathrm{S}\triangleleft p.q = 0.$$

We treat by $\mathrm{S}\triangleleft$. the first of the equations (71), or by $\mathrm{S}\triangleright$. the second. The first gives, $\triangleleft^2 v$ being a scalar

$$\mathrm{S}\Big[\triangleleft^2 p.q + \Sigma \left| \begin{matrix} i & j & k \\ x & y & z \end{matrix} \right| i \triangleleft p \frac{dq}{dx}\Big] = -\triangleleft^2 v.$$

As we have already seen by (60), the first term is $= \Sigma p'_x q'_x$; the second term is $= \mathrm{S}\triangleleft p \triangleright q$. Thus the result is

$$(73) \qquad -\mathrm{S}\Sigma p'_x q'_x + \mathrm{S}\triangleleft p \triangleright q = -\triangleleft^2 v.$$

The first term by (70) is

$$-\frac{1}{2}(\triangleleft v)^2\,;$$

the second term will, by multiplying the equations (71) into one another, and owing to $pq = 1$, give:

$$(74) \qquad \mathrm{S}\triangleleft p \triangleright q = +(\triangleleft v)^2 = \frac{1}{16}\mathrm{V}^2\Pi\,.$$

Thus the equation (73) gives

$$-\frac{1}{2}(\triangleleft v)^2 + (\triangleleft v)^2 = -\triangleleft^2 v\,,$$

from which results (48 *bis*) of § 3.

We remark, that by (70) and (74) we must have

$$(75) \qquad \mathrm{S}[\triangleleft p \triangleright q + 2\Sigma p'_x q'_x] = 0\,,$$

in virtue of the conditions of integrability. This equation, when developed independently of these conditions, will not give an identity *a priori*.

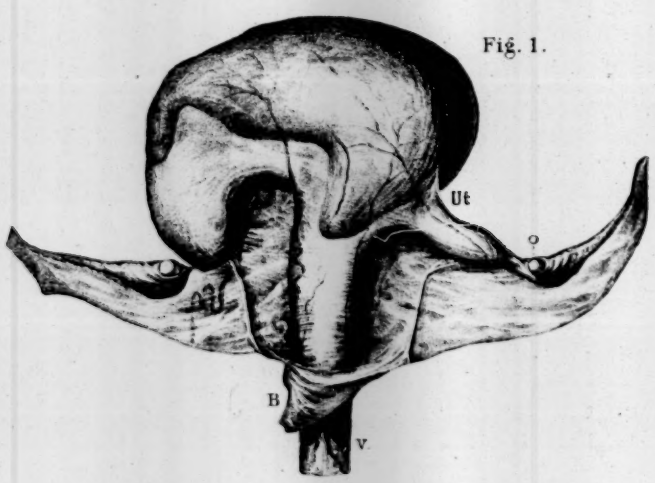


Fig. 1.

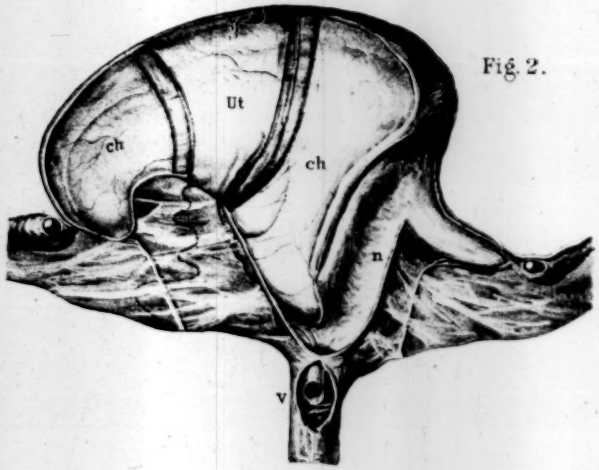


Fig. 2.

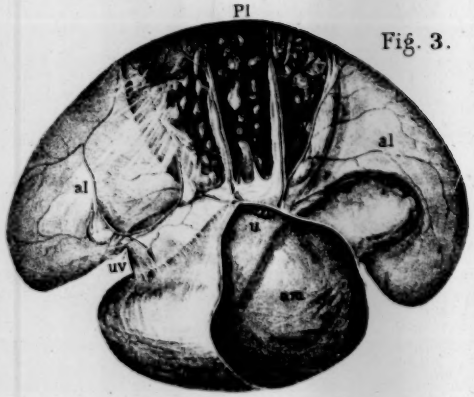


Fig. 3.

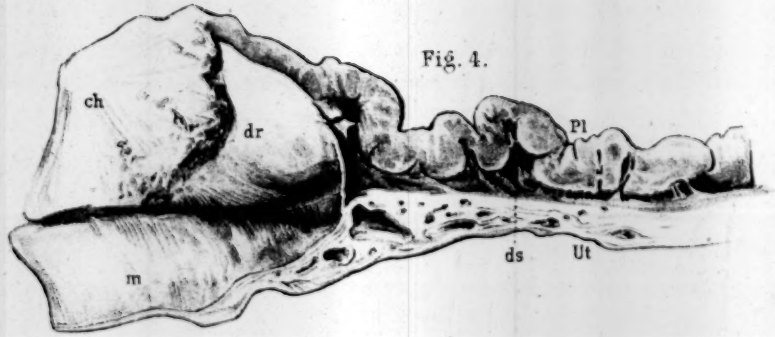
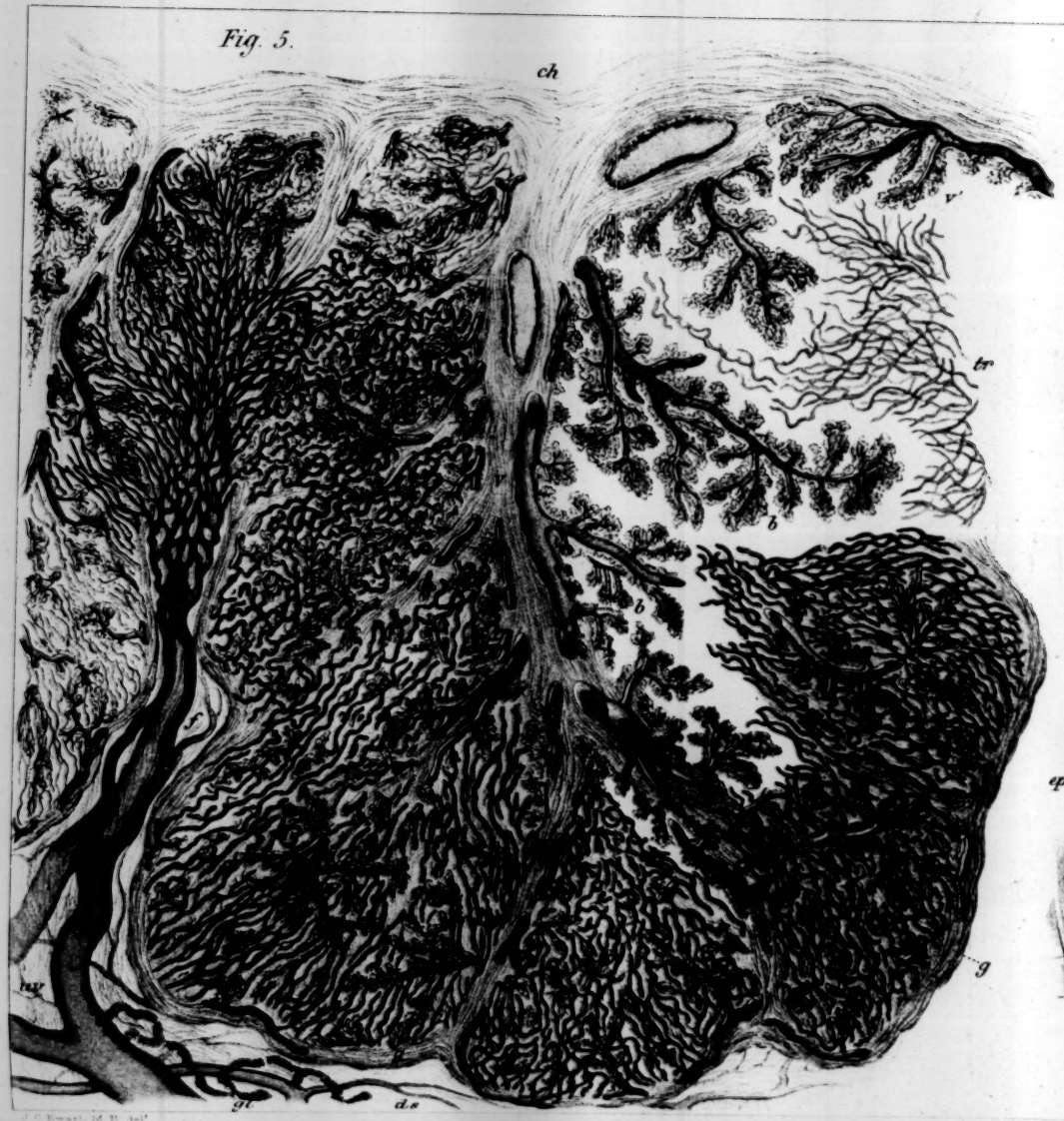


Fig. 4.

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Fig. 5.
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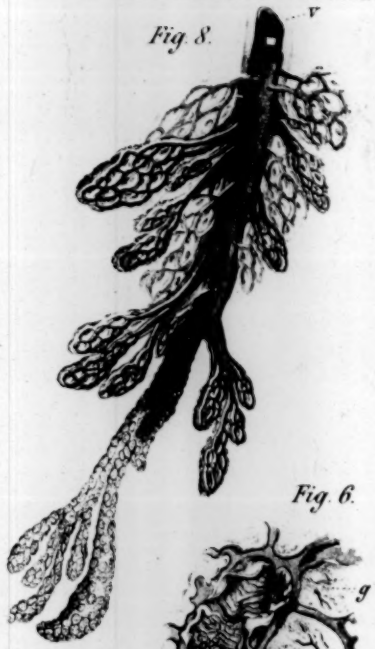
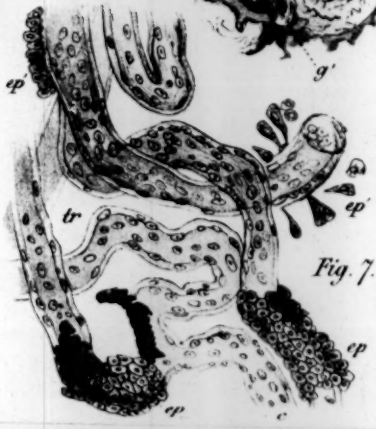
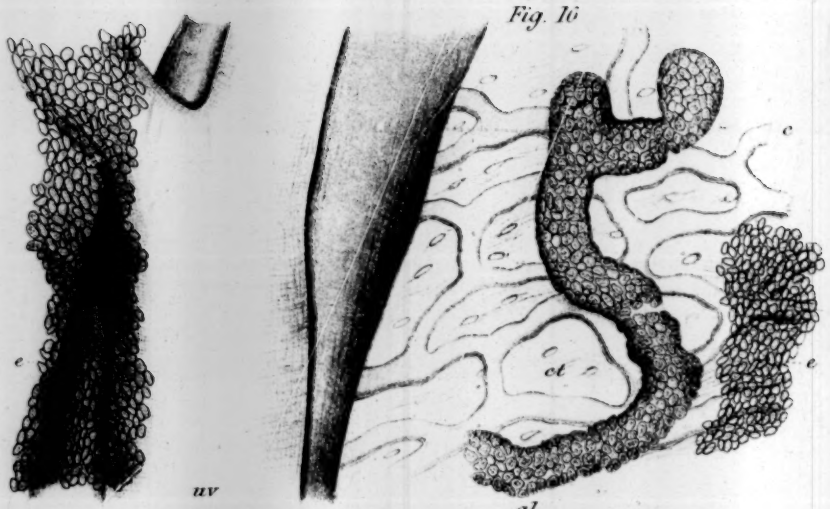

Fig 8.
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Fig. 6.
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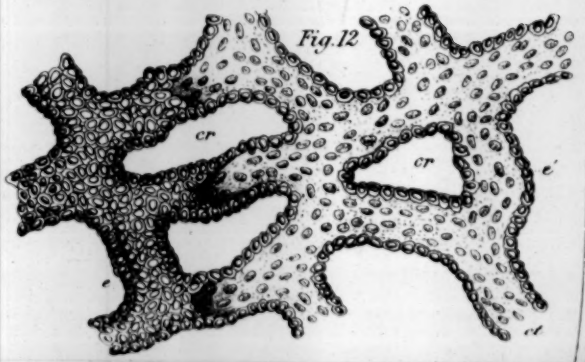

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Fig. 7.
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Fig. 13.

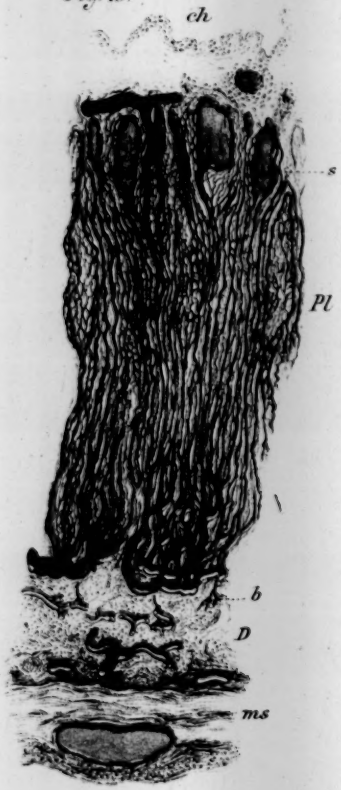


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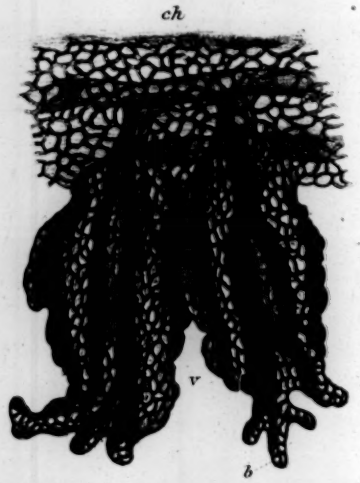


Fig. 17.

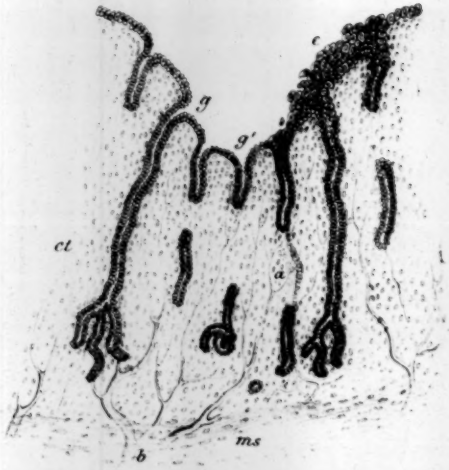


Fig. 14.

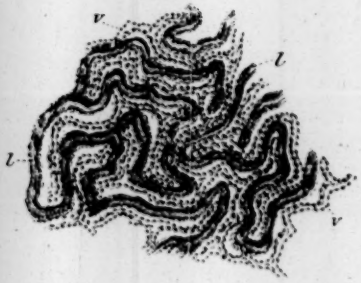


Fig. 16.

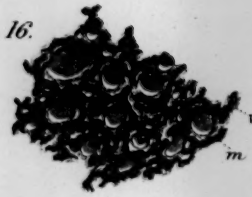


Fig. 18.

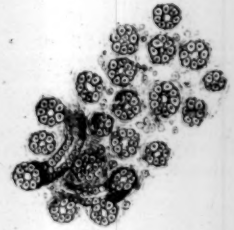


Fig. 19.



XIV.—*On the Placentation of the Seals.** (Plates XVIII.-XXI).

By Professor TURNER.

CONTENTS.

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Description of the Uterus and Placenta, .	276		
On the Appearance of the Fœtus, .	288		

Observations on the form of the Placenta and on the arrangement of the Fœtal membranes in the Seals have been recorded by more than one anatomist. ALESSANDRINI of Bologna† obtained a specimen of the Monk Seal (*Phoca bicolor*) *Monachus albiventer*, in the gravid condition, in which the uterus contained a single fœtus. He recognised the placenta to be zone-like in form, as in the Carnivora. He also described a decidua surrounding the margin of the placenta, and gave many particulars respecting the arrangement of the chorion, the allantois, the amnion, the umbilical vesicle, the umbilical cord, and the general mode of distribution of its blood-vessels. ROSENTHAL stated‡ that in two pregnant seals, which he dissected, the placenta was zone-like, as in the cat and dog, and that each uterus only contained a single fœtus, as is the rule, he believes, in this group of animals. ESCHRICHT considered§ that the examinations which he had made of the membranes of seals preserved in spirit of wine were confirmatory of the correctness of ALESSANDRINI'S observations. BARKOW described‖ the dissection of the uterus of the common seal, *Phoca vitulina*, in which he saw a zone-like placenta. Only a single fœtus was present, and that in the left uterine cornu. Although the right horn was far from equalling in size the left, it was considerably larger than in the non-gravid condition, and the uterine glands were strongly developed in its mucous membrane, but there was no extension of the fœtal membranes into it. He also pointed out that the ramifications of the umbilical vessels extended as far as the ends of the chorion.

* The substance of this memoir was communicated orally to the Royal Society of Edinburgh, June 2, 1873, and a brief abstract was published in the "Proceedings" of that date. Through various causes its expansion into a form suitable for the Transactions of the Society has been delayed much longer than was at the time contemplated by the author. (June 1875).

† Opusc. Scientif. vol. iii. 1819, and Meckel's Archiv. vol. v. p. 604.

‡ Zur Anatomie der Seehunde. Nova Acta Caes. Leop. Akademie XV. 1825.

§ De Organis, &c., p. 19. Hafniæ, 1837.

‖ Zootomische Bemerkungen. Breslau, 1851.

In all the specimens recorded by these anatomists, as well as in the one I am about to describe, the uterus, although with two cornua, contained only a single fœtus; but in a uterus, said to be that of the *Phoca vitulina*, described by A. F. J. C. MAYER,* the left horn is stated to have contained five embryoes, and the right four, each embryo being situated in an oval sac-like dilatation of the uterine cornu. The placenta was zonary, and not only the amnion, but the allantois and umbilical vesicle, were present. This specimen of MAYER's differed so materially in the number of embryoes which it contained from those examined by all other anatomists, that I am disposed to think it could not have been the gravid uterus of a seal, but must have been derived from some true carnivorous animal.†

Seals, like other uniparous mammals, do, however, occasionally produce twins, and I possess twin fœtuses obtained by a former pupil, Mr T. G. KERR, from the uterus of a specimen of *Phoca groenlandica*.

Although most important facts in the naked eye anatomy of the placenta of the seals have been recorded by the anatomists just referred to, yet its minute structure does not seem to have been investigated by any of these observers. This blank in its anatomical history I hope in some measure to supply in this communication.

Uterus and Placenta.

In the month of July 1872 I was invited by my friend Dr JAMES M'BAIN to accompany him on a visit to the "Vigilant," the cruiser to the Board of Fisheries, to see two specimens of the Grey Seal, *Halichœrus gryphus*, which had been shot three days previously by Captain M'DONALD at Sule Skerry, off Cape Wrath. One specimen was a young male, 4 feet 7 inches long, the other a fine female, which measured 6 feet 11 inches from the tip of the nose to the tip of the tail, and 7 feet 10 inches to the end of the hind flippers. From the presence of a white glairy mucus at the orifice of the vagina, and from the distended condition of the abdomen, we were of opinion that the animal was gravid. On opening into the abdominal cavity, the enlarged and pregnant uterus was at once recognised, and through Captain M'DONALD's kindness I was permitted to remove it and several other organs for examination.

The uterus consisted of two horns, a body and a cervix. It was invested by peritoneum, which passed outwards from the horns and the side of the body

* Analecten für Vergleichenden Anatomie, Zweite Sammlung, p. 55. Bonn, 1839.

† Sir EVERARD HOME ("Comparative Anatomy," v. p. 27), refers to a placenta of a seal which had come under his observation, and in vol. vi. plates 26 and 63, gives some figures of the organ. The figures show the villous structure; but they are especially intended by the author to illustrate the "nerves." Professor OWEN ("Comparative Anatomy," iii. p. 745), says that in the seals the placenta is zonular in four or five continuous or connected divisions. In *Phoca vitulina* the diameter of the placental zone, parallel with the long axis of the ovum, is between 2 and 3 inches.

to form the strong broad ligaments, and to be reflected upon the back of the bladder. The horns sprang from the bifurcation of the body. The right contained a large fœtus, and measured 2 feet 6 inches along its anterior convex surface from the angle of bifurcation to the tip of the cornu; whilst the left, similarly measured, was only 9 inches; the circumference of the gravid horn at its middle was 20 inches, that of the non-gravid only 4 inches (fig. 1).

A well-marked "ligamentum rotundum" arose from each horn about one and three-quarter inch from the tip, and extended between the layers of the broad ligament. A strong muscular diaphragmatic band passed from the tip of each horn. A short Fallopian tube also curved outwards and opened by a very wide fimbriated mouth on the inner face of a deep "pavilion," in which the ovary was lodged. The right ovary, about the size of the human testicle, contained a large corpus luteum. The left ovary was scarcely half the size of the right. Tortuous veins and arteries were distributed in the broad ligaments in immediate proximity to the ovaries. Numerous muscular fasciculi extended almost transversely outwards from the body of the uterus into the broad ligament on each side between its two layers. Tortuous veins and elongated serpentine arteries ascended beneath the peritoneal investment of the body of the uterus, and within the right broad ligament, to the gravid right horn. An injecting-pipe was introduced into one of these arteries, and another into a vein, and a transparent injection of carmine suspended in gelatine was passed into these vessels. The cavities of the left horn, of the left side of the body, and of the vagina were then opened by a longitudinal incision. The fore part of the left horn was occupied by a very viscid brown mucus, and was nine inches in length. It was prolonged for nine inches and a half into the left side of the body of the uterus, and the calibre at this part was twice as great as that of the free part of the horn. This portion of the horn contained no appreciable quantity of mucus, and there was no prolongation of the fœtal membranes into it from the gravid side. The cavity of the left horn entering into the corpus uteri was separated from the corresponding portion of the right horn by a broad septum, which terminated in a free sickle-shaped border immediately in front of the os uteri internum. The cervix uteri was $2\frac{1}{2}$ inches in length, and projected into the vagina, where it formed two very prominent lips which surrounded the os externum. Its mucous lining was thrown into longitudinal folds. The vagina was 6 inches in length, and its mucous lining possessed several strongly marked transverse folds. Both the vagina and cervix uteri contained a glairy whitish-brown mucus.

An incision was then made into the gravid right horn where it entered into the formation of the corpus uteri, which was found to be widely dilated, and this incision was prolonged for a short distance into the free part of the horn. A similar incision was also made for a few inches through the uterine wall at the

outer end of this cornu. The presence of a large zonary placenta was then recognised, and the outer envelope of the fœtus, the chorion, was seen to extend from the orifice of the right Fallopian tube as far as the extremity of this cornu, close up to the os uteri internum. A small quantity of reddish-brown fluid, obviously situated between the mucous membrane of the uterus and the outermost fœtal envelope, escaped when the wall of the uterus was cut through. The fœtus lay within its membranes with its snout directed towards the os internum, its tail and hind flippers towards the free end of the horn, its back in relation to the anterior convex border of the uterine cornu, and its abdominal aspect opposite the posterior concave border. The fœtal membranes were then punctured to allow of the escape of a quantity of reddish allantoic fluid, and on slitting them up still further, the fœtus, with its umbilical cord, a little more than two inches in length, was exposed. The umbilical vein was opened into, and a quantity of a transparent blue injection was thrown into the fœtal part of the placenta. Into one of the umbilical arteries some red injection was then passed. The cord was now cut through, and the fœtus removed.

The zonary placenta formed a broad belt, 0·3 to 0·4 inch in thickness, which lay about midway between the tip of the gravid horn and the septum between it and the left cornu. Its transverse diameter was not uniform, for it measured 9 inches in breadth opposite the anterior convex surface of the horn, and not more than 4 inches at the concave posterior surface. Each lateral border of the placenta was free, but continuous with the non-placental part of the chorion. Its uterine surface next this free border, for a breadth of from $\frac{3}{4}$ inch to $1\frac{1}{4}$ inch, varying in different localities, was not adherent to the uterine wall like the great mass of the placenta, but was covered by a prolongation of the mucous lining of the non-placental part of the uterus, which was reflected on to it so as to form a distinct band of decidua reflexa, which was not, however, continued on to the non-placental part of the chorion. This reflected decidua was delicate, and tore down so readily, more especially at its free edge, that portions were stripped away in the act of handling the placenta during its examination. In this manner the fœtal villi were exposed, and formed a fringe around the free border of the placenta. Except in the placental area, the outer surface of the chorion was perfectly smooth.

The fœtal membranes and placenta were then everted, in order to obtain a more complete view of the general disposition of the membranes and of the inner face of the placenta.

The sac of the allantois was co-equal in length with the chorion, to the inner surface of which its membrane was so closely adherent that it was difficult to separate it from the chorion without tearing one or other of the two membranes. The allantois was prolonged over the inner face of the placenta, except at the part opposite the abdominal aspect of the fœtus, where the

umbilical vesicle was in contact with the placenta. The allantois conducted the umbilical vessels to the placenta, and enclosed them in folds, which formed meso-vascular bands, extending for a considerable distance around the inner face of the placenta. Folds of this membrane, also enclosing branches of the umbilical vessels, extended for some distance beyond the margins of the placenta, where they were so arranged as to form the walls of pouch-like recesses, which communicated with the general sac of the allantois. When this sac was first examined it seemed as if it was perfectly closed, and all communication between it and the urachus was cut off. But on dissecting the abdominal end of the cord I found a very slender tubular urachus, which was prolonged as a fine tube to open into the sac of the allantois. This tube possessed thin, semi-transparent walls, and was so fine that it barely admitted a pig's bristle. It extended for $1\frac{3}{4}$ inch between the walls of the amnion and umbilical vesicle, and then for about the same distance between the apposed surfaces of the amnion and allantois, before it opened into the sac of the latter by a narrow funnel-shaped orifice, which was directed so obliquely as to form a valve-like arrangement.

The amnion formed a large elongated bag, in which the fœtus and liquor amnii were contained. It lay in relation to the concave aspect of the uterine horn, and the allantois was reflected around the greater part of its outer surface. Its sac was not equal in extent to the sac of the allantois, and the end which contained the tail of the fœtus was considerably smaller than the part in which the body and head were lodged. It was reflected in the usual way on to the umbilical cord, by which it was conveyed to the belly of the fœtus.

The short funis was cylindrical at its abdominal end, and contained a single vein, two arteries, and the tubular urachus. Two inches from its origin the vein bifurcated, and the cord became flattened, as the branches of the vein, each accompanied by an artery, diverged from each other. The arterial and venous trunks then subdivided into branches to reach the placenta, invested by folds of the allantois as already described. In their course these branches passed between the allantoic membrane and the wall of the umbilical vesicle, and some even were suspended in the cavity of the vesicle. Branches both of the artery and vein ramified between the conjoined chorion and wall of the sac of the allantois, as far as the poles of the chorion.

The umbilical vesicle, though very much smaller than the amnion, formed a well-defined sac which lay between the amnion and placenta, and was elongated laterally into two small horn-like prolongations, which ran parallel and in close relation to the narrower part of the placenta. Its outer surface was in part in contact with the allantois. A funnel-shaped prolongation passed up to the umbilical cord, as far as the angle of bifurcation of the two primary

branches of the umbilical vein from each other, where it terminated in a pointed *cul-de-sac*.

The inner or fœtal face of the placenta possessed a convoluted appearance, with intermediate depressions or sulci, which may be called the primary fissures, and the placental substance was continuous across the bottom of each sulcus from one convolution to the other. The convolutions ran parallel to each other from the narrower to the broader part of the placenta, and were most strongly marked in its median portion. When a vertical section was made through the placenta and adjacent part of the uterine wall, and the placenta gently drawn away from the uterus, its uterine face was also seen to be convoluted, with the convolutions and sulci in reverse order to those seen on its chorionic aspect. A well-defined layer of mucous membrane, which, from its position, represented the serotina, intervened between the muscular coat of the uterus and the placenta, and followed closely the windings of the convolutions, dipping down into the primary fissures in the form of broad laminæ, just as the pia mater dips between the convolutions of the cerebrum. Each convolution was split up into elongated plates by secondary fissures, into which processes of the mucosa, derived not only from the broad laminæ just referred to, but from that in contact with the uterine face of the convolutions, penetrated (figs. 4, 9). Each plate was again subdivided by tertiary fissures into small polygonal lobules, into which more delicate processes of the mucosa entered, and these could be traced through the thickness of the placenta up to the chorion.

The mucosa could readily be peeled off the uterine face of the placenta, and when this was done the laminæ were drawn out of the primary fissures, just as one can draw the pia mater out of the cerebral sulci when the grey matter on the surface of the cerebrum is exposed. The more delicate processes, however, which entered the secondary and tertiary fissures were torn through in the act of peeling, and remained in the substance of the placenta entangled between the fœtal villi. When these processes were seized with a pair of fine forceps, and gentle traction employed, they could be withdrawn without much difficulty from the substance of the placenta. From the ease with which the processes lying in the secondary and tertiary fissures tore across in the act of peeling off the placenta, there could be little doubt that a similar disruption occurs in the separation of the placenta during normal parturition. This opinion was confirmed by an examination of the placenta of a *Phoca vitulina*, shed at the full time, which through the kindness of Professor FLOWER I had an opportunity of inspecting in July 1874, in the Museum of the Royal College of Surgeons of England.* The part of the mucosa, therefore, which is shed

* The birth in the Zoological Gardens of the young seal to which this placenta belonged, is recorded by Mr A. D. BARTLETT in the Proceedings of the Zoological Society of London, June 11, 1868. In his description it is named *Phoca fœtida*, but I believe that the species was *vitulina*, as stated in the text.

along with the placenta consists of the delicate easily torn processes which dip into the secondary and tertiary fissures, and are entangled between the placental lobules and amidst the fœtal villi. The uterine face of the separated placenta is not covered by a continuous layer of decidua, for the greater part of the mucosa is not shed when the placenta is expelled, but remains as a layer of membrane of considerable thickness on the inner surface of the muscular coat, and presents on its placental aspect numerous irregular pits or trenches into which the convolutions of the placenta are received when the organ is *in situ*.

Before describing the minute structure of the serotina, I shall relate some observations on the structure of the mucosa in the non-gravid uteri of some seals, which I have had the opportunity of examining, and also the structure of the uterine mucosa in the gravid uterus of *H. gryphus*, both in the non-gravid horn and in the non-placental area of the gravid horn.

In the non-impregnated uterus of a young seal (species unknown) elongated, tubular, utricular glands were very numerous, and closely packed together in the mucosa. The glands lay perpendicular to the plane of the surface, were tortuous, and apparently branched at their deeper ends; by their opposite extremities they opened by funnel-shaped mouths on the free surface of the mucosa. They were lined by a columnar epithelium, and possessed a central lumen. The interglandular connective tissue contained multitudes of corpuscles.

I obtained in 1870 the non-gravid uterus of an adult grey seal (*H. gryphus*) from a specimen captured in that year off the coast of Fife.* The uterus was empty, and the cavities of its horns contracted, but as the uterine veins were dilated, and a large vascular corpus luteum was present in the left ovary, I concluded that the animal had been delivered of a fœtus not long before her capture. When the uterus was opened the mucous membrane was seen to form strong folds extending in the longitudinal direction. By its deep surface this membrane was connected to the muscular coat by a lax connective tissue. Vertical sections through the mucosa, examined microscopically, displayed numerous tubular glands, which opened freely on the surface. Their main stems lay almost perpendicular to the plane of the surface, but as the glands were somewhat tortuous, and gave off lateral offshoots, they were not unfrequently transversely or obliquely divided. The epithelium did not fill up the gland tubes, but left a central lumen. The exact form of the epithelium cells could not definitely be made out, but the end which lay next the lumen was rounded or somewhat polygonal, like the broad free end of a columnar epithelium cell. The interglandular connective tissue was vascular, and a well-marked capillary plexus ramified immediately beneath the surface of the mucosa around the mouths of the glands.

* The capture of this specimen is recorded in the "Journal of Anatomy and Physiology," vol iv. 1870.

In the uterus of a *Cystophora cristata*, which died in the Zoological Gardens, London, May 19th, 1874, about three and a half months after the birth of a cub, and for which I am indebted to Dr JOHN ANDERSON, I was able to confirm the observations previously made on the uterus of *H. gryphus*. On the free surface of the mucosa was placed a layer of columnar epithelium, the cells of which were large and well formed. The mouths of the utricular glands were seen without difficulty opening on the summits and sides of the longitudinal folds of the mucous membrane; their orifices were circular, closely set together, and each was surrounded by a capillary vascular ring (fig. 19). The free surface of the mucosa was studded with multitudes of minute orifices—the mouths of the glands. The glands were comparatively short both in *H. gryphus* and *C. cristata*, and the capillaries of the mucosa formed a closely-set network around them. The corpus luteum of *C. cristata*, was in the right ovary, which was not more than about half the size of the ovary of *H. gryphus*.

The free surface of the non-gravid horn of the uterus of the pregnant *H. gryphus* possessed no longitudinal folds of its mucous membrane such as were observed in the non-gravid uteri of *H. gryphus* and *C. cristata*. The surface of its mucosa was to the naked eye almost perfectly smooth, but when examined with a simple lens, slight irregularities were seen, partly due to the presence of minute ridges with intervening depressions, and partly owing to a granulated condition of the membrane. When examined with higher powers of the microscope the tubular utricular glands were readily seen. They were more elongated, and less tortuous than in the unimpregnated uterus; the branches at their deeper ends were much more distinctly seen, and they were much less closely crowded together, owing to the increase in the amount of the interglandular connective tissue. The glandular epithelium was abundant, the cells being elongated, though I could not satisfactorily determine that they possessed a precise columnar form. The granulated appearance of the mucosa seemed to some extent due to the presence of these glands in the membrane.

The free surface of the mucous membrane of the non-placental area of the gravid horn of the same uterus was smooth in appearance, both to the naked eye and under a simple lens. With higher powers the tubular utricular glands were also seen without difficulty, but they were more elongated, so slightly tortuous as in many instances to be almost straight, and separated by greater intervals, occupied by the interglandular connective tissue, than in the non-gravid horn. In some of the glands the columnar form of the cells was distinctly recognised, and the almost circular form of the gland orifice on the free surface of the mucosa was in many preparations readily seen. The mucous membrane of the septum between the two horns was smooth on the aspect directed both to the gravid and non-gravid horn. The appearance and form of

the glands, and the proportion of interglandular connective tissue was almost alike on both aspects.

From a comparison of the mucosa of the gravid with the non-gravid horn of the pregnant uterus, and of these with the unimpregnated uteri in these seals, it is evident that the changes which take place in the mucous membrane, in connection with the great distension of the uterus during pregnancy, consist in an obliteration of the strong longitudinal folds of the mucosa; in a large increase in the absolute and relative amount of the interglandular part of the mucous membrane; in an elongation of the tubular glands, which elongation is in great part due to an untwisting of the glands, so that they become much less tortuous, though from the very considerable length which some of these glands possessed, it is possible that they and their branches may have actually grown in length. The similarity in the appearance of the septal mucous membrane on its two aspects was evidently due to the growth of this partition being equal for the non-gravid as for the gravid horn.

The band of mucous membrane reflected on to the border of the placenta was smooth on its free surface like the adjacent part of the uterine mucosa with which it was continuous. When peeled off the placenta, and placed under the microscope, utricular glands were seen in it, which in form and relative numbers closely corresponded with the arrangement just described in the mucosa of the non-placental area of the gravid horn. Many of the glands, however, displayed an appearance such as I had not previously observed; for their lumen, instead of being empty, was occupied by a bright yellow material. It is not improbable that this yellow substance was the secretion of the gland confined within the lumen through some obstruction near the mouth of the gland, which prevented its excretion.

I then proceeded to examine the structure of the mucosa, the general arrangement of which, and relations to the uterine wall and placenta, have already been described. In the non-deciduous serotina, *i.e.*, in the layer of mucous membrane left on the wall of the uterus, after the placenta was peeled off, utricular glands were seen, but they were much more sparingly distributed even than in the mucosa of the non-placental area of the gravid horn. In various of these glands an appearance was observed, indications of which had also been seen in some of the glands both in the mucosa of the non-placental area and in the reflexa, of a breaking up within the gland-tubes of the epithelium into scattered masses, separated by intermediate irregular intervals.

On that surface of the non-deciduous mucosa, which was exposed by peeling off the placenta, irregular scattered patches of cells were seen when examined with a magnifying power of 300 diameters. In some places the patches were so close together that they gave the impression of being portions of the originally continuous epithelial layer of the mucosa, which had become broken up

into patches by the removal of some of the cells in the act of peeling off the placenta. In other localities much wider intervals between the patches existed, so that their probable original continuity with each other was not at first sight so apparent. It was observed that the cells remained in position on those parts of the mucosa immediately superficial to its larger blood-vessels, whilst they were frequently absent from the surface of the membrane situated between these vascular trunks. The cells in a patch were in close contact with each other. They were short columnar cells; their free ends being either circular, or ovoid, or polygonal, and in many cases having the diameter of a white blood corpuscle, though others were somewhat larger (fig. 10).

Both the non-deciduous serotina and the decidua reflexa were much more vascular than the mucosa of the non-placental area of the uterus. The increased vascularity was due to the blood-vessels being larger, and apparently more numerous in a given area. In all these localities vessels of capillary size were present, but the veins and arteries of the serotina and reflexa were considerably larger than those of the non-placental mucosa. This increase in size was not due to the formation of varicosities on limited portions of their walls, but to a general expansion of the vascular tube. No curling or cork-screw like arteries were seen, and the veins presented no unusual tortuosity. In the sub-epithelial layer of the non-deciduous serotina nerves were distinctly seen. The slender nerve trunks gave off fine branches, which ramified in the surrounding connective tissue until the finest branches consisted of but one or two fibres. The sub-epithelial connective tissue contained multitudes of well-marked connective tissue corpuscles.

The broad laminæ of mucosa which dipped into the primary fissures between the convolutions of the placenta had an interrupted layer of epithelial cells on their free surface, similar in shape but somewhat bigger than those of the non-deciduous mucosa just described. The arrangement and relative size of the blood-vessels were also the same, and utricular glands were present, though sparingly distributed in the sub-epithelial connective tissue.

The structure of the delicate bands of deciduous mucosa which passed into the secondary and tertiary fissures in the substance of the convolutions was then examined. The free surface of these bands was covered by an epithelial layer, the cells of which were columnar like those of the non-deciduous mucosa; but their contents were more opaque and yellow, as if in process of fatty degeneration. Flake-like layers of cells were not unfrequently seen lying loose in the fluid in which these specimens were examined, as if they had become detached from the free surface of the decidua. In one or two instances rows of cells, as if the cellular contents of utricular glands, were observed, but no glands were seen in these delicate processes. The bands of decidua, lying in the secondary and tertiary fissures, consisted of a delicate membranous connective tissue, into which the blood-vessels of the non-deciduous mucosa were prolonged. These

decidual bands dipped between the lobules of the placenta, almost up to the chorion, and the maternal vessels branched and formed in them a capillary network. From these bands slender processes passed into the interior of the placental lobules, where they formed a lattice-like arrangement of very slender trabeculæ, winding in a sinuous manner through the lobule. These trabecles could be pulled out of a lobule with a pair of fine forceps, and in many specimens the bud-like processes of the chorionic villi lodged in the interstices between the trabecles were drawn out along with them. Each trabecle was formed of a capillary blood-vessel, surrounded by a thin layer of connective tissue, which again was invested by a layer of columnar epithelial cells similar to those already described on the free surface of the bands of decidua (fig. 7). These cells were very easily detached from the surface of the trabeculæ, and quantities of loose cells floated about the fluid in which the specimens were examined. The deep attached end of a cell was often attenuated into a fine process. The trabecles were therefore delicate bands of the uterine mucosa, and were composed of its several constituents *minus* the utricular glands.

When the uterine face of the placenta, from which the non-deciduous mucosa had been peeled off, was examined, a greyish membrane was seen, which at first sight seemed as if it might have been a portion of the decidua remaining adherent to the surface of the placenta, and the exact nature of which required some care to determine. It lay in contact with the uterine face of the placental lobules; but instead of being prolonged from the uterine surface of one lobule to the corresponding surface of the adjacent lobules, so as to form a continuous layer over the whole uterine surface of the placenta, it was continued for some distance down the side of each lobule into the substance of the placenta, and formed an investment for the individual lobules. Hence the uterine face of the placenta was broken up into polygonal areas, each of which corresponded to a placental lobule, and the areas were separated from each other by the bands of decidua which dipped into the secondary and tertiary fissures of the placenta (fig. 6). I then satisfied myself that the greyish membrane belonged to the fœtal and not to the maternal part of the placenta; for whilst the decidua readily peeled off from the one surface of the grey membrane, the other surface was not only in immediate contact with the chorionic villi, but was continuous with their substance, so that it could not be separated from them without tearing through not only the small blood-vessels which passed from the villi into the greyish layer, but its proper connective tissue substance.

The greyish layer consisted of a tough membrane. When stripped off a placental lobule, and examined microscopically, the surface next the villi was seen to possess many longitudinal folds, often lying parallel to each other, separated by intermediate shallow depressions. This membrane was composed of bundles of the white fibres of connective tissue, the fasciculi of which were

best marked in the longitudinal folds. The corpuscles of the connective tissue were ovoid and fusiform, relatively large in size and granular. Ramifying in the membrane were the small blood-vessels derived from those of the chorionic villi, which had been torn through when the grey membrane was stripped off; hence this membrane derived its vascular supply from the chorionic and not from the maternal system of blood-vessels, and must be regarded as a fœtal and not a maternal structure. On that surface of the greyish membrane which lay next the non-deciduous mucosa, patches of epithelial cells similar to those previously described on the free surface of the mucosa were seen. I believe that these cells, though adhering to the membrane, did not properly belong to it, but to the mucosa, from which they had separated in peeling off the placenta; and in this manner one may explain why the epithelial covering of the mucosa seemed to form an interrupted and not a continuous layer.

Numerous vertical sections were now made through the entire thickness of the placental lobes, and examined with the view of determining the arrangement and structure of the villi of the chorion, their more exact connection with the greyish layer, and their relations to the intra-lobular parts of the deciduous mucosa (fig. 5). The stems of numerous large villi arose at frequent intervals from the placental surface of the chorion, and passed through the placental lobules almost perpendicular to the plane of the chorion and branched in a highly arborescent manner. From the sides of the stems of the villi, from the sides of their branches, and from the extremities of the greater number of these branches much smaller branched villous processes arose which gave origin to multitudes of villous tufts. Some of the larger branches from the parent stem had, however, a different mode of termination: they reached the periphery of the lobule and blended with the greyish layer already described. This layer, therefore, was obviously formed by the junction with each other of the ends of those branches of the villi which reached the periphery of the lobule; by their union a continuous layer of fœtal tissue was formed, not only on the uterine surface of each lobule, but reaching for some distance down its sides. From the placental surface of the chorion, in the intervals between the origins of the stems of the large arborescent villi, numbers of short branching villi arose, which soon subdivided into terminal branching tufts. The terminal branching tufts were, as a rule, slender elongated structures, but some were shorter and more club-shaped.

The matrix substance of the villi consisted of a delicate connective tissue containing multitudes of distinct corpuscles. Where this tissue formed the terminal tufts the corpuscles were very numerous, and appeared in some cases not only imbedded in the substance of the tuft, but as if arranged, after the manner of an epithelium, on the free surface. In some of my preparations the more superficial cells were detached, and were seen to have the form of delicate

scales of nucleated protoplasm. The blood-vessels of the villi were derived from the umbilical vessels. The large trunks lay in the stems of the villi and branched in an arborescent manner, ending in a capillary plexus in the terminal villous tufts (fig. 8). In some of the larger branches a vessel ran parallel and close to the surface of the villus which communicated with the capillaries of the tufts arising directly from the sides of the branches. Those villi which entered into the formation of the greyish membrane conveyed the vessels which formed the capillary plexus situated in it.

The intra-lobular prolongations of the maternal mucosa did not pass directly from the non-deciduous layer of mucosa, investing the muscular coat of the uterus, into the lobules, for the greyish membrane situated on their uterine surface, and on the adjacent part of the sides of the lobule, prevented a direct entrance. The intra-lobular decidua was therefore derived from those processes of the decidua which dipped into the secondary and tertiary fissures. These processes, in the form of slender bands and laminæ, penetrated up to the chorion, and then branched off laterally into the lobules where they formed the sides of the fissures, when they at once broke up into the reticulated lattice-like arrangement of sinuous trabeculæ already described. In sections made through the lobules, where no displacement of the relative position of the fœtal and maternal structures had taken place, the meshes of the reticulum were seen to be occupied by the villous tufts, and not unfrequently the tufts were surrounded by a ring-like arrangement of trabecles. In this manner, throughout the entire lobule, the maternal and fœtal parts of the placenta were so closely intertwined that the two systems of blood-vessels were brought into close juxtaposition with each other: the structures which intervened being, on the maternal side, the epithelial investment of the trabeculæ, and on the fœtal, the flattened scale-like superficial cells of the villi. The greyish membrane also contributed to the production of the juxtaposition, for not only were the folds on its deeper surface vascular, and projected into the lobule so as to have the maternal trabeculæ in contact with them, but it is not improbable that others of the capillaries which it contained were, when the placenta was in position, in relation with the capillary blood-vessels of the non-deciduous mucosa forming the irregular pits or trenches into which the placental convolutions fitted. In the non-deciduous as in the intra-lobular mucosa, a layer of epithelium covered the free surface of the maternal membrane.

From the mode in which the placental lobules were walled in on the uterine aspect by the greyish membrane of fœtal tissue, from the processes of decidua having to penetrate up to the chorion before their capillaries entered the lobules, and from the recurrent course which so large a proportion of the intra-lobular trabeculæ had to take in order to reach the villi situated nearest to the greyish membrane ; the maternal blood-vascular system penetrated throughout the entire

lobule, and was brought into relation with the numerous offshoots of the villi. In the separation of the placenta during parturition a quantity of maternal vascular tissue comes away therefore with the placenta.

The Fœtus.

The fœtus, a male, was well grown. It measured, from the tip of the nose to the end of the tail, 19 inches, and to the end of the hind flipper, 20½ inches. Its girth immediately behind the pectoral flippers was 15 inches. The bladder and urachus were pervious up to the root of the umbilical cord, but I could not pass a bristle from the intra-abdominal urachus into the fine tubular urachus in the cord itself. The umbilical vein did not divide until it reached the under surface of the liver. The umbilical arteries had the usual relation to the side of the bladder. The teeth had not cut the gum, but the outlines of the crowns of the canines and molars could be distinctly felt through the mucous membrane. The palpebral fissure was not closed in by a membrane. The third eyelid was large. Vibrissæ about one inch in length projected from the muzzle and eyebrows.

The fœtus was covered with straight stiffish hairs, the longest of which were about half an inch in length, and neither woolly nor fur-like. The hairy coat was yellowish fawn-coloured, streaked with dark gray bands and spots. The hairs were firmly adherent to the skin, and no loose hairs were found in the bag of membranes. When the hairs were pulled out of the skin no under coat of wool was to be seen. The hairy coat of the mother was lead-grey in colour on the back of the body and head, but on the back of the fore and hind limbs, and at the sides of the neck the general tint was white, with black spots. The belly also was white, and marked with numerous irregular black spots. The young male, shot at the same time, had a different coloration of the hair. The top of the head and back of the body were brown, interspersed with gray irregular patches; down the middle of the forehead was a dark-brown stripe, with a lighter brown stripe on each side. The belly was ash-coloured, with brownish spots at the sides and anal end.

Several observers have directed attention to the shedding of the hairy coat of the fœtus of some species of seal, either *in utero*, or immediately after birth. Wright states* that the young of the *Phoca variegata* of Nillson (*Phoca vitulina*) change the first hair (which is whitish-yellow, long, and as it were curly or woolly) in the uterus in the first half of June, and the hairs of the new-born animals have then the same colour and quality as the hairs of the mother. The moulted woolly hair is found in the uterus alongside of the young animal. This, he says, is consistent with the mode of life of this species of seal, the young of

* Förhand. vid de Skandin. Naturforsk. i Stockholm, July 1842. Abstract by Hannover in Müller's Archiv, p. 38, 1844. I am indebted to Sir James Paget for this reference.

which at once pass into the water. Mr BARTLETT points out that the young seal (*P. vitulina?*) born in the Zoological Gardens, June 8, 1868, a few minutes after its birth rolled and turned about, so as completely to divest itself of the outer covering of fur and hair which formed a complete mat upon which the young animal lay for the first hour or two after its birth. When born it was very active, and within three hours afterwards was swimming and diving about in the water like an adult animal. This young seal was 32 inches long, and weighed 20 lbs. at its [birth. The young of *Halichœrus gryphus*, on the other hand, according to WRIGHT, are born with yellowish-white woolly hairs, and cannot even swim; the mother suckles them on the land for three weeks, and then, when they have changed their woolly hairs, they take to the water.

Captain M'DONALD, who has for many years observed the habits of the Grey Seal (*H. gryphus*) on the west coast of Scotland, has kindly given me some interesting information about this animal. When just born, he says, the hair on the back and belly is yellowish-white, streaked with some faint grey stripes down the back. At the end of a week the hair is whiter than when newly born. When about fourteen days old the hair begins to fall off, and the first coat is entirely shed at the end of twenty-eight days. I saw a young male, nearly five months old, which Captain M'DONALD had alive on board the "Vigilant," to be slate-grey coloured on the back, with scattered black spots irregular both in size and shape. When the skin was wet the slate-grey tint was darker than when dry. The muzzle was a lighter shade of grey. The belly was whitish-yellow, with irregular black spots. The grey colour of the back and the whitish-yellow of the belly shaded into each other along the sides of the animal, where the black spots were more numerous than on the back or belly.

The period of the year in which the seals produce their young varies very considerably in the different species. *Cystophora cristata* gives birth to its young in the month of February, though some writers say about the month of April; *Phoca groenlandica* about the end of March and the beginning ofApril; *Phoca vitulina* in the month of June; *Halichœrus gryphus* again does not, as Captain M'DONALD informs me, bring forth its young until the month of October. On October 1st, 1874, he landed on the Eastern Hysker, a rocky islet near Canna, and found four young grey seals which had just been born. On the 12th October he again landed and found fifteen young animals, all of which had been pupped since his former visit. He has never seen more than one pup to each female. The mother suckles its young about ten weeks, and, if not disturbed, the young animals do not take to the water until the mother ceases to give suck, when the males and females begin again to copulate. There appears to be one adult male to three or even more females. The fœtus described in this memoir was therefore about three months from the completion of its term of intra-uterine life. The period of gestation is about nine months.

Comparison of the Placentation of the Seals with that of the Carnivora.

As the Seals possess, like the true *Carnivora*, a zonary placenta, and as the development of the placenta in the seal, at least in the early stages, is in all probability similar to that in the *Carnivora*, it may not be out of place to examine the placenta in some genera of the latter order, and to compare its structure with that of the seals. I have examined, with this object, the structure of the placenta and gravid uterus in the Cat, Bitch, and Fox, and as I have observed it at various periods of gestation in the cat, I shall commence by describing the placenta in that animal.

In the earliest impregnated Cat's uterus, which I have examined, the compartments were ovoid, and the long diameter of each measured along the arc did not exceed $\frac{8}{10}$ths inch. When a compartment was opened the chorion separated from the uterus with great readiness, and exposed the uterine mucosa. At each pole of the compartment an area $\frac{1}{10}$th inch in its long diameter was smooth, but the rest of the mucosa was hypertrophied, spongy, swollen, and elevated above the smooth polar portions, and formed the placental area. The placental area possessed on its surface an extremely delicate reticulation, many of the strands of which had a sinuous direction. It was thickly studded with minute orifices barely visible to the naked eye, but easily seen with a pocket lens. These orifices were the mouths of the pits or crypts in which the villi of the chorion had been lodged. A few of these openings were two or three times larger than the rest. The appearance which I saw in the cat is evidently similar to that figured by Dr SHARPEY in the bitch (fig. 211),* and by BISCHOFF in the same animal (fig. 48, A),† though, as will be seen further on, I interpret its mode of production in a different manner from those anatomists.

The crypts passed vertically into the spongy substance, and when vertical sections were made through it, they were seen to be separated from each other by trabeculæ; the chief beams of which lay vertically, and when they reached the free surface formed the strands of the reticulum already described (fig. 11). The vertical trabeculæ were connected together by others directed obliquely or in a sinuous manner, and these lateral connections were especially seen about midway in their length. Hence, not only on the surface, but when horizontal sections were made through the placental area, a reticulated arrangement was seen, and the crypts constituted the interstices of the reticulum (fig. 12). As these trabeculæ were formed of the thickened mucous membrane of the placental area, they were necessarily composed of the somewhat modified tissues of that membrane. On the surface was a definite layer of epithelium, the cells of which were short columns, with distinct, circular, or ovoid brightly refracting nuclei.

* Baly's Translation of Müller's Physiology, note p. 1576.
† Entwicklungs-geschichte des Hunde Eies, 1845.

These cells rested on a delicate sub-epithelial connective tissue in which the maternal capillaries ramified. The trabeculæ and the sub-mucous connective tissue were carefully examined, with the object of ascertaining their relations to the tubular glands. In vertical sections the glands were distinctly seen, transversely or obliquely divided, lying in a definite layer of connective tissue situated under the crypts. Sometimes the divided glands were separated by comparatively broad bands of connective tissue from the crypts and trabecular structure, but in other places they were immediately subjacent. They were lined by a well-defined columnar epithelial layer (fig. 11). I looked for the stems of the glands to see if I could ascertain whether they opened into the crypts or passed along the trabeculæ to open on the free surface of the mucosa, but did not succeed in tracing them to their orifices.

As it was important, however, to ascertain if the crypts equalled in number in a given area the glands of the mucosa in the same area, or if the crypts much exceeded in number the glands, I submitted different parts of the mucosa of the gravid uterus of this cat to microscopic examination, and compared the appearances seen with those presented by the mucosa of the non-impregnated uterus. In the non-gravid cat the stems of the glands were almost perpendicular to the free surface of the mucosa. They were so tortuous at their deeper ends as to be repeatedly cut across in a vertical section through the membrane. The inter-glandular connective tissue, containing numerous corpuscles, formed well-marked bands between the glands. Vertical sections made through the mucosa lining the constrictions between the compartments of the uterus of this gravid cat showed the tubular glands to be on the average $\frac{1}{4}$th wider than in the non-gravid condition; the inter-glandular connective tissue was much smaller in quantity, so that the glands were more closely crowded together; but in the placental area of the mucosa of the same cat the inter-glandular tissue was greatly increased in quantity, so that the glands were further apart, and, as in the non-placental area, dilated: but the number of glands seen in the sections did not nearly equal the number of crypts.

In a cat's ovum, which had reached a somewhat more advanced stage of development, where the long diameter of the uterine compartment, measured along the arc, was $1\frac{1}{2}$ inch, I found that the villi of the chorion readily disengaged from the uterine crypts. By far the larger part of the chorion was still villous, not more than $\frac{2}{10}$ths inch at each pole being smooth. The line of demarcation between the placental and non-placental polar areas of the mucosa was very distinct. The placental area, or the hypertrophied and spongy mucosa, possessed a reticulated appearance, the principal strands of which were sinuous, and gave off numerous collateral branching offshoots, which joined adjacent branches to form the walls of the numerous pits or crypts which opened on the surface. The strands and branches were larger, and the pits and crypts were more dilated than in the

younger ovum already described, and on looking down the larger pits their subdivision into smaller crypts could be seen. The crypts were lined by an epithelium, numbers of the cells of which possessed a columnar form, though others were swollen and otherwise altered in shape, so as to be irregularly polygonal. The cell protoplasm was granular and the nucleus was distinct. The sub-epithelial connective tissue was vascular. When vertical sections were made through the placental area, the more dilated size of the crypts and pits, than in the younger specimen, was distinctly recognised, being thus in conformity with the larger size of the chorionic villi. Between the deeper closed ends of the crypts and the muscular coat was a definite layer in which portions of gland tubes, lined by an epithelium, some of which were transversely, others obliquely divided, could be seen. The glands were dilated as in the younger specimen, and not so numerous as the crypts, neither could I obtain satisfactory evidence of the communication of the mouths of the glands with the crypts. I am led therefore to the conclusion that the crypts formed in the early period of gestation in the placental area of the cat are not due to a mere widening of the mouths of the tubular glands, but are produced in the same manner in this animal as I have satisfied myself to be the case in the gravid uterus of the pig and mare, by a great increase in the amount of the interglandular part of the mucosa, which becomes folded, so as to form the crypt-like arrangement which I have just described. In this respect, therefore, my observations agree with those of Professor ERCOLANI on the same animal.*

The interpretation, therefore, which ERCOLANI and I have put on the appearances seen in the placental area of the cat in the early stage of gestation, differs from that given by Dr SHARPEY of the appearances seen in the uterine mucosa of the bitch at a similar stage. As is so well known, Dr SHARPEY held that the pits and "cells" (crypts) seen on the inner surface of the uterus, which receive the villi of the chorion, are the mouths of the utricular glands enlarged and widened. It is possible that in the cat as in the *Orca*,† the utricular glands may open into some of the crypts, so as to seem to justify the inference that they were formed by a widening of the mouths of the pre-existing glands. But this interpretation obviously cannot be given of the formation of those crypts which are interglandular in position. Hence it seems to be more in conformity with the structural arrangements of the organ to conclude that the crypts which arise in the uterine mucosa during pregnancy are new formations, produced by a great hypertrophy and folding of the surface of the mucous membrane.

When the ovum of a cat, which had completed about one-half the period of gestation, was examined, a most important advance in placental formation was observed. The zonary villous band on the chorion was restricted to its middle

* Mem. dell Acad. delle Scienze di Bologna, 1870. Plates 2, 3, 4.
† See my Memoir in Trans. Roy. Soc. Edinburgh, 1871.

third, and an equally large smooth surface was found at each pole. The zone on the chorion was now so completely interlocked with the corresponding zone in the uterine mucosa, that the two surfaces could not be detached from each other. The placenta could only be separated by rupturing the slender marginal band of decidua reflexa, and tearing through or altogether pulling off the placental area of the mucosa, which area was intermediate between the placenta proper and the muscular coat of the uterus, and formed a well-defined decidua serotina.

The villi of the chorion had the form of broad sinuous leaflets, which became attenuated at their uterine ends and gave off bud-like offsets from the free border. When vertical sections were made through the placenta, the villi were seen to pass vertically through the organ up to its uterine aspect. The trabeculæ of maternal tissue, which formed the walls of the pits or crypts in which the villi were lodged, passed between the villi up to the chorion, and closely followed the sinuosities of the villi, so as to form an intimate investment for them; and in horizontal sections through the organ they were seen to be arranged as a series of laminæ, winding in a very sinuous manner between the leaf-like villi (fig. 14). Between the placenta proper and the muscular coat was a well-defined layer of serotina, equal in thickness to the muscular coat itself. It was traversed by the numerous blood-vessels which passed into and out of the placenta, and which formed not unfrequent anastomoses with each other. The decidua serotina consisted not only of the vascular connective tissue, but of the epithelial cells of this part of the mucosa, which were similar in character to those described in the preceding stage of development. In thin sections, tubes, lined by an epithelium, were seen cut transversely or obliquely; they were about equal in diameter to the gland tubes seen in the serotina in a less advanced stage of gestation, and were without doubt the dilated glands of this portion of the mucosa. It may here be stated, that in the non-placental area of the same uterus the tubular glands were distinctly seen separated from each other by comparatively wide intervals of interglandular tissue. The chorionic villi dipped into depressions in the decidua serotina, and were in contact with its epithelium. The trabeculæ and laminæ situated in the substance of the placenta were also continuous with the serotina, and were invested by an epithelial layer, the cells of which were modified columns, like the cells of the decidua serotina. The blood-vessels of the serotina entered the laminæ and trabeculæ, and ramified in them throughout the maternal part of the placenta. In the placenta of one of the embryos where the maternal vessels were injected, they formed a network of capillaries of ordinary magnitude. In the other placentæ from the same uterus, the maternal capillaries, when injected with red gelatine, were dilated to two or three times the size of the capillaries in the fœtal villi, and ascended almost vertically in the trabeculæ (fig. 13). Not unfrequently near the chorionic surface they dilated into sinus-like enlargements, which were crowded with

blood-corpuscles. It is possible that these dilatations may have been to some extent due to the force employed in filling the maternal vessels with injection; but this will not, I think, account for the whole extent of the dilatation. The vessels of the capillary network of the fœtal villi were injected with a blue colour and showed no dilatation; and the contrast between the two systems of vessels within the organ was well seen both in horizontal and vertical sections (figs. 13, 14).

The placenta of a cat, shed in the ordinary course of parturition, was covered on its uterine surface by a layer of soft yellowish-white tissue, which was smooth and uniform in character, and was without any flocculent, ragged processes, projecting from it. This layer was the deciduous serotina, and from it laminæ and trabeculæ passed into the substance of the placenta, which had a similar sinuous arrangement and relation to the fœtal villi as in the placenta at half-time. Examined microscopically, the vascular connective tissue of the serotina, with its epithelial investment, was recognised, but as it was not possible in a detached placenta to inject the maternal blood-vessels, their disposition could not be made out. I examined thin sections through the serotina for the presence of utricular glands. I saw indistinct appearances of tubes transversely or obliquely divided, which might be interpreted as tubular glands; but the aggregation of cells within and around them was so great that it was difficult to speak positively on this point. The chorionic system of fœtal blood-vessels was injected, and the leaf-like villi, with their remarkable compact capillary plexus, were readily seen. On examining with a pocket lens the uterine surface of the serotina, many minute, rounded, scattered holes were seen in it, through each of which a terminal bud of a leaf-like villus projected, so as to reach the uterine surface of the placenta. These buds were often clavate in form, and contained a capillary plexus continuous with that of the body of the villus. It is clear, therefore, that when the placenta of the cat is shed at the time of parturition, a continuous layer of serotina, interrupted only by these minute orifices, is shed along with it.

The presence of a layer investing the uterine surface of the cat's placenta, analogous to the caducous layer of the human placenta, was distinctly recognised by ESCHRICHT, who also described the thin, perpendicular, flexuous laminæ of maternal structure passing through the entire thickness of the organ, and investing the fœtal villi as if with sheaths.* Though ESCHRICHT was at first inclined to the view that the layer investing the uterine surface of the placenta was nothing else than the mucous tissue of the uterus, further consideration led him to state that it altogether differed from that tissue. But he also came to the conclusion that the mucous tissue was left entire in the placental zone, exhibiting only torn and broken-off vessels.

* De Organis, &c., pp. 14, 18.

There can be no doubt, however, from its position and structure, that this layer is the mucosa of that part of the uterus which corresponds to the placental zone; for it and the intra-placental laminæ and trabeculæ are merely a more advanced condition of the crypt-like modification of the mucosa, which I have described in the earlier stages of placental formation in this animal. Is the whole thickness of the mucosa corresponding to the placental zone shed along with the placenta, or is this layer merely the superficial part of the membrane, are questions which may now be asked? These, of course, can only be satisfactorily answered after the uterus of a cat killed immediately after parturition has been examined. But I may state that, in the uterus of the cat in the mid-period of gestation, I found, on peeling off the placenta, that the serotina did not split into two layers,—one, a deciduous serotina attached to the placenta; the other, a non-deciduous serotina remaining connected to the uterine wall, but that the whole thickness of the serotina came away with the placenta, leaving the muscular coat exposed; moreover the uterine surface of the placenta presented a smooth surface precisely similar to that exhibited by the organ when shed at the full time. A similar separation also took place more than once in the process of injecting the vessels of the gravid uterus.

Though the placenta in the bitch, as in the cat, possesses the zonary form, yet its minute structure in the two animals presents sufficient differences to enable the anatomist readily to distinguish one from the other. If the description and figures by SHARPEY and BISCHOFF, of the early stages of formation in the bitch, be compared with the corresponding stages in the cat, a close resemblance is seen; but in the more advanced stages, characteristic differences can be recognised.

In the Bitch, both at half and full time, when the placenta was stripped off the uterine zone, a distinct mucous membrane was left on the uterus, which was continuous at the margins of the zone with the narrow band of decidua reflexa, and through it with the mucosa covering the non-placental area. This zonary mucous membrane was subdivided into numerous, irregularly polygonal, pits or trenches, bounded by folds of the mucous membrane; which folds had a ragged, flocculent appearance. The membrane was very vascular, and at the ragged edges of the folds numerous torn blood-vessels were seen. When examined microscopically the free surface, not only of the pits and trenches, but of the folds, was seen to be covered by a layer of cells—the epithelium of the mucous membrane—which rested on the vascular sub-epithelial connective tissue. When this epithelium was looked at from the surface, a pattern of polygonal cells was seen like the free ends of columnar epithelium; but the cells were bigger than one usually finds this form of epithelium to be, and had, more especially in the uterus at full time, a distinct yellow colour, as if the cells were undergoing fatty degeneration. When the cells were scraped off,

so as to be seen in profile, their columnar form was easily recognised. As this mucous membrane was not detached from the uterus along with the placenta, it is to be regarded as a non-deciduous serotina.

The uterine surface of the placenta also had a ragged appearance, for the numerous folds of the mucous membrane had entered the placenta, and when it was stripped off, their torn ends were seen on its outer surface; but the flocculent appearance was still further increased by the free ends of the chorionic villi, which reached the surface. The prolongations of the mucous folds entered the placenta at a multitude of points in the interspaces between the villi, and as they ascended to the chorion they branched repeatedly, so as to give investments to the branches of the villi of the chorion. These intra-placental prolongations of the mucosa consisted of sub-epithelial connective tissue, in which the maternal vessels ramified, and of an epithelium composed partly of columnar cells, and partly of cells, the regular columnar form of which had been modified into irregular polygons. These cells were larger and more distinct than the cells on the corresponding structures in the cat, and their protoplasm was so very granular as in many cases to obscure the nucleus. These prolongations of maternal tissue constituted a deciduous serotina. The shed placenta of the bitch, whilst possessing in its substance numerous prolongations of maternal tissue, not unlike those previously described in the cat, yet differs from the latter animal, as has also been pointed out by Professor ROLLESTON * in the absence of a continuous layer of deciduous serotina on its uterine aspect.

The chorionic villi in the bitch were arborescent, and not leaf-like as in the cat. They terminated in short villous tufts. The umbilical arteries ended in a compact capillary plexus. The villi were in close contact with the epithelial cells investing the intra-placental prolongations of the mucous membrane.

I may now relate some observations which I have made on the glands in the non-gravid uterine mucous membrane of the bitch. It is well known that two kinds of glands were described by Dr SHARPEY† in the uterine mucous membrane of this animal, viz., short, simple, unbranched tubes, and compound tubes having a long duct dividing into convoluted branches, both kinds opening close together on the surface of the mucosa. These observations were supported by WEBER and BISCHOFF, and generally accepted by anatomists and physiologists; but Professor ERCOLANI, of Bologna, in his first memoir on the structure of the placenta,‡ stated his inability to distinguish more than one kind of gland, and concluded that only the long tubular glands were present. I have felt it necessary, therefore, carefully to examine the uterine mucous membrane of the unimpregnated bitch, with reference to this question. On a surface view, the mouths of the

* Trans. Zool. Soc. v. 1863.

† BALY'S Translation of Müller's Physiology, note, p. 1576.

‡ Memoire sur les Glandes Utriculaires de l'Uterus, p. 22. French Translation. Algiers, 1869.

glands could be distinctly seen closely crowded together, as is so well represented in Dr SHARPEY's figure (fig. 209), and in BISCHOFF's memoir (*Entwicklungsgeschichte des Hunde Eies*, plate xiv. fig. 47). When horizontal sections were made through the membrane near its surface, the glands were seen to be transversely divided, and so closely set together that the interval between any two adjacent glands was in some cases not equal to, in other cases about equal to, the transverse diameter of a gland tube; further, all the gland tubes in any given transverse section exhibited the same structural characters. When vertical sections through the membrane were examined, long compound tubular glands were readily seen passing into the deeper part of the mucosa, and between these, short and simple tubes were also recognised, so that under low magnifying powers, these sections at first sight seemed to confirm the observations of SHARPEY, BISCHOFF, and WEBER, which were made under magnifying powers of 10 and 12 diameters. When magnified more highly, these apparently short simple glands were seen to vary considerably in length, some dipping for only a short distance from the surface of the mucosa, others for a greater distance, and exhibiting, indeed, every gradation in length up to the branched tubular glands themselves. But in the connective tissue immediately deeper than the short glands, portions of tubes were seen extending in line with the short tubes, though apparently not continuous with them; but often with careful focussing a continuity could be traced, though obscured by overlying connective tissue (fig. 17). I am therefore of opinion that the utricular glands in the bitch, as in so many other animals, lie in the mucosa, some almost vertically, others in various degrees of obliquity, so that, when vertical sections are made, some are cut short across, others longer, whilst others again may be seen in almost their entire length. I conclude that all the glands belong to the type of compound tubular glands; that the apparent differences in length are simply due to the mode in which they are cut across in making the section, and that the physiological division proposed by BISCHOFF into simple mucous crypts and proper tubular glands cannot be sustained.

From a dissection which I have made of the gravid uterus of a Fox at about the mid period of gestation, I have satisfied myself that it corresponds in many respects with the bitch, though with specific differences. The uterine mucosa remained on the uterus when the placenta was stripped off, and possessed pits or trenches with intermediate ragged folds. The uterine face of the placenta was flocculent, owing to the prolongations of the folds into the substance of the placenta being torn across in the process of separation. These prolongations entered the placenta at a number of points, and passed with a sinuous course up to the chorion, and gave off many branches, which not unfrequently were arranged as an anastomosing reticulum, in the meshes of which the lateral offshoots of the villi were lodged. They were very vascular, and

their vessels were larger than ordinary capillaries. Compared with the capillaries of the fœtal villi they were from twice to four times as big, so that they may be regarded as indicating an early stage of a dilatation into maternal sinuses, such as is still more clearly seen in the sloth, and reaches its maximum development in the human placenta. Many of these vessels ran vertically through the placenta, so that when horizontal sections were made through the organ they were seen in transverse section. In many cases these transversely divided vessels were surrounded by a ring of cells, the epithelial investment of the process of maternal tissue in which the vessel lay, which showed that the process only contained a single dilated capillary (fig. 16). The epithelial cells investing the intra-placental prolongations of the decidua were remarkably large and distinct, and on the average about $\frac{1}{4}$th or even $\frac{1}{3}$d as large as the corresponding cells in the bitch. The fox, therefore, like the bitch, has no continuous layer of modified mucosa, such as is seen in the cat, on the uterine face of the separated placenta. The villi of the chorion had an arborescent arrangement, and gave off both lateral and terminal offshoots, in which a network of capillaries ramified.

The placenta of the Seal has a closer affinity in its arrangement and structure to that organ in the canine, than in the feline *Carnivora*. In the seal, as in the dog and fox, the decidua serotina, or mucous membrane of the placental zone, does not form a continuous layer on the uterine face of the separated organ. A definite layer is, however, left on the uterine zone itself, when the placenta is shed, which is subdivided into pits or trenches, by projecting folds. When the organ is *in situ*, these folds dip into the substance of the placenta, but are torn through in the process of separation, so that the only portions of the maternal tissue which are shed in the act of parturition, are the intra-placental prolongations of the mucous membrane. That the membrane left on the inner surface of the uterus in the placental zone is the mucous membrane, is proved by its vascular structure, by the layer of columnar epithelial cells on its free surface, and by the presence of utricular glands. But the intra-placental prolongations, whilst consisting of the columnar epithelium, and of vascular sub-epithelial connective tissue, contain no utricular glands. The serotina, therefore, both in its deciduous and non-deciduous portions, is nothing more than the modified mucous membrane. In the feline *Carnivora*, again, as illustrated by the comn ɹ cat, the mucosa not only sends prolongations into the substance of the pl. ɹta, but forms a continuous layer on the uterine face of the shed placenta, and there is a consequent deficiency in the corresponding zone in the uterus itself.

Hence, thoughall the *Carnivora* part with a considerable portion of the maternal mucosa in the separation of the placenta, yet they exhibit differences

in the degree in which the shedding takes place. The *Felidæ* have a higher grade of deciduation than the *Canidæ*, and with the latter the *Phocidæ* correspond. Hence the Dogs and Seals, in their placental affinities, are less removed from the *Cetacea*, the *Suidæ*, and the *Solipedia* than are the Cats. The pits and trenches of the mucosa, which one sees on the uterine zone, after the separation of the placenta in a seal, a fox, or a dog, are obviously similar in their morphological characters to the crypts of the mucosa of a mare, a cetacean, or other animals with a diffused placenta. In the seal, the pits and trenches possess a precision of form more than is seen in the dog and fox, a circumstance which is undoubtedly due to the subdivision of the placenta of the seal into definite minute lobules. The higher grade of deciduation in a cat may perhaps be accounted for by the broadly laminated villi, their very sinuous form, and the depth in the mucosa to which their terminal bud-like offshoots penetrate, giving to the fœtal part of the placenta a "grip," if I may so term it, over the maternal part, so as to interlock the latter more firmly with the villi, and thus to cause the mucosa to be more completely shed in the process of separation. For, as I have already pointed out in a previous memoir,* the shedding or non-shedding of maternal tissue, along with the fœtal, during the act of parturition is determined by the degree of interlocking of the fœtal and maternal portions of the organ with each other, and not from the presence in the deciduata of a structure or structures which do not exist in the non-deciduata.

In the fox and seal the intra-placental prolongations of the mucosa are subdivided into a reticulated arrangement of slender trabeculæ, each bar of which contains only a single dilated capillary; but in the seal this subdivision is carried out to a greater extent than in the fox. In the seal occurs that very remarkable anastomosis of the distal ends of the primary branches of the chorionic villi, which gives to the placenta its precise lobular subdivision, and walls in each lobule at its uterine periphery with the greyish membrane. From a somewhat cursory examination of the placenta of a *Phoca vitulina* in the Museum of the Royal College of Surgeons of England, it appeared to me that a similar membrane existed also in that animal, so that I am disposed to consider the arrangement as one which is of more than generic, indeed of ordinal value.

From the general correspondence in shape and structure between the placenta of the *Pinnepedia* and that of the true *Carnivora*, there can be no doubt that in both orders the early stage of formation is marked by the production of crypts in the placental area of the uterine mucosa, and that these crypts are formed, quite independently of the utricular glands, by a great growth and folding of the interglandular tissue. In the grey seal, the villi of the

* Trans. Roy. Soc. Edin. 1871, p. 426.

chorion, which are lodged in these crypts, acquire not only a considerable length, but a highly arborescent form, and give origin to multitudes of villous tufts. As the branching and growth of the villi proceed in the course of development, the crypts will necessarily become divided into smaller compartments; and as the villous tufts increase in number and size, the walls of the crypts will become, no doubt, thinned, until at length they will lose their uniformly continuous surface, and become subdivided into the reticulated arrangement described in this memoir, in the meshes of the network of which the villous tufts are lodged. That the increased area of the uterine mucosa during pregnancy is due to a great increase in the interglandular part of the membrane, is proved by the much wider separation of the glands seen in both the non-placental and placental areas of the uterus of *H. gryphus*, as compared with the non-gravid uterus.

Neither in the true *Carnivora* nor in the *Pinnepedia* do the utricular glands appear to play an important part in fœtal nutrition in the fully formed placenta. Not only is the number of glands small in relation to the size of the placental area, but their epithelial lining has obviously undergone changes which do not seem to be consistent with the possession of functional vigour. In the cat, indeed, the lumen of the gland tube seems to be quite occluded. But whilst these changes have taken place in the utricular glands, other cells have been developed in the gravid mucous membrane, which have all the characters of functionally active structures. I refer to the columnar epithelium cells lining the crypts, and investing the free surface of the laminæ and trabeculæ, with which the chorionic villi are in opposition. On the theory that the walls of the crypts, the laminæ, and trabeculæ, are produced by a great growth of the interglandular part of the mucosa, these cells would be descended from the epithelial covering of the non-gravid mucous membrane. Professor Ercolani, who has described the arrangement in the cat, considers that by the great growth and folding of the mucous membrane during pregnancy, multitudes of glandular follicles (crypts) are formed, of which these cells are the ephithelial lining; and there can be no doubt that the same conclusion must be come to respecting the similar cells which I have found in the placenta of the bitch, the fox, and the seal. The crypts or follicles are therefore to be regarded as secreting structures, which in the *Pinnepedia* and *Carnivora* have replaced the utricular glands. From the maternal blood-vessels which lie immediately subjacent to this epithelial layer the cells elaborate a secretion to be poured into the follicles or crypts, where it is absorbed by the villi, and applied to the nutrition of the fœtus. It would appear, therefore, as has been suggested by Ercolani, that in these and in all other mammals in which a similar glandular organ is produced, the nutrition of the fœtus is effected, not by a direct interchange of materials between the maternal and fœtal

systems of the blood-vessels, but by the production of a secretion in the maternal placenta which is absorbed by the fœtal placenta. The great vascularity of the maternal placenta is in relation therefore to the activity of this new formed glandular organ, and to the amount of the pabulum required for the production of the secretion.

EXPLANATION OF PLATES.

Figures 1, 2, and 3 were drawn from nature under my superintendence by Mr Peter Thomson; Figures 4 and 9 by Mr John R. Reid. For the series of beautiful microscopic drawings from which the remaining figures have been engraved, I am indebted to my former assistant, Mr J. C. Ewart, M.B.

Plate XVIII.

Figure 1. Gravid uterus, vagina, broad and round ligaments, ovaries, and bladder of *Halichœrus gryphus*. *Ut*, uterus; *V*, vagina; *O*, left ovary; *B*, bladder. Figures 1, 2, 3 much reduced.

Figure 2. Gravid uterus of the same animal, the cavities of which and of the vagina have been opened into. *Ut*, the wall of the uterus in the region of the zonary placenta; the shaded fringe on each side is to represent the free edge of the placenta. *Ch*, *ch*, the non-placental part of the chorion; *n*, the non-gravid horn of the uterus, into which the fœtal membranes do not extend; *V*, the vagina, into the cavity of which the cervix uteri may be seen projecting.

Figure 3. A view of the fœtal membranes which have been everted. *Pl*, the inner convoluted surface of the placenta, which is traversed by folds of the allantois, containing branches of the umbilical vessels; *al*, *al*, the inner surface of the allantois; *am*, the sac of the amnion opened into, out of which the fœtus has been removed; *u*, the short umbilical cord; *uv*, one of the two horns of the umbilical vesicle, the opposite horn is concealed between the right pole of the amnion and the placenta.

Figure 4. Vertical section through the wall of the uterus and placenta at and near the margin of the latter. *Ut*, wall of the uterus, in which the large uterine vessels may be seen; *Pl*, the placenta; *ds*, decidua serotina, passing from the inner surface of the uterus into the sulci between the convolutions of the placenta; *m*, the mucous membrane of the uterus in its non-placental area: *dr*, the mucous membrane reflected on to the placenta at its free border, —the shaded band to the left of the letters represents the line where the placental villi, through tearing down of the reflected decidua have been exposed; *ch*, the non-placental part of the chorion. Natural size.

Plate XIX.

Figure 5. Vertical section through one lobule and a portion of an adjacent lobule of the placenta of the Grey Seal. *Ch*, chorion; *VV*, stems of the large arborescent villi; *V'*, smaller villi arising directly from the chorion. The blue-coloured vessels in the chorion and villi are the ramifications of the umbilical vein. *g.g.*, greyish membrane at the periphery of the lobule in which the blue-coloured vessels of the villi ramify; *b.b.*, bud-like offshoots of the finer branches of the villi; *ds*, uterine mucosa forming the non-deciduous serotina in relation with the placental lobules; *gl*, utricular gland; *uv*, uterine blood-vessels, coloured red, passing into *f*, a tertiary fissure between the two lobules. At the upper end of this fissure these vessels form a network continuous with the intra-lobular maternal capillaries. At *tr* the intra-placental trabecular arrangement of the mucosa is shown isolated and drawn away from the finer branches of the villi. In the greater part of this figure the maternal trabeculæ are shown *in situ* intertwined amidst the fœtal villi. × 40.

Figure 6. Uterine surface of four of the lobules of the placenta. *g.g.*, greyish membrane forming the periphery of the lobules; at *g'* the membrane has been dissected off; the blue-coloured vessels are branches of the umbilical vein; *t*, processes of the uterine mucosa, with the vessels coloured red, dipping into the tertiary fissures between the lobules. × 4.

Figure 7. Intra-placental maternal trabeculæ, *tr*. At *ep*, *ep* the columnar epithelial covering is seen *in situ;* at *ep'*, *ep'* partially shed. Elsewhere the epithelium has been entirely removed, so as to show the sub-epithelial tissue with its corpuscles, and the single capillary *c*, in each trabecula. × 200.

Figure 8. Branch of a villus, *V*, with its terminal bud-like offshoots. The vascularity of the villus is shown in the upper part of the figure, whilst at the lower end its cellular structure is represented. × 200.

Plate XX.

Figure 9. Portion of placenta, *Pl*, of the Grey Seal partially dissected off the uterus. *mm*, the uterine mucosa forming the non-deciduous part of the serotina; folds of this membrane may be seen entering the sulci or primary fissures, *s*, *s*, between the convolutions of the placenta. The broader red lines on the exposed surface of the placenta are intended to represent the secondary fissures of the placenta, and the finer lines the tertiary fissures, by which it is subdivided into the ultimate lobules, *l*. *np*, *np*, non-placental portions of the mucous membrane. Natural size.

Figure 10. Surface view of the uterine mucosa forming the non-deciduous serotina. At *e*, *e* the broad ends of the columnar epithelium cells, still *in situ*, are represented. In the rest of the figure the epithelium has been removed. *uc*, larger trunks of the blood-vessels of the mucosa; *c*, capillary network; *ct*, corpusculated sub-epithelial connective tissue; *gl*, portion of one of the utricular glands. × 250.

Figure 11. Vertical section through the placental area of the mucosa of the Cat, described on p. 290. *cr*, the layer of branching crypts; the epithelial lining of the crypts, *ep*, *ep*, and their highly corpusculated connective tissue walls, *ct*, *ct*, are represented; *gl*, the glandular layer; the glands are seen in section, much less numerous than the crypts, and surrounded by connective tissue; *ms*, the muscular coat. × Hartnack 3 obj. 4 Oc.; tube out.

Figure 12. Horizontal section through the crypt layer of the same uterus. *cr*, *cr*, cavities of the crypts with their epithelial lining. *At e* the epithelium covering the free surface of the walls of the crypts is seen; at *e'* the walls are in section, and the sub-epithelial connective tissue, with its corpuscles, *ct*, is exposed. × Hartnack 7 obj. 3 Oc.; tube out.

Plate XXI.

Figure 13. Vertical section through the placenta, *Pl*, of a Cat, about half time (p. 293). *Ch*, the chorion, the vessels of which are coloured blue, so that the blue network which passes through the thickness of the placenta represents the vessels of the villi; *D*, the decidua serotina; the red-coloured vessels are the vessels of the uterine mucosa, which ascend in the walls of the crypts as far as the chorion, where they not unfrequently show considerable dilatations, *s*. *b*, bud-like terminal offshoot of a villus penetrating into the serotina; *ms*, the muscular coat. × Hartnack 3 obj. 3 Oc.

Figure 14. Horizontal section through the placenta of the same Cat. *VV*, transversely divided sinuous villi, the capillaries in which are coloured blue; *l*, *l*, laminæ of uterine mucosa, forming the walls of the crypts. The red colour represents maternal vessels, as in figure 13.

Figure 15. Villi, *V*, with a portion of the chorion, *ch*, of the shed placenta of a Cat at full time; *b*, a terminal bud, such as in figure 13, *b* penetrates deeply into the serotina. × 40.

Figure 16. Horizontal section through the placenta of a Fox (p. 297). *V*, the blue-coloured vessels of the fœtal villi; *m*, the transversely divided colossal maternal capillaries.

Figure 17. Vertical section through the non-gravid uterine mucosa of a Bitch (p. 297). *e*, ends of columnar epithelial cells covering free surface of mucosa; *g*, tubular gland shown in its entire length; *g'*, a tubular gland cut short. At *a* the continuity of an apparently short gland, with the deeper end of a tube, is shown. *ct*, interglandular connective tissue, with its corpuscles; *b*, arteries passing into the mucosa; *ms*, muscular coat. × 100.

Figure 18. Horizontal section through the non-gravid uterine mucosa of a Bitch, near the free surface. The close relation which the glands have to each other is shown; also the small proportion of interglandular tissue, in which rounded cells, not unlike lymph or white blood corpuscles may be seen. × 100.

Figure 19. Surface view of the non-gravid uterine mucosa of the Crested Seal (*Cystophora cristata*). At *e* the columnar epithelium is *in situ*, elsewhere it has been removed; *g*, mouth of a tubular gland. The capillary network of the mucosa is coloured red. × 100.

ADDENDUM, November 4.

Since the preceding pages were printed off, I have succeeded in obtaining for examination the uterus of a cat, killed five hours after giving birth to four kittens, so that I am now able to supplement my description of the placentation of this animal by stating what is the *post-partum* condition of its uterus.

The uterus was contracted, and the mucous lining thrown into well-defined rugæ. Each placental area was a narrow zonular trench, bounded at each margin of the zone by a fold of the mucosa. The surface of the non-placental part of the mucosa was unbroken and covered by epithelium. The surface of the placental zone was blood-stained, and with a number of shreds of membrane hanging from it, so that it had a torn and flocculent appearance. When thin flakes were removed from the surface of the placental zone, and examined microscopically, they were seen to consist of multitudes of free red blood corpuscles, of very delicate fibres of connective tissue, intermingled with which were fusiform and lymph-like corpuscles, and here and there a patch of cells, evidently epithelium. A series of vertical sections was then made through the placental area and adjacent non-placental part of the mucosa, and examined with low and high magnifying objectives. The free edge of the section in the non-placental area was covered by a well-defined layer of columnar epithelium, deeper than which was a thick layer of sub-epithelial connective tissue, intervening between the epithelium and the muscular coat. Lying vertically in this connective tissue were numerous utricular glands, which opened on the free surface of the mucosa, and were lined by columnar epithelium.

In the placental area itself the surface epithelium was absent, and the free edge of the section had not a smooth outline, but was irregular, and with slender filaments of connective tissue projecting from it. The thickness of the connective tissue layer on the surface of the muscular coat was appreciably less (on the average about one-third) than in the non-placental area. In this connective tissue sections through utricular glands were seen. Some of these sections were transverse to the tube of the gland, others oblique, others almost longitudinal. The epithelial lining of the glands was present, and it is not unlikely that the occasional patch of cells found on the surface of the placental area may have belonged to the glands and not to the surface epithelium. In more than one of the sections I saw in the placental area gland structures, which had not the form of cylindrical tubes, but were somewhat irregularly dilated. Numerous blood-vessels, which were the vascular trunks going to the placenta, were also seen plugged with collections of blood corpuscles.

From this description it will be seen that in the normal separation of the

placenta at the time of parturition so complete a shedding of the mucosa of the placental zone does not take place as was effected by artificially tearing off the placenta in an earlier period of gestation. The muscular coat is not exposed, but is covered by a layer of connective tissue, forming the deeper part of the sub-epithelial connective tissue of the placental area, in which portions of gland tubes and blood-vessels may be seen. This layer forms a non-deciduous serotina. The surface epithelium of this area and the more immediately subjacent tissue are, however, shed along with the placenta, and form a well-defined deciduous serotina, so that although the entire thickness of the serotina is not shed during parturition, yet a larger proportion falls off with the placenta than in the bitch, fox, and seal.

XV.—*Essay towards a General Solution of Numerical Equations of all Degrees having Integer Roots.* By H. F. Talbot, F.R.S.

(Read 17th May 1875.)

The method of solution which I have the honour to lay before the Society is founded upon an old and well-known arithmetical process called "*casting out the nines.*" It consists in substituting for any number the remainder which that number leaves when divided by *nine.* As this remainder contains only a single figure, whereas the original number may contain any number of figures, it is obvious that if the remainder can be employed instead of the number itself, a great saving of time and trouble must follow.

Thus, for example, let 274511358777 be the proposed number. In order to find the remainder, it is not necessary to divide it by 9, which might occasionally be troublesome. It is sufficient to add the figures together by 2 or 3 at a time, till the sum equals (or surpasses) *nine;* then erase those figures, and employing the surplus only, continue the process. Thus, in the given example, 2 + 7 making 9, are erased or cast aside. Then come 4 + 5 = 9, which are similarly dropped; then come 1 + 1 + 3 + 5 = 10, which is to be accounted as 1 only (9 being omitted.) This 1, with 8 following, make 9, and are therefore cast aside. Finally, 7 + 7 + 7 = 21, which exceeds a multiple of 9 by 3, and therefore 3 is written for it. The remainder of the given number is accordingly *three.*

In future I shall call the remainder of a number, when divided by 9, its *reduced form,* or more simply, its *reduct.*

Another mode of obtaining the *reduct,* but less simple, is to add together *all* the figures of the given number (which in this example would make 57): then to treat this result as a new number and find its *reduct,* thus: 5 + 7 = 12. Again, treating 12 as a new number, we have 1 + 2 = 3, therefore the final *reduct* is 3, as we found before

Theorem I.—If numbers are added together, the reduct of their sum equals the sum of their (partial) reducts.

Theorem II.—If numbers are multiplied together, the reduct of their product equals the product of the (partial) reducts.

This is only an extension of Theorem I.

Examples: Add together 84147, whose *reduct* is 6,
and 13201 . *reduct* 7
97348 . *reduct* 4.

Here reduct of sum is 4, and sum of reducts is $6 + 7 = 13 \equiv 4$ (if the sign $\equiv$ denote *equivalence*).

Add together	8746795 . (reduct 1)
and	4893781 . (reduct 4)
	13640576 . (reduct 5) .
Add together	379 . reduct 1
	911 . reduct 2
	772 . reduct 7
	487 . reduct 1
Sum	2549 . reduct 2 .

Here the reduct of the sum is 2, and the sum of the reducts is $1 + 2 + 7 + 1 = 11$, whose reduct is 2. This theorem is of great utility, for suppose a very long column of figures has to be added up, and the calculator does not feel certain that he has done it correctly, then take the *reduct* of each number and add them together, neglecting the *nines*. If the result is not equal to the reduct of the sum, the calculator is thereby warned that he has made a mistake, and must recommence the operation. But this is only a *negative* test; for if the results agree, that is no proof that the calculation is right, but only that there has been a kind of compensation of errors.

Examples of Theorem II.—Multiply 89 by 17, the product is 1513, whose *reduct* is 1. But the reduct of 89 is 8, and that of 17 is 8, therefore multiply 8 by 8, and the product 64 has for its reduct 1, which is correct. But instead of multiplying 8 by 8, it is simpler to multiply -1 by -1, which gives 1 for the reduct. And, in general, we may always substitute for a number its complement to 9.

Multiply together 87 . 11 . 13, the product is 12441, whose reduct is 3. But the reduct of 87 is 6, that of 11 is 2, and that of 13 is 4. Multiply therefore 6 . 2 . 4 and we get 48, whose reduct is 3, which is correct.

Theorem II. is as useful as Theorem I., for if the calculator has many long numbers, which are all to be multiplied together, he can try if the *reduct* of his product equals the product of all the partial reducts, and if not, he has committed an error of calculation.

But all the factors may be equal, and this leads us to the consideration of the powers of numbers.

Example.—The cube of 23 is 12167, whose reduct is 8, and since that of 23 is 5, that of 23^3 will be 5^3 or $125 \equiv 8$.

The cube of 43 is $79507 \equiv 1$, and the reduct of 43 being 7, that of 43^3 is 7^3 or $343 \equiv 1$.

The construction of a few tables is very useful, of which the following may be taken as a specimen:—

x	x^2	x^3	x^4	x^5	x^6	x^7
1	1	1	1	1	1	1
2	4	8	7	5	1	2
3	–	–	–	–	–	–
4	7	1	4	7	1	4
5	7	8	4	2	1	5
6	–	–	–	–	–	–
7	4	1	7	4	1	7
8	1	8	1	8	1	8
0	0	0	0	0	0	0

This means that if the reduct of any number is (for example) 4, that of its fifth power will be 7. *Vice versâ*, if a fifth power is given, whose reduct is 7, that of its root can only be 4; in other words, the root will be some number of the form $9n + 4$. But if the fifth power have 0 for its reduct (or be divisible by 9) then the root may have any one of the three forms—

$$9n,\ 9n + 3,\ 9n + 6.$$

The above table is formed in this way, and may be continued to any extent. Suppose we have calculated the first four columns, and want the column headed x^5. Multiply together x and x^4, and we get x^5, thus:

1 . 1	1	5 . 4	2
2 . 7	5	6 . 0	0
3 . 0	0	7 . 7	4
4 . 4	7	8 . 1	8

Or else, multiply together the columns x^2 and x^3, thus:

1 . 1	1	7 . 8	2
4 . 8	5	0 . 0	0
0 . 0	0	4 . 1	4
7 . 1	7	1 . 8	8

being the same result as before.

Let us now calculate the *reduct* of a complex polynomial, such as, for example,

$$x^5 - 15\ x^4 + 85\ x^3 - 225\ x^2 + 274\ x - 120.$$

First, replace the coefficients by their reducts: this gives

$$x^5 - 6x^4 + 4x^3 - 0\,.\,x^2 + 4x - 3.$$

And the result of this depends upon the value which we assign to x. First, then, let $x = 1$, the reduct will be

$$1 - 6 + 4 + 4 - 3 = 0.$$

Next let $x = 2$. The table gives, if $x = 2$, the reduct of $x^5 = 5$, of $x^4 = 7$, of $x^3 = 8$.

Therefore substituting these values we have

$$\begin{aligned} &5 - 6.7 + 4.8 + 4.2 - 3 \\ &= 5 - 6 \quad + 5 \quad + 8 \quad - 3 = 0. \end{aligned}$$

Next let $x = 3$, then all the superior powers of x give 0, so that the polynomial

becomes simply $4x - 3 = 12 - 3 = 9$, or 0. If we go on, we find with $x = 4$ the reduct is 0; with $x = 5$ the same; but with $x = 6$ the reduct is 3. With $x = 7$ it is 0, and with $x = 8$ it is 0. That shows that if x is of the form $9a + n$, and n has any value (except 6 or 0) the polynomial will be divisible by 9 (its reduct being 0).

To show the great utility of these various theories, I will take the following example. In Barlow's Mathematical Tables the number 380204032 is given as being a *perfect fifth power*.

Let us therefore calculate its root. It may be said there is no rule given in books of arithmetic for the finding of fifth roots, and therefore we must have recourse to some tentative method. But that is not necessary; the following simple considerations lead directly to the desired result:

1. Fifth powers end in the same digits as their roots.
2. One-figure numbers have 1 to 5 figures in their fifth powers.
   Two-figure numbers have 6 to 10.
   Three-figure numbers have 11 to 15.
   Four-figure numbers have 16 to 20.
3. The little table, which I have given in a former page, shows that if the reduct of a number is 1, the reduct of its fifth power is 1, the other digits giving the following results:

Reduct of x	1	2	3	4	5	6	7	8	0
Reduct of x^5	1	5	0	7	2	0	4	8	0

Application to the given example. The given fifth power is 380204032, whose reduct is 4.

1. This being a number of 9 figures, its root will have 2 figures.
2. Since the fifth power ends in 2, the root ends in 2 likewise. Hence the root is one of the following series of numbers:

$$12 \,.\, 22 \,.\, 32 \,.\, 42 \,.\, 52 \,.\, 62 \,.\, 72 \,.\, 82 \,.\, 92 \,.$$

3. Since the reduct is 4, the reduct of its root is 7; in other words, the root is of the form $9n + 7$. Hence the root is one of the following series of numbers:

$$16 \,.\, 25 \,.\, 34 \,.\, 43 \,.\, 52 \,.\, 61 \,.\, 70 \,.\, 79 \,.\, 88 \,.\, 97 \,.$$

But the only number common to the two series is 52. Hence 52 is the root required.

It is rather curious to observe that it is much easier to find the fifth root of the given number than it would be to construct the fifth power if the root were given. This is opposed to the ordinary opinion that inverse processes are more difficult than direct ones.

Another Example.—Find the fifth root of 1, 350, 125, 107, a number whose

reduct is 7. As before, the root will consist of 2 figures. Since the power ends in 7, the root ends in 7. It is therefore comprised in the series:

$$17 \,.\, 27 \,.\, 37 \,.\, 47 \,.\, 57 \,.\, 67 \,.\, 77 \,.\, 87 \,.\, 97 \,.$$

Since the reduct of the power is 7, the reduct of the root is 4. Hence the root is comprised in the series:

$$13 \,.\, 22 \,.\, 31 \,.\, 40 \,.\, 49 \,.\, 58 \,.\, 67 \,.\, 76 \,.\, 85 \,.\, 94 \,.$$

But the only number which is found in both series is 67. Therefore 67 is the root required.

These theories enable us to effect the solution of numerical equations of any degree, whose roots are integers, to an apparently unlimited extent; but I will not enter upon that subject in all its generality at present, because from the late period of the Session at which this paper was commenced, it has been impossible to give it the desired extent and generality. I will confine myself on the present occasion to the solution of a particular case which occurs when the roots of the given equation are small numbers, less than $9^2 + 9 = 90$. In that case these roots can be found, if I am not mistaken, whatever be the degree of the equation, by a direct and very simple process.

My first example shall be a cubic comprised in what is called the Irreducible Case.

Let the given equation be

$$x^3 - \mathrm{A}x^2 + \mathrm{B}x - \mathrm{C} = 0 \,.$$

Instead of the coefficients write their *reducts* a, b, c, &c. This gives

$$x^3 - ax^2 + bx - c = 0 \,,$$

which I call the *reduct equation*.

Let the three roots be x, x', x'', and let x be of the form $9y + n$. We can determine n without knowing y. n is one of the nine digits $1 \,.\, 2 \,.\, 3 \,.\, \ldots \,.\, 9$. Substitute these digits successively for x in the *reduct equation*, and it will be seen *which* of them (generally *three*) satisfy that equation, viz., by giving a reduct = 0. This is most conveniently done by the help of the little table given before.

TABLE A.

x	x^2	x^3
1	1	1
2	4	8
3	0	0
4	7	1
5	7	8
6	0	0
7	4	1
8	1	8
0	0	0

Thus, for instance, if we are trying whether $n = 4$, we have the *reduct equation* $1 - 7a + 4b - c$, and if a, b, c are such as to satisfy this by giving a reduct = 0, then we may suppose 4 to be one of the values of n.

These things being premised, I find that if the roots are each of them less than 90, the solution reduces itself to the following very simple form.

Rule.—Put $x = 9y + n$, and give n such a value as to satisfy the reduct equation $x^3 - ax^2 + bx - c = 0$.

Determine P from the formula $p = 3n^2 - 2an + b$, making P the reduct of p. Substitute n for the unknown quantity x in the *original* equation (not in the reduct equation), and let the result be $9k$. I put the result in this form, because it will always be divisible by 9 (since by hypothesis n satisfies the *reduct equation*). Therefore divide by 9 and we get k. And now let the *reduct* of k be called Q.

Solve the equation $Py = Q$, where P and Q are of course numbers not exceeding *nine*, and thus y becomes known. Finally, put $x = 9y + n$ and x will be one of the roots of the given cubic.

This will best be illustrated by examples.

First Example.

Let the given equation be $x^3 - 162\ x^2 + 8531\ x - 146370 = 0$. Its reduct will be $x^3 - 0x^2 + 8x - 3 = 0$, or $x^3 + 8x - 3 = 0$, so that $a = 0$, $b = 8$.

Substitute for x the numbers *one* to *eight*, and by help of Table A we get

	x^3	+	$8x$	−	3		
With number 1	1	+	8·1	−	3	= − 3	for the reduct.
„ 2	8	+	8·2	−	3	= 3	„
„ 3	0	+	8·3	−	3	= 3	„
„ 4	1	+	8·4	−	3	= 3	„
„ 5	8	+	8·5	−	3	= 0	„
„ 6	0	+	8·6	−	3	= 0	„
„ 7	1	+	8·7	−	3	= 0	„
„ 8	8	+	8·8	−	3	= − 3	„

This shows that the *reduct equation* can only be satisfied by taking either 5, 6, or 7 as the value of n, and in fact we shall find that the three roots of the cubic are of the forms $9y + 5$, $9y' + 6$, $9y'' + 7$. To test the accuracy of this, multiply together $(x - 5)\ (x - 6)\ (x - 7) = 0$, and we get the reduct equation $x^3 + 8x - 3 = 0$, as before.

Let us now calculate the three roots of the cubic in succession.

First, take the root $x = 9y + 5$ so that $n = 5$, then the formula $p = 3n^2 - 2an + b$ becomes $p = 3n^2 + 8$ (since $a = 0$, $b = 8$). Hence $p = 3.25 + 8 = 83$, whose reduct is 2. Therefore P = 2. Next, substitute 5 for the unknown quantity in the original equation $x^3 - 162\ x^2 + 8531\ x - 146370$, and the result is *nine times* 11960, the reduct of which number is 8: we have therefore Q = 8. We now take the equation $Py = Q$ or $2y = 8$, which gives us $y = 4$. Finally, we have $x = 9y + n = 9y + 5 = 9.4 + 5 = 41$. Therefore 41 is one of the roots of the equation $x^3 - 162\ x^2 + 8531\ x - 146370 = 0$. It is evident that it would be difficult to obtain this root by mere guessing, and it cannot be found by CARDAN's rule, therefore a direct mode of solution is to be welcomed.

Let us now proceed to find the second root. Let $n = 6$, then the formula $p = 3n^2 + 8$ becomes p 116 whose reduct is 8, therefore P = 8. Substitute

6 for x in the original equation and the result is *nine times* 11200, the reduct of which number is 4, therefore Q = 4. Solve the equation $Py = Q$ or $8y = 4$. To do this we must add successive *nines* to 4 till it becomes divisible by 8. Thus, 4, 13, 22, 31, 40, which last number is divisible. Hence $Py = Q$ means $8y = 40$, whence $y = 5$. Finally

$$x = 9y + n = 9y + 6 = 9.5 + 6 = 51.$$

Therefore the second root of the equation is 51.

Let us now proceed to find the third root. Let $n = 7$, then the formula $p = 3n^2 + 8$ becomes $p = 3.49 + 8 = 155$, whose reduct is 2, therefore P = 2. Substitute 7 for x in the original equation and the result is *nine times* 10,472, whose reduct is 5, therefore Q = 5. Solve the equation $Py = Q$ or $2y = 5$, which, by adding 9, is $2y = 14$, and we obtain $y = 7$. Finally, $x = 9y + n = 9y + 7 = 9.7 + 7 = 70$. Therefore the third root of the equation is 70.

Thus we have determined the three roots of this cubic to be 41, 51, and 70.

It is now easy to verify these roots by multiplying together the three factors $x - 41$, $x - 51$, and $x - 70$, thus—

$$\begin{array}{l} x - 41 \\ x - 51 \\ \hline x^2 - 41.x \\ \quad - 51.x + 41.51 \\ \hline x^2 - 92\ x + 2091 \\ \qquad x - 70 \\ \hline x^3 - 92\ x^2 + 2091\ x \\ \quad - 70\ x^2 + 70.92\ x - 70.2091 \\ \hline x^3 - 162\ x^2 + 8531\ x - 146370 \end{array}$$

which was the given equation.

Second Example.

Let the given equation be

$$x^3 - 174\ x^2 + 9749\ x - 177276 = 0.$$

The *reduct equation* will be

$$x^3 - 3x^2 + 2x - 3 = 0.$$

Hence the coefficients $a = 3$, $b = 2$. Proceeding as before to try the nine digits. 1 does not satisfy this equation, for it gives $1 - 3 + 2 - 3$, which is not equal to 0. Neither does 2 satisfy it, for it gives $8 - 12 + 4 - 3$, which is not equal 0. Trying the others, we find that the equation is only satisfied by the numbers 6, 7, and 8. We verify this result by multiplying together $(x - 6)$ $(x - 7)$ $(x - 8) = 0$, which gives the *reduct equation* $x^3 - 3x^2 + 2x - 3 = 0$ as before. Let us now calculate the three roots of the cubic, which we have found to be of the forms $9y + 6$, $9y' + 7$, $9y'' + 8$.

First try the form $x = 9y + 6$, so that $n = 6$. Put $x = 6$ in the original equation and the result is *nine times* 13870, whose reduct is 1 = Q.

The formula $p = 3n^2 - 2an + b$ becomes (since $a = 3$, $b = 2$) $p = 3.36 - 2.3.6 + 2$, the reduct of which is $2 = \mathrm{P}$.

Then $\mathrm{P}y = \mathrm{Q}$ or $2y = 1$, add *nine* and $2y = 10$, whence $y = 5$.

Then $x = 9y + n = 9y + 6 = 9.5 + 6 = 51$, therefore 51 is a root of the given equation.

Second Root.

Now try the form $x = 9y + 7$, so that $n = 7$. Put $x = 7$ in the original equation and the result is *nine times* 13024, whose reduct is $1 = \mathrm{Q}$. The formula gives

$$p = 3n^2 - 2an + b = 3.49 - 2.3.7 + 2,$$

whose reduct is $8 = \mathrm{P}$. Then $\mathrm{P}y = \mathrm{Q}$ or $8y = 1$, whence $y = 8$ (since the reduct of 8.8 is 1). Finally

$$x = 9y + 7 = 9.8 + 7 = 79.$$

Therefore 79 is a root of the given equation.

Third Root.

Let x be of the form $9y + 8$, so that $n = 8$. Put $x = 8$ in the original equation and the result is *nine times* 12212, whose reduct is $8 = \mathrm{Q}$. The formula $3n^2 - 2an + b$ becomes $3.64 - 2.3.8 + 2$, whose reduct is $2 = \mathrm{P}$. Then $\mathrm{P}y = \mathrm{Q}$ or $2y = 8$ whence $y = 4$. Finally

$$x = 9y + 8 = 9.4 + 8 = 44.$$

Therefore 44 is a root of the given equation. The three roots of the cubic are therefore 51, 79, and 44.

The preceding pages contain only a mere outline of the method proposed, but it would not be doing justice to it if I did not give an example of the solution of an equation of the fifth degree.

Supposing, for the sake of simplicity, the same conditions as before, namely, that all the roots are positive integers less than 90, the solution of the fifth and also of all higher degrees is the same as for the cubic, the only change being in the formula for p, which in the cubic is $p = 3n^2 - 2na + b$, but in the fifth degree $p = 5n^4 - 4n^3a + 3n^2b - 2n.c + d$, and in the higher degrees it differs for each degree.

Example of Solution of Equation of Fifth Degree.

Let the given equation be

$$x^5 - 168.x^4 + 10831.x^3 - 335412.x^2 + 5000188x - 28791840 = 0,$$

he *reduct equation* will be

$$x^5 - 6.x^4 + 4.x^3 - 0.x^2 + 4.x - 3 = 0,$$

so that the coefficients are

$$a = 6,\ b = 4,\ c = 0,\ d = 4,\ e = 3,$$

and the formula will be

$$p = 5n^4 - 24.n^3 + 12.n^2 + 4,$$

or

$$p = 5n^4 - 6n^3 + 3n^2 + 4,$$

whose value depends on the value given to n.

Resuming the *reduct equation*

$$x^5 - 6.x^4 + 4x^3 + 4x - 3 = 0,$$

we find that it is satisfied by each of the digits 1, 2, 3, 4, 5.

Thus, for instance, the digit 1 gives $1 - 6 + 4 + 4 - 3$, which is plainly $= 0$. The supposition $x = 2$ gives the same result (remembering that for x^3 we must write 8, for x^4 16, that is 7, for x^5 32, that is 5). And so on for the other digits. It is therefore probable that the roots of the equation have the form

$$9y + 1,\ 9y + 2,\ 9y + 3,\ 9y + 4,\ 9y + 5,$$

whose *reducts* would be 1, 2, 3, 4, 5.

Multiply together the factors

$$(x - 1)\ (x - 2)\ (x - 3)\ (x - 4)\ (x - 5)$$

and the result is

$$x^5 - 6x^4 + 4x^3 + 4x - 3,$$

which agrees with the reduct equation. We may therefore conclude that the roots have the forms above specified.

Let us now calculate the five roots. I will adopt the following notation.

p_1 denotes the value which p acquires when the root has the form $9y + 1$ (or has 1 for its *reduct*). Similarly $p_2\ p_3\ p_4\ p_5$ denote its values for the other roots, whose *reducts* are 2, 3, 4, 5. The formula is

$$p = 5n^4 - 6n^3 + 3n^2 + 4,$$

and the following little table will be useful :—

Values of n	n^2	n^3	n^4
1	1	1	1
2	4	8	7
3	0	0	0
4	7	1	4
5	7	8	4

whence we deduce (putting P as before), for the reduct of p,

$$p_1 = 5 - 6 + 3 + 4 = 6 = \mathrm{P}_1$$
$$p_2 = 8 - 3 + 3 + 4 = 3 = \mathrm{P}_2$$
$$p_3 = \quad\quad + 4 = 4 = \mathrm{P}_3$$
$$p_4 = 2 - 6 + 3 + 4 = 3 = \mathrm{P}_4$$
$$p_5 = 2 - 3 + 3 + 4 = 6 = \mathrm{P}_5$$

We now proceed, as in the case of the cubic, to substitute the numbers 1.2.3.4.5 successively in the given or original equation. The results $q_1\ q_2\ q_3\ q_4\ q_5$ will all be divisible by *nine*. Let them be so divided, and the *reducts* of the quotients taken and called respectively $Q_1\ Q_2\ Q_3\ Q_4\ Q_5$.

Number substituted for x in the original Equation.	Result, when divided by *nine*.	Reduct of that number.
One . .	2679600	$3 = Q_1$
Two . .	2227680	$0 = Q_2$
Three . .	1836768	$3 = Q_3$
Four . .	1500720	$6 = Q_4$
Five . .	1213800	$6 = Q_5$

Having now found the values of P and Q for each root, we proceed, as in the case of the cubic, to solve the equation $Py = Q$, and having found y, $9y + n$ will be the root of the given equation.

First Root.—Here $P_1 = 6$ and $Q_1 = 3$, therefore $6y = 3$ whence $y = 2$, and $x = 9y + 1 = 19$, which is the root required.

Second Root.—Here $P_2 = 3$ and $Q_2 = 0$ $\therefore 3y = 0$, which gives $y =$ either 3 or 6, whence $x = 9y + 2$ is either 29 or 56. This ambiguity will be removed hereafter.

Third Root.—Here $P_3 = 4$ and $Q_3 = 3$, $\therefore 4y = 3$ whence $y = 3$ and $x = 9y + 3 = 30$, which is the root required.

Fourth Root.—Here $P_4 = 3$ and $Q_4 = 6$, $\therefore 3y = 6$ whence $y = 2$ and $x = 9y + 4 = 22$, which is the root required.

Fifth Root.—Here $P_5 = 6$ and $Q_5 = 6$ $\therefore 6y = 6$ which gives $y =$ either 1, 4, or 7, and $x = 9y + 5$ is either 14, 41, or 68.

But it cannot be 68, because no root of the given equation can end in 8.* Therefore this last root is either 14 or 41.

Thus we have determined three roots of the given equation to be 19, 30, and 22. This sum is 71, and since the coefficient of the second term of the given equation is 168, the sum of *all* the roots must be 168. Subtracting 71, the sum of the known roots, we have 168 — 71 or 97 as the sum of the two roots which are not yet determined. But we know that they are some two of the four numbers 29, 56, 14, 41, and the only two of these numbers whose sum is 97 are 56 and 41. This simple consideration dispels the ambiguity of the first calculation, and we conclude finally that the five roots are 19.56.30.22.41.

I propose to demonstrate the convenient rule by which these equations have been solved in the next part of this memoir.

* This appears by a simple process, but too long to explain here.

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Penicillium Glaucum.



Fig. 1.

Torula Cerevisiæ.



Fig. 2.

Bacteria
from various sources.



Fig. 3

Bacteria growing



Fig. 4

Fig. 5. ***Granuligera***

In boiled Urine. | *In unboiled Urine.* | *Growing in Urine diluted with Water*

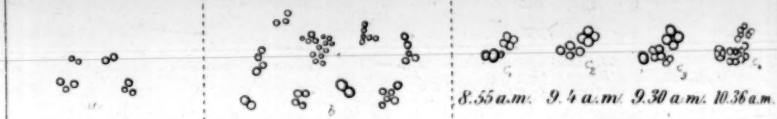
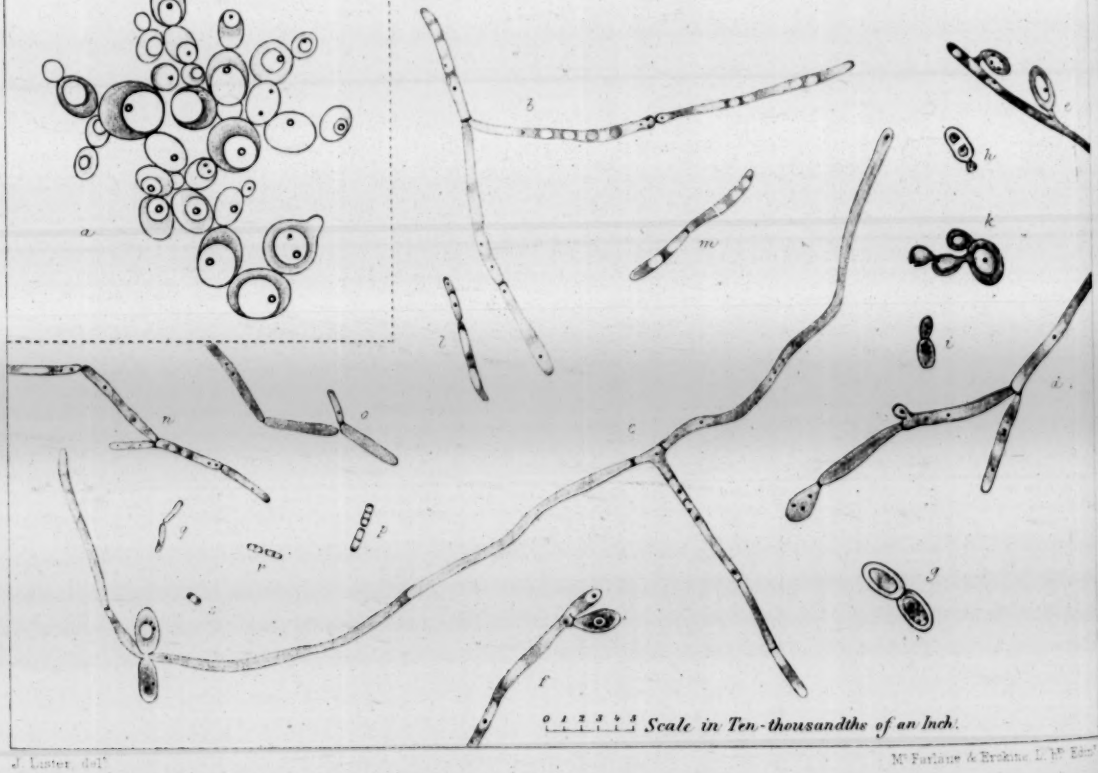


Fig. 6. ***Torula Ovalis.***

In Urine 18th Decr 1871.

In the same glass of Urine 9th Aug. 1872.

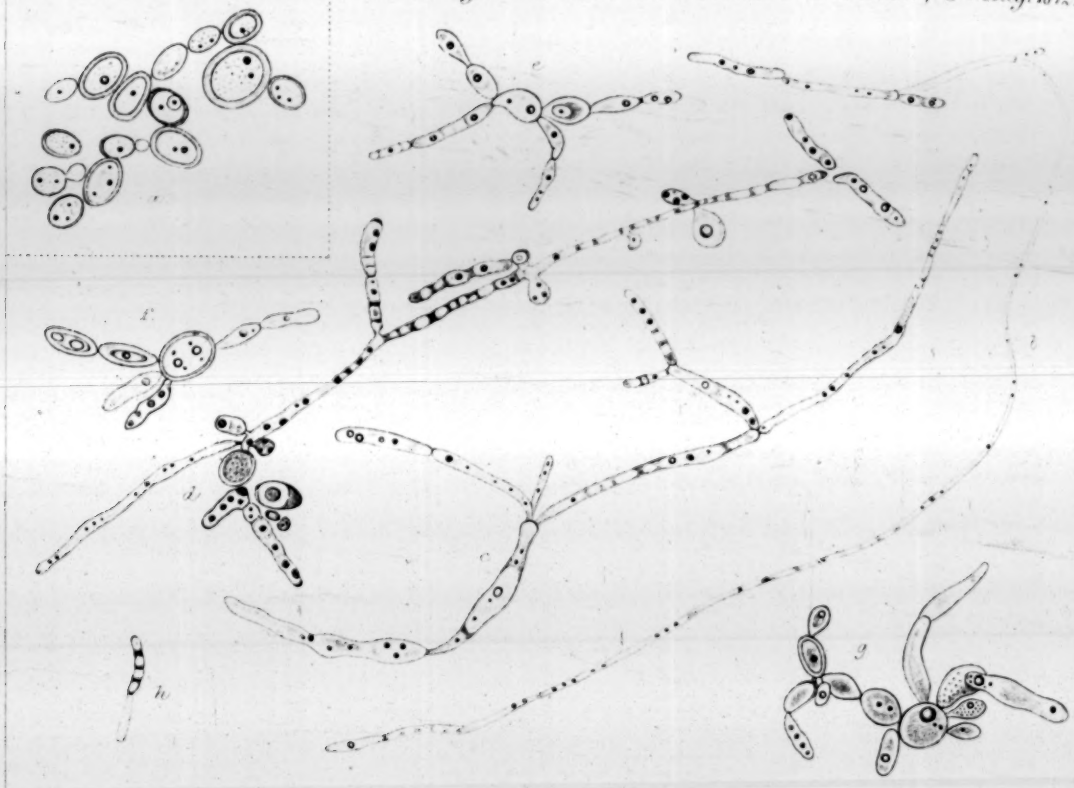


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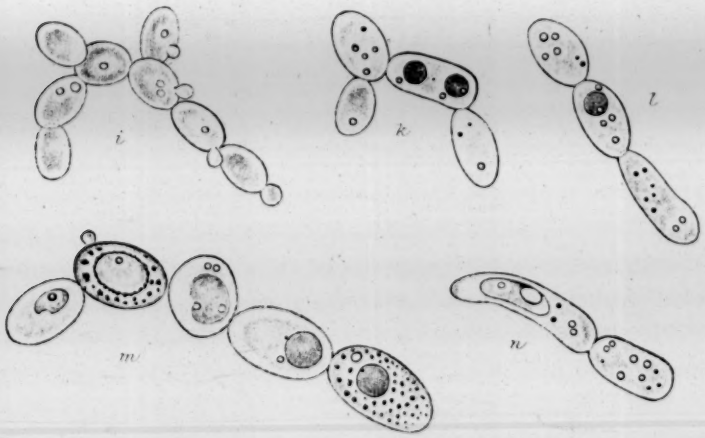
Torula Ovalis continued.

In Pasteur's Solution 30th Dec. 1871.

After eight month's in Pasteur's Solution, 9th Aug. 1872.



Offspring of the same specimen in fresh Urine, 14th Nov. 1873.



In fresh Pasteur's Solution, 18th Nov. 1873.



Scale in Ten-thousandths of an Inch.

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Oidium Torutoides

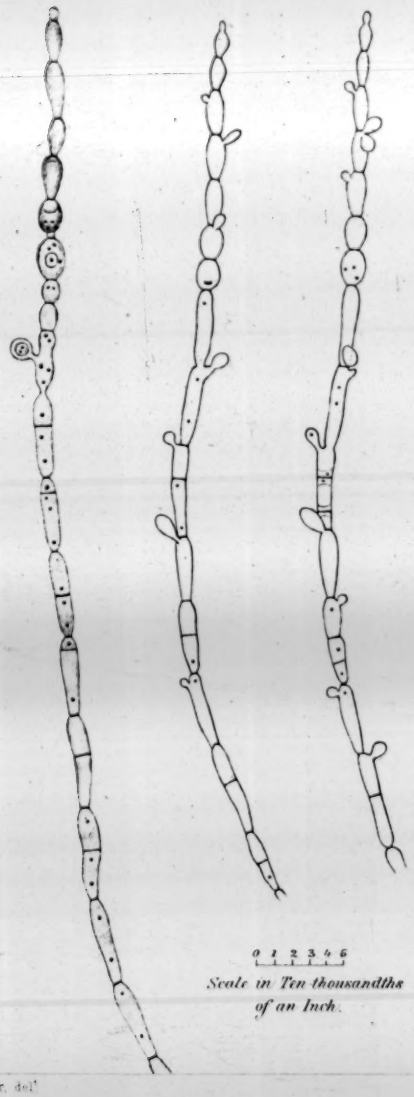
Fructifying Filament

From a glass of stale Pasteur's Solution examined in Water.

15ᵗʰ Aug. *16ᵗʰ Aug.*

7. 25 p.m. *11. 45 a.m.* *1. 45 p.m.*

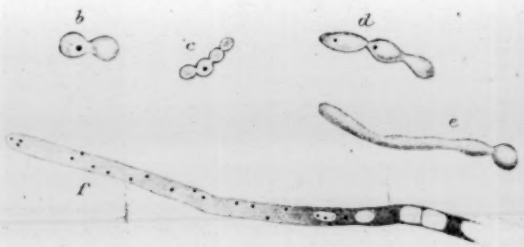
a_1 a_2 a_3



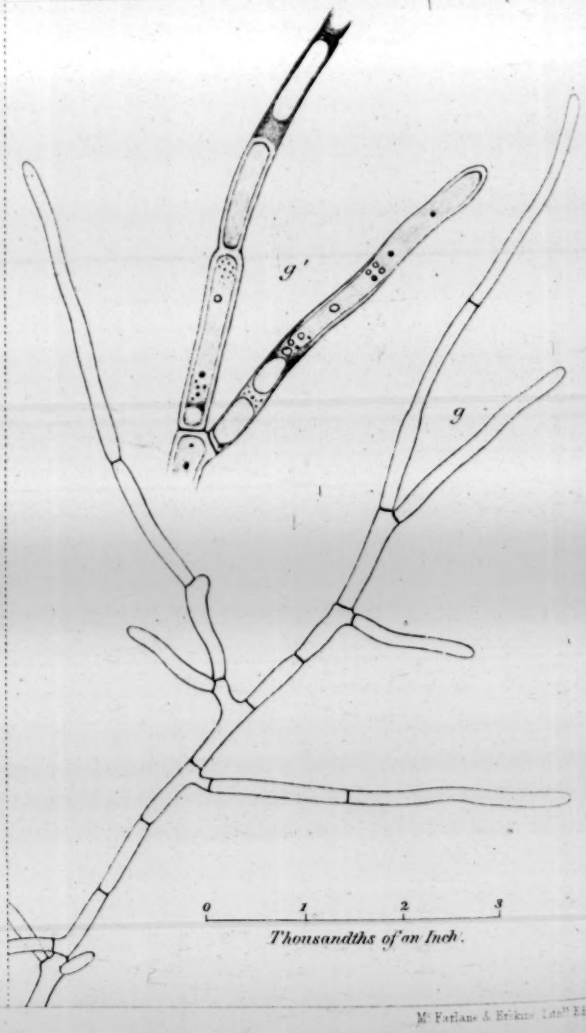
0 1 2 3 4 5 6

Scale in Ten thousandths of an Inch.

In fresh Pasteur's Solution, Glass Nº 1.



In fresh Pasteur's Solution, Glass Nº 2.

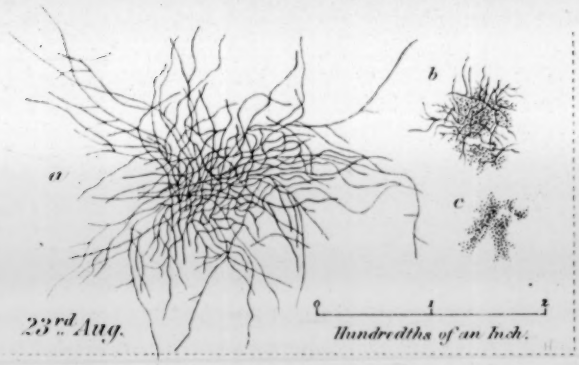


0 1 2 3

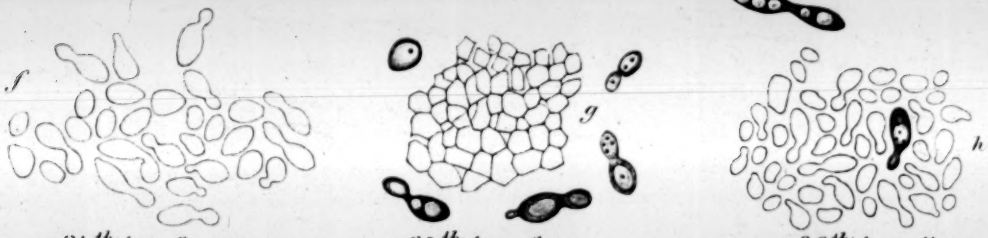
Thousandths of an Inch.

Oidium Toruloides continued

In Urine, Glass N.º 1 inoculated 21st Aug.

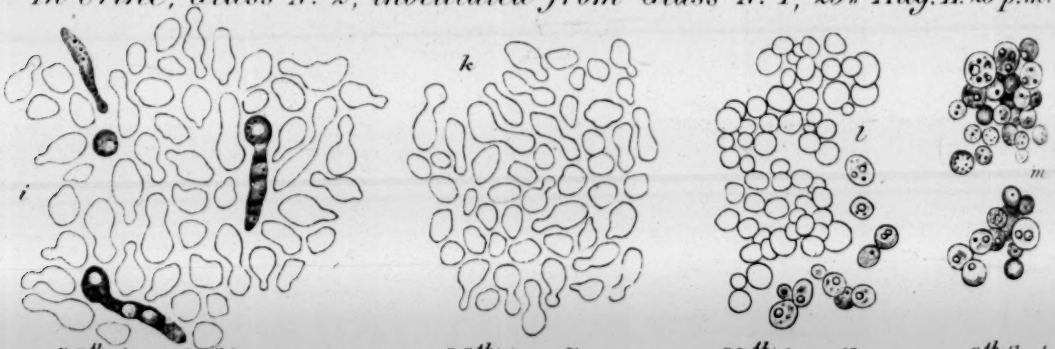






24th Aug. 8 p.m. 25th Aug. 6 p.m. 26th Aug. 11 a.m.

In Urine, Glass N.º 2, inoculated from Glass N.º 1, 25th Aug. 11.23 p.m.



26th Aug. 9.30 a.m. 26th Aug. 7 p.m. 27th Aug. 12 p.m. 6th Sept.

The Toruloid Form changing when transferred to Pasteur's Solution

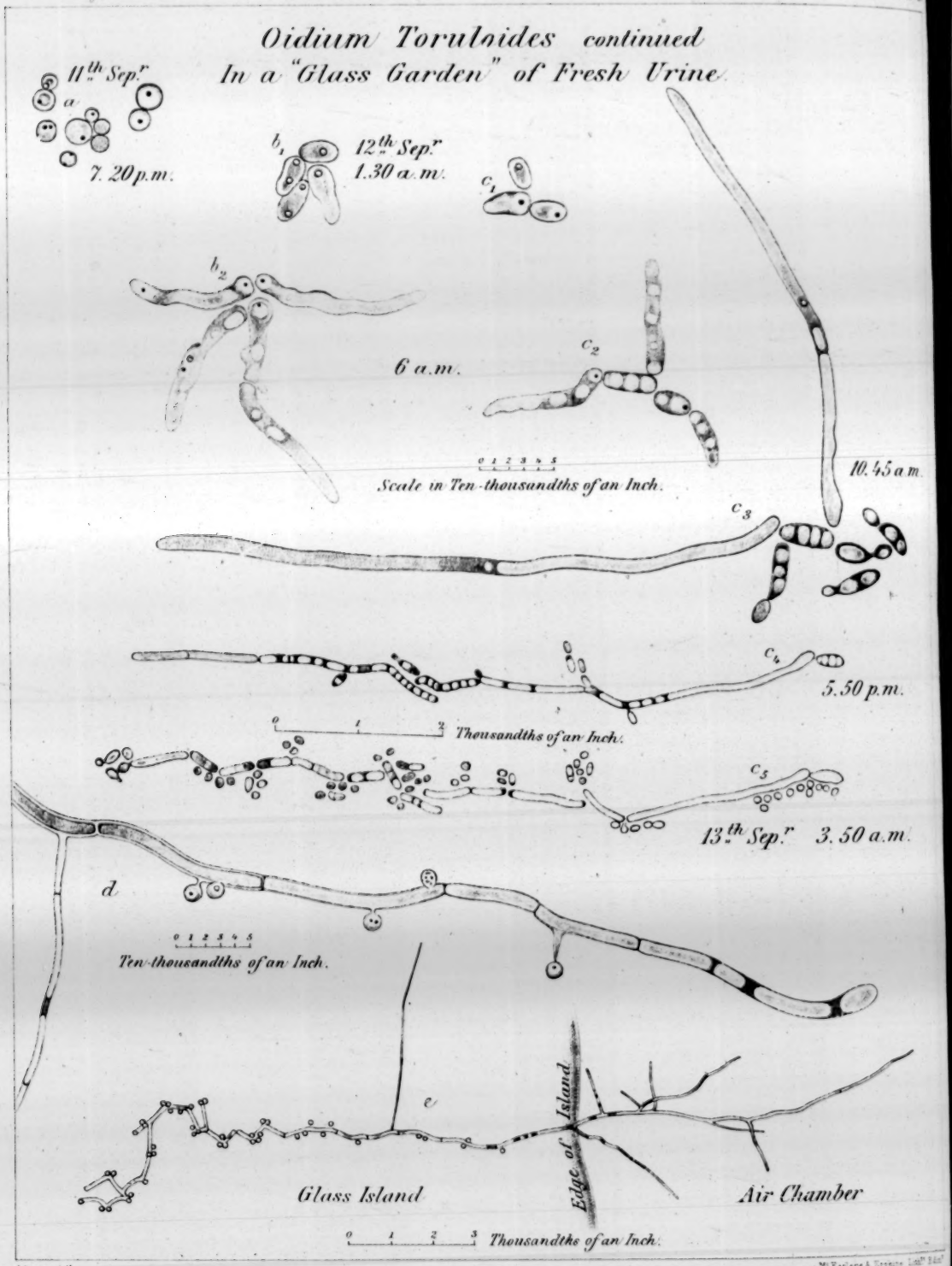


Aug. 30th 5.50 p.m. 6.8 p.m. 11 p.m. 12 p.m. 31st Aug. 7.45 a.m.

J. Lister del.

Mc Farlane & Erskine Lith. Edin.

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J. Lister, del.r

Mc Farlane & Erskine Lith.rs Edin.r

XVI.—*A Contribution to the Germ Theory of Putrefaction and other Fermentative Changes, and to the Natural History of Torulæ and Bacteria.* By Joseph Lister, F.R.S., Professor of Clinical Surgery in the University of Edinburgh. (Plates XXII.—XXVI.)

(Read 7th April 1873.*)

Part I.

Although the subject of the following communication has of late years attracted a great deal of attention among the public generally, it may, nevertheless, be well for me to preface my statements by a few elementary remarks.

It is well known that organic substances, when left exposed under ordinary circumstances, undergo alterations in their qualities. For example, an infusion of malt experiences the alcoholic fermentation; a basin of paste prepared from wheaten flour becomes mouldy; or, again, a piece of meat putrefies when so treated. The microscope shows that each of these changes is attended by the development of minute organisms. In the fermenting sweet-wort the yeast which falls to the bottom of the containing vessel is found to consist of budding cells, constituting the yeast-plant, *Torula Cerevisiæ*, represented in Plate XXII. fig 2.† In the mouldy paste the blue crust which is the most frequent appearance, owes its colour to the spores of a species of filamentous fungus, *Penicillium Glaucum*, the commonest of all moulds, of which fig. 1 in Plate XXII. represents a pencil of fructifying threads; and the putrid flesh will be probably found teeming with bodies which, in the most typical form, consist of two little rods, connected endways as by a joint, such as are seen at *a*, fig. 3, Plate XXII., characterised by astonishing powers of locomotion, and, from their rod-like form, termed *Bacteria*.

The Germ Theory supposes that the organisms are the causes of the changes; that the germs of these minute living things, diffusible in proportion to their minuteness, are omnipresent in the world around us, and are sure to gain access to any exposed organic substance; and, having thus reached it, develope if it prove a favourable *nidus*, and by their growth determine the chemical changes; and further, that these organisms, minute though they appear to us, form no

* This communication was originally made orally to the Royal Society on the 7th of April 1873. In preparing it for the press I have introduced various details which I was unable to enter upon at the time. I have also added facts ascertained at subsequent periods; but the dates of the observations being always mentioned, there will be no difficulty in distinguishing between those made before and after the delivery of the original address.

† In the present state of uncertainty regarding the true affinities of the yeast-plant, it seems justifiable to retain for it the old name *Torula Cerevisiæ*, a practice which has the advantage of enabling us to apply to similar budding cells the generic name Torula, and the adjective toruloid.

exception to the general law of living beings, that they originate from similar beings by parentage.

Of those who oppose this theory, some attribute the changes to the oxygen of the air; others, while convinced of the insufficiency of the oxygen theory, hold the doctrine of so-called chemical ferments, and ascribe the alterations we are considering to organic principles destitute of vitality, the organisms being regarded as accidental accompaniments; while others, admitting perhaps the fermentative agency of the organisms, believe that they do not necessarily spring from parents like themselves, but may arise, *de novo*, from the inorganic world by spontaneous generation.

The philosophical investigations of PASTEUR long since made me a convert to the Germ Theory, and it was on the basis of that theory that I founded the antiseptic treatment of wounds in surgery. The results of the treatment pursued constantly on this guiding principle have convinced me more and more of the truth of the theory upon which it was based; and if I were to put together the facts which I have had presented to me in surgical practice, proceeding on the antiseptic system, I should be able to present an array of evidence in favour of the Germ Theory as good and convincing as experiments performed in a laboratory.

But whilst I was thus for my own part thoroughly convinced of the truth of the Germ Theory of fermentative changes, I was led about a year and a half ago to direct my attention again to the subject by a remarkable paper by Dr BURDON SANDERSON, which appeared as an appendage to a report by the Medical Officer of the Privy Council.* Dr BURDON SANDERSON produced evidence, of which the following may be taken as a specimen:—If a vessel like a miniature ale-glass was heated considerably above the boiling point of water, to destroy any organisms adhering to it, and, when cooled sufficiently, was charged with boiling PASTEUR'S solution,—a fluid ingeniously devised by that eminent chemist to provide suitable pabulum for organisms, consisting of a solution of cane-sugar, some ammoniacal salt, and earthy materials derived from the ashes of yeast,—the liquid being left freely exposed to the air, fungi developed in it, but no bacteria. If, on the other hand, a drop of water, say water from the tap, was introduced into the PASTEUR'S solution, within a few days the originally transparent liquid was rendered milky by the presence of abounding bacteria. Another very remarkable fact is mentioned by Dr BURDON SANDERSON in the paper referred to. These bacteria, which have been commonly regarded as tough-lived organisms, difficult to kill, were found by him to be deprived of vitality altogether by simply drying them thoroughly at a temperature no higher than that of an incubator for the hatching of eggs, about 100° F.

* This paper will also be found in the Quarterly Journal of Microscopical Science, vol. xi. 1871.

By this second fact he explained the first. If bacteria are deprived of all vitality by dryness, then it seemed comprehensible that the dust of the air should contain no living bacteria, and, therefore, that none should have grown in the PASTEUR'S solution exposed to the atmosphere in the experiment first mentioned.

Further, Dr SANDERSON was led to conclude that bacteria were the sole causes of putrefaction; that fungi could only cause mustiness, or a comparatively insignificant alteration in organic substances.

Now, if these conclusions were strictly correct, they would affect my surgical practice in a most important manner. If it were true that the air does not contain the causes of putrefaction, then it would not be necessary for me, in carrying out the antiseptic system of treatment, to provide an antiseptic atmosphere. All that would be needful would be to purify the surface of the skin of the part to be operated upon by means of some efficient antiseptic, to have my own hands, and those of my assistants, and also the instruments, similarly purified; and then the operation might be performed without the antiseptic spray which we now use, and no one would rejoice more than myself to be able to dispense with it.

At the same time, striking as Dr SANDERSON'S facts were, I could not believe the truth to be exactly as he stated—that "no amount of exposure has any effect in determining the evolution of microzymes" (bacteria).* Various considerations, including circumstances that I had witnessed in surgical practice, made me fear the news was too good to be true. I determined, therefore, to put the matter to the test by a very simple experiment.

The fluid which I used was urine, which has so often been made the subject of experiments by PASTEUR and others; but instead of employing boiled urine for the purpose, I thought that in all probability the fluid might be obtained unboiled, yet uncontaminated, by a very simple procedure. According to a principle which I enunciated about two years ago before the Royal Medical Society here,† and of which I must not now give any more evidence than the fact that will immediately follow,—the healthy living tissues are capable of preventing the development of these low organisms in their immediate vicinity. If that were true, although undoubtedly the skin in the neighbourhood of the *meatus urinarius* must contain such organisms, yet supposing the urethra to be in a state of perfect health, the tissue of the lining membrane should prevent the entrance of those organisms, even for the thousandth part of an inch, within the mucous canal. The urethra, of course, contains putrescible materials, whether it be residual urine or the mucus secreted by the lining membrane;

* See Microscopical Journal, vol. xi. page 338.

† In an address delivered after the author had been elected an honorary member of the Society.

and the intervals between acts of micturition would afford time for the organisms to spread extensively inwards if it were a tube of indifferent matter; but I hoped, in accordance with the principle which I had had reason on other grounds to believe in, that the organisms would prove unable to develope in this putrescible material, however favourable a *nidus* for their growth. If this were really the case, instead of having the urine drawn off with a catheter, with special precautions, as was done by a surgeon at PASTEUR'S request, if the skin round the orifice of the urethra were treated with an efficient antiseptic, say with a solution of carbolic acid in forty parts of water, the urine might then be passed from the patient from whom it should be obtained, perfectly uncontaminated, though unboiled, free from any living organisms. Accordingly, on the 16th November 1871 I performed the following experiment:—Six wine-glasses were heated far above the temperature of boiling water by means of a spirit-lamp. I may here remark that in the rest of this communication, wherever I use the word "heated" (in quotation marks), I shall wish to be understood as meaning that the thing spoken of is not hot when used, but that it has been heated far above the boiling point of water, and then allowed to cool. Six glasses, then, were thus prepared, "heated" by means of a spirit-lamp. A glass plate large enough to cover them all, and overlap them considerably, was also similarly "heated." Urine was then passed into these six glasses with the antiseptic precaution that I have mentioned. Two of the glasses, before being covered, received each a minim of water from the tap; and into a third a much smaller quantity of water was introduced. To the rest no water was added, but one was left exposed for twenty-four hours to the air of my study, while the other two were put at once under the cover of the glass plate. After the lapse of forty-eight hours, quite in accordance with Dr SANDERSON'S statement, the two to which the drops of water had been added were turbid from the development of large and active bacteria; and the one which received a very minute quantity of water was similarly affected, though in a less degree, while the other glasses showed no change. But when twelve more hours had passed, the glass which had been exposed to the air, without the addition of any water, presented spots of opacity in the cloud of deposited "mucus," and on examining a portion of the cloud with the microscope, I found in the first field several bacteria in full activity. But the other two which had been covered by the glass plate from the first were perfectly clear. I should say that after twenty-four hours these glasses, instead of being covered with the glass plate, were put under a glass shade common to them all; an exceedingly rude method of experimenting, merely intended to obtain rough evidence of whether exposure to the air would or would not lead to the development of bacteria. Considering, therefore, how imperfect were the means of excluding dust, I was

not at all surprised to find, in the course of a few days, that the two glasses which had remained clear longer than the rest also exhibited organisms of different kinds, into the details of which I need not enter further than to say that those of one of the glasses included distinct bacteria.

This experiment, rude as it was, showed clearly that exposure to the air might lead to the development of bacteria, provided always that the urine was free from contamination to begin with. And, further, the comparative slowness of any change in the two glasses which were neither treated with water nor intentionally exposed to dust, led me to think that in all probability, if the experiment had been performed more rigorously, I should have had no development at all in them; or, in other words, that the method of obtaining uncontaminated urine was really trustworthy. If so, the fact was not only valuable as affording a ready means of performing experiments on the question at issue, but also exceedingly interesting in itself, as a strong corroboration of the view that the healthy living tissues prevent the development of these organisms.

Accordingly, it seemed worth while to perform another similar experiment somewhat more rigorously, and this was done on the 21st November of the same year. Wine-glasses were "heated" as before, but each was provided with a separate cover, which was also "heated." Two of these covers were inverted porcelain evaporating dishes, which had the advantage of preventing the direct effect of lateral currents of air; but as I had only two such dishes at hand, I used for the rest of the glasses square pieces of glass plate, overlapping well in all directions; and a glass shade was put over all as an additional protection from dust. Further, instead of having the urine passed directly into the several glasses in succession, which was an inconvenient procedure, I had it introduced, in the first instance, into a flask provided with a porcelain cap, the flask having been heated over a red fire and allowed to cool under protection of the cap, which had also been thoroughly heated. The glasses were then successively charged from the flask with as little exposure as possible. The residual urine in the flask was boiled for nine minutes, and two additional "heated" and covered glasses were charged with the boiled urine, and to one of these a drop of tap water was added. I shall speak of those again by and by. As regards those charged with the unboiled urine, one was exposed for forty minutes to the air of the room; one was exposed for nine and a-half hours; and the other two (those with the porcelain covers), were, in the first instance, not exposed at all. The one exposed for nine and a-half hours to the air, showed, in four days, besides some minute plants of filamentous fungi, opaque spots in the cloud of mucous deposit, and next day the liquid was turbid with perfectly characteristic and abundant bacteria, and had acquired a rank, strong odour. The urine exposed for forty minutes showed indeed no bacteria, nor any torulæ or other organisms except three plants of filamentous fungi, which appeared to be of

three different species, judging from their differences in density and rate of growth. They continued to grow until at last they almost filled the wine-glass, the fluid above them retaining its transparency unimpaired. When they had grown too large for their wine-glass, I transferred them to a large goblet into which urine had been passed, with the same sort of antiseptic precautions as were before described, after the goblet had been heated along with its saucer-like cover, and allowed to cool under a glass shade, packed round its base with cotton-wool to exclude dust. In this goblet the fungi continued to develope; and one growing more rapidly than the rest at length overlapped and smothered them, and then continued to grow alone till, by the end of January, ten weeks after the commencement of the experiment, the goblet was almost full of the delicate white filamentous mass, which, with the bright unaltered amber-coloured liquid above, presented a very beautiful appearance. At length, in the early part of February, I observed that the whole urine had become turbid, and at the same time the fungus, which before had continued to grow steadily upwards, had suddenly collapsed into about a third of its former volume. On examination I found that the liquid had a strong smell, and contained multitudes of minute granules grouped irregularly in a different manner from that which prevails among bacteria. In bacteria, where more than two constituent elements are connected together, they are commonly arranged in a linear series, constituting what are termed leptothrix filaments, as seen in Plate XXII. fig. 3 *b* and fig. 4. But in the case of these granules, when three or four were associated, they never showed themselves in a line, and when only two were together the members of the pair were often dissimilar in size. Yet, though unlike bacteria, there could be little doubt that these granules were some species of organism, and the natural interpretation was that it had found its way into the glass, and, developing in the urine, had rendered it poisonous for the fungus, just as is commonly seen when bacteria grow along with *Penicillium Glaucum* in urine. The bacteria occasion putrefaction in the fluid, and when this has advanced to a certain degree the growth of the Penicillium is arrested.

I had before met with granules of similar size and grouping. They occurred in one of the two glasses of boiled urine in this experiment. To one of those glasses, it may be remembered, a drop of tap water was added, while the other was simply covered with a glass plate. In the former glass bacteria of usual appearance showed themselves, as was to be expected; but it was five days before they occurred, whereas a specimen of the same urine unboiled presented bacteria in abundance in two days when similarly treated. This, I may remark, implied that the unboiled urine was a much more favourable *nidus* for the development of these organisms than the boiled liquid, and therefore a more sensitive medium to experiment with. The other glass of boiled urine, to which no water was added, continued unchanged for three weeks, which was

more than could have been expected, as it was covered merely with a plate of glass, there being no room for it under the glass shade. But at the end of that time the urine became turbid, and I found under the microscope multitudes of granules, of which samples are represented at *a* in Plate XXII. fig. 5, resembling what I have described as occurring in the goblet. Plate XXII. fig. 5 *b* represents another specimen of similar bodies which occurred in a glass of unboiled urine about the same period. I have introduced this sketch because it shows the peculiar irregular groups formed when several are together, as well as the variety of size of the individual granules.

That these granules were really organisms I had once an unexpected opportunity of proving. On the 5th February of the same year, I was examining some of them which had grown in a glass of unboiled urine, diluted with twice its bulk of distilled water which had been boiled and allowed to cool, and as I proceeded to sketch the group represented at c_1 in Plate XXII. fig. 5, I saw that it grew under my eyes. When I began the sketch, the lower three members of the group were a pair. About ten minutes later, at 9.4 A.M., the three had become four, as seen at c_2, where also the constituents of the other group of four are seen to have increased in bulk. By 9.30 the lower four had grown to seven, as is shown at c_3,* where also the left hand granule is seen to be greatly swollen. At 9.50 the upper four granules were observed to be each faintly marked by a transverse line, and finally by 10.36 those four had become developed into eight, as shown at c_4, while the large granule most to the left was marked by a cross, indicating that it was undergoing division into four. The "fissiparous generation" thus observed to take place was clear proof that these little bodies were really organisms; while the manner in which the divisions occurred appeared to mark the species off from bacteria, in which the only recognised segmentation is in a line transverse to the longitudinal axis, as is illustrated by the sketches given in fig. 4 (see explanation of the Plates). This mode of growth explained also the peculiar arrangement of the granules, which serves to distinguish it from bacteria, viz., that when three or four are present in a group they are not, as a rule, arranged in a straight line. I suggest provisionally the name Granuligera for this little organism, of which there may, for aught I know, be various species. Its distinction from bacteria is a matter of considerable interest, because, although destitute of anything like vital movement, it often renders fluids as turbid as bacteria, and like them produces a rank smell in urine, followed in a few days by strong ammoniacal odour. So far as urine is concerned, therefore, it seems to be an instance of an organism different from bacteria giving rise to putrefaction.

About this time my study suffered from an epidemic of Granuligera. I

* There were, no doubt, in reality eight; one of them being obscured by lying beneath the quadruple granule just formed out of one of the single ones.

could not now perform the same experiments with the same success as in the first instance: any that I tried was sure to be followed by the development of this pervading organism. I eluded it, however, by continuing the investigation in a room at the top of the house, which had been for a considerable time unoccupied. Here the results of experiments corresponded with those originally obtained in the study.

But I have not yet spoken of the two glasses of the second experiment which were not exposed, but were kept covered with the evaporating dishes under a glass shade. The liquid in both these glasses having remained unaltered for nearly a fortnight (thirteen days), I exposed one of them to the air for nine hours in my study, which is a warm room (over the kitchen), the weather being dry and frosty, and then replaced it, covered as before, under the glass shade, having previously ascertained that the odour was that of perfectly fresh urine. Two days later the cloud of mucus presented a multitude of vertical white streaks, and the side of the glass was also similarly marked, and when another day had passed the whole liquid was manifestly turbid, and there were also two little patches of scum upon the surface. On microscopic examination I found that the scum was composed of a species of torula, and that the turbidity was due to a small organism which, while motionless like granuligera, resembled bacteria in its mode of segmentation and arrangement. It is represented in the sketches given at *c* in Plate XXII. fig. 3, where it will be observed that when three elements exist together they are in a straight line, and that some of those which are in pairs present a transverse line of incipient division through each constituent portion. Occasionally this organism was met with in the form of long chains (Leptothrix), and it is plainly referable to the bacteric group. But no filamentous fungus occurred from first to last in this glass, which, in that respect, was the exact converse of the one which was exposed to the atmosphere in the first instance for forty minutes, and in which, it will be remembered, filamentous fungi occurred without either torulæ or bacteria—the obvious explanation of the difference being that different organisms happened to prevail in the air of the room at the two periods of exposure.

The other glass was left permanently covered; and the urine in it remained permanently free from organic development or putrefactive alteration. After the lapse of many weeks, when its bulk had been considerably reduced by evaporation, it became turbid, leading me to suspect bacteria. But on applying the microscope I found the appearance was occasioned merely by saline deposit, and the contents finally dried up into a solid residue, without undergoing any other perceptible change.

I need hardly point out how entirely such a fact as this disposes of the oxygen theory as regards this particular fluid at ordinary temperatures. Neither cover nor shade fitted closely, so that a constant interchange was

taking place by diffusion between the air in the wine-glass and the oxygen and other gases of the external atmosphere; yet no putrefaction or other fermentative change occurred. Nor is the fact less significant in its bearing upon the theories of chemical ferments and spontaneous generation. The vesical mucus has been commonly regarded as the special chemical ferment of urine: but it was here present, unaltered by boiling or any other treatment, yet failing for weeks together to produce any fermentative change. And the mere fact that the liquid was received into a vessel which had been heated so as to destroy all life within it, and afterwards protected from the access of dust, ensured the absence from first to last of all organic development. It is, therefore, certain that this urine contained no materials or principles capable at ordinary temperatures of evolution into living beings.

At the same time the behaviour of the glasses which were exposed to the air in this experiment indicates that the foreign element which gives rise to bacteria, like that which occasions the growth of filamentous fungi and torulæ, may enter in the form of atmospheric dust.*

But the results of this simple experiment were valuable in other respects. In the first place, it afforded ample proof that urine may be obtained perfectly free from organisms by merely applying an efficient antiseptic as a preliminary measure to the *meatus urinarius;* and I have before referred to the high interest which attaches to this point.

Secondly, it shewed that if an organic liquid is obtained in an uncontaminated state to begin with in a "heated" wine-glass, covered with a "heated" cap shaped like an evaporating dish, and further protected by a glass shade, we are secure against the introduction of any organism from without, so long as the arrangement is left undisturbed.

Further, the permanent freedom from contamination in this glass was particularly satisfactory, because, seven days after it was charged, I had removed a drachm of the liquid from it by means of a "heated" pipette, in order to ascertain the effect of water upon the unboiled urine as above alluded to (see p. 310). If no organic development resulted from the sudden entrance of so considerable a volume of air as then passed into the glass to take the place of the liquid withdrawn, it follows that, various as are the organisms which float in the atmosphere, they constitute but a very small proportion of the

* It may be urged that the particles of dust which give rise alike to the development of organisms and to fermentative changes in a fluid like urine are not necessarily organisms, but may possibly be little bits of so-called chemical ferments which occasion chemical alterations, that in their turn lead to the evolution of organisms by spontaneous generation. Such a view, plausible as it may appear, will be shewn in the sequel to be utterly destitute of scientific basis. Meanwhile we must be content with the sure step mentioned in the text, viz., the fact that neither fresh healthy urine nor its mucus contains any such evolutionary particles. I feel justified in stating this as a general truth regarding urine, since it has been found to hold not only in numerous other experiments with this liquid derived from the same source, but also when it was obtained by the same method from two other individuals.

abounding particles of dust which a beam of sunlight reveals in an occupied apartment.

A similar inference must be drawn from the circumstance before mentioned, that the sole result of forty minutes' exposure of one of the glasses of this experiment to the air was the development of three plants of filamentous fungi, whereas the particles of dust which fell into it during that time must have been very much more numerous.

If, then, the withdrawal of a drachm of liquid, or exposure for more than half-an-hour had so little effect, it was plain that the removal of one or two minims, executed nimbly so as to involve little more than momentary exposure, must be practically free from the risk of accidental contamination.

I thus became possessed of a means of making observations upon these minute but highly important organisms, which promised to yield results of a more definite character than any which had been hitherto obtained.

Various detailed accounts have been given of late years, not only of the spontaneous generation of animal and vegetable forms of more or less complexity, such as large ciliated infusoria from an infusion of hay, or torulæ and penicillia from milk globules, but also of the transition of one form of organism into another. But in the latter class, as in the former, the liability to deception is so extremely great, in consequence of microscopic organisms accidentally present developing side by side with the minute objects investigated, and presenting the appearance of growing out of them, that, without the slightest doubt being thrown upon the good faith of the observers, the so-called facts are justly received with the gravest suspicion. But with the means now at our disposal the grand source of error in former similar inquiries might be eliminated, and results of a more satisfactory character might therefore be anticipated. I was thus led to prosecute the investigation far beyond what I had at first intended, and will now proceed to give a selection from the results.

That which I will mention first has reference to the origin both of torulæ and of bacteria.

On the evening of the 13th December 1871, during a drizzling rain which had been falling all afternoon, I took a "heated" wine-glass with its cover out into the street, and, raising the cover, allowed a few drops of rain to fall into the glass, and having covered it again and brought it back into the house, I charged it with unboiled urine from a "heated" flask, the arrangements for obtaining the liquid being the same that have been before described. In the course of two days I noticed a tiny opaque streak proceeding vertically downwards from a point on the inside of the glass; and on the following day the streak had increased, and the cloud of mucus was speckled with numerous white points. On the fourth day, while the speckling of the cloud had increased, and the streak had become coarsely granular, two little plants of filamentous

fungi were also seen floating in the clear liquid. By the fifth day the specks in the mucous deposit had assumed the appearance of coarse grains of white sand, and similar granules were sprinkled over the lower part of the inside of the glass. I removed one of these granules with "heated" pipette, and examined it microscopically. It proved to be a very beautiful torula, composed of pullulating oval cells of great delicacy, disposed in groups, of which one is represented in Plate XXII. fig. 6 *a*. Though not very different in size from the yeast-plant, it proved itself to be a totally distinct species, not only by the more delicate and less granular character of the cells, but by the fact that it grew thus luxuriantly in non-saccharine urine, in which the *Torula Cerevisiæ* will only grow with extreme difficulty. For the sake of distinction I may term it *Torula Ovalis*, on account of the oval form of its cells. When ten days had elapsed after the mingling of the rain water with the urine, the white granular deposit had greatly increased, and some scum was also present on the surface, which the microscope showed to consist of the same oval torula. But the two plants of filamentous fungi had subsided and had apparently ceased to grow; the liquid, though still brilliantly clear and but very slightly affected in odour, having doubtless become unfit for their development through chemical changes induced by the torula. Another small fungus plant, observed several days before upon the side of the glass below the level of the liquid, seemed, however, to be still increasing. At this time having occasion to go into England for a few days, and being desirous of continuing the investigation, I took some of the liquid with me, decanting a drachm of it with "heated" pipette into a "heated" test-tube about five inches long, which I covered with an inverted test-tube of about the same length (of course also "heated"), and packed the tube vertically in a box with cotton-wool. Five days later (on the 28th December), having prepared some PASTEUR'S solution in a manner which I hoped would ensure absence of living organisms at the outset,* I inoculated about an ounce with half

* In preparing the liquid I deviated to some extent from PASTEUR'S formula, which is 100 parts distilled water, 10 parts pure sugar candy, 1 part tartrate of ammonia, and the ashes of 1 part of yeast. I employed lump-sugar instead of sugar candy, and reduced its proportion by one half, as it seemed to me likely to prove somewhat too strong to suit some organisms. Further, as I had not at hand a reference to enable me to ascertain how much of the mineral salts PASTEUR employed, I used what seemed to me about a suitable amount for a fungus to consume, judging from the quantity that I got by incinerating a certain weight of yeast; and this, as I afterwards found, was a little more than PASTEUR'S proportion. My solution, then, had the following composition :—

Distilled Water, ...	5000 grs.
Lump-Sugar, ...	250 grs.
Crystallised Tartrate of Ammonia, ...	50 grs.
Dry Ash of Yeast, ...	5 grs.

making rather more than half-a-pint. The liquid was introduced through a "heated" funnel into a "heated" Florence flask provided with a "heated" glass cap, and was boiled and allowed to cool in the pure and covered vessel. A better method of procedure will be described in a later part of this communication.

a minim of the urine in the test-tube, including some of the white deposit at the bottom. The glass, which was of course "heated," as well as its porcelain cap, was placed under a glass shade in a room varying in temperature from about 60° to 70° F. It is necessary to state, that before raising the inverted test-tube which covered that containing the urine, I carefully wiped the mouth of the former with a rag dipped in a strong watery solution of carbolic acid; without this precaution there would have been a risk of contamination of the urine-tube with some portion of cotton or dust adhering to the covering tube.* The urine still continued quite bright, and on examining with the microscope the residue in the pipette after the inoculation, I found it to consist of the oval torula unmixed with anything else.

Thirty-six hours after the inoculation I found the inside of the glass that contained the PASTEUR'S solution sprinkled over from top to bottom with a fine granular deposit resembling white sand under a pocket-lens, and about a third of the surface of the liquid was occupied by a dense white scum which microscopic examination on the following day showed to consist of oval torula cells, closely resembling those in the urine of inoculation. A group of these from the PASTEUR'S solution is represented at *a*, Plate XXIII. On the 3d January 1872, I inoculated a second "heated" and covered glass of the same stock of PASTEUR'S solution by introducing into it a drop from the former glass of the same fluid containing the growing organism, and in the course of the next twenty-four hours the cells of *Torula Ovalis* were again seen under the microscope in a deposit on the side of the glass. Next day, being about to return to Edinburgh, I introduced some of the contents of this second glass of PASTEUR'S solution into a "heated" test-tube provided with an inverted test-tube cover, and packed the tube with cotton-wool in a box along with that containing the urine. Meanwhile, although eleven days had elapsed since the urine was decanted into the test-tube for the journey south, the liquid remained perfectly transparent, and showed no appearance of any other organism besides the *Torula Ovalis*; so that it may be assumed that the plants of filamentous fungi present in the original urine-glass had been avoided in the process of decanting, and that the *Torula Ovalis* existed in the test-tube unmixed with any other organism.

Being occupied with other matters, I did not look at these test-tubes again until eight months had passed, during which time they had remained undis-

* The efficacy of a strong watery solution of carbolic acid for the destruction of minute organisms was familiar to me from experience in antiseptic surgery; and it is also well illustrated by the method of obtaining uncontaminated unboiled urine described in the text. The fact is of great value in experiments on this subject, as it affords a simple and sure mode of purifying portions of apparatus which it would be inconvenient or impossible to subject to heat. And the extensive experience which this investigation has involved, enables me to state with confidence that wiping a piece of glass with a rag moistened with a solution of carbolic acid in twenty parts of water as efficiently destroys adhering organisms as heating to redness in a flame.

turbed in the cotton-wool in which they were packed. This proved to have been a very fortunate arrangement, the long narrow form of the vessels and their covers, and the mass of cotton about them, having so interfered with evaporation, that a considerable proportion of the liquid remained in the glasses. On closely inspecting them on the 6th of August 1872, I saw that in both the part of the glass that had been left dry by the slow evaporation was studded over with little round whitish gelatinous-looking bodies, smaller than pins' heads, which I thought might perhaps be a fungus related to the torula, a surmise which was at once verified by examination of the glass containing the urine. Having raised the test-tube cover, after wiping its lower part with 1 to 20 watery solution of carbolic acid, I succeeded in picking up with a mounted needle (passed through the flame after washing the wooden handle with carbolic solution), a portion of one of the little gelatinous bodies, and submitted it to the microscope. It proved to be made up of plants of an exquisitely delicate filamentous fungus, of which *b*, in Plate XXII. fig. 6, represents one young plant entire, giving off a branch, and *c* a somewhat larger plant, bearing two oval bodies considerably thicker than the thread from which they spring, which must be looked upon as spores (conidia). In *d*, *e*, and *f* are given portions of other filaments bearing similar conidia. Such conidia were also seen free and pullulating, either in pairs, as in *g*, *h*, and *i*, or more rarely in somewhat larger groups as at *k*, which, in fact, constituted a torula undistinguishable from the original *Torula Ovalis*. But while some of the buds proceeding from the filaments had thus the character of toruloid conidia, differing from ordinary branches not only by their form but by their thicker and more substantial character, it was more common to see sprouts presenting the opposite condition of extreme slenderness, as at *n* and *o*, and similar delicate bodies were often seen free, commonly in pairs, as represented in the series *l*, *p*, *q*, *r*. Of these, *l* resembles in its thicker half a very young plant such as *m*, while its more slender portion corresponds with *p*. This again, as well as the still more delicate *q* and *r*, seemed to be neither more nor less than bacteria, as was shown not only by their form, but by the fact that precisely similar bodies were not unfrequently seen exhibiting active and perfectly characteristic movements. Further, there were many motionless bodies, such as *s*, which previous experience enabled me to recognise as young bacteria multiplying by segmentation, while they were fully equal in thickness to sprouts, such as *o*, proceeding from the filaments. The identity of the bacteria with the filaments was further indicated by the precise similarity of the delicate transverse markings often observed in the former (as in *p* and *r*) with those of young plants, such as *m*.

That bacteria should originate from filamentous fungi was an idea entirely opposed to the preconceived notions with which I entered upon this inquiry; for, in common with those authorities on the subject whose observations ap-

peared entitled to greatest weight, I had regarded these organisms as a separate and altogether distinct group. But the contrary conclusion was forced upon me not only by the observation which I am now recording, but by various others, some of which will be described in the sequel. I need hardly remark that, if correct, it is of the very highest interest.

In the present instance it is certain that the bacteria moving in the liquid were identical, morphologically, with buds derived from the fungus; and this fact receives additional weight from the circumstance that the glass had been left untouched for eight months, having been previously securely guarded against the entrance of organisms from without; and even if bacteria, as such, had been accidentally introduced when the vessel was last exposed, it is in the highest degree improbable that they would have remained in an active condition for such a protracted period. If, therefore, we set aside the idea of spontaneous generation, which I trust before this paper is concluded the reader will see that we are justified in doing, it is difficult to conceive how these bacteria could have arisen, except from a gradual alteration in the character of the original organism under the influence of progressive changes in the medium which it inhabited.*

I next proceeded to examine the PASTEUR's solution. The liquid was still perfectly transparent and colourless, contrasting remarkably with the jet black colour which I had observed to result in a much shorter period from the action of yeast upon the same fluid.† There was, however, a good deal of white deposit, partly in the form of a loose sediment, partly as a delicate incrustation upon the side of the tube, and some white patches were floating free, probably in consequence of the disturbance of the vessel: there was also a little scum on the surface. Only about a sixth part of the liquid had evaporated; and, as before mentioned, the part of the glass which had been left dry was studded over with little gelatinous bodies like those in the tube of urine. The tube being longer in the present case, I failed to pick out any of those little bodies with a needle. I was therefore obliged to content myself with examining a drop taken with "heated" pipette from the upper part of the liquid, including some of the white floating particles. These, however, proved all that I could desire, being composed of the same organism that I had found in the urine, and all the better seen because it had not been disturbed by the needle. *b*, *c*, and *d* of Plate XXIII. represent three entire plants, of which *b* fully equals in slenderness any seen in the urine; and some idea of its exquisite delicacy may be given by saying that ten such threads might lie abreast in the diameter of a single red

* It is indeed conceivable that a bacterium incapable of growing in fresh urine may have lain dormant in the liquid till it had become so altered under the influence of the torula as to be a suitable nidus for it. Meanwhile the fact of the morphological identity of this bacterium with buds from the filamentous fungus must be taken for what it is worth.

† I am not prepared to say whether the black colour which I have invariably found to be caused by the prolonged action of yeast upon PASTEUR'S solution is due to the *Torula Cerevisiæ* or to other organisms accompanying it.

corpuscle of human blood. *d* is introduced as a good example of the production by such filamentous plants of substantial conidia having the characters of the cells of *Torula Ovalis*, while in *c* we have a plant which in some parts is as delicate as *b*, while in others it looks as if composed of elongated cells of the torula. Other obviously transitional forms, between the filamentous fungus and the torula, are represented by the groups *e*, *f*, and *g*. Comparing the appearances of the organism as it occurred in the two glasses, the cellular element predominated over the filamentous in the PASTEUR'S solution, while the converse was the case in the urine. The toruloid groups, rare in the latter liquid, were abundant in the former, in which also the filamentous plants were as a rule of a coarser character, and were invariably small; that is to say, not extending to any great length, as they did in the other medium. The granules of the filaments and the nuclei of the cells were also much more marked in the PASTEUR'S solution. Along with this deficiency of the filamentous element, the bacteric form was absent in the PASTEUR'S solution. Some of the buds were indeed as slender as the bacteria of the urine, as is illustrated by the plants *b* and *c*, and here and there such buds were seen floating free in pairs such as *h*, but no bacteric movement was to be seen. This puzzled me at the time; but I afterwards found that it was no reason for surprise, and I shall hereafter have occasion to mention cases of bacteria of ordinary form and active movement in urine, assuming a motionless character and at the same time a very different appearance in other media.

Although the proof already afforded of the identity of the *Torula Ovalis* with the filamentous fungus may appear sufficiently ample, yet, as the point is of extreme interest, I have been well pleased to obtain further confirmation of the fact while preparing this communication for the press. On the 9th November 1873 I once more removed the test-tube containing the PASTEUR'S solution from its cotton packing to see what change it might have undergone. I found about half of the original volume of the liquid still remaining unevaporated. It was still transparent, but it was now of a pale brownish yellow colour, and the sediment had a similar tint. A delicate incrustation existed on the interior of the glass, but did not reach up to the level of the liquid, and the gelatinous lumps had disappeared from the dried part above. Raising the test tube cover with careful antiseptic precautions, I removed a few drops, taking up at the same time a little of the crust, which I detached from the side with the "heated" pipette; and, after inoculating a glass of PASTEUR'S solution with about half a minim, I proceeded to investigate the remainder. Under the microscope the solid constituent proved to be composed in the main of granular masses, looking like confused aggregations of the organism in an effete and degenerate state; but projecting from the edges of these masses were plants and corpuscles, which, from their translucent and fresh appearance, made me hope that they were alive.

The filaments closely resembled those seen in this glass a year and a quarter before, except that they were invariably very short, and the corpuscles, while sometimes in groups more or less resembling the original torula, were often of a more elongated form and strongly nucleated. During the first five days after the inoculation there was no distinct appearance to the naked eye of any growth taking place in the new glass of PASTEUR's solution. At the end of that time, however, thinking that a speck of delicate scum, which existed from the first, appeared slightly increased, I examined a portion microscopically, and found it to consist entirely of cells which appeared of new formation, some of them presenting transitional forms between the elongated bodies common in the test-tube and the constituents of the oval torula. The growth afterwards continued, both as a very delicate scum, and as a fine white deposit; but its rate was extremely slow, and the product for the most part on a much smaller scale than the original torula, and more resembling the elements found in the test-tube.

Very different was the behaviour of the organism in unaltered urine. Two days after the inoculation of the PASTEUR's solution, I introduced half a minim of the liquid from the test tube into a "heated" and covered glass, containing unboiled urine from a flask which had been charged on the first of March, but, though it had furnished the material for many successive experiments, retained its original characters unimpaired.* For two days there was no appearance of growth; but on the third day a small patch of scum, which had been the immediate result of the inoculation, was considerably increased in size, and had acquired a much coarser character, and several small detached specks of similar aspect were floating on the surface. The side of the glass was also sprinkled with minute particles like grains of white sand, often disposed in vertical streaks, while other similar granules were deposited at the bottom, the liquid retaining its brilliant clearness. In short, the naked eye appearances were an almost exact reproduction of those which resulted from the introduction of the rain drops into the original urine nearly two years before, and on applying the microscope to a portion of the scum taken up with "heated" pipette, I was delighted to find it composed exclusively of the *Torula Ovalis* in all its original beauty, the constituent cells pullulating freely, as shown at *i*, Plate XXIII., which represents, for convenience of sketching, a small specimen of the groups, which were commonly much larger, like those of the yeast plant when in full activity. In some fields the cells were peculiarly large, as at *m*, and here and there, as at *l* and *n*, a cell was somewhat longer than usual, just as occurs in *Torula Cerevisiæ*; but there was no appearance of filamentous growth. It was a torula pure and unmixed; yet its identity with the *Torula Ovalis* and its distinction from the yeast-plant were declared not only by the form and aspect of the cells,

* The method by which this flask was prepared, and the mode of decanting into the experimental glasses, will be described in a later part of this paper.

but still more by the fact that, just like the original specimen, it grew freely in non-saccharine urine, in which *Torula Cerevisiæ* develops only with extreme difficulty.

The organism, having thus, after many months of slow growth in the filamentous form in the altered PASTEUR'S solution, recovered its purely toruloid and luxuriant habit in the medium in which it presented those characters at the outset, retained them when transferred to uncontaminated PASTEUR'S solution. For having, with the touch of a "heated" pipette, introduced a speck of the rapidly growing scum from the urine into a second glass of PASTEUR'S solution, which had been charged along with the former six days before, but had hitherto remained unchanged, I found the morsel of scum increased in fourteen hours to four times its original diameter, and on the following day it nearly covered the surface of the liquid, and the side of the glass was sprinkled with white granular specks, which after another day were disposed in vertical streaks, just as they had been in a glass of PASTEUR'S solution, inoculated from the original urine-glass nearly two years previously. And on examining the scum microscopically, I found it to consist of the torula unmixed with any filamentous element, as seen in *o*, *p*, *q*, *r*, and *s*, Plate XXIII.

Those who have the patience to follow me through these minute details, inseparable from so minute a subject, will acknowledge the importance of having it clearly demonstrated that an organism, which, for weeks together and in different media, showed itself as an unmixed torula, was in reality a conidial development from a filamentous fungus. For one such instance rigorously proved, leads to the suspicion that the same is in all probability the case with the whole group of torulæ, and that though BERKELEY appears to have been deceived when he thought he traced a direct connection between *Torula Cerevisiæ* and *Penicillium Glaucum*,* yet his belief that the yeast plant is derived from some filamentous form will turn out to have been sound when the mode of investigation which I have been describing shall have been applied to that case. Without some such method, permitting us to study an organism for a protracted period, unmixed with others, in different media or in the same medium altered under its fermenting influence, the true affinities of the *Torula Ovalis* would have remained as obscure as those of *Torula Cerevisiæ* are at present. Further, without entering here upon all the bearings of this observation, it may be remarked that for an organism so humble as a torula, though modified by varying circumstances, to retain its specific morphological and physiological characters unimpaired for two years together, is a fact fraught with the deepest instruction.

I next unpacked and examined the test-tube containing the urine. I found

* See DE BARY, Morphologie und Physiologie der Pilze, &c., Leipzig, 1866, p. 184.

the fluid all evaporated except about two minims above a considerable crystalline mass. The part of the glass, about an inch high, left exposed by the drying was studded over as before with round gelatinous specks, those on the upper half inch being largest, viz., about $\frac{1}{60}$th inch in diameter. Breaking the tube with antiseptic precautions, I examined one of the little transparent lumps with the microscope, and found it to consist almost exclusively of the filamentous form of the fungus, the conidial element being, as before, much less marked in this tube than in that of PASTEUR'S solution. There was a somewhat larger proportion of conidia in the liquid residue, which, however, was thick from the abundance of the fungus filaments in it; but there was no longer any appearance of bacteria. I introduced a portion of the gelatinous lump into a glass of uncontaminated urine, which had been charged along with the one inoculated from the tube of PASTEUR'S solution (viz., nine days previously); but as no growth showed itself in the course of the next eleven days, I concluded that the organism had, in the highly concentrated and altered urine, at length lost its vitality. Yet the examination of this urine-tube proved not devoid of interest. For although the bacteria which were seen in it when it was last examined had the ordinary rod shape, and did not differ in appearance from those commonly seen in putrefying urine, yet the liquid in this glass had no ammoniacal odour, but a very peculiar smell resembling musty cheese rather than urine, and it was sharply acid to test paper, even when diluted with several times its bulk of water. Here, then, we have an example of what we shall see abundantly illustrated in the sequel, viz., that bacteria of similar morphological characters may differ entirely as regards the fermentative changes to which they give rise, being, like the torulæ, as specifically distinct as the fungi from which some of them at least appear to take their origin.

The observations to which I have next to direct attention were made upon a filamentous fungus, which I was induced to investigate in the hope that it might prove to be the parent of the *Torula Cerevisiæ*, occurring as it did in circumstances analogous to those under which the filamentous form of the *Torula Ovalis* had been met with. I had introduced into a "heated" and covered glass of PASTEUR'S solution a morsel of German yeast, with the effect of inducing the usual evolution of gas that accompanies the alcoholic fermentation, followed by the gradual supervention of the black colour before alluded to. Some minute plants of filamentous fungi, seen in the course of the first few days, had apparently ceased to grow, and no penicillium or other ordinary fungus appeared; but after the lapse of two months I observed, upon the surface of the liquid and upon the part of the glass left exposed by evaporation, a low white mould, which, under the microscope, was seen to be composed of branching septate filaments and fructifying threads, the latter in somewhat irregular forms, but most frequently producing moniliform terminal chains of spores; the fungus,

though apparently too insignificant to have attracted the notice of mycologists, being referable to the genus oidium. The largest of the spores were not unlike those of yeast; and other similar spores were seen in toruloid groups in the scum that existed on the surface of the liquid. Hoping that I had discovered the filamentous form of the *Torula Cerevisiæ*, I was anxious to investigate this mould further; but having used all the scanty growth for the examination already made, I set the glass aside to allow further development, and circumstances prevented me from looking at it again till nearly four months more had elapsed. I then found the sour liquid blacker than ever, and further reduced by evaporation, the only other change visible to the naked eye being that the same low white mould had grown again in small amount upon the side of the glass. Finding that it still retained the same characters under the microscope, I hoped that by transferring it to a saccharine solution I might get it to reproduce the *Torula Cerevisiæ*, just as I had got back the *Torula Ovalis* by placing its filamentous form in fresh urine. Accordingly, having taken up a portion of the mould with a "heated" knife, I introduced a morsel of it into a "heated" and covered glass containing freshly prepared PASTEUR'S solution, and placing the remainder in a drop of water between plates of glass, made a further examination with the microscope. a_1, in Plate XXIV., represents a fructifying filament, the segments of which are some of them in the form of a moniliform chain of spores, while others present a transverse line indicating tomiparous division into gemmæ, and one has given off a conidial bud, the last being an appearance comparatively rarely seen in this fungus when first removed from the wine-glass. But on examining again, after fifteen hours, the same specimen, which had been kept in a moist atmosphere to prevent evaporation, I found free spores in considerable numbers about the filament previously sketched, and the filament itself was studded with numerous fresh conidial buds, as shown in outline at a_2, the one previously present having dropped off. The great rapidity with which this conidial budding took place under the influence of the water is further indicated by the sketch at a_3 taken only two hours later, where all the buds present at the former examination are seen to have either grown larger or to have dropped off, while several fresh ones have made their appearance.

This abundant formation of conidia in the new medium increased my hopes that I should get back the *Torula Cerevisiæ* in a saccharine fluid. This hope, however, was doomed to disappointment. So far from the organism exhibiting in the glass of PASTEUR'S solution a toruloid development, it assumed there the opposite condition of a filamentous growth, in which any appearance of conidial formation was a rare occurrence. *b*, *c*, and *d* in Plate XXIV. represent sprouting conidia, *e* a very young plant, and *f* the extremity of a filament. The entire distinction of this fungus from the yeast plant was further shown physiologically by the fact that it grew extremely slowly in the saccharine

liquid, and failed to cause any evolution of gas in it, though kept under observation more than two months. I was thus led to conclude that this oidium had been merely an accidental concomitant of the yeast plant, having sprung, perhaps, from one of the adventitious filamentous plants noticed during the first few days in the glass, and having survived the chemical changes in the fermenting liquid under which the yeast plant itself had succumbed.

But though disappointed of the results which I had hoped to have obtained from this oidium, I made some other observations upon it which proved to be of considerable interest. The remarkable conidial development which took place from it in water seemed such a striking instance of change of habit in the plant induced by a new medium, that I thought it worth while to try what effect would be produced upon it by various other liquids, and among the rest by unboiled and uncontaminated urine; and on the 21st August 1872, I introduced into one of a series of "heated" and covered glasses of that fluid, prepared on the 10th of the month, and as yet unaltered, a minute portion of the organism from the glass of fresh PASTEUR'S solution, where, as before mentioned, it was growing slowly in a filamentous condition; the delicate threads becoming broken up in the process, and diffused in an invisible form in the liquid. At the same time, for the sake of comparison, I inoculated from the same source another glass of PASTEUR'S solution, as well as other liquids to which I need not here allude. In the fresh glass of PASTEUR'S solution the growth proceeded, as in the previous one, in the form of branched and septate filaments, one of which is represented in outline at *g*, Plate XXIV., on a smaller scale than the rest of the plate, while *g'* gives in detail a portion of the same filament as seen under the usual higher power: and in the course of two days the naked eye detected white specks upon the side of the glass, which the pocket-lens showed as little woolly tufts. Meanwhile, in the urine the glass had also become sprinkled with white specks, but under the pocket-magnifier, while some of them were filamentous, as in the PASTEUR'S solution, many presented a granular appearance.

In examining the growth microscopically, I availed myself of an arrangement which I have often found advantageous. At the time of inoculation I had introduced into the urine a small plate of glass, with pieces of fine silver wire connected with its ends in the form of hooks, by which it could be suspended from the rim of the glass; so that, lying horizontally in the liquid, it might arrest as they fell organisms diffused through it. The little apparatus had of course been previously purified by heat. Such a plate being carefully removed with "heated" forceps after development has advanced to any desired degree, and covered with a slip of thin glass, permits the examination of any growth that may have formed upon it, in a comparatively undisturbed condition. Thus, in the present instance, I was enabled to see *in situ* with the microscope the plants

which the pocket magnifier had revealed. Under a low power they presented appearances such as are shown at *a*, *b*, and *c*, Plate XXV., *a* being a purely filamentous growth, *c* a granular group, and *b* one exhibiting both characters in combination. Under the high power the granular parts were found to be composed either of groups of free pullulating cells of oval form, generally disposed in pairs, as shown at *e*, Plate XXV., or of plants of a most imperfect description, consisting of cells of a similar character to the free ones, or slightly more elongated, connected end to end, and often producing conidial buds, as in the specimen figured at *d* in the same plate.

On the following day the difference between the two glasses was still more marked. The filamentous plants in the PASTEUR's solution had considerably increased, but those in the urine had almost all fallen to the bottom, their places being taken by abundant specks and streaks of granular aspect, and even the few plants that still remained adhering had lost their purely filamentous character and had become granular. There were also little patches of scum upon the urine, whereas the surface of the PASTEUR's solution presented only some floating filamentous plants. I removed a portion of the scum with "heated" pipette, and submitted it to the microscope, and found it to consist exclusively of free oval cells, like those seen in the granular specks the day before, as shown in outline at *f*. In the course of the next twenty-four hours all appearance of filamentous growth disappeared from the urine; but while the liquid, which was now for the first time observed to have a slightly offensive smell, had become unsuited for that mode of development of the organism, it had stimulated the corpuscular form in a most remarkable manner, the scum having increased with amazing rapidity. Thus, between 8 P.M. on the 24th and 5.30 A.M. on the 25th, it grew from a loose patch, about half an inch in diameter, to a dense film that covered almost the entire surface of the liquid in the urine glass, and eight hours later, the cell growth had been so great that the scum had become pushed up upon the glass to about a quarter of an inch above the level of the liquid, while the urine was rendered cloudy by the subsidence of detached cells. In the course of the afternoon the liquid had become turbid throughout, and the air in the glass shade was still more decidedly offensive; yet, under the microscope, the only organism discoverable was that represented by the pairs of cells before described, so that we have here another clear example of fermentative change of putrefactive character induced in urine by other agency than bacteria. Samples of the cells are given in *g*, Plate XXV., where they are seen to resemble those of *d* and *e* in having vacuoles, but no nuclei, merely, in some cases, inconspicuous granules. In *g* is also given in outline a portion of the scum, showing how densely packed the constituent cells were, corresponding with the remarkable naked eye appearance, which was that of a dense white layer, like a film of paraffin.

On the same day (August 25), I introduced a small portion of this scum into a second glass of urine, prepared along with the former one fifteen days previously, but as yet retaining its brilliant clearness, and in other respects also unaltered, the transference being effected by a touch with the tip of a slender glass rod previously "heated." The results of this inoculation differed from those seen in the former glass of urine in this, that no filamentous plants at all now made their appearance, while, on the other hand, the corpuscular mode of development proceeded with great rapidity. Thus, eight hours after the inoculation, the side of the glass already presented streaks having a granular aspect under a pocket-lens, and a portion of scum which had remained at the surface had increased to four times its original extent, presenting the same dense white character as in the former glass, like a film of wax or paraffin. I examined a portion of the scum microscopically, and found it to consist in the main of cells, free or in pairs, formed by pullulation, as shown (for the most part in outline) in the sketch given at *i*, Plate XXV. But there was besides frequently seen an appearance of somewhat longer sprouts, like an abortive attempt at the formation of filamentous plants, of which also specimens are given in the sketch *i*. Twelve hours later the inside of the glass looked as if sprinkled over with coarse white sand, while the scum had grown so rapidly as to be more than eight times as large as when last observed. A portion of scum is represented in outline at *k*, where it is seen that there is no longer any appearance of long sprouts, the filamentous tendency having entirely disappeared, while the constituent cells are of smaller dimensions than before. Another point of much interest was the fact, that now, within twenty-four hours of the inoculation, the liquid, which, when inoculated, had the odour of perfectly fresh urine, was already markedly offensive; and after the lapse of twenty-four hours, while the scum had almost covered the surface of the liquid, the rank smell was strong. Now it may be remembered that in the former glass no offensive smell was observed during the first three days, though the filamentous growth had proceeded luxuriantly, and that it was only after four days, when the filamentous form had given place to the corpuscular and the scum had made its appearance, that the rank odour was perceived. Hence we are led to infer that the same organism may differ in its effects as a fermentative agent according to its habit, the toruloid form in the present instance being a much more energetic ferment than the filamentous. I had the opportunity of verifying this observation seventeen days later, when another glass of urine being inoculated with the same scum, there was again a rank smell in twenty-four hours.

But to return to the glass under consideration. On the 27th August, two days after inoculation, it stank as strongly as the first glass; and now on examining a portion of the scum with the microscope, I was surprised to find a very remarkable change in its constituent cells, which, instead of being oval bodies

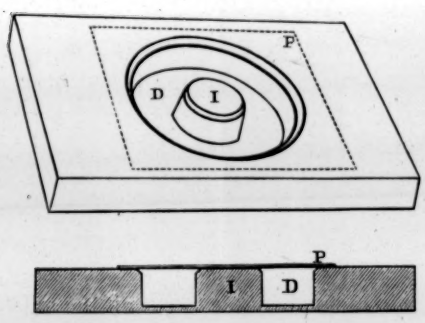
with mere vacuoles and inconspicuous granules, and either free or in pairs, were now of spherical form destitute of vacuoles, but strongly nucleated, as shown at *l*, Plate XXV., and disposed in considerable irregular groups, as seen in the outline portions of the sketch. The same character was maintained by the scum in this glass during the rest of the time (fifteen days) that it was kept under observation, *m* being a sketch of its appearance after the lapse of ten days; so that the organism had assumed completely the appearance of a spherical torula.

But it may be asked, Was I not deceived in supposing that the new toruloid form in the second inoculated glass had anything to do with the oidium? May it not have been a totally different species accidently present, just as the oidium itself was apparently adventitious in the yeast-glass? That all the oval cells should have disappeared within twenty-four hours, and given place to another species producing a scum of the same remarkable naked-eye appearance, seemed indeed very improbable; but, on the other hand, the difference of character in the cells was so remarkable, that if it was really only due to a modification of the same organism, it was desirable, if possible, to place the fact beyond doubt. With this object, on the 30th August, I mingled a morsel of the scum, by means of a "heated" glass rod, with a drop of PASTEUR'S solution* on a "heated" slip of glass, and placed upon it a "heated" piece of thin covering glass, and over this a larger plate of thin glass also "heated," overlapping the former one well on all sides, and luted down the margins of the upper glass with melted paraffin, applied with a hot steel pen. The object of this arrangement was that, while evaporation should be prevented by the paraffin luting, the interval between the thin glass plates might contain a supply of air to permit the growth of the fungus. I then selected for observation a group of the spherical cells near the edge of the liquid, and therefore near the air between the plates, and sketched them with camera lucida, as shown at n_1, Plate XXV. This was at 5.50 P.M. At 6.8 P.M., I noticed a change in the nuclei of the cells, such as I have often observed in spores preliminary to germination, as indicated at n_2, and by 11 P.M., the object being still undisturbed under the microscope, the lowest of the cells had not only increased in size, but had produced a considerable elongated sprout (see n_3), while the other cells were all markedly changed in the character of their nuclei. At midnight the sprout from the lowest cell had itself produced another sprout, also of oval form (see n_4), and by 7.45 next morning, when I next looked at the object, two other cells had been produced from the last sprout, while some, if not all the other cells of the group, had also germinated, giving rise to the appearance shown at n_5. And it will be observed that the products of this growth of the cells of the scum were not spherical and nucleated like them, but had the oval and vacuoled character of

* The PASTEUR'S solution contained 1 per cent. of alcohol, for reasons with which I need not trouble the reader.

the scum of the earlier period, so that the specific identity of the two growths was no longer doubtful.

I afterwards obtained still more satisfactory evidence on the same point. The long sprouts observed in the scum of the second glass of urine, a few hours after inoculation, seemed to indicate that the very liquid which, when altered by fermentation, induced the change of the organism to the toruloid condition, favoured, when perfectly fresh, a return to the filamentous form. I therefore resolved to watch, if possible, the earliest growth of the spherical cells of the scum in uncontaminated urine. For this purpose I proceeded on the same principle as before; but experience having shown that the thin layer of atmosphere between the glass plates was exhausted within a few hours, I tried a new arrangement for providing a larger supply of air, that which I ultimately arrived at being as follows:—A piece of plate-glass about $\frac{3}{8}$ths of an inch thick, and about $2\frac{1}{2}$ inches by $1\frac{1}{2}$ in the other dimensions (shown in diagram in the accompanying woodcuts, the lower of which represents it in section), is excavated by the lapidary into a circular ditch, D, round a central island, I, the island being $\frac{3}{8}$ths of an inch in diameter, and the ditch or air chamber of about the same breadth, and as deep as the thickness of the glass will conveniently permit, viz., about a quarter of an inch. A piece of thin covering glass, P, sufficiently large to cover the ditch as well as the island, but not quite so broad as the glass plate, so that it can be conveniently sealed down with paraffin, completes the "glass garden,"* which is stocked as follows:—The glasses must first be heated and allowed to cool, without access of dust to the air-chamber. The glass plate with the cover *in situ*, and covered further with a rather larger slip of ordinary glass, is placed upon a broad plate of metal on a retort stand, and over this a metal lid, such as that of a tin biscuit-box. Heat is then applied to the metallic plate by means of a BUNSEN'S burner or large spirit-lamp, till a drop of water sprinkled on the tin lid above passes off at once by ebullition. The lamp is then removed, and cooling is allowed to take place completely. The object of the metal plate and lid is to diffuse the heat, and thus prevent cracking of the thick and irregularly-shaped plate of glass. The lid above aids in keeping out dust during cooling, and this is further effected by the thin covering glass and the overlapping glass slip



* Such glass gardens may be obtained of Messrs SANDERSON, lapidaries, 92 Princes Street, Edinburgh.

above that. This slip is also useful in the stocking of the glass garden. Having been taken up from the other glasses, it is placed inverted on the table, so that the surface which was downwards during the cooling, and therefore free from dust, may be directed upwards. A few drops of the new liquid medium, in which development is intended to occur, are then placed upon it with "heated" pipette, and to these a minute portion of the organism is added and diffused thoroughly among the fluid by stirring with a "heated" glass rod. The thin covering glass being now raised by means of "heated" forceps, aided in the manipulations by a "heated" needle, a very small drop of the mixture of organism and medium is placed, by means of the pipette, upon the central island of the garden, and, in order to ensure a moist atmosphere in the air-chamber, a drop of water, which has been boiled, and cooled under protection from dust, is introduced with a clean "heated" pipette into the ditch.* The thin covering glass, which has been still held in the purified forceps, is then accurately replaced, after which its margins are luted down with paraffin, which is conveniently melted in an egg-spoon, and applied with a clean steel pen heated from time to time in the spirit-lamp. This process requires considerable delicacy and quickness of manipulation, and constant watchfulness; but with these conditions it may be conducted with most satisfactory results; and I have watched one and the same organism continuing to grow unmixed in such a garden for several weeks together, though carried about with me in a journey made in an autumn holiday.

As soon as the stocking of the garden is completed, it is placed under the microscope, and some individual specimens of the organism are sketched by camera lucida,—a map, on a smaller scale, being also made with the camera to enable the observer to find the objects again.

On the 11th September I stocked such a garden with a little of the scum from the second urine glass, mixed with uncontaminated urine from one of the glasses charged on the 10th August, the liquid still retaining its original brightness and fresh odour. The cells of the scum thus introduced between the island and the covering glass were all of the spherical character, as is illustrated by the groups at *a* in Plate XXVI., sketched at 7.20 P.M., within a few minutes of their introduction. At 9.50 P.M. the nuclei were found more conspicuous and altered in position, but there was as yet no change of form in the

* The actual order of proceeding is to introduce the boiled water into the air-chamber first, after which the same pipette, being clean, may be at once used for the liquid medium. I have found the most convenient form of pipette for these experiments to be a small syringe, having its nozzle connected, by means of a short piece of caoutchouc tubing, with a glass tube very narrow and thin, so that it is almost instantaneously heated nearly to redness by passing it through a flame, and cools with corresponding rapidity. The tube is bent near its middle at about a right angle; so that neither the syringe nor the hand is held over the experimental glass, while the yielding nature of the caoutchouc junction allows the end of the glass tube to be pressed, without risk of breaking, against any object, such as the side of a wine-glass, from which an organism is being picked up.

cells. Early next morning I found that the cells generally were sprouting; but it happened that those which I had drawn had shifted their position slightly, so that I could not distinguish them in their now altered shape from others in their vicinity, but I selected two groups for further observation, represented at b_1 and c_1, sketched at 1.30 and 1.35 A.M. respectively. It will be observed that, while this early stage of germination has changed them from the spherical to an oval form, they still retain their nucleated character. Five hours later, growth had advanced in both groups so as to give the appearance represented at b_2 and c_2. In both groups the nuclei have almost disappeared, while the sprouts have much increased; and in c_2, while the highest of the three cells has produced a short filament, the lowest has formed two oval vacuoled cells, and the other, after growing an oval cell, has gone on to the development of a short filament.

After four more hours had passed, I was rejoiced to find the experiment crowned with complete success. The longer sprouts of c_2 had become extended to threads of considerable length, as represented in c_3; while the progeny of the other original cell was in the form of pairs of oval vacuoled bodies destitute of nuclei, exactly resembling the constituents of the first scum, or of the granular deposits which accompanied the woolly tufts on the first urine-glass. And just as in that glass, at an early period, some plants exhibited the filamentous, and others the corpuscular form of growth, so was it with the offspring of the three spherical cells whose development we have followed.

Such was the effect of uncontaminated urine upon this organism. Afterwards, however, as the liquid gradually became vitiated under its fermenting influence, the filamentous form of growth which first appeared began to give place again to the corpuscular, a change which the "glass garden" afforded opportunity of watching with perfect precision. c_4 shows the lower of the two filaments of c_3 at 5.50 P.M. on the same day, represented on a smaller scale. It will be observed, that, while the filament has increased considerably in length, it exhibits a tendency to break up into segments, and here and there along its course it has produced oval corpuscles. And a further progress of the same alteration of habit is exhibited in c_5, where the same filament is again sketched on the same scale after the lapse of ten hours more, viz., at 3.50 A.M. on the 13th September. The filament has only increased very slightly in length, but the terminal portion has broken up into segments, and assumed a zig-zag form in consequence, while a multitude of corpuscles have been produced in the course of the filament, partly by budding of the segments of the thread, and partly by the pullulation of the corpuscles themselves, many of which are already of the spherical form. And the spherical cells, when examined with a high power, were found to be nucleated like those of the last scum. Here and there a plant was found in which, in consequence, I presume, of

greater vigour, the filamentous growth had proceeded further before the corpuscular development occurred, and formed septate branches, reproducing exactly the original filamentous form of the organism. This is illustrated by *d*, which represents part of another plant, drawn under the high power in the evening of the same day, and introduced not only on account of the delicate septate branch which it presented, but because nucleated spherical cells were seen to spring directly from little stalks on the thicker portion.

Next day I found one plant so beautifully illustrative of the whole subject that I took a sketch of it, which is represented at *e*, Plate XXVI., the drawing being on a much smaller scale, to enable me to include the whole. The plant had sprung from a spore situated not far from the edge of the island, and had grown towards the air chamber, and, arriving there, had continued to spread itself upon the under surface of the thin glass that formed the roof of the chamber. It will be observed that the part of the plant which is most distant from the air chamber has assumed the zig-zag form resulting from a tendency to break up into segments, and has produced a considerable number of spherical spores. Nearer to the air, again, the plant retains its original form, and has very few conidia; while the part in the air chamber presents the characters of a branched filamentous fungus entirely destitute of conidial formation, and this in the very same plant which in another part of its course has the loosely jointed character with spherical spores.

But how were these differences in different parts of the plant to be explained? Why did the portion in the air-chamber retain the purely filamentous and compact character, while the part on the island and other plants situated there became broken up, and produced conidia? The conidial development upon the island could not be the result of deficiency of oxygen; for this mode of growth occurred in greatest profusion in the scum of the urine-glass, which was freely exposed to air which was being constantly changed. And in point of fact, the air in the glass garden was not nearly exhausted at this period; for on examining it again on the 3d October, I found that the filamentous form of the fungus had by that time grown rampantly over the roof of the air-chamber, and had even grown down its walls in some places, and spread upon its floor. The obvious explanation appeared to me to be, that the agent which exercised the modifying influence upon the growth of the organism was some volatile product of fermentation, probably that which assailed the nostrils with a pungent stink, and that, where it was evolved in a limited space confined between the two plates of glass, it accumulated and produced its effect upon the plants. When, on the other hand, it was formed in the very thin film of liquid, which alone accompanied the plant on the roof of the air-chamber, it escaped into the air as fast as it was produced, and left the fungus unchanged. And this view is strongly confirmed by another fact, which I observed at the

time when the glass garden was stocked (on the 11th September), viz., that in the first urine glass the filamentous form of growth, which had been entirely suspended four days after inoculation, was again present in abundance, forming little woolly tufts, which studded the side of the glass. In other words, the urine had been restored to a condition compatible with the filamentous mode of development; and the natural explanation of this occurrence is, that the substance which exerted the modifying influence upon the organism, stimulating the corpuscular while checking the filamentous formation, was a volatile product of fermentation of some constituent of the liquid present in limited amount, and that when this constituent was exhausted, and the volatile product had escaped, the organism was again at liberty to form filaments, as it would have done if placed in fresh urine.

The investigation with the "glass garden" had thus abundantly proved that the filamentous fungus seen in the glass of PASTEUR'S solution, the pairs of oval vacuoled corpuscles of the primary scum in urine, and the spherical nucleated cells of a later period, were one and the same organism, modified by circumstances; while in the last-named variety we have another example of a plant presenting for weeks together the character of a pure and unmixed Torula, which, had I seen it only in that condition, I should have considered as much entitled to that generic name as the yeast plant, yet rigidly demonstrated to be a conidial development of a filamentous form. Comparing it with the *Torula Ovalis*, there is this curious difference between them, that whereas in the latter fresh urine is a medium in which the toruloid form especially flourishes, the filamentous growth making its appearance in it only when the liquid has been altered by the fermenting influence of the organism, the converse is the case with this plant. The present species, like the *Torula Ovalis*, failed to effect the ammoniacal fermentation of urea, the contents of the second urine glass being found still sharply acid on the 5th of November, ten weeks after inoculation. Yet it is, as we have seen, an energetic putrefactive ferment of some of the urinary constituents, and on this account is attended with considerable interest. And as the remarkable naked eye appearance of the scum which it forms in that liquid when altered under its agency, and the toruloid character of the constituent cells, appear to furnish sufficiently definite specific characters, it seems desirable that it should be named, and I have suggested for it the title *Oidium Toruloides*.

Some other points observed in the investigation of this plant appear of sufficient interest to be placed on record. One is, that the spherical toruloid cells of the scum of the second urine glass, when introduced into a fresh glass of PASTEUR'S solution, produced none of the purely filamentous growth such as resulted from the inoculation of the two previous glasses of that liquid with the filamentous form of the organism, any threads met with being only of a very

loose and imperfect character, like that represented at *d*, Plate XXV., while the chief product of the development was pairs of oval vacuoled corpuscles, resembling those of the scum of the urine at an early period. And the result was not only a granular deposit on the side of the glass, but a scum upon the surface, whereas neither of the other glasses of PASTEUR's solution had shown any scum. This difference between the glasses continued as long as they were kept under observation; that inoculated with the toruloid scum still presenting a growth mainly of scum, without any filamentous appearance visible to the naked eye till the 14th of September, eighteen days after inoculation, while the other two glasses had still no scum whatever, and exhibited abundant conspicuous woolly tufts. This fact is of itself proof of a very important general truth, viz., that a particular habit of growth impressed upon an organism by temporary residence in a new medium may sometimes be retained for a long period after it has been restored to its former habitat. The effect of the stale urine upon this plant was to substitute the corpuscular for the filamentous mode of development; and although, when returned to the PASTEUR's solution, there was a degree of recovery, as indicated by the change from the spherical nucleated cells to the oval vacuoled corpuscles, and still more by the occasional appearance of coarse imperfect threads, yet the original character was not restored during the eighteen days of observation. And this circumstance is the more interesting, when it is remembered that the corpuscular variety appeared to differ from the filamentous in fermentative power, the former being more energetic in its effect on urine than the latter. Facts of this kind may tend to elucidate points of great importance in the history of contagious diseases, such as the greater virulence of such disorders at some periods than at others. For it seems highly probable from analogy that the *materies morbi* may be of the nature of minute organisms; and if this be the case, we can understand, from what we have seen of the plant under consideration, that differences of energy in the virus may be occasioned by varying circumstances.

The failure of the plant to resume the filamentous habit when returned to PASTEUR's solution, makes it the more remarkable that it should have recovered that power in fresh urine, implying that this secretion, when in a perfectly unaltered condition, is a still more favourable medium for the organism, permitting a degree of recovery which was impossible in PASTEUR's fluid.

The last fact which I have to mention regarding this plant, is its behaviour in an albuminous liquid. This medium, which also proved valuable in experiments to be described in a later part of this paper, was prepared on the same principle as the unboiled urine, by taking the material uncontaminated from its natural receptacle, by aid of antiseptic measures. An egg, known to have been

laid within the last twenty-four hours, was steeped for a while* in a solution of carbolic acid in twenty parts of water, to destroy any organisms adhering to the shell, and was then broken in a fine spray of carbolic acid solution of the same strength, and about an ounce of the white of the egg was introduced into a flask containing ten ounces of water, which had been boiled and allowed to cool, the air which entered during cooling having been filtered of dust by a mass of cotton wool tied tightly over the mouth of the vessel before boiling. The flask was agitated occasionally during the next twenty-four hours, to promote diffusion of the albumen in the water, after which the liquid was passed through a boiled filter placed in a "heated" funnel, protected with a "heated" glass cover, under a large glass shade.† It was thus cleared of the shreddy residue of the white of egg, and also of the opaque floccules resulting from the action of the carbolic acid spray upon the albumen, and was obtained of crystal clearness in the "heated" flask into which it was received, and in which it was kept protected from dust by a "heated" glass cap and a glass shade. A "heated" wine-glass, provided with cover and glass shade as usual, being charged with some of this liquid, I inoculated it with a little of the toruloid scum from the second wine-glass on the 3d September. The result was a corpuscular development of a delicate inconspicuous character, the growth proceeding so slowly that the little patch of scum, in which alone any increase was observed, had not doubled its diameter in ten days. I now introduced with a "heated" needle a little piece of the fungus, in the filamentous form, from the first glass of PASTEUR'S solution. This retained the filamentous mode of growth in the new habitat, but increased so slowly that after the lapse of six weeks the little woolly mass which lay at the bottom of the glass had only grown to the height of an eighth of an inch, while the patch of scum was but very slightly larger than before, and a mere trace of granular deposit was seen upon the glass.

But though the growth of the organism in this medium had been so extremely languid, it had effected a very remarkable change in its constitution, the liquid, though still clear, having been altered from its original crystal purity to a deep rich brown colour, like that of porter.

It happened that I had inoculated another glass of this same albuminous fluid

* The actual time was much longer than I had intended, viz., two days. A subsequent experiment, in which one hour and twenty minutes was the period of immersion, was equally successful. Even after the two days of the present occasion, the carbolic acid did not seem to have affected the albumen, which was free from coagulation to the surface.

† This was a most troublesome procedure to carry out. I afterwards simplified the process very much, so as to dispense with both the spray and the filter, extracting the albumen with "heated" pipette passed into a hole made in the carbolised shell with "heated" forceps, a piece of carbolised cotton wool being wrapped round the pipette and egg to prevent entrance of dust, filtration of the mixture of albumen and water being effected by decanting through a boiled syphon, which had a piece of sponge tied over the end in the flask.

seven weeks previously with another very delicate filamentous fungus, which I must not here describe. The species had developed very luxuriantly, so as to occupy the greater part of the liquid with its white woolly growth, and clamber some distance up the inside of the glass above. Yet the colour of the fluid was scarcely altered at all, having a barely perceptible pale brownish tinge, and this circumstance made the great effect of the scanty growth of the *Oidium Toruloides* the more striking. At the same time, the dark brown liquid was entirely destitute of odour, and thus I obtained for the first time demonstration of what I have long suspected, as the result of experience in antiseptic surgery, viz., that an albuminous fluid may be affected with a fermentative change without the occurrence of smell. I have seen, for example, a psoas abscess furnish merely a slight oozing of serous discharge under antiseptic management, till a single careless application of the dressing admitted, as I believed, some fermentative organism, which, without giving rise to any odour, so altered the character of the discharge, as to stimulate the diseased part to profuse suppuration, leading to death by hectic. I have also observed erysipelas occur in spite of antiseptic treatment, and occasion profuse suppuration without smell, although from analogy there is reason to suspect that the virus of that disorder is of the nature of an organism, operating as a ferment upon the animal fluids. Facts such as these had often led me to express the view which at the time might be regarded as transcendental, but which the above observation proved to be a truth.

Thus this single insignificant species, when subjected to the precise method of investigation which I have described, afforded proof of several important general truths, which may thus be recapitulated.

1*st*, It shows how greatly such organisms may vary under the modifying influence of different media.

2*d*, It affords another clear example of the origin of a torula from a filamentous fungus.

3*d*, It shows that the corpuscular form of such an organism may differ in fermentative energy from its filamentous parent.

4*th*, That the corpuscular habit of growth acquired in one medium may be retained for a considerable time after the organism has been restored to a habitat in which the corpuscular form did not originally present itself.

5*th*, That when placed in a more favourable medium, the toruloid variety may reproduce the purely filamentous.

6*th*, This plant is another instance of an organism which is not bacteric, giving rise to a putrefactive fermentation in urine.

7*th*, It proves that an albuminous liquid may be affected with a fermentation with inodorous products.

Lastly, The trustworthiness of the method of investigation is strikingly confirmed by the fact, that in none of the glasses of PASTEUR'S solution, urine,

or albuminous fluid inoculated with this oidium, and in neither of the glass gardens, did bacteria, or any other kind of fungus besides the one intentionally introduced, make their appearance during the entire month in which the observations were made.

EXPLANATION OF THE PLATES

Illustrating Professor Lister's Paper on the Germ Theory of Putrefaction and other Fermentative Changes.

Plate XXII.

Figure 1. A pencil of fructifying threads of the common blue mould *Penicillium Glaucum*.

Figure 2. A group of cells of the yeast plant, *Torula Cerevisiæ*.

Figure 3. Bacteria from various sources. The pair below the letter *a* are examples of a granular appearance of the protoplasm, and the presence of a distinct nucleus in each segment. *b* a Leptothrix filament, some of the segments being nucleated.

Figure 4. Illustrates the ordinary mode of growth of Bacteria, viz., by increase of the segments lengthwise and transverse segmentation. When first sketched, at 7.30 A.M., the object consisted of three segments a_1, c_1, b_1. During the few minutes that elapsed between the completion of this sketch, and that at 7.42 A.M., the uppermost segment b_1 is seen to have increased in length to the size shown at b_2, and the two lower ones are not only longer, but each presents a transverse line of segmentation, while the middle segment is bent at this new place of division, c_2. Three minutes later the three lowest of the five segments of which the object now consisted separated from the other two, and in the sketch taken at 7.48 they are shown thus detached, the lower two obviously increased in length. Two minutes later one of these three was found to have separated and moved off, and the remaining pair were observed to swim away as an ordinary double bacterium.

Figure 5. Represents a minute organism, consisting of granules grouped in a different manner from that which commonly prevails among Bacteria. The difference of arrangement is explained by difference in the mode of growth, as is illustrated by the sketches c_1, c_2, c_3, and c_4 which represent the same granules in process of fissiparous generation. It will be observed that the granules, instead of increasing like ordinary Bacteria in one direction only, swell up in all dimensions and afterwards undergo segmentation, either into pairs or into fours, as indicated in the letterpress, page 319.

Figure 6. Represents a form of Torula which resulted from the mingling of a drop of rain with fresh uncontaminated urine. Appearing in the first instance as an unmixed Torula (*a*), it changed in course of time to a delicate filamentous fungus (*b* and *c*) bearing buds, some of which were more or less toruloid in aspect and habit of growth, while others were morphologically identical with Bacteria, as described in detail in the text.

Plate XXIII.

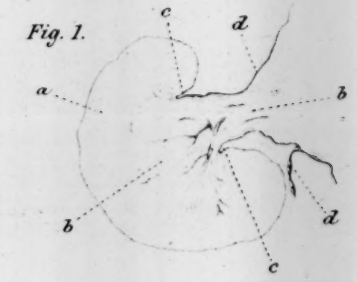
Represents the same organism (*Torula Ovalis*), varying in character according to the medium in which it grows, and the period during which it has inhabited it. For a detailed description, see letterpress.

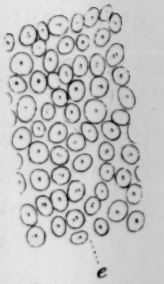
Plates XXIV., XXV., and XXVI.

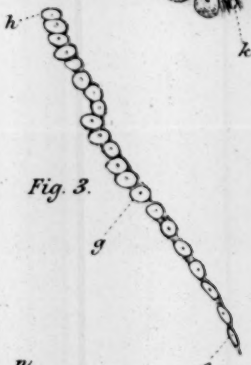
Show a minute fungus varying according to its habitat, from a filamentous growth to Torulæ of very different characters, all distinctly traced to one and the same organism. For a detailed description, see letterpress.

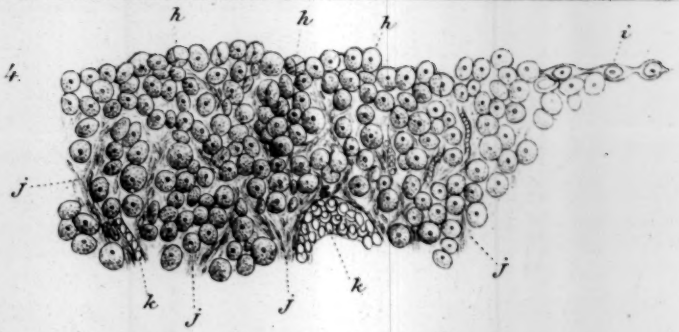
These illustrations are all taken from camera-lucida sketches, the magnifying power being 1140 diameters, except in some cases where it is lower, as indicated by the scales on the plates.

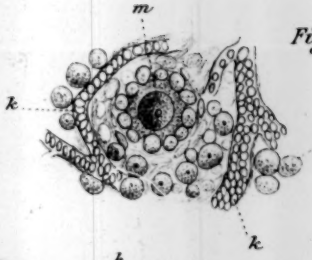
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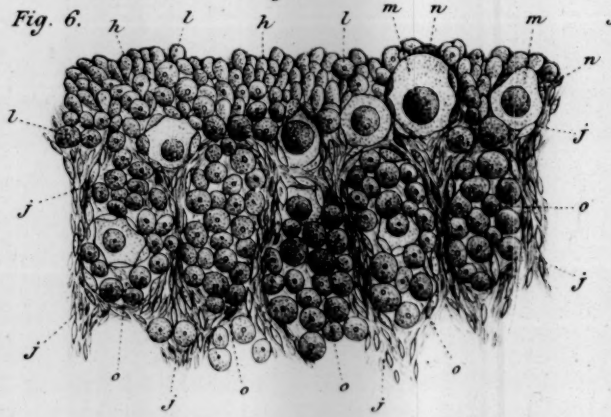


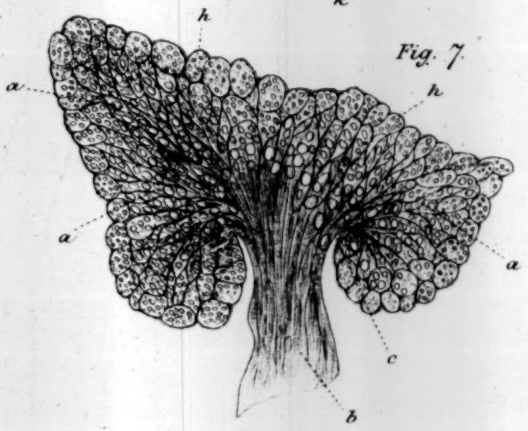








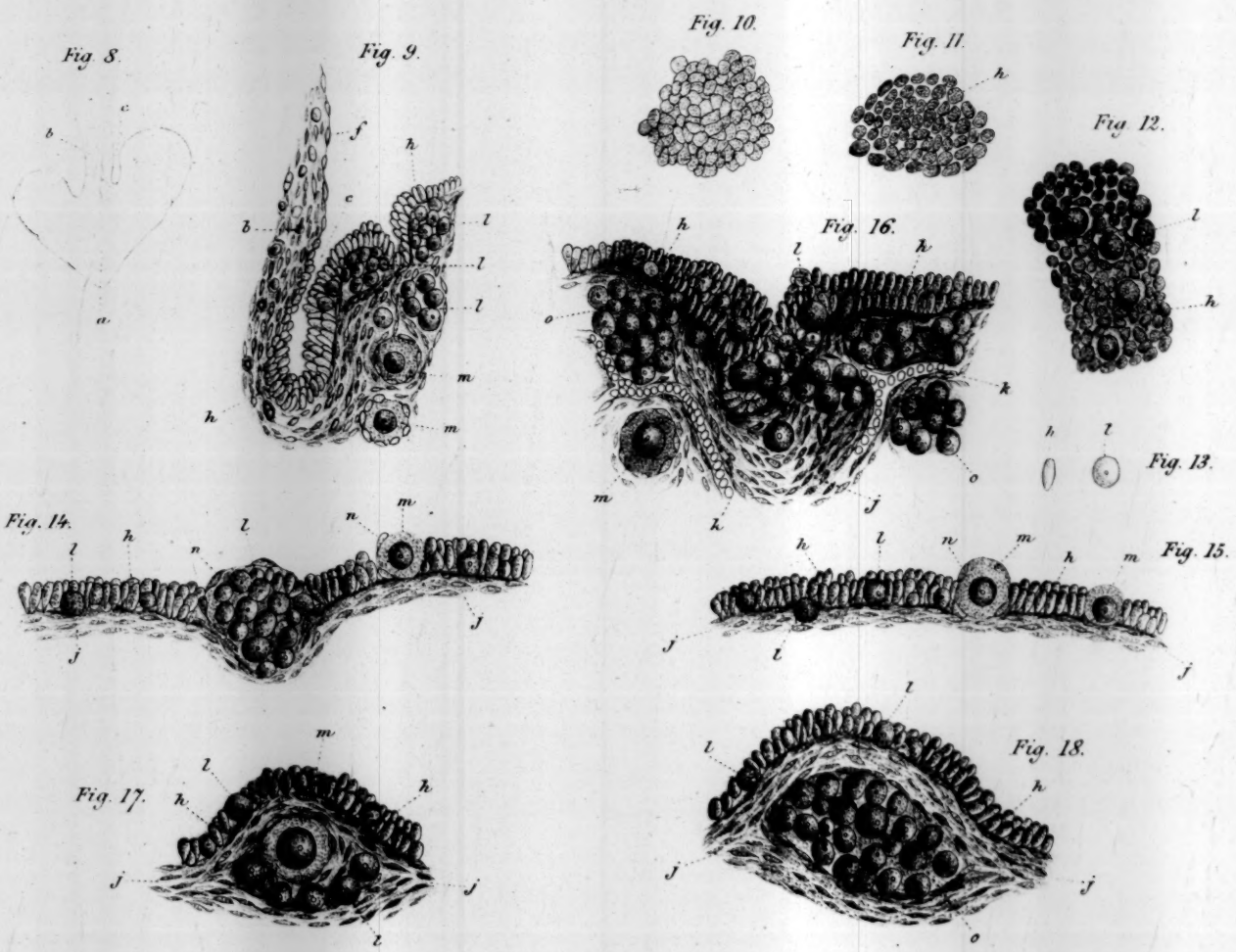




Jas Foulis, M.D. del.

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Fig. 8.
Fig. 9.
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Fig. 19.

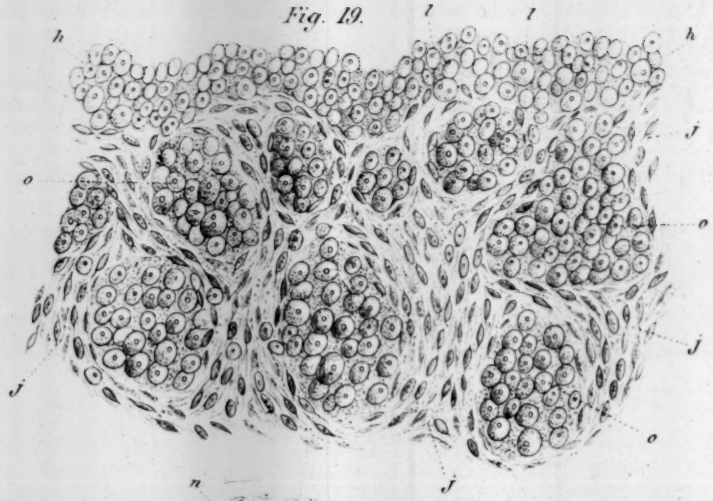


Fig. 20.



Fig. 21.



Fig. 22.

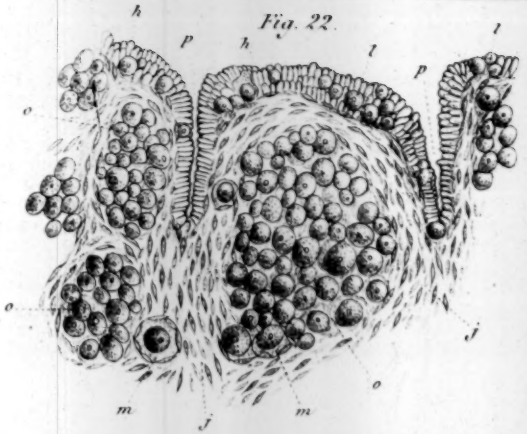
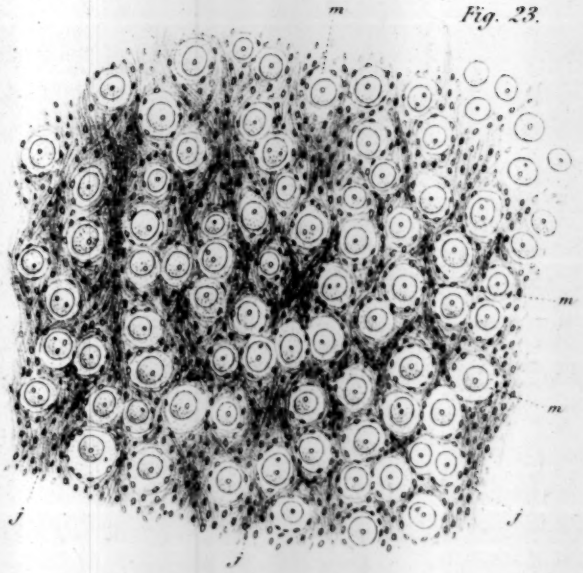


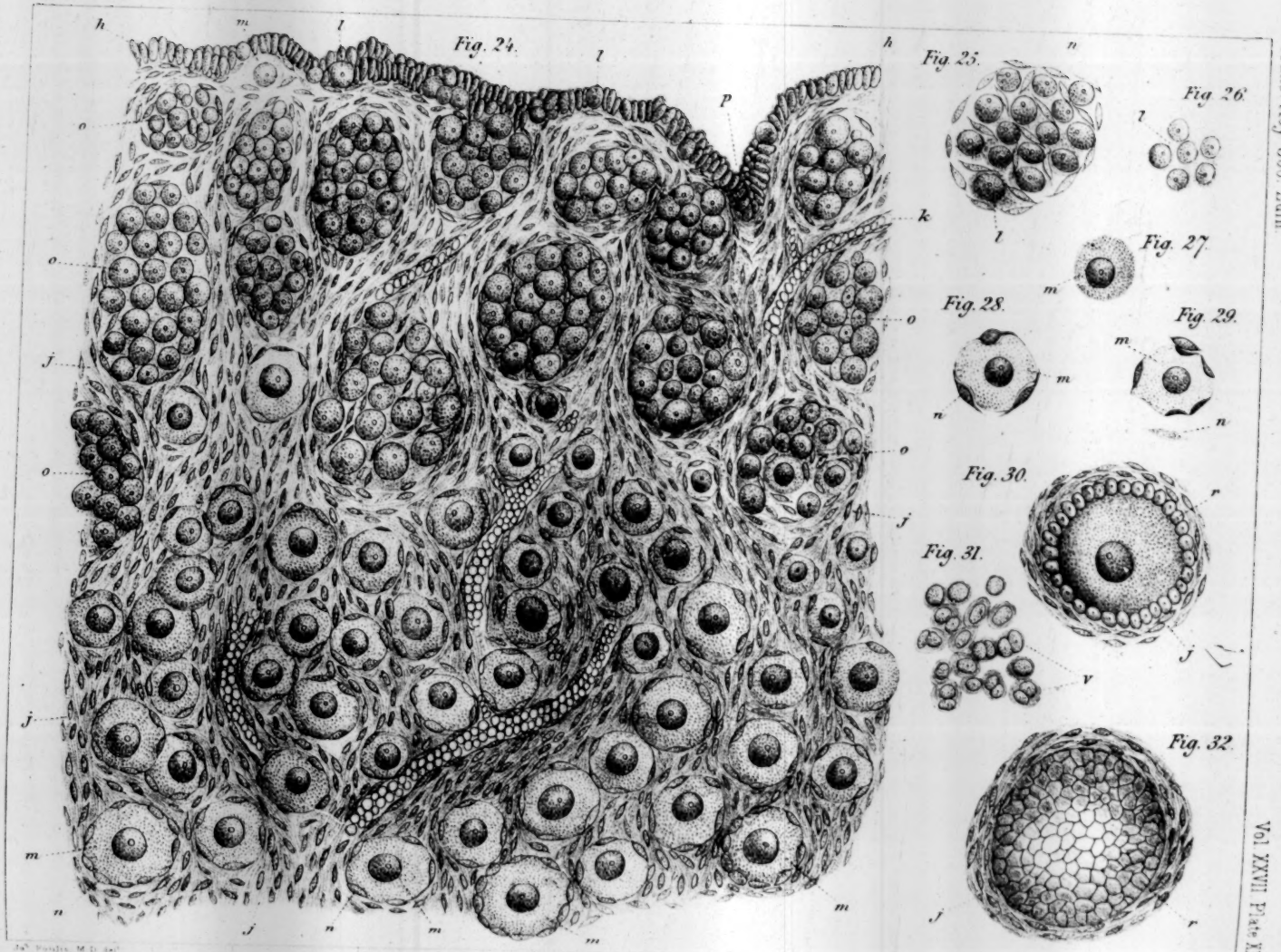
Fig. 23.



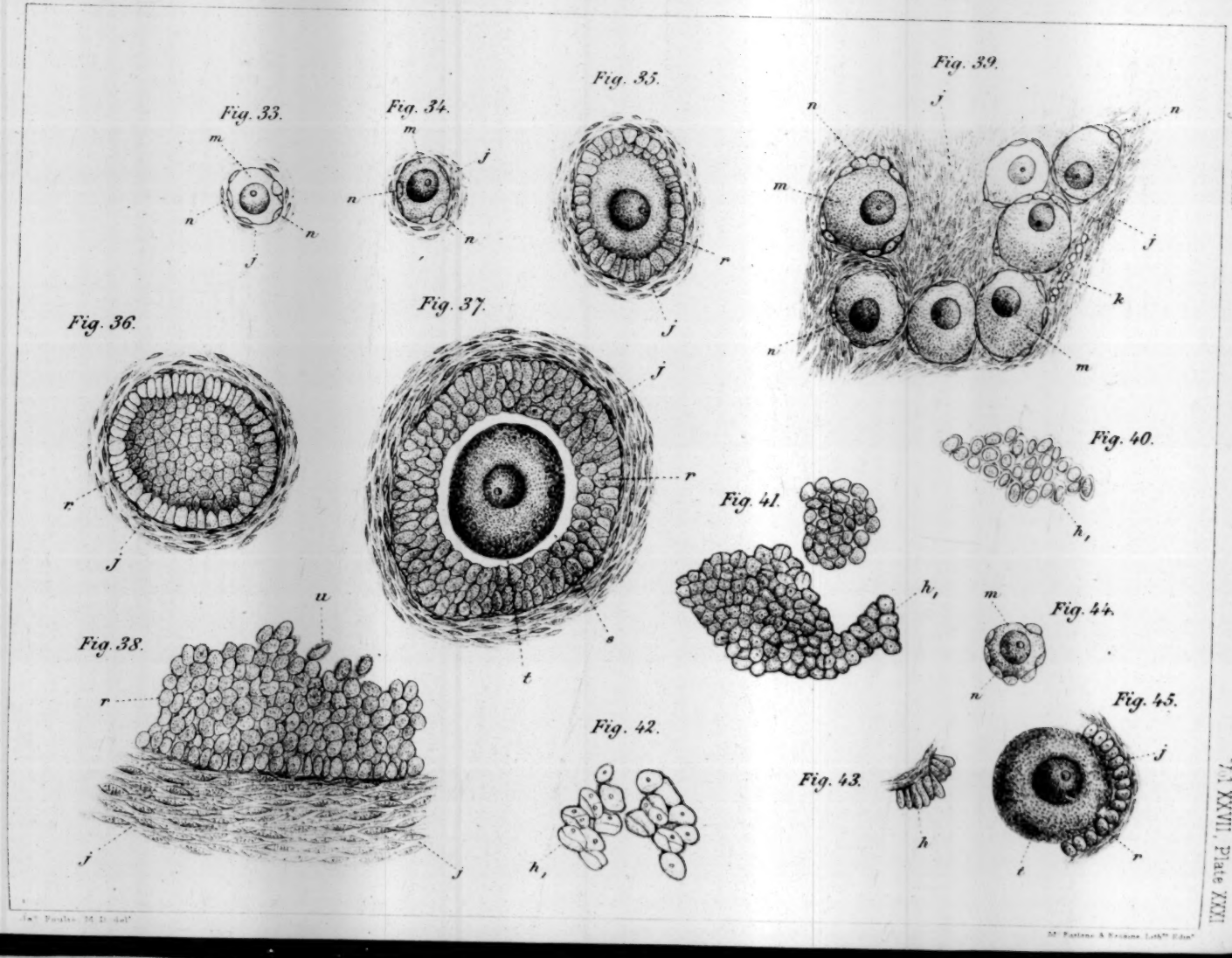
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Fig. 24.
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Fig. 26.
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Fig. 33.
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XVII.—*On the Development of the Ova and Structure of the Ovary in Man and other Mammalia.* (Plates XXVII.–XXXI.)

By James Foulis, M.D. (Edin.) Communicated by Professor Turner.

(Read 21st December 1874.)

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In the month of August 1872, Professor Turner suggested as a subject of investigation the structure of the ovary, and development of the ova, more especially with reference to the recently published observations of Waldeyer.

In the month of April 1874, I handed in to the Medical Faculty of the University of Edinburgh, as my graduation thesis for the degree of M.D., an essay entitled "Contributions to the Normal and Pathological Anatomy of the Ovary and Parovarium."

Accompanying this thesis, were numerous microscopic preparations of the human fœtal ovary, by means of which I was able to demonstrate my descriptive remarks on the anatomy of the organ. Since that date I have made many additional observations on the anatomy of the ovary, especially in connection with the development of the ova, and the formation of the membrana granulosa; and in the present memoir the result of my observations is given. During the last eighteen months, I have made numerous microscopic preparations of the ovaries of various fœtal and adult animals, most of which I now possess, and a careful examination of these verifies my earlier statements on the development of the ova, and the structure of the ovary.

It is not my intention to give a lengthened account of the views of earlier observers on the development of the ova and the structure of the ovary. It may, however, be interesting to notice the following points in the history of this subject.

Previous to the time of De Graaf, the ovary and testicle were considered

identical in structure. In 1671, DE GRAAF introduced the word ovary, and described the Graafian vesicles as ova, which he said were produced in the ovary. The distinction between the true ovum and the Graafian vesicle was not made out till the year 1824, when Messrs PREVOST and DUMAS, by numerous observations, were led to the conclusion that the ova were contained in the Graafian vesicles before conception. The mammalian ovum itself was discovered by VON BAER in 1827, and in 1834 COSTE and others discovered the germinal vesicle in the mammalian ovum. In 1835, WAGNER demonstrated the germinal spot and established the doctrine of a general uniformity in the structure and mode of origin of the ova of animals.

The discussions in the numerous works which appeared between the years 1835 and 1838 turned more or less on the question whether the germinal vesicle, or the ovum itself was first formed, or whether the Graafian vesicles were first produced, and the eggs subsequently developed in them. In MÜLLER'S "Archiv. für Anatomie und Physiologie," 1838, there is a paper by VALENTIN, on the development of the Graafian follicle in the ovary of mammals. He described the Graafian follicles as being formed within long tubes which originally constituted the special structures of the ovary, as the seminiferous tubes do of the testicles, but which become obliterated by the increasing growth of the Graafian follicles.

MARTIN BARRY, by his numerous observations, was led to the conclusion that the germinal vesicle is the part which first makes its appearance in the stroma of the ovary at the commencement of the formation of the ova.

VALENTIN, BISCHOFF, and others, held the view that the Graafian follicles may be detected in the stroma of the ovary before any part of the ovum can be distinguished.

It was with the appearance of PFLÜGER'S work that the views of the structure of the ovary and development of the ova took a new turn. VALENTIN, in 1838, as already observed, first demonstrated the branched and tubular glandular structure of the ovary; an observation which BILLROTH corroborated. Little attention, however, was paid to this observation until it was rediscovered by PFLÜGER, who in a detailed monograph developed his views respecting the structure of the ovary.

In the first place, PFLÜGER showed that the Graafian follicles with the eggs do not appear individually and independently in the stroma of the ovary. He described tubes as passing from the surface of the ovary down into the stroma, composed of a structureless membrana propria, by the inflections of which the several Graafian follicles are successively divided off.

PFLÜGER, (as quoted by WALDEYER),* stated that the tubes lying close beneath the surface of the ovary terminate by cœcal extremities, and in these

* STRICKER'S "Handbuch der Gewebelehre," article Ovary, by WALDEYER, translated by Mr POWER.

the germinal vesicles originate surrounded by a diffuse protoplasm, that forms a more definite investment for each as they become more deeply situated with the tube. Thereupon a number of the cells become conspicuous by their more vigorous growth, whilst the rest remain unaltered, and form the epithelial lining of the tubes. The larger cells, which are primordial eggs, occupy the axis of the tube.

These subsequently increase in number by fission and budding, the products constituting the definite eggs that for a time remain connected with one another in the interior of the tubes in the form of a chain, by processes of protoplasm, constituting the "egg chains" of PFLÜGER. The membrana propria outside the epithelial lining of the tubes sends in processes between the individual eggs, in consequence of which the latter are gradually separated from each other along with a portion of the tubular epithelium; the latter forms the epithelial lining of the Graafian follicles in this way produced. PFLÜGER, in one of his figures (plate iii. fig. 1), has represented a connection existing between his tubes and the superficial columnar epithelium, and has frequently remarked that the contents of the egg tubes must proceed from the ovarial epithelium, which he always considers to be a serous epithelium.

The most important work published within the last few years on the development of the ova and ovary is that by W. WALDEYER, entitled "Eierstock und Ei." (Leipzig, 1870).

According to WALDEYER, the first appearance of the ovary consists of a thickened germ epithelium investing a small outgrowth rich in cells, which projects from the interstitial tissue of the Wolffian body on its median side. The thickened epithelium investing this outgrowth gradually forms the rudiments of the Graafian follicles and ova, and of the subsequently appearing epithelium of the ovary, whilst the outgrowth itself is destined to furnish the vascular stroma of the ovary.

In the embryo of fowls, WALDEYER states, the interesting observation may be made as early as the fourth day of incubation, that some among the germ epithelial cells have become conspicuous by their round form, their size, and the size of the nucleus they contain. We may conclude, from the regular arrangement of these structures and the constancy of their position, that they represent the youngest primordial ova, which thus, even during embryonic life, are formed by a simple process of growth from the epithelial cells of the germ organ. The further development of the ovaries depends on a peculiar mode of growth of the superficial epithelium on the one hand and of the vascularised stroma on the other. Certain more or less delicate processes of the connective tissue now shoot forth from the stroma, whilst coincidently the epithelium increases by the continual production of new cells. The processes then penetrate between the epithelial cells, enclosing a variable number of them, which

thus by degrees come to be more or less deeply imbedded in the vascular stroma. From the plan and mode in which these changes take place it is evident that the several epithelial masses must be connected with each other in a plexiform fashion, and consequently that the ovary at this period of development is composed of a framework of connective tissue, the meshes of which communicate freely as in a cavernous tissue. Some, and sometimes many, among the imbedded epithelial cells become conspicuous by their size and the size of their nuclei, as we have already seen to occur amongst the superficial epithelial cells. Other cells remain of small size, and surround the larger cells as a kind of epithelium. The connective tissue stroma between the imbedded masses of epithelial cells constantly undergoes increase, and especially grows in between the several egg cells with their epithelial investment. Thus each epithelial ball is divided by these ingrowing vascularised trabeculæ into as many cavities as it contains egg cells.

In describing the ovary of a newly born child, WALDEYER thus states, in reference to its tubular structure, "One sees long branching formations in the form of tubes, anastomosing with each other, as VALENTIN first described, and lying separate from each other at considerable distances. They pass upwards opening with narrow mouths into the epithelium, and appear as direct tubular gland-like processes of it.

"At the time in which the tubes described by PFLÜGER exist, that is, as far as I can find, from the ninth month till a short time after birth, they present the structure ascribed to them by PFLÜGER, with the exception already mentioned, that there is as little of membrana propria in them as there is in the primary follicles. In the tubes, and mostly in the middle of them, as PFLÜGER described, we meet with egg cells distinguished by their size and form, often immediately concatenated one behind the other. Whether in the tubes new egg cells are formed, I cannot decide; but I think it likely, because here, as well as on the surface epithelium, some epithelial cells may develope into egg cells. Division of the egg cells in the tubes PFLÜGER seems to have observed, but I have not seen it in fresh specimens."

Follicles are formed from the tubes as well as from the egg compartments, directly through the growth of interstitial tissue. At the lower end of the tube, as may be well explained from the want of a membrana propria, interstitial tissue grows into the tubes and encloses the individual egg cells along with a portion of the not fully developed epithelial cells which surround them, and in this way primary follicles are produced.

In summing up WALDEYER thus remarks:—

"As the chief result of my investigations, it must be stated that both the egg and the follicular epithelial cells are derived directly from the germ epithelium. There is a reciprocal growth of vascular connective tissue and germ

epithelium cells, in consequence of which large and small masses of the latter become imbedded more and more in the stroma of the ovary. The imbedded cells present a variety. Some of them, by simple increase in size, grow into ova, viz., primordial ova, while others keep to their original size, and by numerous divisions, at least as it appears to me, produce still smaller cells, viz., the follicular epithelial cells. A genetical distinction between primordial ova and follicular epithelial cells has consequently no existence. The germ epithelium is the common source of both. The first origin of eggs cannot be looked for in the ovary, but dates much farther back, even to the beginning of embryonic life.

The tubes of VALENTIN and PFLÜGER can lay claim only to a secondary importance, and are not essential for the egg and follicle formation; the greater part of the follicles have undoubtedly an earlier existence, long before these tubes are formed."

In the following description of my observations on the development of the ova and structure of the ovary, I shall in the first place state what I have seen in the fœtal calf, then describe the ovary in young kittens, afterwards give an account of what I have observed in the human ovary, make some general observations on the development of the membrana granulosa in adult ovaries, and conclude with a few remarks on the structure of the ovum.

The drawings in illustration of this paper are all original, and have been carefully prepared, by the aid of the camera lucida, from microscopic preparations in my possession.

THE OVARY OF THE CALF.

If thin sections are made through the ovary of a fœtal calf of about nine inches in length, the following appearances may be recognised in them. A section of such an ovary presents for examination two parts,—an external cortical or parenchymatous part (fig. 1, *a*), and an internal fibro-vascular part (*b*, *b*). The latter is directly continuous with the peduncle or stalk to which the ovary is attached at the hilum (*c*, *c*). On the surface of the peduncle, above and below, is a layer of epithelium (*d*, *d*) directly continued from the peritoneal epithelium towards the ovary; and when the layer is detached from the stalk or peduncle, and examined under high powers of the microscope, it is found to consist of small corpuscles, with large, clearly-defined, oval-shaped nuclei (fig. 2, *e*). Many of the nuclei contain granular matter in their interior. Each nucleus presents a sharply defined wall. In their largest diameter the nuclei measure about $\frac{1}{250}$th of an inch. The protoplasm round each nucleus is very clear and transparent. Between the nuclei are seen delicate lines, indicating apparently the surfaces of contact of the contiguous corpuscles. The clear substance round some of the nuclei is so small in quantity that the nuclei are almost in

contact; but in other instances it is in considerable quantity, and the nuclei are proportionally distant from each other.

When such a layer of epithelium is seen in profile (fig. 3, *f*, *g*, *h*), it consists of nuclei as before described, placed close to each other. Around each is a quantity of clear protoplasm, which abuts against that of its neighbour. The nuclei are arranged in a single layer, and as we approach the ovary they become placed closer and closer together, and more columnar in form, and the protoplasm investment of each becomes gradually less and less in quantity till we reach the actual surface of the ovary at the lower border, where we find the nuclei crowding together and continued on to the surface of the ovary in the form of the germ epithelium. As the nuclei pass on to the surface of the ovary they are no longer arranged in a single layer, but crowd one on the top of the other, and it is then not possible to say where peritoneal epithelium ends and germ epithelium begins.

All round the ovary the germ epithelium consists of a thick layer of corpuscles. Each corpuscle is a nucleus surrounded with a thin film of protoplasm.

On the upper and lower surfaces of the stalk, as seen in section, the peritoneal epithelium is continuous with the germ epithelium at the lower borders of the ovary in the manner described. The germ epithelium consists of corpuscles (fig. 4, *h*, *h*, *h*), arranged in the form of a thick layer passing round the ovary from one lateral border to the other, and gradually thinning off as it passes into and becomes continuous with the peritoneal epithelium.

The corpuscles of the germ epithelium vary considerably in size and shape; some are round, others oval or columnar, some are twice as large as their neighbours. The greater number are spherical, and within the larger ones a distinct spot or nucleolus is seen. Many of the corpuscles may be seen undergoing division into two or more parts. By this process of fission the corpuscles are ever producing new elements, which in their turn produce similar ones.

The largest germ epithelial corpuscles measure about $\frac{1}{2000}$th of an inch in diameter. In several instances among the germ epithelial corpuscles on the surface of the ovary, I have observed some individuals flattened from above downwards, and the protoplasm round the nucleus drawn out at both ends in such a manner as to present the appearance of a fusiform corpuscle, or peritoneal epithelial cell seen in profile (fig. 4, *i*.)

Below the germ epithelium the cortical zone of the ovary consists for the most part of corpuscles which resemble very closely the germ epithelial corpuscles. A careful examination of this part of the ovary shows that from the deeper or fibro-vascular zone delicate bundles of young connective tissue with blood-vessels (fig. 4, *j*, *j*, *j*) pass upwards among the corpuscles in a radiating manner towards the germ epithelium; offshoots proceed in various directions from these bundles, and give rise to the formation of irregular-sized

meshes of vascular connective tissue, in which are enclosed large and small groups of corpuscles.

In the cortical zone such groups of corpuscles are found under the germ epithelium all round the ovary, and in many situations they are in connection superiorly with the corpuscles of the germ epithelium. At this stage of development the germ epithelium cannot be separated by any distinct line of demarcation from the masses of corpuscles which are below it, but at a later stage the germ epithelium rests on a thin irregular stratum of young tissue. This stratum is part of the fibro-vascular stroma of the ovary, and is formed by the growth of that tissue round the groups of corpuscles, which at an earlier stage of development are found in connection with the germ epithelial corpuscles all round the ovary.

Tracing downwards the bundles of vascular tissue which lie between the groups of corpuscles, we come to the fibro-vascular zone of the ovary (fig. 1, *b*). It must be understood that there is no boundary line between the cortical and fibro-vascular zone; the division is more or less artificial, but it is certain that the cortical, and by far the greater part of such a young ovary, consists of corpuscles with a few bundles of vascular tissue intermingled, while the deeper fibro-vascular zone, as its name implies, consists of very vascular tissue arranged in the form of a mesh-work, and in these meshes are included large corpuscles in various sized groups.

In this fibro-vascular zone we find numerous blood-vessels anastomosing freely with each other (fig. 5, *k*, *k*); the blood-vessels everywhere lie in the midst of bundles of young connective tissue. In the smallest meshes of fibro-vascular tissue generally one large corpuscles is found (fig. 5, *m*). The largest corpuscles consist of a central, large, bright nucleus with a nucleolus; the nucleus presents a sharply-defined double-contoured wall, and is surrounded by a considerable quantity of protoplasm, in contact with which are several very small corpuscles, forming a sort of wreath round the large corpuscle. Immediately outside this wreath is the vascular tissue of the stroma. Many of such large bodies as are now described lie imbedded throughout the fibro-vascular zone of the ovary, especially in its deeper parts; more superficially, many large bodies, but generally without the wreath of smaller corpuscles, are found in the meshes of the stroma; and as we pass upwards in our examination of the cortical zone, we find the fibro-vascular tissue decreases in quantity, but the corpuscles become more numerous, though smaller, till we reach the germ epithelial layer, which consists entirely of corpuscles.

From this description of the appearances presented in a section of the ovary of a very young fœtal calf, we learn that the greater part of the parenchymatous zone of such a young ovary consists of corpuscles very similar in appearance to the corpuscles of the germ epithelium. From the deeper parts

of the ovary delicate bundles of vascular tissue proceed upwards in all directions among the corpuscles, which thus become included in groups in the meshes of the stroma. All round the ovary, under the germ epithelium, groups of corpuscles from the germ epithelium are being imbedded in the stroma by the growth of vascular connective tissue round them. The largest corpuscles are situated most deeply in the stroma, and are in an advanced stage of development, while those more superficially imbedded are in a less advanced stage of development; the youngest corpuscles are immediately under, and in connection with the germ epithelium. The most deeply imbedded corpuscles have already assumed the character of primordial ova, and among the others we can trace many in course of development into similar bodies. In each primordial ovum the nucleus becomes the germinal vesicle, and the protoplasm which surrounds it gradually forms the yelk of the mature ovum.

The Ovary of the Kitten.

I shall now describe the structure of the ovary and the development of the ova in a kitten of two or three weeks; and I may here remark, that I know of no animal better suited than the kitten to show the relation of the germ epithelium to the stroma of the ovary. A section of a young kitten's ovary presents a form similar to that of the young calf's ovary.

In a thin vertical transverse section of a two weeks' old kitten's ovary we may distinguish

1st, The germ epithelium.
2d, The zone of egg clusters.
3d, The fibrovascular stroma.

The germ epithelium consists of distinct corpuscles arranged in a layer which passes round the ovary from one lateral border to the other, and becomes continuous with the peritoneal epithelium which covers the stalk or peduncle.

The corpuscles of the germ epithelium (fig. 6, *h*, *h*) consist of clearly defined nuclei, all of which have a thin investing film of protoplasm. In some instances this protoplasm is very clearly made out, and is in considerable quantity, but in other cases it can scarcely be seen, even under very high powers of the microscope. As in the ovary of the fœtal calf, there is a constant proliferation of the germ corpuscles by a process of fission. They are somewhat granular, and vary considerably in size; some are oval, but the greater number are spherical. In the ovary of a kitten of four weeks the corpuscles of the germ epithelium appear columnar in form and compressed laterally. In the round or spherical corpuscles (*l*, *l*), which are generally larger than the others, the nucleus is extremely well marked, and frequently possesses a bright nucleolus. The spherical form of the larger corpuscles appears to be produced

by the swelling out of the nucleus. In the two weeks' old kitten the corpuscles of the ovarian germ epithelium are several deep, and the layer itself is of irregular thickness.

As we examine the layer of germ epithelium as it passes round the ovary, we are at once struck by the fact that the corpuscles present a great variation in size and degree of development. Here and there we see large spherical nuclei (*l*, *l*, *l*), having round them a thin investing layer of protoplasm, while in other situations certain individual corpuscles stand forth prominently among their neighbours, and are conspicuous by their size and the size of their nuclei (*m*, *m*.) In these latter the protoplasm surrounding the nuclei is in the form of a thick layer. Between these largest corpuscles and the ordinary small ones every variety in size and form is to be met with. The largest corpuscles, which present so much protoplasm round the nucleus, are evidently individuals which have reached an advanced stage of development. These have been termed primordial ova, and there can be no doubt that a great number of the larger or spherical germ epithelial corpuscles are developing into similar bodies.

The ordinary sized germ epithelial corpuscles measure about $\frac{1}{2500}$th of an inch, while the primordial ova measure about $\frac{1}{1000}$th of an inch, but these measurements vary.

On carefully examining the primordial ova, in many instances two or more small fusiform corpuscles are seen in close contact with their yelk or protoplasm.

These fusiform bodies are about the size of the smallest germ epithelial corpuscles, and appear like connective tissue corpuscles, and in more than one case I have traced them in direct continuity with delicate bundles of minute fusiform bodies, which will presently be described as passing up from the deeper parts of the ovary towards the germ epithelium. In other cases, however, it appears to me that the bodies which lie in contact with the protoplasm of the primordial ova are germ epithelial corpuscles which have been displaced, pushed aside, or flattened out during the growth of the primordial ova. No cell wall can be made out round these primordial ova, but in certain sections one frequently sees a thin irregular stratum of a hyaline substance passing round the young ovum in contact with its protoplasm, apparently growing up from the young connective tissue subjacent to the germ epithelium. In this hyaline substance two or three oval-shaped nuclei are seen. In some preparations the primordial ovum has tumbled out of its hyaline girdle, which remains like a ring standing up among the germ epithelial corpuscles.

In favourable specimens, in a single section of a three to four weeks old kitten's ovary, I have counted as many as from thirty to forty large primordial ova among the ordinary germ epithelial corpuscles on the surface of the ovary. They may be recognised at once, for they appear like giants among their neighbours. Each consists of a large spherical nucleus, surrounded with a con-

siderable layer of protoplasm; it is this latter substance which gives magnitude to the young ovum, as may be seen by the fact that though many of the nuclei of the larger germ epithelial corpuscles are nearly as large as the nuclei of the primordial ova, yet, being without the extensive investment of protoplasm, they do not appear markedly conspicuous among the rest like the primordial ova. Occasionally, two large spherical nuclei or germinal vesicles are found in a single large primordial ovum among the germ epithelial corpuscles on the surface of the ovary.

Among the germ epithelial corpuscles in the deeper parts of the germ epithelial layer all round the ovary we meet with many large primordial ova, and in contact with their yelk or protoplasm are small fusiform bodies, each consisting of an oval-shaped nucleus, around which is an investment of protoplasm drawn out at either end in a fusiform manner. Besides these primordial ova, numerous germ epithelial corpuscles, in various stages of development into the same, are found in this situation.

Immediately subjacent to the germ epithelial layer is the zone of egg clusters. In osmic acid preparations this can be seen with the naked eye as a well-marked thick layer. The egg clusters are large oval-shaped and spherical collections of round corpuscles (*o*, *o*, *o*). The oval-shaped clusters lie close together, with their long axes directed from the centre of the ovary in a radiating manner towards the germ epithelium. Between the clusters, and separating them, are delicate bundles or strings of small fusiform corpuscles, with blood-vessels (*j*, *j*, *j*), which may be traced growing upwards from the deeper parts of the ovary towards the germ epithelium. By far the greater number of the egg clusters are oval, but spherical-shaped groups or clusters are also met with, generally deeper in the ovary. Each egg cluster consists of a collection of corpuscles, most of which are spherical, and resemble very closely the larger corpuscles of the germ epithelium. If we direct our attention to any one egg cluster, it will be seen that the corpuscles vary in size, just as we described the corpuscles of the germ epithelium. At the lower part of each egg cluster, that is, farthest away from the germ epithelium, we find many large primordial ova, similar to those already described among the corpuscles on the surface of the ovary, and besides these are numerous large spherical nuclei, having round them protoplasm in layers of varying thickness. In each egg cluster it is possible to trace the corpuscles in all stages of development into primordial ova. Every large corpuscle in each egg cluster is potentially a primordial ovum. In contact with each primordial ovum in the egg clusters are small fusiform corpuscles, which may be traced as offshoots from the bundles of similar corpuscles which lie between and separate the egg clusters from each other. It is easy to compare the corpuscles in each egg cluster with the corpuscles of the germ epithelium, and to follow the steps of their development into primordial ova.

In such a young kitten's ovary many of the egg clusters are still in connection superiorly with the corpuscles of the germ epithelium, but in most cases, we find the bundles of young connective tissue which lie between the egg clusters have grown completely round them, separating them not only from each other, but also from the germ epithelium above. While the corpuscles of the germ epithelium are thus being enclosed in the meshes of young connective tissue, the nucleus of each swells out into a spherical body, around which is gradually produced that protoplasm which later constitutes the yelk of the primordial ovum.

When we trace the bundles of young connective tissue downwards, we find they are offshoots of similar tissue of which the central part of the ovary consists. In the deeper parts of the ovary we find a very vascular young connective tissue forming the stroma of the organ, in which are imbedded numerous primordial ova, some in a far advanced stage of development. This stroma consists of minute fusiform corpuscles and blood-vessels. As the vascular bundles or strings of this tissue grow upwards between the egg clusters, delicate offshoots of the same insinuate themselves between the primordial ova and corpuscles in the clusters, and in this manner nourishment is brought within reach of these developing bodies. This interstitial growth begins at the lower part of each egg cluster, and gradually the primordial ova become separated from each other as the connective tissue thickens in between and around them, and they become at last included in separate meshes or primordial follicles. Where the egg clusters have not been completely shut in by bundles of connective tissue, the fusiform corpuscles of the latter may be distinctly followed up as far as the corpuscles of the germ epithelium, and, indeed, seem to disappear among them.

By the growth of the vascular tissue of the stroma among the imbedded corpuscles, the egg clusters in all parts of the ovary are gradually subdivided or broken up into single egg-containing meshes or follicles; and while this process is going on, the primordial ova are rapidly advancing in development. In the more superficial parts of the stroma subjacent to the egg clusters, and in the fibro-vascular zone of the ovary, the above-described process has already taken place. The primordial ova in some of the follicles are of very large size, and in the ovary of a four weeks' old kitten it is of great interest to compare the original germ epithelial corpuscles on the surface of the ovary with these large primordial ova now imbedded deep in the stroma, and we are thus able to observe what an extraordinary change has taken place in them during their development; and, what is of more importance, we recognise the nature of the change. Examined under a magnifying power of 1000 diameters, the ordinary germ epithelial corpuscle on the surface of the ovary appears as if it were of such a size that at least three of them would lie side by side on a threepenny piece without overlapping it; whereas the highly developed ovum imbedded deeply in

the stroma appears as large as a florin or a half-crown piece, and the nucleus or germinal vesicle as large as a threepenny or fourpenny piece. Between these two extremes young ova in all stages of development may be seen in the stroma of such a young ovary. The change which the germ epithelial corpuscle undergoes during its development is the following:—

As soon as the corpuscle is imbedded in the stroma, its nucleus swells up into a round or spherical body, within which generally appears a spot or nucleolus. The nucleus presents a sharply defined limiting membranous wall, and becomes the germinal vesicle of the future ovum. Immediately around the nucleus is gradually produced that protoplasm which afterwards constitutes the yelk of the mature egg. *In all parts of the ovary, wherever we examine such young developing ova, we find fusiform corpuscles, like the fusiform corpuscles of which the stroma is composed, lying in contact with the protoplasm which surrounds the nucleus or germinal vesicle.*

The youngest egg clusters are immediately below the germ epithelium, and many of them are in connection with it. These latter have not, as yet, been completely closed in and separated from the germ epithelial layer by the connective tissue bundles of the stroma.

The clusters of corpuscles in connection at their upper parts with the germ epithelium differ considerably from each other, both in size and form. Some are oval and elongated, others are spherical. Many of them are larger at their lower and middle parts than at the part in connection with the germ epithelial corpuscles. If a vertical section passes down through such clusters as these, such as are swollen out at their lower and middle parts but narrow at their upper parts, we have presented under the microscope the appearance as if bottle-shaped tubes full of round corpuscles extended from the germ epithelium downwards into the stroma of the ovary. These appearances are often seen, and it is important to study them well. It will be remembered that PFLÜGER* described in the young kitten's ovary numerous tubular processes passing downwards from the surface of the ovary into the stroma, in which the germinal vesicles originated, and by the successive divisions of which Graafian follicles containing eggs were formed. I have very carefully examined the ovaries of kittens and puppies, but have failed to find any tubular processes of epithelium passing into the stroma from which Graafian follicles are formed, and it appears to me that PFLÜGER is incorrect in stating that such exist in the ovary of the kitten.

In section, the ovary of a kitten at birth presents a structure very similar to that of the ovary of the two to three weeks' old kitten, but we find in the germ epithelium layer very few primordial ova. Many large spherical nuclei, with a thin film of protoplasm round them, are seen among the germ epithelial corpuscles, and

* PFLÜGER, "Die Eierstöcke der Saügethiere und des Menschen," Liepzig, 1863, p. 4.

the germ epithelial layer itself is much thicker as a stratum all round the ovary. Most of the large egg clusters below it, at this stage of development, are in communication superiorly with the corpuscles of the germ epithelium, and separating these clusters delicate bundles of young connective tissue, made up of minute fusiform corpuscles, may be traced growing up between and around them. In the deeper part or fibro-vascular zone of such a young ovary the stroma is rich in blood-vessels, and numerous young eggs are imbedded in its meshes, but no large eggs, such as we described in the four weeks' old kitten, are found.

The Human Ovary.

In section, the ovary of a human fœtus of about seven months presents a somewhat triangular form (fig. 7) The ovary (*a*) is attached to a stalk or peduncle (*b*) consisting of fibro-vascular tissue, which passes into the ovary at the hilum (*c*), and from the direct prolongations of which the whole stroma is derived. On examining a thin section of such an ovary under low powers of the microscope, direct prolongations from the stalk are seen proceeding in a radiating manner towards the periphery in all directions, and communicating with each other in such a manner that the whole stroma becomes arranged in the form of a mesh-work consisting of fibro-vascular tissue. In the meshes of this stroma are large and small collections of corpuscles. At the periphery of the organ the meshes are large, and the contained groups of corpuscles are correspondingly large; but as we pass deeper into the ovary the meshes with the included groups of corpuscles become smaller, till at last we find small meshes containing but one or two large corpuscles.

The surface of such an ovary is very irregular, presenting numerous small fossæ-like grooves and furrows, seen clearly under a low magnifying power.

Investing the ovary, and passing round it from one lateral border to the other, is a layer of columnar corpuscles. This layer is the germ epithelium (*h*, *h*), and as it invests the ovary it dips down into and lines certain tubular structures and tubiform depressions which, when seen in section, appear as passing down from the germ epithelial layer into the substance of the ovary.

(*a*). *Nature of the Germ Epithelium.*—In a fœtus of 7½ months the germ epithelium consists of columnar-shaped corpuscles placed side by side and arranged as an investment to the whole ovary.

The germ epithelial layer rests on a thin irregular stratum of connective tissue which is part of the general stroma of the ovary, and is formed by delicate processes of the same, which at an earlier stage of development grew upwards from the deeper parts of the ovary to surround and inclose in meshes those large groups of corpuscles found immediately under the germ epithelium in its entire extent. This young connective tissue stratum is the forerunner of the tunica albuginea.

On tearing to pieces in a little water a fragment of this ovary and examining the debris under high powers of the microscope, many small portions of the germ epithelium will be found. When the deep surface of the epithelial layer is examined, the corpuscles are seen to be placed close together, and present in the membrane a tessellated appearance (fig. 10). The chief part of each corpuscle is the nucleus. Around each nucleus is a small quantity of protoplasm which acts as a cement substance holding the nuclei together. When the germ epithelial membrane is looked down upon from above, immediately on bringing the corpuscles into view a clear space is seen round the nucleus of each corpuscle (figs. 11 and 12, *h*, *h*). This space is occupied by clear protoplasm. The corpuscles in this early stage of development appear as little nucleated pieces of protoplasm; the nucleus is always the conspicuous part of each corpuscle, but the protoplasm round it may vary in quantity. In each piece of epithelial membrane examined, the corpuscles are of different sizes. Some of the nuclei are swollen up into large spherical bodies (fig. 12, *l*), and around them is an increased quantity of clear protoplasm. In the larger nuclei a nucleolus is generally seen. The largest corpuscles are undoubtedly primordial ova. Between these and the smallest germ epithelial corpuscles every variety in size is met with.

When seen in profile (figs. 13, 14, 15, *h*, *h*, *h*), the germ epithelial corpuscles are columnar, but many of them are assuming an oval and spherical form (figs. 13, 14, 15, *l*, *l*, *l*). In the spherical ones the nucleus is clearly defined, and shows distinctly its well-marked membranous wall. Within the nucleus is a nucleolus, and around it is a thin film of protoplasm. In some instances this film is so fine as scarcely to be made out. When a section of a 7½ months' fœtal ovary is examined under high powers of the microscope, in many situations among the ordinary germ epithelial corpuscles all round the ovary, we find individuals standing forth prominently and conspicuous by their size and the size of their nuclei (figs. 14, 15, *m*, *m*) similar to those bodies we described as conspicuous among the corpuscles of the germ epithelium in the kitten's ovary. Such have been termed primordial ova. On comparing these with the smaller round corpuscles, and these latter with the ordinary germ epithelial corpuscles, it is easy to see that they are germ epithelial corpuscles in a far advanced stage of development. In the progress of growth the nucleus of the ordinary germ epithelial corpuscle first swells out and enlarges, becoming oval, then spherical, and around it is gradually produced that protoplasm which assumes such dimensions in the primordial ova. In contact with these primordial ova we often see small fusiform corpuscles (*n*, *n*), and as in the case of the kitten's ovary, some of them appear to have grown up among the germ epithelial corpuscles from the stratum of young connective tissue (*j*,*j*) on which the germ epithelium rests.

It is a fact of great interest, that as the germ epithelial corpuscle becomes a

primordial ovum, the nucleus, which at first appears ill-defined and semi-solid, and is surrounded by a comparatively small quantity of protoplasm, now shows a remarkably clear definition, and at last appears as a spherical vesicular body with a fine double-contoured wall, and within it one or two nucleoli are usually seen. The nucleus of each germ epithelial corpuscle becomes the germinal vesicle of the primordial ovum, and the nucleolus corresponds to the germinal spot.

The ordinary germ epithelial corpuscles measure in their longest diameter about $\frac{1}{1800}$th of an inch, and in their shortest about $\frac{1}{3500}$th of an inch, but both these measurements vary considerably.

On referring to WALDEYER's work "Eierstock und Ei," plate ii. figs. 9, 11, 13, I find that he represents the germ epithelial corpuscles as little bodies in which the nuclei are comparatively small, while the protoplasm round these is in all cases very extensive. This is not in accordance with my observations. An examination of the ovaries of numerous fœtal and newly-born animals clearly shows that in each ordinary germ epithelial corpuscle the nucleus constitutes by far the greater part of the corpuscle, the protoplasm around it being in the form of a fine film. In the primordial ova, however, the enlarged nucleus has around it a correspondingly large quantity of protoplasm, and then these bodies present the appearance as described by WALDEYER.

(*b*.) *The Relation of the Germ Epithelium to the Peritoneal Epithelium.*—In a section of the ovary of a 7½ months' fœtus, the stalk or peduncle (fig. 8, *b*) is directly continued into the ovary to form the stroma, which we described as arranged in the form of a mesh-work. If we now direct our attention to the epithelium covering this stalk (fig. 9, *f*), we find it is directly continuous with the peritoneal epithelium, and on the other hand passes without a break into the epithelium which covers the ovary, but a gradual change in its character takes place as it slides into and becomes continuous with the latter.

In the peritoneal epithelium (fig. 9, *f*), as seen in profile, the nuclei of the cells are oval and flattened from above downwards, and are placed at a considerable distance from each other, and the protoplasm around them is extensive. Tracing this epithelium towards the ovary, as we approach the latter, we observe the nuclei of the epithelial cells to become round and columnar, and gradually to lie closer together until at last they are almost in contact. They then crowd together, and in the form of a thick layer pass on to the surface of the ovary as the corpuscles of the germ epithelium (*h*). As the corpuscles of the peritoneal epithelium gradually slide into those of the germ epithelium, we find the protoplasm which invests the nuclei of the former gradually becomes less and less in quantity, and the nuclei themselves become gradually larger and more columnar, until at last the nuclei pass on to the surface of the ovary as distinct columnar bodies, each having round it a fine investment of protoplasm. In thus tracing the peritoneal epithelial corpuscles into

germ epithelial corpuscles we see the nuclei undergoing fission, and by this process crowds of germ corpuscles are produced. The whole germ epithelial layer must be regarded as a thick layer of proliferating corpuscles. From the first appearance of the ovary as an organ until its development is completed, there is a constant proliferation of the germ epithelial corpuscles; and, in growth of the ovary, as fast as some corpuscles become imbedded in the stroma, others are produced to take their place.

It is interesting to recall to recollection the statements of SCHENK*, WALDEYER, and others, that in the first instance the whole peritoneal cavity is lined by a layer of columnar corpuscles, and when the Wolffian body appears in connection with REMAK'S middle plate, it likewise has an investment of similar corpuscles. The first appearance of the ovary is the thickening of this columnar epithelium on the median side of the Wolffian body (WALDEYER.) At the commencement of this thickening there is seen under it a small outgrowth of young connective tissue, continuous with and proceeding from the interstitial tissue of the Wolffian body. There can be little doubt that the thickening of the columnar epithelium is brought about by a proliferation of its corpuscles, and the proliferation itself is due to the influence on the corpuscles of the vascular young connective tissue subjacent to them. While the columnar corpuscles constantly proliferate to form the germ epithelial corpuscles, from which all the future ova are derived, the rest of the peritoneal chamber becomes lined by a layer of flat epithelium, forming the peritoneal epithelium.

From the very first appearance of the germ epithelium of the ovary as a distinct structure, processes of the subjacent young connective tissue grow upwards among the corpuscles as the first step in that process of imbedding, whereby the corpuscles become surrounded in meshes of vascular stroma, and ultimately form ova. From its earliest condition till late in the development of the ovary, the germ epithelium cannot be stripped off as a layer from the ovary, because delicate processes of the ovarian stroma are constantly growing up between the corpuscles as fast as they are produced.

PFLÜGER states his opinion that the ovum is simply a peritoneal epithelial cell. With this statement I cannot altogether agree. Both the ova and the peritoneal cells are undoubtedly evolved from a common ancestral source—the columnar layer of the great peritoneal cavity; but whilst in one limited region the columnar corpuscles form the germ epithelium and become converted into ova, in the greater part of their distribution they become converted into the endothelium lining the peritoneal cavity. Hence the ovum is no more a modified peritoneal epithelial cell than is the peritoneal epithelial cell a modified ovum.

From the corpuscles of the germ epithelium all the ova are derived. We

* Beiträge Zur Lehre von den Organ-Anlagen im motorischen Keimblatt.

recognise the germ epithelium by the fact that immediately below it all round the ovary are clusters or groups of corpuscles contained in meshes of the ovarian stroma, but under the peritoneal epithelium covering the stalk no such groups are found.

It is worthy of remark, however, that sometimes under those epithelial corpuscles, which, as it were, form a connecting link between peritoneal and germ epithelial corpuscles, we find small groups of round corpuscles apparently consisting of abortive ova. It is only under and in connection with the true germ epithelium that groups of true primordial ova are formed.

(*c.*) *The manner of Inclusion of the Primordial Ova and Germ Epithelial Corpuscles in the Stroma of the Ovary.*—In a section of the ovary of a human fœtus of 3½ months, one sees large strings of connective tissue corpuscles (fig. 19, *j,j,j*) growing upwards in a radiating manner from the deeper parts of the ovary toward the germ epithelium (*h, h.*) These strings or bundles communicate with each other and form meshes, and in these meshes are round groups of corpuscles (*o,o,o*) which resemble very closely the corpuscles of the germ epithelium (*h, h*) which invests the ovary. Immediately under the germ epithelium such groups of corpuscles may be seen partially imbedded in meshes of the stroma. In the deeper parts of the ovary we find large primordial ova (fig. 20, *m,m*) lying separate from each other, and in contact with the protoplasm which surrounds the germinal vesicle of each are small connective tissue corpuscles (*n, n*) exactly similar to those which make up the strings or bundles of young tissue in other parts of the ovary. These little connective tissue corpuscles are quite different in appearance from the corpuscles of the germ epithelium on the surface of this young ovary, or those imbedded in groups in the stroma.

The stroma of the human fœtal ovary of 7½ months consists almost entirely of fusiform connective tissue corpuscles, and in a section of such an ovary we find the whole stroma arranged in the form of a mesh-work, in the meshes of which are large and small groups of corpuscles, just as we described in the case of the ovary of the calf, the kitten, and human fœtal ovary of 3½ months. In the bundles of connective tissue corpuscles which (as seen in section) lie between the groups of corpuscles we find blood vessels, and on close examination it is seen that the walls of such blood-vessels consist of connective tissue corpuscles. Wherever the bundles of connective tissue proceed, in the midst of them are blood-vessels. On tracing the bundles of vascular tissue upwards, we find they arch round large and small groups of corpuscles immediately under the germ epithelium, and completely inclose them in meshes. In some situations under the epithelium the connective tissue bundles have not yet grown round the groups of corpuscles, but may be traced up as far as the germ epithelial corpuscles on either side of the groups. Under the germ epithelial layer the youngest connective tissue is found, and in this situation it

is the forerunner of the tunica albuginea. This youngest connective tissue appears as a transparent jelly-like substance, and in it are numerous fusiform corpuscles or nuclei. Fine homogeneous points of this young tissue may be seen insinuating themselves among the corpuscles of the germ epithelium, and in some preparations of 6 and 7½ months' fœtal ovaries in which the germ epithelial layer is partially detached such fine points of the jelly-like young tissue may be seen in considerable numbers.

In many places immediately under the germ epithelium, small groups, consisting of a few germ epithelial corpuscles (fig. 21, *q*, *q*, *q*), are found in the act of being surrounded by this same jelly-like tissue; some of the groups are completely surrounded and separated from the germ epithelium layer, while others are partially surrounded, and are still in connection with the germ epithelium superiorly. The youngest connective tissue can always be traced in direct continuity with vascular bundles of tissue which completely surround the large groups of imbedded germ epithelial corpuscles, and is part of the general stroma of the ovary. This imbedding of germ epithelial corpuscles takes place under the germ epithelium all round the ovary.

After being thus included in meshes of the stroma, the germ epithelial corpuscles increase in number and in size, and there results the formation of those large egg clusters which are found under the germ epithelium in all parts of the ovary.

Each imbedded corpuscle undergoes the following change:—The nucleus enlarges, gradually becoming a spherical vesicular body, and the protoplasm which surrounds it is at the same time increased in quantity. As the result of the enlargement of each corpuscle in the group, the whole group as a cluster expands and becomes more or less spherical. As these egg clusters expand, those lying immediately under the germ epithelium push the latter structure before them, and in this manner the surface of the young ovary is made to present a very irregular appearance. Between the prominences or irregularities thus produced are depressions or furrows. These originate as simple depressions between two or more adjacent expanding egg clusters, and they become deepened by the growth and expansion of new egg clusters under the germ epithelium in connection with those already formed.

In describing the appearance of a seven and a half months' human fœtal ovary in section, I stated that the germ epithelium passed round the ovary from one lateral border to the other, dipping into and lining certain tube-like structures and tubiform depressions which appear to pass from the surface downwards into the organ. Now, if vertical sections are made through a young ovary such as above described, whose surface presents numerous irregularities, we have the appearance presented as if tube-like structures (fig. 22, *p*, *p*) passed down from the surface of the ovary into the substance of the organ, and

these are all lined by germ epithelium. Similar tube-like structures are produced when sections are made through the convoluted surface of a brain, and the grey matter might even be compared to the germ epithelium, inasmuch as it lines these sulci. But such structures are not tubes, nor are the similar appearances seen between the irregularities of the surface of a young fœtal ovary.

An examination of the young ovary, looking down on its surface from above, soon convinces us that there are no real tubular structures passing from the epithelium into the organ. On bringing the germ epithelium into view, and then slightly depressing the tube of the microscope, one sees large and small round groups of spherical corpuscles separated from each other by the connective tissue of the stroma, as was described by WALDEYER. Some of these groups are still in communication with the germ epithelial corpuscles, as may be seen in vertical sections where the knife has passed through a group of corpuscles not yet completely included in a mesh of the upward growing stroma.

In the germ epithelial layer which dips down into and lines the depressions of the surface of the ovary (fig. 22, *p, p*), we find the corpuscles undergoing changes during their development similar to the changes which the corpuscles of the germ epithelium in other situations undergo. At the bottom and sides of the sulci, among the corpuscles of the germ epithelium, we find large spherical nuclei with a thin investment of protoplasm, and large primordial ova, such as may be found in all parts of the germ epithelial layer, whether it lines depressions or passes over prominences of the surface of the ovary.

Frequently large egg clusters are formed under the germ epithelium which lines the furrows or sulci, and it often happens that the walls of these furrows come in contact; pushed together, as it were, by laterally situated expanding egg clusters. At the bottom of the furrows, where the epithelial walls come in contact, connective tissue passes through among the corpuscles, and in this way large egg clusters become formed immediately below the germ epithelium at the bottom of the sulci. A vertical section passing down through such a sulcus and the group of corpuscles immediately below it, produces the appearance as if a tubular prolongation of germ epithelium was dilated at its lower part into a large sac, full of developing corpuscles of the germ epithelium.

During my investigations on the structure of the ovary and development of the ova, I have never found any real tubular structures, neither in the human ovary, nor in any other mammal that I have examined, such as the cat, dog, calf, sheep, guinea-pig, rabbit, and in no instance have I found Graafian follicles formed out of such structures in the manner described by PFLÜGER, VALENTIN, SPIEGELBERG, WALDEYER, and others.

(*d.*) *Development of the Egg Clusters.*—Each egg cluster is a group or collection of germ epithelial corpuscles enclosed in a mesh or capsule of the ovarian stroma.

The germ epithelial corpuscles on the surface of the ovary constantly produce new elements by the process of fission, and when included in a vascular mesh of stroma the corpuscles increase greatly in number by division, and from a few imbedded corpuscles a large group or cluster may be derived. It appears to me a most interesting and remarkable observation that these corpuscles, after a certain increase in number, expand or swell out into spherical bodies, and a careful examination of the egg clusters has convinced me that this change is brought about by the nucleus in each corpuscle swelling out into a spherical vesicular body, which afterwards becomes the germinal vesicle of the primordial ovum, and in close contact with the wall of the nucleus is gradually produced that protoplasm which afterwards forms the yelk of the ovum.

In each egg cluster we find certain individuals much farther advanced in development than the rest, and these appear exactly like the large primordial ova which we described as found among the corpuscles of the germ epithelium on the surface of the ovary. Each corpuscle in the cluster is potentially a primordial ovum. At the first there is but a small quantity of protoplasm round the nuclei of the corpuscles in each egg cluster, but as the nuclei enlarge and expand, the protoplasm round them is gradually produced in considerable quantity.

This development of the germ epithelial corpuscles into primordial ova takes place in the egg clusters in all parts of the ovary. All the imbedded germ epithelial corpuscles do not reach the stage of primordial ova, many of them abort and disappear, and perhaps furnish a pabulum for the more vigorous and healthy ones.

In the ovary of a puppy at birth, one sees, in a beautiful manner the egg clusters under the germ epithelium in all stages of development. Some of the clusters appear to consist entirely of large primordial ova, while in others we can trace the growth of the germ epithelial corpuscles into ova, and immediately under the germ epithelium are little groups of corpuscles in the act of being included in meshes of the stroma.

In following the further development of the ovary and ova, we notice that around each egg cluster is a well-marked capsule or mesh of vascular connective tissue. This connective tissue consists almost entirely of fusiform corpuscles and young blood-vessels. Just as at first, when delicate processes of the young stroma grew upwards among the ordinary germ epithelial corpuscles to include them in meshes, so now delicate processes of the young connective tissue with blood-vessels proceed from the walls of these meshes, and insinuate themselves in among the developing corpuscles in each egg cluster. As these processes thicken, the primordial ova gradually become separated from each other, and at last each is included in its own mesh or capsule of the stroma.

These single egg-containing meshes or capsules are the primordial follicles.

This formation of young Graafian follicles takes place in the egg clusters in all parts of the ovary, and at the same time new egg clusters are being formed under the germ epithelium in the manner described. In the ovary of a 7½ months' human fœtus we find many newly-formed egg clusters immediately under the germ epithelium, but below these, earlier formed egg clusters are in various stages of alteration into single egg-containing follicles, while deeper still we find a great number of young follicles all produced from the first formed clusters in the way we have described (fig. 24).

In the first formed Graafian follicles, which are situated most deeply in the stroma of the ovary, the young ova are of large size, and we are at once struck by the fact that the germinal vesicles in all are about the same size, although the protoplasm around them may vary considerably in quantity.

In each young Graafian follicle the ovum fits tightly; it occupies the whole cavity of the follicle; there is no space between it and the wall (fig. 24, *m*, *m*, *m*). The mesh of stroma closely embraces the protoplasm of the ovum, and in almost every case we find fusiform connective tissue corpuscles (*n*, *n*) in the wall of the mesh lying in close contact with and indenting the yelk of the young ovum. Wherever we examine the primordial follicles, we see such fusiform corpuscles of the stroma lying in contact with and indenting the yelk of the contained ova. I called attention to the circumstance that among the germ epithelial corpuscles on the surface of the ovary primordial ova were found, having in contact with their protoplasm small fusiform corpuscles, which in some instances could be traced growing as offshoots from delicate bundles of similar bodies which formed part of the ovarian stroma.

(*e.*) *The Development of the Membrana Granulosa.*—The stroma of the human fœtal ovary is remarkable for the great number of connective tissue corpuscles it contains. Wherever we examine the stroma and its processes in all parts of the ovary, we find in it well-formed connective tissue corpuscles. In the middle parts of the ovary, where the stroma is well developed, the connective tissue corpuscles show very distinctly a central oval nucleus with nucleolus. Around the nucleus is a small quantity of protoplasm, drawn out at either end in a fusiform manner. Besides these we see naked nuclei and many small round bodies; in these latter the nucleus is in a state of division into two or more parts. These small round bodies appear to be swollen out connective tissue corpuscles. In a well-formed connective tissue corpuscle the nucleus is comparable to the germinal vesicle of the ovum, and like it, at a certain stage of its development, it shows a sharply-defined double contoured wall. In all parts of the ovary we find the connective tissue corpuscles dividing.

In various parts of the stroma we find delicate fibres developing into connective tissue corpuscles. The central part of such fibres becomes swollen out, and in this swollen out part a distinct oval nucleus appears; sometimes

these fibres are direct prolongations of the protoplasm surrounding the nuclei of well-formed connective tissue corpuscles. The nucleus of a connective tissue corpuscle at first often appears as a solid or semisolid body, but it may become distinctly vesicular, like the nucleus or germinal vesicle of the ovum.

It will be observed that, in my description of the germ epithelium and of the development of the ova, I have avoided the use of the word cell, and substituted the term corpuscle, and my reason for not using the term cell in connection with the germ epithelial corpuscles is, that the germ epithelial corpuscles, and the so-called columnar epithelial cells which line the pleuro-peritoneal cavity of the embryo, are nuclei which have a thin film or investment of protoplasm round them. These nuclei are the homologues of the nuclei of the peritoneal epithelial cells. When a corpuscle divides, each half of the nucleus carries with it a small investment of protoplasm. The protoplasm round the nuclei varies considerably in quantity during the development of the germ epithelial corpuscle. The term cell is employed in somewhat different significations by biologists. Some, for example, holding that a cell must have a definite wall, whilst others look upon the wall as of secondary and minor importance, and hold that a cell essentially consists of a nucleated mass of protoplasm. The germ epithelial corpuscles and connective tissue corpuscles do not possess a cell wall. When once a cell wall has formed round a corpuscle, the nucleus and cell contents may divide. The cell wall does not participate in the division, but incloses the products of the division. The cell wall may burst and liberate the contents of the cell.

Every egg cluster is included in a mesh of the stroma. This mesh consists of connective tissue corpuscles and minute blood-vessels whose walls consist of such corpuscles. Delicate processes of this vascular young tissue from the wall of the mesh grow inwards among the corpuscles which are developing into primordial ova. On tearing to pieces small fragments of a 7½ months' fœtal ovary, and placing the debris in a little water under the microsrope, we find small groups of primordial ova (figs. 25, 26), and single individuals (figs. 27, 28, 29), which have been torn away from the egg cluster, and in connection with some of the largest of these we frequently find fusiform corpuscles (figs. 28, 29, *n*, *n*) similar to those which lie in the walls of the meshes. In good specimens the fusiform corpuscles are found lying in indentations in the yelk substance (figs. 28, 29) which surrounds the germinal vesicle of the primordial ovum, and sometimes we see primordial ova whose yelk is indented in many places, but the fusiform corpuscles have been displaced from these indentations. No zona pellucida is found round such young primordial ova. The connective tissue corpuscles must therefore be in contact with the yelk of the primordial ova.

Directing our attention to the youngest follicles, we find these vary in size, but in every case the young ovum fills up the whole follicle in such a manner

that its protoplasm presses against and distends the follicular wall. Little fusiform corpuscles in the walls, *and belonging to the stroma of the ovary*, indent the protoplasm of each young ovum as it lies in its follicle. (Fig. 24, *m*, *m*.)

In an empty follicle from which the young ovum has been removed, these little connective tissue corpuscles appear as minute buds projecting into the cavity of the follicle from its wall.

In the deeper parts of the ovary numerous young egg-containing follicles are seen. In the youngest follicles but two or three fusiform corpuscles are found in contact with the yelk of the contained ovum. As the follicles increase in size and become older, the number of small corpuscles in contact with the contained ova also increases. In some we find seven or eight, and in still older follicles a perfect wreath of minute corpuscles is formed round the yelk of the young ovum. The oldest follicles are found in the deepest parts of the ovary, and in most of them there is a perfect wreath of minute corpuscles (fig. 30, *r*) lining the follicle. Now, from these young connective tissue corpuscles in the wall of the young follicles which lie in contact with and indent the yelk of the primordial ova, the corpuscles of the membrana granulosa are derived.

In describing the growth of fibre-like bodies in the stroma into connective tissue corpuscles, I stated that the middle parts of such fibres become swollen out, and in the swollen out part a distinct nucleus appears. This nucleus, though at first appearing semi-solid, may become distinctly vesicular, and within it a nucleolus is afterwards seen. By a careful examination of a whole series of young Graafian follicles, we trace the development of the corpuscles of the membrana granulosa in the following way:—Around the young ovum in each follicle the connective tissue corpuscles increase in number by division. As a single fusiform corpuscle divides, its nucleus appears to commence the division, and each half of the nucleus carries with it a small quantity of the protoplasm which originally invested the single nucleus. When a little wreath of such corpuscles is formed round the young ovum the nucleus of each corpuscle swells out and becomes a distinctly vesicular little body, presenting a very fine double contoured wall, around which is a small quantity of protoplasm. Within the nucleus generally a minute spot is seen. The protoplasm which surrounds the vesicular nuclei acts as a sort of cement substance, holding them together in the form of a capsular membrane round the young ovum. This capsular membrane is the first appearance of the membrana granulosa.

Only those connective tissue corpuscles which lie in actual contact with the yelk of the ovum in the follicle develope into corpuscles of the membrana granulosa; while those in the wall outside the membrana granulosa remain as connective tissue corpuscles, and follow the usual developmental process into ordinary fibrous tissue, of which the main part of the follicular wall at last consists. We shall afterwards see that in some of the oldest Graafian follicles in the adult

ovary, in man and other mammals, which are about to burst to liberate the contained ovum, a great part of the wall of the follicle outside the membrana granulosa becomes again converted into large connective tissue corpuscles.

When first formed, the membrana granulosa consists of a single layer of minute corpuscles arranged in the form of a capsule round the ovum (fig. 30, *r*). As the young ovum enlarges, which it rapidly does after the formation of the membrana granulosa, it distends its follicle, and the corpuscles of the membrana granulosa increase greatly in number. In this membrane, when first formed, the corpuscles lie in close contact with each other, and when looked down upon from above they present a polygonal form from mutual compression (fig. 32, *r*). By the constant multiplication by division of its corpuscles, the membrana granulosa at last comes to consist of several layers. As a further stage in development, certain of the corpuscles, generally those in the middle parts of the membrana granulosa, break down, and, it appears to me, become dissolved in a fluid which afterwards forms the liquor folliculi. By the breaking down and solution of these corpuscles, a cavity, the follicular space occupied by fluid, is formed. In section, this space appears semilunar in form. After the formation of this space the ovum is not entirely separated from the membrana granulosa, but remains connected with the wall of the follicle by a heap of corpuscles which surrounds it. In good specimens a layer of corpuscles remains in contact with the zona pellucida round the ovum for a long time after the formation of the follicular space. When first formed, the corpuscles of the membrana granulosa in the human fœtal ovary of 7½ months measure about $\frac{1}{3000}$th inch.

For the complete demonstration of the development of the corpuscles of the membrana granulosa, the ovary of an adult rabbit is admirably suited. In a section of such an ovary, we first direct our attention to the structure of the stroma of the organ, and we find it consists entirely of very minute attenuated fusiform corpuscles. At first sight these appear as simple fibres, but each fibre is an elongated nucleus, having round it a minute quantity of protoplasm. The stroma of the ovary of an adult cat has an exactly similar structure. In the adult rabbit's ovary, in a single section, we may find young ova in various stages of development. In a very young ovum (fig. 33, *m*), we notice first the large germinal vesicle, with its germinal spot. Around the germinal vesicle is a small quantity of protoplasm, immediately in contact with which are several small fusiform corpuscles (*n*, *n*). These lie flattened against the ovum round its yelk, and are exactly similar to fusiform corpuscles, of which the stroma (*j*) is composed. Around an ovum slightly farther advanced in growth we find one or two of the fusiform corpuscles (*n*, *n*) have assumed a swollen condition, and in some instances individual corpuscles are in the act of dividing. In the case of an ovum still farther advanced in develop-

ment, the corpuscles round the yelk substance have now assumed a spherical form, and as they increase in number and press against each other, they gradually become columnar (fig. 35, *r*).

As in the case of the human fœtal ovary of 7½ months, the corpuscles of the membrana granulosa thus formed consist of vesicular nuclei placed close together, their walls being almost in contact, but a very minute quantity of cement material lying between them. When this young membrana granulosa is looked down upon from above, the corpuscles present a beautiful pavemented appearance. The nucleus of each corpuscle is polygonal from pressure by its neighbours (fig. 36, *r*). By the constant division of the corpuscles, the membrana granulosa soon consists of several layers (fig. 37, *r*), and a follicular space is formed by the breaking down and solution of some of the corpuscles. In the nearly ripe Graafian follicle of the rabbit's ovary one frequently finds several small follicular spaces in different parts of the thick membrana granulosa; and in a section made through such a follicle, bands or straps of membrana granulosa cells appear to pass from the ovum to the wall of the follicle. These bands were called "Retinacula" by MARTIN BARRY. They are simply appearances produced when the section has passed through the walls or septa which separate several follicular spaces lying near each other.

In the ovary of the adult or old cat (fig. 39), we can trace in a beautiful manner the growth of the corpuscles of the membrana granulosa from the fusiform corpuscles of the stroma, which lie next to the protoplasm or yelk of the imbedded young eggs. In a section, we have first the epithelium on the surface. This consists of small flat polygonal nucleated cells (fig. 40, h_1), about $\frac{1}{2000}$th or $\frac{1}{2500}$th part of an inch in diameter. This layer of epithelium is all that remains of the germ epithelium, and it can be stripped off from the ovary without difficulty. Below the epithelium, and passing round the ovary, is a stratum of connective tissue, consisting of elongated fusiform corpuscles. In the lower part of this stratum the fibres decussate freely. Immediately below this layer we come upon the remains of the large egg clusters which we described in the young kitten's ovary. A perfect zone, consisting of young eggs, lies immediately under the stratum of tissue which passes horizontally round the ovary, under the epithelium. In this zone the eggs lie very close together, many of them are in actual contact. In each egg we recognise the central germinal vesicle, with its spot or nucleolus—the germinal vesicles are all about the same size. Below this zone of eggs is the general stroma of the ovary, processes of which consisting of elongated fusiform corpuscles and fibres, grow in between and around all the eggs in the egg zone. These processes of stroma then become continuous with the horizontal zone of tissue which lies external to the egg zone, and with it grow round the ovary to form the tunica albuginea. As the processes of the stroma of the ovary grow in between and around the young eggs in the egg

zone, fusiform connective tissue corpuscles (n, n) may be seen lying in contact with the yelk substance of the eggs, and these fusiform corpuscles are exactly similar in appearance to the corpuscles which make up the stroma (j, j) in other parts of the ovary. Around many of the young eggs the corpuscles of the membrana granulosa may be traced as they develope from these fusiform connective tissue corpuscles. The nuclei of these connective tissue corpuscles divide, swell up, and gradually form a wreath of little nuclei round the ovum. Each little vesicular nucleus has around it a small quantity of protoplasm, and in the nucleus a spot is generally found. By a constant division of these corpuscles the membrana granulosa at last consists of a thick layer, just as in the case of the Graafian follicle in the rabbit's ovary.

In the adult human ovary an exactly similar development of membrana granulosa corpuscles can be followed out. As the eggs lie imbedded in the stroma, the nuclei of those elongated fibres of the stroma which are in contact with the yelk substance swell up, and by constant division produce a wreath of little corpuscles round the ovum.

On comparing the epithelium on the surface of these adult ovaries with the fusiform corpuscles which lie round the young eggs imbedded in the stroma of the ovary, from which the corpuscles of the membrana granulosa are produced, we at once see how altogether different they are in appearance, and how impossible it is that there can be any connection between them. The epithelium on the surface of the adult cat's ovary is separated from the deeply imbedded eggs by a thick layer of connective tissue, and while the corpuscles of the epithelium are flat, polygonal bodies, with oval nuclei, the corpuscles in contact with the imbedded young eggs are elongated fusiform bodies, similar to those which make up the stroma in all parts of the ovary. By carefully examining these fusiform bodies as they lie on the surface of the egg, and as they lie round the egg, as in a profile view, it is seen that they are entirely different from the epithelial corpuscles on the surface of the ovary, and they are parts of the ovarian stroma. These observations appear to me to prove very conclusively that WALDEYER's view as to the development of the cells of the membrana granulosa is untenable.

After a single layer of membrana granulosa corpuscles is produced round the ovum, the wall of the follicle outside this capsular layer becomes fibrous and vascular. The wall of a nearly ripe Graafian follicle is very vascular. In the rabbit's ovary, in a very young follicle, the corpuscles around the egg, which give rise to the membrana granulosa corpuscles, are at first minute fusiform bodies, and lie flattened against the ovum, when seen in profile. As they develope into the corpuscles of the membrana granulosa they swell up, and by pressure against each other become columnar. Now, immediately outside this layer of columnar corpuscles the tissue consists of minute fusiform corpuscles (figs. 35, 36, 37, j,

j, *j*) and blood-vessels. In a nearly ripe Graafian follicle, just before bursting to liberate the ovum, these minute corpuscles in the wall of the follicle outside the membrana granulosa swell up and enlarge, producing large fusiform corpuscles (fig. 38, *j*, *j*) very similar to the corpuscles of the membrana granulosa at a certain stage of development. This condition of the wall of the ripe follicle is also well seen in the human ovary. In the wall of the nearly ripe Graafian follicle of the rabbit's ovary, we can trace these large fusiform corpuscles becoming more and more like the ordinary small corpuscles of the stroma, as we examine them in the more external parts of the follicular wall. I have ascertained by careful observation that from the very first appearance of the Graafian follicles to their bursting, no blood-vessels pass into the cavity of the follicle to reach the ovum, and yet this body increases enormously in size in a short space of time. This great increase in size is brought about by the production of protoplasm or yelk round the germinal vesicle, even after the latter has reached a definite size.

The chief function of the membrana granulosa is to nourish the ovum during its development. From the first appearance of the ovum as an ordinary germ epithelial corpuscle, until the development of the embryo and its extrusion from the uterus, it exists as a parasite. On the surface of the ovary at first it is an ordinary germ epithelial corpuscle. In the ovary, during a certain stage of its development, it is simply surrounded by vascular nutritious tissue, of which, however, it forms no part, but is simply resting on it. Here it imbibes nourishment until it is thrown off from the ovary. It then passes into the uterus, where changes due to impregnation occur in it, and processes of its surface become imbedded in the vascular mucous membrane of the uterus, and by the agency of these nourishment is absorbed for the germ till it reaches a certain stage of development, when it is thrown off from the parent. In the uterus, as in the ovary, the ovum is simply surrounded by vascular nutritious tissue of which it forms no part, but simply rests on it, and absorbs nourishment from it for its own development.

In the old follicles, some of the cells of the membrana granulosa show a distinct but fine cell-wall round the protoplasm which invests the nucleus. In some of my preparations of the rabbit's ovary, the division of the nuclei of the corpuscles in the membrana granulosa is excellently seen (fig. 31, *v*.) In these older follicles the protoplasm round the nucleus of the membrana granulosa corpuscle is sometimes very extensive, forming a thick layer which may be found drawn out in a fusiform manner (fig. 38, *u*) at one or more points.

In certain parts of the adult cat's and rabbit's ovary, we find large patches of granular cells, and running in between the cells are processes of connective tissue. These large granular cells have nuclei, but they are indistinctly seen because of the granular nature of the substance which surrounds them. In most cases the

nuclei appear as round or oval bodies with nucleoli. An examination of the patches shows that the granular cells are swollen up individuals of the membrana granulosa from old and probably ruptured follicles, and are undergoing a fatty degeneration, while at the same time connective tissue corpuscles from the wall of the follicle are growing in between them. In almost every old ovary we find large yellow patches, consisting of the cells we have now described.

At birth the stroma of the human ovary is well developed in every part, and is arranged in a perfect network throughout the whole organ. The ovary might be compared to a piece of sponge in whose meshes ova are contained. The ova in the human ovary at birth are very numerous. They may be fairly estimated at about 35,000 in each organ. In favourable specimens a section of such an ovary presents under the microscope a splendid object. In it the stroma appears to be actually saturated with young ova (fig. 23, *m*), in all of which we can recognise the germinal vesicle as a distinct spherical body, and sometimes two or more germinal spots are seen in it, and around each germinal vesicle is the yelk substance filling up the entire cavity of the Graafian follicle. In all parts of the stroma the minute connective tissue corpuscles crowd together, and in the deeper parts of the ovary, where the oldest Graafian follicles are situated, the connective tissue corpuscles in the walls of the follicles may be seen in contact with and indenting the yelk substance of the included young ova, and in these cases the development of the corpuscles of the membrana granulosa may be clearly followed out. In such a section, under the germ epithelium, the last formed egg clusters have not as yet been completely subdivided by the connective tissue stroma into the ultimate egg containing meshes or Graafian follicles, but this is rapidly approaching completion.

After birth, the inclusion of germ epithelial corpuscles by the ovarian stroma becomes less and less, until at the age of about two years the process has entirely ceased; for at this period of development the tunica albuginea has assumed a special character as a complete investment to the whole ovary under the germ epithelium. Most superficially its fibres are arranged in a stratified manner, and run horizontally round the organ; and in sections the general stroma of the ovary is seen in connection with the tunica albuginea in all parts.

From the earliest appearance of the ovary we have on the one hand a growth of germ epithelial corpuscles, and on the other a growth of vascular connective tissue. The germ epithelial corpuscles, in groups or clusters, become gradually surrounded by the vascular connective tissue, and, as development proceeds, the connective tissue grows into the groups between the corpuscles, and these become at last separated from each other, and by the thickening of the tissue in, between, and around them, they are ultimately included in separate

meshes of the stroma. At the same time the imbedded corpuscles are developing into primordial ova. This intercorpuscular growth of connective tissue takes place in all parts of the ovary wherever groups of germ epithelial corpuscles become imbedded, and it continues until the inclusion of germ epithelial corpuscles from the surface of the ovary has ceased. It is in this way, and in this way only, that we have Graafian follicles formed. As already explained, the Graafian follicles are the ultimate meshes of the stroma, formed by the growth of the connective tissue around the developing primordial ova. As these young ova become surrounded by the vascular tissue, the connective tissue corpuscles in the wall of the follicles in contact with their yelk develope into the corpuscles of the membrana granulosa in the manner described. According to my observations, tubular structures have no existence in the ovary at any period of its development, and I have never detected the formation of Graafian follicles and the corpuscles of the membrana granulosa in any other manner than that I have indicated.

By the continued free growth of the vascular stroma throughout the whole organ, the walls of the Graafian follicles become greatly thickened. The oldest and largest Graafian follicles are situated in the deepest parts of the stroma; this is what we would naturally expect, for they were the first formed, and lie surrounded by a great number of blood-vessels which are branches of the large trunks entering the ovary at the hilum.

Thus, in the ovary of a rabbit of about six weeks, one sees the largest Graafian follicle situated deeply in the organ; on the surface of the ovary is the germ epithelium. Between the deeply situated Graafian follicles and the germ epithelium on the surface of the ovary, the Graafian follicles with the contained ova become gradually less and less in size. The youngest Graafian follicles, the last formed, are found immediately under the layer of tissue known as the tunica albuginea on which the germ epithelium rests.

Although the eggs are so numerous in an ovary at birth, very few of them come to maturity.

In the ovary of a cat about five months old is a thick zone of young eggs immediately under that stratum of tissue which passes round the ovary constituting the tunica albuginea. This egg zone is all that remains of the egg clusters which I described as forming so large a part of the ovary of a kitten of two to four weeks.

Below the egg zone the ovary consists of fibro-vascular tissue. This part forms about two-thirds of the whole organ, and thick processes of it pass upwards among the eggs in the egg zone. These thick processes have developed from the long strings or delicate bundles of fusiform corpuscles, which in the young kitten grew up between the egg clusters separating these latter from each other. Secondary offshoots may now be traced growing from the larger bundles in

between the eggs, and as these processes thicken all traces of single large egg clusters become obliterated, and a complete zone of eggs, passing round the ovary takes their place. In this egg zone a great number of eggs become destroyed. In some cases large vacuoles form in the yelk or protoplasm which surrounds the germinal vesicle, and this latter structure is often pushed aside or pressed against the wall of the follicle. The germinal vesicle itself does not seem to be involved in this process of vacuolation. The vacuoles form in the yelk and not in the germinal vesicle. In the human ovary at birth I have also seen a large number of eggs distorted or destroyed by the formation of large vacuoles in the yelk substance.

I do not attempt to explain the meaning of this formation of vacuoles in the yelk substance of the young eggs. I have observed it in the ovaries of several mammals, and I have also seen it taking place in the protoplasm which surrounds the nuclei of large cancer cells, obtained from cancerous ovaries and other malignant growths. I may also mention, that I have seen vacuoles in the protoplasm around the nuclei of pus and renal epithelial cells.

In the central part of this five months' cat's ovary large Graafian follicles are found in various stages of development.

In the growth of these large Graafian follicles, many young and smaller follicles are destroyed by pressure; and it would appear that many of the imbedded eggs simply atrophy and form a pabulum for the connective tissue corpuscles which surround them.

It may be asked, What becomes of the germ epithelium after its egg-forming character has disappeared?

After birth the corpuscles of the germ epithelium gradually become smaller in size, losing their columnar form, till at the age of six years, in the case of the human ovary, they present the appearance of small flattened corpuscles (fig. 41, h_1). They are very small, measuring in their longest diameter not more than the $\frac{1}{3000}$th part of an inch. The epithelial membrane composed of such corpuscles can be stripped off the surface of the ovary without difficulty. In the human ovary at twelve years of age, the epithelium has preserved an almost identical appearance to that above described, the epithelial corpuscles still remaining of very small size. In the ovary of an adult, the epithelial corpuscles do not measure more than the $\frac{1}{3000}$th to the $\frac{1}{2500}$th part of an inch in diameter.

In the ovary of old rabbits, the germ epithelial corpuscles remain as a well-marked layer of small oval corpuscles, and in the ovary of old cats the corpuscles are flattened from above downwards, and show very beautifully their epithelial character (fig. 40, h_1). The corpuscles show well-marked oval nuclei, and the protoplasm round them is extended out and abuts against the protoplasm around neighbouring nuclei; at the lines of contact there is the appearance of a cell wall.

Structure of the Ovum.

In man and in the cat the mature ovum measures about the $\frac{1}{150}$th part of an inch in diameter.

The zona pellucida, so named by Von Baer, appears when examined under low powers as a strong, perfectly transparent, homogeneous lamella sharply differentiated from the yelk. The membrane possesses considerable elasticity, and when torn to pieces with a needle, the yelk substance with the germinal vesicle escapes in a stream.

Generally, the external surface is exactly parallel to the internal, but in certain specimens I have seen fine processes of the membrane projecting outwards in a radiating manner between the epithelial corpuscles which invest the ovum.

In the zona pellucida investing the ovum in the rabbit's ovary, when examined under very high powers, fine radiating lines may be seen running through it from its external to its internal surface. I have not seen these lines in the zona pellucida in the cat. They were first described by J. Müller and Remak in the vitelline membrane of the ova of fish. In an extremely thin section of the zona in the rabbit's ovum I have not been able to see them, but in thick sections they are readily found. I believe these radiated lines do not indicate any special structure of the zona, but are produced by the membrana granulosa cells which lie in contact with each other, the lines of contact being seen through the zona as fine linear marks.

The zona resists the action of acetic acid, and its chemical characters are not accurately known.

It is extremely difficult to say how the zona is formed. By most continental observers it is considered to be a product of the follicular epithelium, but from the circumstance that in many instances in the cat's ovary I have discovered it as a well-marked structure round the ovum when little or no trace of the follicular epithelium existed, I am inclined to the belief that it is formed by a hardening of the external part of the yelk of the ovum. I regard the ovum as a large cell, of which the zona is the cell wall and the germinal vesicle is the nucleus.

The parts of the ovum within the zona pellucida consist of the yelk substance and the germinal vesicle. The yelk substance forms by far the greater part of the mature ovum, and constitutes a large spherical mass in which a number of minute granules are suspended. In the eggs of young cats these granules and small bright bodies are found collected together at the external part of the yelk near the zona pellucida. As already mentioned, the primordial eggs do not possess a cell wall at first, but lie in the young Graafian follicles with their yelk substance in close contact with the fibrous wall of the stroma which

surrounds them. It is not uncommon to find two primordial ova in a single Graafian follicle, and as these eggs develope and lie in close contact with each other they are flattened one against the other, and a distinct zona pellucida may be seen round the yelk of each, although no follicular epithelial cells intervene between the eggs. Follicular epithelial cells, however, line the follicle in which these eggs are contained.

I have failed to detect any membrane between the yelk and the zona pellucida. In the mature eggs, as well as in the primordial ova, large vacuoles are occasionally found.

The germinal vesicle appears as a sharply defined globular body placed somewhat excentrically in the interior of the yelk, and is about a fifth to a sixth of the whole egg in diameter. It possesses a delicate membranous wall clearly defined under high powers of the microscope. The germinal vesicle is a highly refractile body, and is at once conspicuous within the ovum. Within it a few granules and one or two distinct nucleoli are found. In a human ovary at birth, very frequently primordial ova with two germinal vesicles are detected, and I have one specimen of the ovary of a child of two years in which a large single ovum contains four well-formed distinct germinal vesicles. In this case the yelk substance is very extensive, and completely surrounds the four vesicles. The wall of the Graafian follicle is in close contact with the yelk, and small fusiform corpuscles around this substance may be traced developing into the corpuscles of the membrana granulosa, but no zona pellucida is to be seen. In the ovary of an adult woman I have also seen a single ovum with two germinal vesicles, but the egg was comparatively young and no zona pellucida or complete epithelial investment was around it.

The nucleolus or macula germinativa is said to be always present in the primordial egg. The presence of two or three germinal spots in a single germinal vesicle has often been noted. Besides the germinal spot a few minute corpuscles are occasionally seen in the germinal vesicle. By some the germinal spot is thought to be a solid body, and by others it is considered to be of the nature of a vesicle, and I have seen round it an extremely fine linear investment. In the cat's ovum very frequently a minute bright spot or vacuole is noticed in the germinal spot. SCHRÖN described this as a solid body, and termed it the "granule." (KORN).

GENERAL CONCLUSIONS.

The following general conclusions have been arrived at in the course of my investigations :—

The corpuscles of the germ epithelium are derived by direct proliferation from those columnar corpuscles which invest the median side or surface of the Wolffian body, and which are continuous with the layer of columnar corpuscles

that lines the pleuro-peritoneal cavity of the embryo in the early stages of development. The stroma of the ovary in the early stages of development is produced by a direct growth out from the interstitial tissue of the Wolffian body immediately beneath the germ epithelium on the median side of the Wolffian body.

The germ epithelial corpuscles proliferate by fission. In the human fœtal ovary of $7\frac{1}{2}$ months they measure $\frac{1}{2500}$th to $\frac{1}{2000}$th of an inch in their longest diameter, and about $\frac{1}{3500}$th of an inch in their shortest diameter. Each germ epithelial corpuscle is a nucleus surrounded by a thin film or investment of clear protoplasm. In the act of becoming primordial ova, the nucleus of each germ epithelial corpuscle swells up into a spherical body, within which is generally seen a nucleolus, and around which is produced clear homogeneous protoplasm which subsequently forms the yelk of the ovum. Germ epithelial corpuscles are seen on the surface of the ovary in all stages of development into primordial ova. In each primordial ovum the spherical germinal vesicle presents a sharply defined limiting membranous wall. Within the germinal vesicle is the nucleolus or germinal spot. All the ova in the ovary are derived from germ epithelial corpuscles.

In all parts of the germ epithelium processes of vascular connective tissue stroma grow in between and around certain of the germ epithelial corpuscles, whereby the latter become more and more imbedded in the stroma of the ovary. Germ epithelial corpuscles are being constantly produced on the surface of the ovary, to take the place of those already imbedded in the stroma. The imbedded corpuscles increase in number by division, and the nucleus of each swells up into a spherical germinal vesicle, around which is gradually produced the yelk of the ovum. In all parts of the young ovary under the germ epithelium, groups of germ epithelial corpuscles become imbedded in meshes of the stroma. As each individual in the group swells up the nucleus or germinal vesicle becomes very distinct as a round or spherical body. From the swelling out of each germ epithelial corpuscle in the group, the whole group expands and becomes more or less spherical. Such groups of developing corpuscles are called egg clusters. Each egg cluster is inclosed is a mesh or capsule of vascular stroma of the ovary. Each imbedded germ epithelial corpuscle is potentially an ovum.

The stroma of the young ovary consists for the most part of fusiform connective tissue corpuscles and blood-vessels. The walls of the young blood-vessels in the young stroma consist of connective tissue corpuscles. The connective tissue corpuscles are direct offshoots from the ovarian stroma, and are found in contact with the yelk or protoplasm of each primordial ovum situated among the germ epithelial corpuscles on the surface of the ovary. Wherever we find primordial ova we see connective tissue corpuscles in con-

tact with the yelk of each. In all parts of the ovary we find the nuclei of connective tissue corpuscles dividing. Sometimes these corpuscles are swollen out into round bodies containing three to four nuclei. In each egg cluster several of the included germ epithelial corpuscles are in a much farther advanced stage of development than their fellows. From the walls of the meshes inclosing the egg clusters, delicate processes of vascular connective tissue grow in, between, and around individual corpuscles in the egg clusters, and by a continued intergrowth of the young stroma in this manner each individual of the group becomes at last enclosed in a separate mesh or capsule. These last formed meshes are the Graafian follicles.

As a rule, each Graafian follicle is occupied by one young ovum. The protoplasm or yelk of each ovum is in close contact with the wall of each Graafian follicle. In contact with the yelk of each young ovum, and indenting it, are connective tissue corpuscles, which form part of the wall of each Graafian follicle. In the formation of the membrana granulosa, these connective tissue corpuscles in the wall of the Graafian follicle, and in contact with the yelk of the contained ovum, increase in number by division, their nuclei swell out into little vesicles, and at last a perfect capsule of such corpuscles is produced round the ovum. This capsule is the membrana granulosa or follicular epithelium of the follicle. At first the membrana granulosa consists of a simple layer of corpuscles lining the follicle. The individual corpuscles of the membrana granulosa in the human ovary measure about $\frac{1}{3000}$th inch. As the ovum becomes mature, the corpuscles of the membrana granulosa proliferate, and then many layers of small corpuscles are produced between the ovum and the follicular wall. The cells of the membrana granulosa are thus derived from the corpuscles of the connective tissue stroma, and not, as WALDEYER states, from the germ epithelial corpuscles. The follicular space is formed by a breaking down and probable solution of certain of the corpuscles of the thickened follicular epithelium in the middle parts of the same. The discus proligerus consists of follicular epithelial corpuscles, which are in contact with the zona pellucida of the ovum. The zona pellucida or vitelline membrane is formed by a hardening of the outer part of the yelk or protoplasm of the ovum, and is not, as REICHERT, PFLÜGER, and WALDEYER stated, a product of the follicular epithelium. At birth the human ovary contains not less than 35,000 ova, few of which reach maturity.

In the human ovary at birth the germinal vesicles measure $\frac{1}{1500}$th to $\frac{1}{1200}$th of an inch. Most of them are about the same size, and present a sharply-defined membranous wall. In some germinal vesicles two or three germinal spots are seen. The tunica albuginea is the thickened stroma growing round the ovary. At the age of $2\frac{1}{2}$ years all formation of ova from the germ epithelium has ceased.

Graafian follicles are not formed from tubular structures in the manner

described by PFLÜGER, SPIEGELBERG, and WALDEYER. The appearance of tubular structures passing into the stroma of the ovary are produced by sections through furrows and depressions between irregular prominences on the surface of the fœtal ovary. The irregularities of the surface of the fœtal ovary are produced at first by the expansion of egg clusters upwards under the germ epithelium. Where the walls of furrows and depressions come in contact, egg clusters are formed by the imbedding of germ epithelial corpuscles in that situation, just as in other situations. Egg clusters are formed in connection with the germ epithelium lining the furrows and depressions. Among the germ epithelial corpuscles lining the furrows, &c., we find large primordial ova, and corpuscles in all stages of development into the same, just as in other situations among the ordinary germ epithelial corpuscles.

In the human fœtus, round and oval-shaped groups of germ epithelial corpuscles are found in connection with the germ epithelium all round the ovary. When vertical sections are made through these they present the appearance as if tubular structures filled with developing germ epithelial corpuscles passed from the germ epithelium downwards into the stroma of the ovary.

The development of the corpuscles of the membrana granulosa, from connective tissue corpuscles of the stroma, can be well followed out in the ovaries of adult rabbits and cats.

At the age of six years the epithelium on the human ovary consists of very small flat hexagonal-shaped corpuscles, measuring $\frac{1}{2500}$th to $\frac{1}{3200}$th of an inch. The corpuscles are undergoing division. This layer can be stripped offwithout difficulty. At the age of twelve the epithelium has little difference in appearance from the above, the small size of the epithelial corpuscles being remarkable. The epithelium is beautifully seen in old cats, and must be regarded as homologous with the peritoneal epithelium. In old cats the epithelium on the surface of the ovary consists of very small distinct cells, measuring from $\frac{1}{1800}$th to $\frac{1}{2000}$th inch, with granular oval nuclei.

EXPLANATION OF PLATES.

In these plates the same letters have been employed to mark corresponding structures in the whole series of figures.

a. The parenchyma of the ovary.
b. The fibro-vascular peduncle or stalk of the ovary.
c. The lower border of the ovary at the hilum.
d. The layer of epithelium on the stalk of the ovary.
e. The layer of epithelium stripped off from the stalk, and seen from a surface view.
f. Peritoneal epithelial corpuscles.
g. Corpuscles intermediate between peritoneal and germ epithelial corpuscles.
h. Germ epithelial corpuscles.
h_1. Corpuscles from the surface of adult ovary.
i. Fusiform shaped corpuscles among germ epithelial corpuscles.
j. Connective-tissue stroma of the ovary.
k. Blood-vessels.
l. Large spherical germ epithelial corpuscles.
m. Primordial ova.
n. Fusiform corpuscles in contact with the yelk-substance of the primordial ova.
o. Large egg clusters.
p. Tubiform depressions of the surface of the ovary.
q. Small groups of germ epithelial corpuscles being included in the stroma of the ovary.
r. The membrana granulosa.
s. The zona pellucida.
t. The yelk substance of the nearly mature ovum.
u. Individual corpuscles of the membrana granulosa, showing the nucleus surrounded with a considerable quantity of protoplasm.
v. Division of the nuclei of the corpuscles of the membrana granulosa.

PLATE XXVII.

Figure 1. Vertical section through the ovary of a fœtal calf, about 9 inches long, showing the parenchyma (*a*) and the fibro-vascular stalk or peduncle (*b*) prolonged into the ovary at the hilum (*c*). On the surface of the peduncle is a layer of epithelium (*d*). (Page 349.)

Figure 2. Surface view of this layer of epithelium. (Page 349.)

Figure 3. Profile view of the same layer of epithelium, showing the gradual change of peritoneal epithelial corpuscles (*f*) into germ epithelial corpuscles (*h*). (Page 350.)

Figure 4. The parenchyma of the fœtal calf's ovary, showing the germ epithelial corpuscles (*h*) and the growth of delicate connective-tissue of the stroma (*j*) and blood-vessels (*k*) among the corpuscles below the germ epithelium. (Page 350.)

Figure 5. A portion of the fibro-vascular zone of the fœtal calf's ovary, showing a primordial ovum (*m*), surrounded with a wreath of small corpuscles, outside of which are blood-vessels (*k*) and young connective-tissue. (Page 351.)

Figure 6. Portion of the ovary of a kitten about three weeks old, showing the germ epithelium (*h*), large spherical germ epithelial corpuscles (*l*), and primordial ova (*m*.) In contact with the yelk substance of these primordial ova are fusiform corpuscles (*n*). Bundles of young connective-tissue (*j*) are seen growing upwards, between, and around large egg clusters (*o*), situated immediately below the germ epithelium. (Page 352.)

Figure 7. Semidiagrammatic drawing of the ovary of a human fœtus of $7\frac{1}{2}$ months, showing the parenchyma (*a*), and the fibro-vascular peduncle (*b*), prolonged into the ovary at the hilum (*c*). From the peduncle the whole stroma of the ovary is derived. The germ epithelium (*h*) rests on the youngest connective-tissue, which is part of the stroma, and is the forerunner of the tunica albugineæ. (Page 357.)

PLATE XXVIII.

Figure 8. Section of a human fœtal ovary of $7\frac{1}{2}$ months, seen under a low magnifying power, showing the stalk or peduncle (*b*), the parenchyma of the ovary (*a*), and the lower border of the ovary (*c*).

Figure 9. A portion of the same ovary highly magnified showing the gradual change of peritoneal epithelial corpuscles (*f*) on the peduncle into germ epithelial corpuscles (*h*) at the lower border of the ovary (*c*). (Page 359.)

Figures 10, 11, 12. Surface view of the germ epithelium from the same ovary. In figure 12, among the ordinary germ epithelial corpuscles (*h*) large spherical germ epithelial corpuscles (*l*) are seen. (Page 358.)

Figure 13. The ordinary germ epithelial corpuscle (*h*) and the larger spherical corpuscle (*l*). (Page 358.)

Figures 14, 15, 16. Profile views of the germ epithelium. Among the ordinary columnar corpuscles (*h*) large spherical corpuscles (*l*) and primordial ova (*m*) are seen. Fusiform corpuscles (*n*) are in contact with the yelk of the primordial ova. (Page 358.)

Figure 17. A large primordial ovum (*m*), with several large spherical corpuscles (*l*) included in a mesh of the stroma (*j*) under the germ epithelium (*h*).

Figure 18. An egg cluster (*o*), consisting of large sperical corpuscles (*l*) included in a mesh of the stroma (*j*), under the germ epithelium (*h*).

Plate XXIX.

Figure 19. A portion of the ovary of a human fœtus of 3½ months, showing the germ epithelium (*h*) and groups of large germ epithelial corpuscles or egg clusters (*o*) imbedded in the stroma. The stroma (*j*) consists almost entirely of connective-tissue corpuscles. (Page 361.)

Figure 20. A portion of the same ovary, showing large primordial ova (*m*) imbedded in the stroma (*j*). Fusiform corpuscles (*n*) of the stroma are seen in contact with the yelk of these primordial ova. (Page 361.)

Figure 21. A portion of the ovary of a human fœtus of 7½ months, showing the manner of inclusion of the germ epithelial corpuscles in groups (*q*) in the stroma of the ovary. (Page 362.)

Figure 22. A portion of the ovary of a human fœtus of 7½ months, showing in section the furrows (*p*) which lie between the prominences on the surface of the ovary. (Page 363.)

Figure 23. A portion of the ovary of a child at birth, showing the stroma (*j*) crowded with primordial ova (*m*).

Plate XXX.

Figure 24. Section through the ovary of a human fœtus of 7½ months, showing the germ epithelium (*h*), large spherical corpuscles (*l*), and groups of similar corpuscles or egg clusters (*o*) imbedded in meshes of the stroma (*j*). In the lower part of the figure many primordial ova (*m*) in various stages of development are seen. In contact with each primordial ovum are fusiform, connective-tissue corpuscles (*n*), similar to the fusiform corpuscles of which the stroma consists. Numerous blood-vessels (*k*), whose walls consist of connective-tissue corpuscles, ramify throughout the whole ovary. (Page 365.)

Figure 25. A small egg cluster consisting of large spherical germ epithelial corpuscles with fusiform connective tissue corpuscles (*n*) intermingled. (Page 366.)

Figure 26. A small group of spherical germ epithelial corpuscles.

Figure 27. A primordial ovum. It has no zona pellucida.

Figures 28, 29. Primordial ova (*m*). In contact with and indenting the yelk substance of each, are fusiform connective-tissue corpuscles from the wall of the young Graafian follicle.

Figure 30. The membrana granulosa (*r*) when first formed seen in section. Outside the membrana is the ovarian stroma (*j*). From the human ovary at birth.

Figure 31. The corpuscles of the membrana granulosa from the nearly ripe Graafian follicle of the rabbit's ovary. The letter *v* points to the division of the nuclei of the corpuscles.

Figure 32. Surface view of the membrana granulosa (*r*) when first formed. From the human ovary at birth.

Plate XXXI.

Figures, 33, 34, 35 36, 37, 38, illustrate the development of the membrana granulosa in the ovary of an adult rabbit. (Page 368.)

Figure 33. A primordial ovum (*m*). Fusiform connective-tissue corpuscles (*n*) of the stroma (*j*) lie in contact with the yelk substance of the young ovum.

Figure 34. A young ovum farther advanced in development than the last. Some of the fusiform corpuscles (*n*) in contact with the yelk are swollen out.

Figure 35. An ovum still farther advanced in development, completely surrounded by a wreath of corpuscles, which have been derived from the fusiform corpuscles of the stroma (*j*). This wreath is the membrana granulosa (*r*) seen in section.

Figure 36. A Graafian follicle from which the ovum has been removed. The membrana granulosa (*r*) is seen in section.

Figure 37. A section through a Graafian follicle, and the nearly ripe ovum contained in it. The membrana granulosa (*r*) consists of several layers of corpuscles. The zona pellucida (*s*) is well developed, and the yelk substance (*t*) at its peripheral part contains numerous bright granules and particles.

Figure 38. A section through the membrana granulosa and wall of a ripe Graafian follicle just before bursting. The membrana granulosa (*r*) consists of several layers of corpuscles. Each corpuscle is a large nucleus surrounded by protoplasm (*u*). In the wall of the follicle outside the membrana granulosa the fusiform corpuscles of the stroma (*j*) are very large. (Page 371.)

Figure 39. A section through the egg zone of an old cat's ovary. The stroma (*j*) consists almost entirely of fusiform corpuscles. Around the young eggs (*m*) the fusiform corpuscles (*n*) of the stroma may be traced in their development into the corpuscles of the membrana granulosa. (Page 369.)

Figure 40. The epithelium (h_1) from the surface of the same cat's ovary.

Figure 41. The epithelium (h_1) from the surface of the ovary of a child six years of age.

Figure 42. The same epithelium more highly magnified showing the "grooved" appearance of the corpuscles.

Figure 43. Germ epithelial corpuscles (*h*) seen in profile. From the ovary of a $7\frac{1}{2}$ months' human fœtus.

Figure 44. A primordial ovum (*m*) surrounded with fusiform corpuscles (*n*). From the stroma of the same ovary.

Figure 45. An ovum (*t*) from the deeper parts of the same ovary, showing the appearance of the membrana granulosa (*r*) when first formed round the ovum.

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Fig. 1.

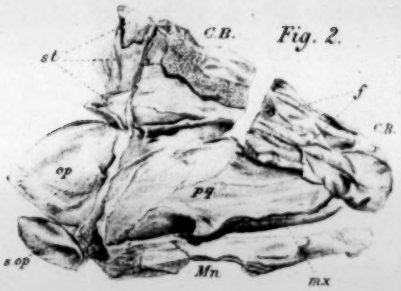


Fig. 2.



Fig. 4.



Fig. 5.



Fig. 3.



Fig. 6.



Fig. 7.

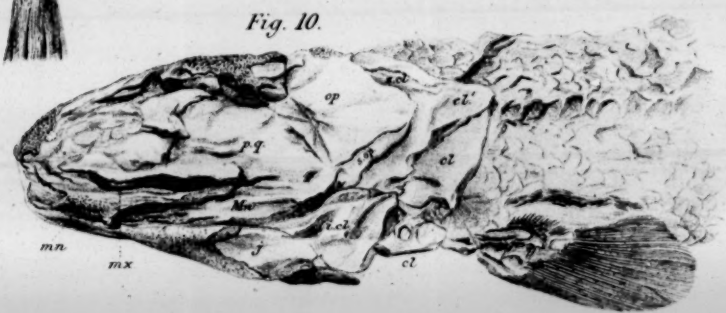


Fig. 8.



Fig. 9.

Fig. 10.



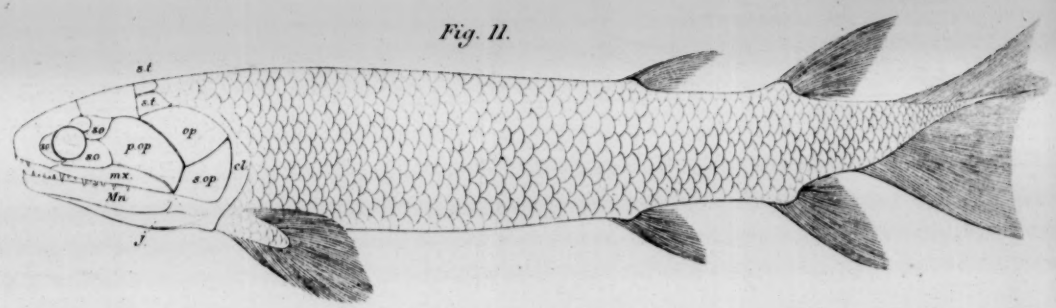


Fig. 11.

XVIII.—*On the Structure and Affinities of* Tristichopterus alatus, *Egerton.*

By Ramsay H. Traquair, M.D., F.G.S., Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh. (Plate XXXII.)

(Read April 5, 1875.)

Concerning the affinities and systematic position of this very remarkable Devonian fish, there has hitherto prevailed very great uncertainty. The two original specimens, discovered by Mr C. W. Peach, in the Old Red Sandstone of John O'Groat's, Caithness, and described by Sir Philip Egerton,* left us in complete ignorance as to the osteology of the head and the dentition, while the evidence they afforded as to the structure of the pectoral fins was by no means so clear as might have been wished for. To quote from Sir Philip's description:—"The bones of the head, with the exception of a small fragment of the operculum, are wanting, but the impressions left upon the matrix show that they were sculptured in rather a bold pattern, not unlike the ornament on some of the cranial bones of some of the *Holoptychii*, and consequently differing in this respect from the corresponding parts in *Dipterus*. The pectoral fins are very indistinctly seen. They appear to have had a short obtuse lobe forming the base, and extending therefrom a set of numerous fin-rays more elongated than those forming the pectoral fin in *Dipterus*." To Dipterus, however, in Sir Philip Egerton's opinion, its affinities pointed, as far as could be gathered from the structure of the body as displayed in the specimens, his description concluding as follows:—"The absence of all evidence as to the dental apparatus of Tristichopterus is much to be regretted. On other points the affinities between this genus and Dipterus are so striking that they cannot be classified in separate families.† Accordingly he assigned to *Tristichopterus* a place along with Dipterus in the family of "Cœlacanthi," the term being used in its former extended sense, not as now restricted to the peculiar genera *Cœlacanthus*, *Undina*, *Holophagus*, and *Macropoma*.

Professor Huxley, at the conclusion of his Essay on the Classification of the Devonian Fishes,‡ published in the same Decade of the Geological Survey, makes the following statement regarding the genus in question:—"In the absence of a full knowledge of the head, of the paired fins, and of the dentition,

* Dec. Geol. Survey, x. 1861, pp. 51–55, pl. v.
† *Loc. cit.* p. 55.
‡ Dec. Geol. Survey, x. 1861, p. 40.

it would be hazardous to form any decided opinion as to the affinities of *Tristichopterus;* I strongly suspect, however, that it will turn out to be the type of a new family allied to the Ctenododipterini and Cœlacanthini." However, at page 24 of the same essay, he places it along with *Dipterus* in PANDER's family of Ctenododipterini, though with a mark of interrogation.

But when, in 1871, Dr GÜNTHER* pointed out the close relationship between *Dipterus* and the recent *Ceratodus* and *Lepidosiren,* and the consequent desirability of transferring the first named genus to the group of Dipnoi, the question of course came up as to whether *Tristichopterus* should also accompany it thither. But to this no satisfactory answer could be given, so long as our knowledge on so many essential points of its structure was deficient. After referring to the manner in which the innumerable fine and closely-placed rays of the fins overlapped "with their proximal ends, the extremities of the interspinous bones, as in the Dipnoi," and the peculiar form of the tail, which "represents a most curious intermediate condition between the diphycercal tail of the *Sirenidæ* and the heterocercal of *Dipterus,*" Dr GÜNTHER concludes:—"Unfortunately, the head and the base of the paired fins are destroyed in the only two specimens known; and it is chiefly the last-named character which prevents me from associating this genus with the *Dipnoi.*"

No further description of the structure of *Tristichopterus* having appeared since the publication of the Tenth Decade, I felt great satisfaction when Mr PEACH communicated to me a number of additional specimens, collected by him in the years 1864–65, and which throw a very great amount of the light desired on those points of its structure previously unknown to us.† These specimens exhibit in a clear and unmistakable manner the character of the dentition, the structure of the paired fins, and the leading features of the osteology of the head, and completely prove that *Tristichopterus* has no special affinity either with *Dipterus* or *Cœlacanthus.* Before, however, passing on to discuss the question of its real affinities and systematic position, I shall first proceed with the description of the new facts disclosed.

General Proportions.—One of the specimens, the counterpart of which is in the British Museum, is quite entire, though the axis of the body is so curved as to render the dorsal margin considerably concave, the ventral correspondingly convex. The entire length of this specimen, carefully ascertained with a flexible measure, is $10\frac{1}{2}$ inches, of which the head occupies about $\frac{1}{3}$th part; the greater depth of the body, just behind the subacutely lobate pectorals, being 2 inches. The general form of the fish is thus rather slender, and the fins are crowded towards the posterior aspect of the body,—the first dorsal commencing 6 inches, and the second $7\frac{1}{4}$ inches from the tip of the snout,

* Description of Ceratodus, Phil. Trans. 1871.

† These specimens are now in the Museum of Science and Art, Edinburgh.

while opposite these two dorsals are respectively placed the ventrals and the anal. The lower lobe of the very peculiarly shaped caudal fin commences at 8¼ inches from the front, and on the dorsal aspect the rays of the upper lobe begin to be apparent a few lines further back.

Another specimen, crushed vertically, and lying on its back,—or more correctly, the counterpart of a specimen in that position, is also very nearly entire; the front and a considerable part of the right side of the head being unfortunately cut off by the edge of the slab, and the extremity of the tail being rather distorted and indistinct. If we add half an inch to complete the head in front, the length of this example would also be about 10 inches; the pectorals arising 2¼ inches, the ventrals 6 inches, and the anals 7½ inches from the supposed extremity of the snout.

These two specimens thus closely correspond with each other, and with the more complete of the two examples figured by Sir PHILIP EGERTON, and show the accuracy with which he allowed for its missing portions. Some of the more fragmentary specimens before me show, however, that the fish must sometimes have attained a considerably larger size, one head measuring, from the tip of the snout to the hinder margin of the gill cover, no less than 3¼ inches, which would give over 15 for the entire length of the fish.

The Head.—The head was protected above by a cranial "buckler" (Plate XXXII. figs. 1 and 2, C.B.), which in the leading features of its configuration recalls to our minds that of the *Saurodipterini*, though its external sculpturing is very different. As in that family, it tends to divide across into two portions,—a posterior or parietal, and an anterior or fronto-ethmoidal; here, however, the anterior moiety is the longer, exceeding the other by nearly ⅓ of its length. The hinder division of this buckler is on the whole quadrate in form, but broader behind than in front, the posterior and wider margin being also somewhat concavely excavated. The front portion forms anteriorly a rounded depressed snout, and seems on each side to be excavated to take part in the formation of the upper boundary of the orbit, though this part of the margin is not so clearly defined as might be wished; nor are the nasal openings seen in any of the specimens, which is not strange, taking into account their position in *Osteolepis* and *Diplopterus*, so close to the margin of the upper lip. It is not possible to map out the ossifications entering into the composition of this buckler; probably their arrangement would not depart much from that which is to some extent traceable in the *Saurodipterini*. As far as the anterior portion is concerned, the impression of two distinct *frontals*, entering largely into its composition, is distinctly seen in the specimen represented in fig. 2, and the presence of a small conical tooth on the labial margin of the snout in another (fig. 1), leads us to conclude that the premaxillary (*p. mx*) was also here represented. The entire outer surface of the buckler, as indeed of all the external bones of the head,

is sculptured with tolerably fine, irregular, angularly contorted, and interrupted rugæ, with intervening furrows and pits, the pattern assuming sometimes almost a tubercular aspect.

Along the posterior margin of the cranial shield are three plates (*s. t*, figs. 1 and 2), one mesial, somewhat polygonal in form, and two lateral, each apparently of a triangular shape. These are obviously the representatives of the three plates, which occur in a similar position in *Osteolepis*, *Glyptolæmus*, *Megalichthys*, &c., and of which different interpretations have been given by different authors. In Professor HUXLEY's description of *Glyptolæmus*,* the mesial one is marked "supra-occipital," the two lateral, "epiotic." Mr PARKER† has, however, pointed out that they are dermal bones, and not to be considered homologous with those other deeper ossifications of the cranial cartilage. By PANDER‡ they are in *Osteolepis* simply designated "Hautknochen," and considered to be equivalent to the five little plates, which in the recent *Polypterus* occur immediately behind the transverse row of supra-temporals, and between the pair of upper supra-claviculars (supra-scapulars), being in reality the first scales of the back. On the other hand, he considered the transverse chain of small plates (supra-temporal) which lie immediately behind the parietals of Polypterus, to be represented in *Osteolepis microlepidotus* by the narrow portion of the cranial shield, which in that species is marked off near the hinder margin by a more or less interrupted superficial transverse groove. I am myself very much inclined to the belief that the three dermal bones in question are in reality equivalent to the transverse supra-temporal chain in *Polypterus* and *Lepidosteus*, and which have their representatives as well in the amphibian Labyrinthodonta as in most Teleostean fishes; the transverse grooving across the posterior part of the cranial shield in many Saurodipterines being probably only of the nature of superficial markings.

Regarding the condition of the *side walls*, or of the *base* of the cranium, not the smallest information is yielded by any of the specimens.

The facial bones in their general form and arrangement also remind us very much of those in both the "Saurodipterini" and "Glyptodipterini." The gape extends very far back, so that the lower jaw is as long as the cranium proper, and the anterior margin of the operculum comes to be inclined obliquely downwards and backwards. The *operculum* (*op*) is of a somewhat trapezoidal form, the anterior and inferior margins being the longest, the upper the shortest, while the posterior superior angle is obtuse and rounded. Below it is in contact with the *suboperculum* (*s. op*), which is also trapezoidal, but not quite so large; its anterior superior angle is produced a little way upwards into a narrow sharp-pointed process overlapped by the operculum. In front of the

* Dec. Geol. Survey, x. p. 2.
† Shoulder Girdle and Sternum, p. 19.
‡ Ueber die Saurodipterinen, &c., des Devonischen Systems, St Petersburg, 1860, p. 11–12.

opercular bones, and covering a great portion of the cheek, is a large oblong plate (*p. op*), which comes into close contact below with the hinder part of the maxilla, and articulates in front with two smaller plates separating it from the orbit, while above it is in contact with the side of the posterior part of the cranial shield. Its outer aspect is granulated; the inner is smooth, and shows, along its oblique and slightly curved posterior margin, a narrow shallow groove, evidently for articulation with the hyomandibular element of the suspensorium. The correspondence of this plate with the similarly placed one in *Osteolepis*, *Diplopterus*, *Gyroptychius*, &c., and with the great cheek plate of Polypterus, is at once evident. In those fishes it is by most authors reckoned to be the *præoperculum*, though by Professor HUXLEY it has been marked "supra-temporal," seeing that by its remarkable extension forwards on the cheek it differs so greatly from the true præoperculum in the Lepidosteid Ganoids, and in ordinary Teleostei. The *orbit* is situated very far forwards, and has connected with it a distinct chain of osseous plates (*s. o*, figs. 1 and 11). Of these there is a small one placed at the posterior superior angle of the orbit, lying for a small distance along the outer margin of the anterior division of the cranial shield, immediately in front of the transverse line, which marks that off from the posterior division. This is followed by two plates of considerable size, forming the posterior boundary of the orbit, and interposed between it and the great cheek plate. Of these the lower one is the larger; it is in contact below with the maxilla, and sends a process forwards forming a portion of the inferior boundary of the orbit, when it apparently joins another, or *præ-orbital*, plate, whose conformation is not, however, very distinctly seen. The *maxilla* (*mx*) forms a long and narrow bar, slightly expanded posteriorly, placed below the great cheek plate and the suborbitals, and extending forwards to the small *premaxilla*, which is firmly united with the anterior, or ethmoidal, part of the cranial shield. The oral margin of the maxillary bone is distinct enough, though in no specimen is its upper boundary shown with the clearness that might be desired. Internally, along its anterior two-thirds, it was certainly very firmly united to the outer margin of the palato-quadrate arch, an interval being left posteriorly for the passage of the muscles of the lower jaw.

The *palato-suspensory* apparatus (*p. q*, figs. 2, 3, and 10) is formed by a broad and extensive bony lamina, extending forwards from the opercular bones, and articulation of the lower jaw to the ethmoidal part of the skull. Its upper margin is connected posteriorly with the squamosal region of the cranium; in front of this its plane becomes a little twisted, so that its outer surface comes to look more upwards, and the previously upper margin is directed inwards to the base of the skull. Its posterior margin, gently curved with a posteriorly directed convexity, passes obliquely downwards and backwards to the very posteriorly situated articulation of the lower jaw; this margin, apparently corresponding to the hyo-

mandibular element, articulates behind with the operculum and subоperculum, and externally with the great "præ-opercular" cheek plate along the narrow groove, already described, on the hinder edge of the inner aspect of that bone. Lastly, by the anterior two-thirds of its outer margin, it is immovably fixed to the maxilla, and then recedes a little inwards from that bone before passing back to the quadrate articulation, so that, as already mentioned, an interval is left posteriorly for the passage of the masticatory muscles. In spite of the most careful examination, I have not been able to discover the lines of demarcation between the probable constituent elements of this apparatus, which in its general relations, though considerably broader in form, corresponds closely with that in the recent *Polypterus*.

The *lower jaw* (*Mn*) is long, and, though pretty robust, is more slender than in the *Saurodipterini*; near the front, its lower margin is slightly excavated. Seen from the side it is tolerably straight; from below it is, as we might expect, gently curved inwards towards the symphysis. Not much can be made out regarding its component elements, though the *dentary* is evidently powerfully developed, and a small narrow detached plate (fig. 5) lying on one of the slabs near a head, and having on one margin a few minute conical teeth, is in all probability the *splenial*.

The space between the two mandibular rami, on the under surface of the head, is occupied by two very narrow jugular plates (*j*), each of which is acutely pointed in front; there is no trace either of an azygos jugular, or of lateral ones.

Dentition.—The dentition of *Tristichopterus* is clearly enough exhibited in several of the specimens. The lower jaw of one shows two stout sharp conical teeth, each measuring $\frac{1}{8}$ inch in length by a little more than $\frac{1}{16}$ inch in diameter at the base; they are both so split that the external surface is seen only towards their apices. Another specimen exhibits three mandibular teeth of nearly the same dimensions, and entire except at their bases, besides one maxillary tooth with its apex broken off, and the complete impression of another. A detached tooth, quite entire (Pl. XXXII. fig. 8, magnified), found in the same bed with the other remains, and from its shape and markings undoubtedly belonging to the same species, measures nearly $\frac{3}{16}$ inch in length by $\frac{1}{16}$ in diameter at the base. Besides these larger teeth, the presence of smaller ones, some being very minute, is obvious; some of these are seen on the premaxillary of the head represented in fig. 1. All these teeth are of an acutely conical form, slightly incurved, and very sharp; their external surface is brilliantly polished, but not smooth in every sense, being closely fretted all over with very minute and short longitudinal indentations, passing, indeed, into fine striæ at the base. A distinct fluting is also observed at the bases of the larger teeth.

In the detached palato-quadrate arch represented in fig. 3, a portion of the upper jaw has remained attached to its outer margin, showing the bases of some

of the maxillary teeth cut in transverse section. But a still clearer illustration of the structure and arrangement of the teeth is seen in another specimen (fig. 7), being a horizontal section of a portion of a jaw in a piece of grey limestone from the same locality, and which was found naturally polished by the waves. The internal structure of the teeth here exhibited corresponds so closely with that seen in the last-mentioned unquestionable specimen of *Tristichopterus*, that we are tolerably well justified in assuming the specimen to belong to the same species.* Here the bases (*A. A.*) of two of the larger teeth are seen in section, each having the empty socket of another beside it, and in addition we find no less than 26 smaller teeth cut through, the latter becoming gradually smaller towards the middle of the interval between two larger ones. In both of those specimens the transverse sections of the bases of the larger teeth measure from $\frac{1}{12}$ to $\frac{1}{8}$ inch in diameter; in some the section is quite round, in others slightly oval. A small central pulp cavity is shown with the dentine around it arranged in a few simple plications, most of which, though not all, reach the central cavity; in this manner the pulp cavity appears in the section to send out a number of narrow radiating prolongations towards the periphery, a few of which are seen to bifurcate. Further up in the body of the tooth the dentinal folds become shorter, and the pulp cavity proportionally larger till towards the apex the latter has become perfectly simple. The pulp cavities of the smaller teeth appear to be perfectly simple throughout.

Whether or not *Tristichopterus* was possessed of palatal teeth is not discoverable from any of the specimens under description.

The Shoulder-Girdle and Paired Fins.—None of the specimens show very distinctly the upper attachment of the shoulder-girdle to the skull, or the form of the supra-claviculars. In one, evident traces are seen of a powerful *second supra-clavicular* (*s. cl*, fig. 10) extending downwards and backwards to articulate with the next or clavicular element, but too crushed and indistinct for special description. The *clavicle* (*cl*, figs. 1, 6, 9, and 10) is well marked in all; it is a stout, broad, oblong plate, expanded and produced a little forwards below; its outer surface is marked with the characteristic ridged-granular ornament, while its smooth internal aspect shows three peculiar rounded impressions, probably for the attachment of the coraco-scapular elements of the base of the pectoral fin. Articulated with the front of the lower end of the clavicle is another smaller plate (*i.cl*), the *interclavicular* of PARKER,—the "accessorisches Clavicularstück" of GEGENBAUR.

The pectoral fin itself (Pl. XXXII. figs. 9 and 10) is large, obovately fan-shaped, terminally rounded, and consists of very numerous slender rays, attached

* Since the above lines have been in type, the Museum has acquired from Mr Peach an additional and nearly perfect specimen of *Tristichopterus*, in which, near the front of the head, the base of one of the large teeth is seen broken, or cut, across in transverse section. The transverse section of this tooth is $\frac{1}{8}$ inch in diameter, and displays a structure absolutely identical with that described above.

to each side, and round the extremity, of a central scaled lobe, which is less than half the length of the fin, the latter being thus subacutely lobate. In a specimen measuring $2\frac{2}{3}$ inches from the tip of the snout to the posterior margin of the operculum, the entire pectoral fin measures $1\frac{3}{4}$ inch, and the lobe $\frac{5}{8}$ inch in length. The rays are slender and closely set, short at their commencement on each side of the lobe, but becoming very rapidly elongated; they commence to bifurcate very soon after their origin, and the process is frequently repeated, till at their terminations the longitudinal divisions are very delicate. They are also divided all along their course by transverse articulations; the joints being, however, seldom less than four or five times as long as they are broad. The pectoral fin is not absolutely complete in any one example; a comparison, however, of its appearance in the various specimens of the suite shows that it would have pretty much the form shown in the restored figure given in Pl. XXXII. fig. 11. The lobe is covered externally with scales similar to those to be presently described as covering the body, though smaller, and which generally completely obscure the supporting skeleton within. Nevertheless, in the specimen represented in fig. 9, some insight is derived into the nature of this internal skeleton, though, unfortunately, not to so full an extent as might be desired. There is first, at the lower and posterior part of the clavicle (*cl*), some obscure bony matter (*a*), which may possibly represent the remains of ossification in the scapulo-coracoid cartilage. This is followed by a central basal stem for the fin, consisting of at least two flattened oblong pieces (*b* 1, *b* 2) articulated end to end. On one side (from the possible twisting round of the otherwise displaced fin, it is hard to say if it really be the medial or lateral) are two distinct *radials* (*r* 1, *r* 2) set at acute angles to the axial stem, and of which the second arises at the place of junction of the first with the second axial segment. Attached to the extremity of the second axial segment are two additional pieces, one of which (*b* 3) seems to represent a third division of the axis, the other (*r* 3), slightly diverging, may be considered as a third radial. There is no clear evidence of radials on the other side of the axis, but as fin rays are also here present it is hard to suppose that they were destitute of similar supporting elements, and it seems meanwhile probable that defective preservation is the cause of their not being distinctly recognisable. Were radial elements present on both sides of the axis, the skeleton of the pectoral of *Tristichopterus* would present an abbreviated form of that of the Ceratodus limb, in which we have an elongated segmented central axis (*Archypterygium* of GEGENBAUR) set with segmented radials on both sides. Dr GÜNTHER has suggested an analogy between the doubly fringed acutely lobate pectoral of *Ceratodus*, and the diphycercal tail with elongated axis in that genus, and many fossil ones; the pectoral of *Tristichopterus*, and probably of all other Crossopterygidæ with subacute and obtusely lobate structure, would in like manner correspond with the diphycercal tail with shortened axis

seen in *Polypterus*.* On the other hand, the pectoral limb in the Selachii, in the Sturionidæ, Lepidosteidæ, and Amiadæ, and in modern Teleostei, offers analogies with the heterocercal tail with shortened axis, the axis being abbreviated, and the radials developed principally, or (in most cases) entirely, on one side of it.

The ventral fins, whose rays are similar in structure to those of the pectorals, are smaller than the latter, and are very slightly lobate at the base. They are better shown in one of the specimens from which Sir PHILIP EGERTON's figures were taken, than in any of the present suite. I have not observed the pelvic bones.

The Internal Skeleton of the Trunk.—An osseous vertebral column is traceable from the cranium to the extremity of the attenuated prolongation of the body through the upper part of the caudal fin. The bodies of the vertebræ were certainly ossified, but whether they were completely so, or remained more or less in the condition of "ring-vertebræ," is not discoverable from any specimen I have seen, as they are in every case so compressed that no view is afforded of their anterior or posterior surfaces. A series of distinct neural arches, passing upwards into spines, the latter flattened laterally in the fore part of the body, are seen appended to the dorsal aspect of the vertebral centra. Corresponding haemal arches and spines are seen in the caudal region, though anteriorly I have not been able to detect any trace of ribs, and the extreme caudal termination of the column seems to be formed by centra alone.

The interspinous bones of the azygos fins have been well described by Sir PHILIP EGERTON. Those of the first dorsal are small and obscure, but there are three very prominent ones supporting the second dorsal and anal fins respectively, and which are in turn supported above and below by a large flattened bony piece, considered by Sir PHILIP EGERTON to be probably a composite spinous apophysis, "formed by the union of three or more spines." Similar interspinous ossicles are seen supporting the lower lobe of the caudal, disappearing, however, before the extreme termination of the vertebral column is reached; their number is given by Sir PHILIP EGERTON as eight or ten. The ossicles supporting the anterior rays of the upper lobe of the caudal fin are certainly neurapophyseal in their nature.

The Azygos Fins.—The first dorsal, placed opposite the ventrals, is small and narrow, being somewhat elliptic-lanceolate in shape, and frequently found more or less adpressed towards the back; the second is larger and of a more expanded triangular form, closely resembling the opposed anal in general proportions and form. The bases of all three are very short, and slightly lobate; their most anterior rays are short, but rapidly increase in length till the apex

* The tail of *Polypterus* is not, however, absolutely diphycereal, though conforming more to that type than to any other.

of the fin is reached, from which they become gradually shorter, but increase in delicacy and in obliquity of direction—the most posterior ones being quite horizontal in position. The form of the caudal is indeed remarkable, presenting as Dr GÜNTHER observes, "a most peculiar intermediate condition between the diphycercal tail of the Sirenidæ and the heterocercal of Dipterus." It is large and fan-shaped, nearly truncated posteriorly, the hinder margin being only slightly excavated. A prolongation of the body axis, becoming very rapidly attenuated, and then tapering to a fine point, runs right through it, but as the lower margin of this prolongation slopes much more rapidly upwards from the commencement of the anterior rays of the lower part of the fin, its termination comes to the posterior margin of the latter much above the middle, so that, as in heterocercal forms, the larger part of the caudal fin is developed on the lower aspect of the vertebral column. The rays of the upper part of the caudal commence a little further back than those of the lower; in both cases they are short at first, but become rapidly longer till the upper and lower apices are reached, from which they become gradually shorter, finer, and more oblique in their origin from the body axis. Those arising from the extreme point of this axis are very delicate, and project beyond the margin of the rest of the fin, so as to produce the appearance described by Sir PHILIP EGERTON as "forming a kind of supplemental fin, projecting beyond the terminal margin of the true caudal fin." It must, however, be observed that the rays in question, though projecting in that remarkable manner, form a perfectly continuous series with those of the rest of the caudal (see restored figure, Pl. XXXII. fig. 11). All these fins are composed of slender, closely-set rays, repeatedly dichotomising, and divided by frequent transverse articulations, the joints being, however, as in the case of the paired fins, always considerably longer than broad. Those of the anterior part of the lower lobe of the caudal are, as Sir PHILIP EGERTON has pointed out, stouter than the others; they are, in fact, the stoutest fin rays in the entire structure of the fish. There are no traces of fulcral scales on the anterior margins of the fins, and, in this respect, *Tristichopterus* differs from *Gyroptychius* and from the *Saurodipterini*, where such scales are present, though differing rather in form from the pointed imbricating fulcra of the *Palæoniscidæ* and *Lepidosteidæ*. The statement of Sir PHILIP EGERTON, borne out by one of his figures, that the anterior rays of the upper lobe of the caudal are "short and *fulcral*, the anterior ones being short, and forming a *marginal fringe* along the upper edge of the fin," I do not find corroborated by a very beautifully preserved tail in the series of specimens from which the present description is taken. The fin-rays overlap the extremities of the supporting ossicles, and are more numerous than the latter elements—characters occurring also in many other Crossopterygidæ, and in the Palæoniscidæ.

Scales of the Body.—The scales are of moderate size, rounded, thin, and

deeply imbricating—the exposed area being smaller than that which is overlapped. The exposed surface is ornamented by very fine closely-set raised striæ, often interrupted, and branching and anastomosing; their general direction is from before backwards, though they are usually seen to converge slightly towards the median line of the scale. On very close examination these striæ may frequently be observed to be decussated obliquely by still more delicate lines, seen in the intervals between them, and which seem to radiate from the central point of the scale. The covered area is very minutely granular, a concentric linear arrangement of the granules being also usually to be observed. The impression of the under surface of a scale, in one specimen, shows evidence of a small elevated central boss or elevation. The scales of the lateral line seem to be perforated by a slime-canal, and to be slightly notched posteriorly.

Conclusion.—The structural characters of *Tristichopterus*, as far as they have as yet been ascertained, may be summed us as follows:—

Body slender, elongated; scales cycloidal, thin, imbricating, delicately striated. External bones of the head sculptured; cranial roof bones united into a buckler of two principal parts, anterior and posterior; snout depressed, rounded; orbit placed far forward; a large præopercular plate covering the cheek in front of the well-developed operculum and suboperculum, as in the *Saurodipterini*. Gape extending far back; maxilla narrow, closely united internally to the bony palate; mandible stout; jugular plates two, narrow; no median or lateral jugulars. Teeth in both jaws conical, sharp, slightly incurved, of different sizes; the larger teeth having the dentine thrown internally into a few simple folds at the base, which is also fluted externally. Vertebral column with ossified centra, attenuated posteriorly. Shoulder-girdle provided with interclaviculars; pectoral fins subacutely lobate; ventrals very slightly lobate. Dorsal fins two, placed far back and opposite the ventrals and anal respectively, the posterior dorsal and anal being each supported by three prominent interspinous bones. Caudal intermediate in general form between the heterocercal and diphycercal type; large, pointed above and below; the rays affixed to extreme termination of body-axis projecting beyond the line of the nearly vertical posterior margin of the rest of the fin. Rays of all the fins slender, articulated, closely set, overlapping their supporting ossicles; fulcral scales absent.

This assemblage of characters renders it evident that *Tristichopterus* has, as stated in the introduction, no affinity with *Dipterus*, nor any special relationship with *Cœlacanthus*, but that, on the other hand, its place in Professor Huxley's classification of the Crossopterygian Ganoids is in the cycliferous division of his family of Glyptodipterini. This group may, however, be very advantageously subdivided, as there are structural differences between some of its members, which, as it seems to me, are of importance enough to rank as family

distinctions. Dr LÜTKEN* has already proposed to unite those genera having rhombic scales with the *Saurodipterini*, under the name of *Rhombodipterini*, an arrangement in which I am much inclined to concur—the remainder with rounded imbricating scales being designated *Cyclodipterini*. But among these Cyclodipterini there occur fishes with two somewhat different types of pectoral fin—those in which that member is very acutely lobate, like that of *Dipterus*, or of the recent *Ceratodus* (*Holoptychius*, *Glyptolepis*), and those in which it is only subacutely so, the lobe terminating considerably before the end of the fin, which consists of more elongated rays, especially in the central part (*Tristichopterus*, *Rhizodus*, *Rhizodopsis*). Following Dr GÜNTHER in associating the Ctenododipterini (*Dipterus*, *Ctenodus*) with the Dipnoi, I would therefore, provisionally at least, propose the following modification in the arrangement of the remaining Crossopterygidæ:—

I. Caudal diphycercal, but with shortened body-axis. Dorsal fin multifid, pectorals obtusely lobate, scales rhomboidal.
FAM. 1.—Polypteridæ (*Polypterus*, *Calamoichthys*).

II. Caudal with elongated attenuated body-axis, heterocercal or diphycercal.
A. Pectorals obtusely lobate, tail diphycercal, dorsal fins two, scales cycloidal, air bladder ossified.
FAM. 2.—Cœlacanthidæ (*Cœlacanthus*, *Macropoma*, *Holophagus*).
B. Pectorals subacutely lobate, dorsal fins two, tail heterocercal or diphycercal.
a Scales rhomboidal.
FAM. 3.—Rhombodipteridæ.
* Scales sculptured,
Sub-fam.—Glyptolæmini (*Glyptolæmus*, *Glyptopomus*).
** Scales smooth.
Sub-fam.—Saurodipterini (*Osteolepis*, *Diplopterus*, *Megalichthys*).
β Scales cycloidal, sculptured.
FAM. 4.—Cyclodipteridæ (*Tristichopterus*, *Gyroptychius* (?), *Rhizodus*, *Rhizodopsis*, *Strepsodus*, *Archichthys* (?)).
C. Pectorals acutely lobate, scales cycloidal.
a Dorsal fins two, ventrals subacutely lobate, scales thick, sculptured.
FAM. 5.—Holoptychiidæ (*Holoptychius*, *Glyptolepis*, *Dendrodus* (?), *Cricodus* (?)).
β Dorsal fin elongated, continuous with the upper part of the caudal; ventral fins acutely lobate; scales thin.
FAM. 6.—Phaneropleuridæ (*Phaneropleuron*, *Uronemus*).

I have placed the genus *Gyroptychius*, M'COY, in the family of Cyclodipteridæ, next to *Tristichopterus*, with a mark of interrogation, because, according to PANDER's elaborate description,† it seems to approach the subject of the present memoir nearer than does any other genus hitherto described, in the sculpturing of the head plates, and of the scales, which, according to that author, "sind elliptisch und bedecken sich dachziegelartig." The fulcration of the fins, the rhomboidal diphycercal form of the caudal, and the greater relative

* Ueber die Begrenzung und Eintheilung der Ganoiden; Palæontographica, Bd. xxii., Erste Lieferung, 1873, p. 47.
† *Op. cit.* pp. 55–61.

size of the exposed area of the scales, which are also mentioned as "an manchen Stellen des Körpers ins Rhomboidale übergehend," are, however, very obvious marks of distinction. Yet, as to the true nature of M'Coy's *Gyroptychius*, there seem to be very considerable doubts. Classified by Pander as one of the "Dendrodontes," a family afterwards merged by Professor Huxley in the cycliferous Glyptodipterini,* and originally described by M'Coy as a "Cœlacanth"† (*i.e.*, cycliferous Crossopterygian), *Gyroptychius* was, however, placed by Sir Philip Egerton among the Saurodipterini,‡ and by Huxley in the rhombiferous section of the Glyptodipterini.§ And it must be owned that, although the elliptical scale, figured by Professor M'Coy from the back of *Gyroptychius angustus*, "very much resembles," as Sir P. Egerton says, "a scale of Tristichopterus,"‖ yet, in the definition of the genus, its founder states that the scales of the flanks are "sub-rhomboidal;" and those figured by him from the side of *G. diplopteroides* are most decidedly Saurodipterine in form and arrangement, the surface sculpturing also very closely resembling the markings seen on many saurodipterine scales when divested of their external ganoine layer. Not having, however, yet seen the original specimens on which the genus was founded, I must be content, in the meanwhile, to leave this question as it is.

* Dec. Geol. Survey, x. p. 23. † British Palæozoic Fossils, pp. 596–597, Plate 2, C, figs. 2, 3.
‡ Qu. Journ. Geol. Soc. xvi. (1860) p. 126. § Dec. Geol. Survey, x. p. 23.
‖ *Ibid.* p. 54, *note.*

EXPLANATION OF PLATE.

In all the figures the matrix has been omitted to save space. The same letters refer to the same bones throughout.

C.B, cranial buckler; *f*, frontal; *p.mx*, pre-maxillary; *Mn*, mandible; *sp*, splenial; *op*, operculum; *s.op*, sub-operculum; *p.op*, præopercular cheek-plate; *s.t*, supra-temporals; *s.o*, sub-orbitals; *j*, jugular plate; *p.q*, palatoquadrate arch; *cl*, clavicle; *s.cl*, supra-clavicular; *i.cl*, inter-clavicular; *b*, basal segments of archypterygium; *r*, radials.

Figure 1. Head of *Tristichopterus alatus*, the facial bones of the left side exhibited from the *internal* aspect. The granulated impression of the outer upper aspect of the cranial buckler is shown, a good deal of the bone of its anterior division still adhering to the matrix; and in the upper part of the figure the præopercular plate of the right side is shown, as also the operculum, the latter partly in impression.

Figure 2. Another head, seen from the right side. The præopercular plate and circumorbital bones are gone, the palatoquadrate apparatus being thus exposed, with a portion of the maxilla attached to the anterior part of its outer margin; the supra-temporals, operculum, part of the sub-operculum, and the cranial buckler, are seen in impression of their internal surfaces, part of the externally granulated bone of the latter still remaining in its posterior division. The mandible is rather injured along its inferior margin.

Figure 3. Palato-quadrate apparatus detached. A portion of the edge of the maxilla, showing bases o numerous teeth cut transversely, is seen anteriorly and externally.

Figure 4. Detached præopercular plate, seen from the inner aspect.

Figure 5. Detached splenial of mandible (†).

Figure 6. A group of facial and shoulder-bones lying detached on a slab of stone.

Figure 7. Horizontal section of a detached dentigerous bone (mandible?), showing the structure and arrangement of the teeth. Magnified a little more than 3 diameters. A. A. Transverse sections of the bases of two of the larger teeth.

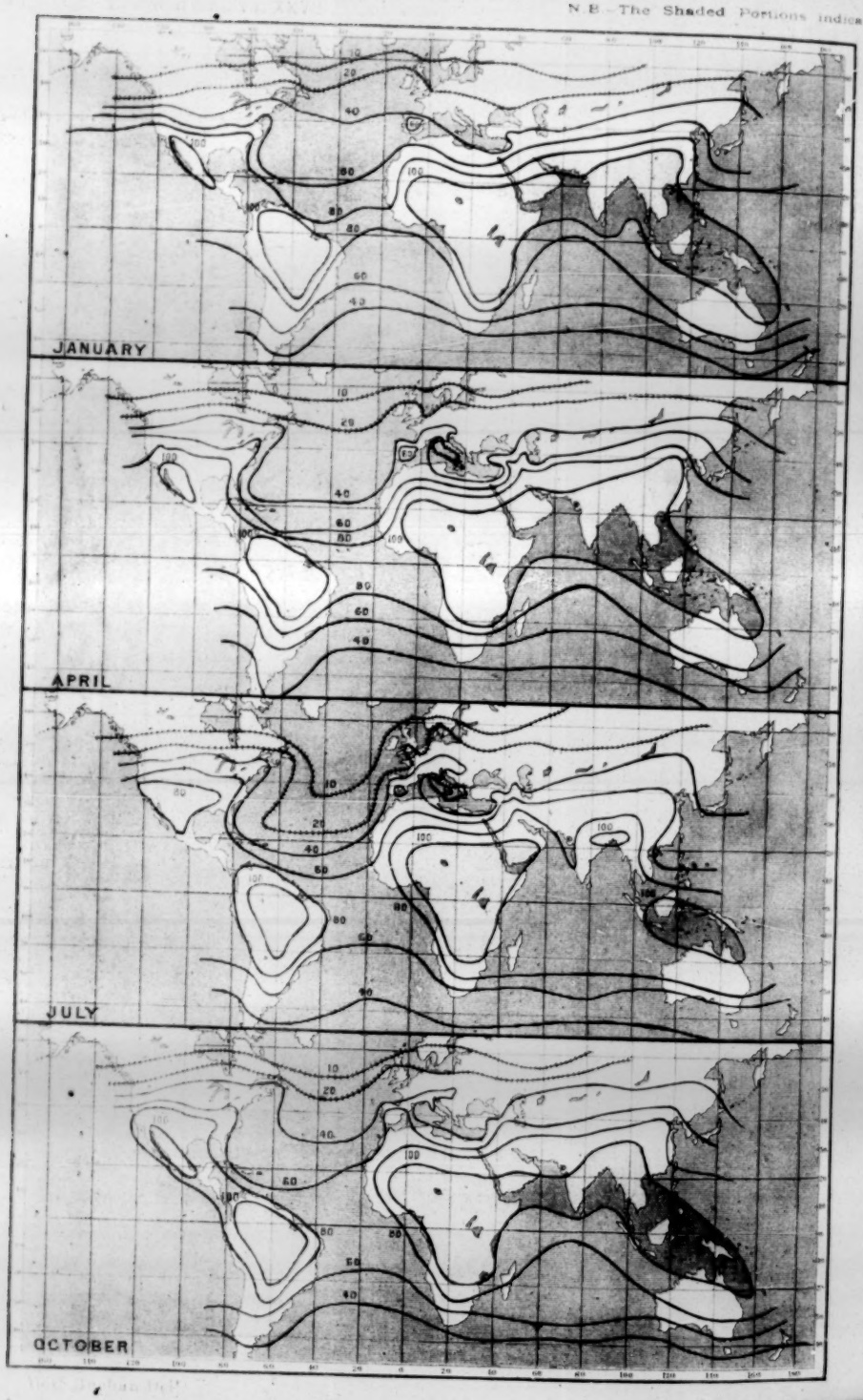
Figure 8. One of the large teeth of *Tristichopterus*, magnified 7 diameters.

Figure 9. Clavicle and part of the pectoral fin, the latter displaced downwards, but showing part of the internal skeleton of the lobe.

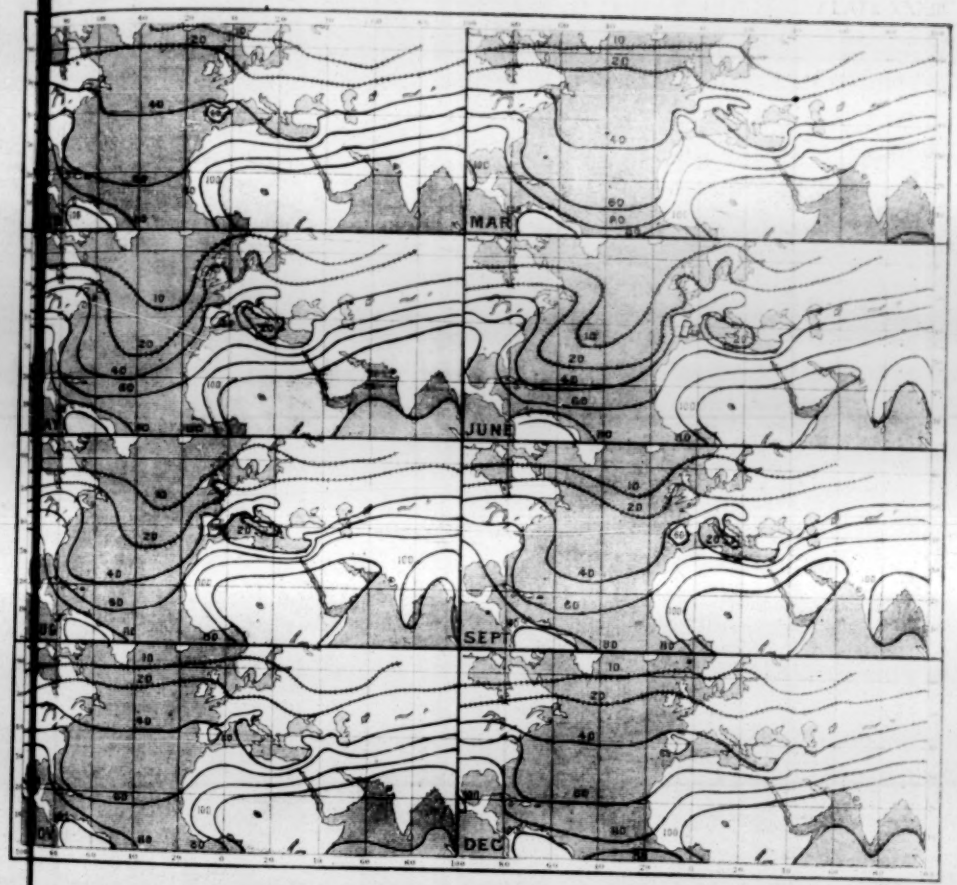
Figure 10. Anterior part of a specimen of *Tristichopterus*, principally to show the form of the pectoral fin and of its basal lobe. The rays of the inner side of that fin are injured and broken up, except at their origins. *s.cl*, Left supraclavicular; *cl'*, portion of left clavicle; *cl*, right clavicle, seen from the inner side; *i.cl*, right interclavicular.

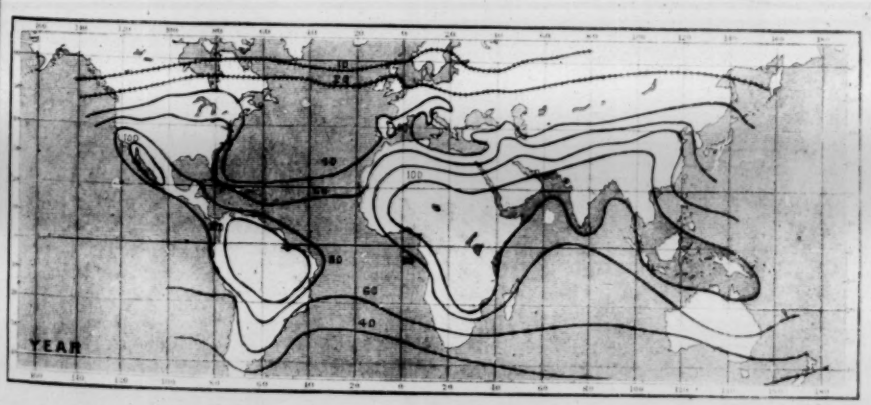
Figure 11. Restored figure of *Tristichopterus alatus*.

CHARTS showing the mean Monthly and Annual Amount
from the A.M. Maximum to the P.M. Minimum, by Lines of 10, 20, 40, 6
N.B.—The Shaded Portions indicate



mount of the Diurnal Oscillation of the Barometer over the Globe,
, 40, 60, 80, & 100 (and upwards) thousandths of an inch (0·010, 0·020 inch, &c.)
ate the Oscillation of 0·100 inch and upwards.





XIX.—*On the Diurnal Oscillations of the Barometer.* PART I. By ALEXANDER BUCHAN, Secretary of the Scottish Meteorological Society. (Plate XXXIII.)

(Read 15th March 1875.)

Of the periodical variations of atmospheric pressure the best marked are the diurnal oscillations which in tropical and sub-tropical regions are among the most regular of recurring phenomena. In higher latitudes these oscillations become from day to day gradually more and more masked, owing to the frequent fluctuations to which atmospheric pressure is there subjected. If, however, hourly observations continue to be regularly made for some time, the diurnal oscillation becomes apparent in the averages deduced from them.

At Edinburgh, where observations were made at 9.30 A.M. and 3.30 P.M. during 109 months, the morning exceeded the afternoon means in every one of the months except six, and on these six occasions the barometric phenomena were similarly abnormal at neighbouring stations. Thus the oscillation between these hours occurs with very considerable regularity at Edinburgh; but at stations in north-western Europe more under the influence of the ocean, such as Guernsey, Helston, Valentia, Helder, and Copenhagen, though situated in lower latitudes, it is much less regularly marked. In the course of this investigation all clearly abnormal, or otherwise doubtful means were, whenever possible, inquired into, with the result that in most cases they were due to errors of observation of an inch or half an inch; to errors arising from including very high or low readings in the averages of one hour, while observations on the same day were wanting for the other hour; and lastly, to clerical and printers' errors. All means evidently erroneous have been rejected.

The general character of the daily oscillations of atmospheric pressure is shown by the two curves of the accompanying figure, which represent by the

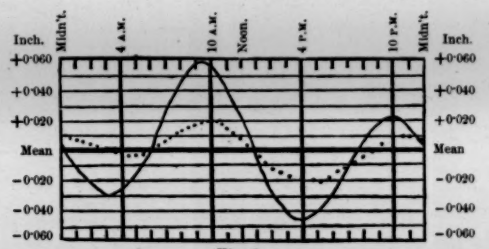


Fig. 1.

solid line the mean oscillation at Bombay for six years, and by the dotted line that at Vienna for ten years.

The following are the extreme variations from the daily mean pressure for January, April, July, and October, at Bombay for six years:—

	A.M.				P.M.			
	Min. Inch.	Hour.	Max. Inch.	Hour.	Min. Inch.	Hour.	Max. Inch.	Hour.
January, . .	−·028	3–4	+·068	9–10	−·051	4	−·022	10
April, . . .	−·028	3	+·063	9	−·055	4	−·019	10
July, . . .	−·031	3–4	+·037	10	−·030	4–5	−·024	10
October, . .	−·029	3	+·063	9	−·031	3–4	−·022	9

Similarly, the maxima and minima at Vienna for ten years, with the hours of their occurrence, are as follows:—

	A.M.				P.M.			
	Min. Inch.	Hour.	Max. Inch.	Hour.	Min. Inch.	Hour.	Max. Inch.	Hour.
January, . .	−·008	6	+·018	10	−·020	3	+·012	10
April, . . .	−·003	5	+·021	10	−·027	5	+·014	11
July, . . .	+·003	3	+·022	9	−·028	5	+·009	11
October, . .	−·010	6	+·020	10	−·015	4	+·008	10

These illustrations may be regarded as typical, to a large extent, of the diurnal barometric fluctuations in tropical and temperate regions. At Bombay the amounts are large, and the times of occurrence of the maxima and minima pretty regular from 3 to 4 and 9 to 10 A.M. and P.M. On the other hand, the oscillations at Vienna are much smaller in amplitude, and the times of occurrence of the critical phases take place through wider intervals, being from 3 to 6 and 9 to 11 A.M. and P.M.

If only the amount of the whole of these diurnal oscillations be taken into consideration, which at Bombay average 0·155 inch, they might well be classed among the most remarkable of atmospherical phenomena, but when the regularity of their occurrence from day to day, and their *quasi* tidal circuit round the globe are considered, they will be at once recognised as the most important of all atmospheric fluctuations. It is remarkable that, though they are among the best marked of meteorological phenomena over at least two-thirds of the globe, yet none of these phenomena, with the exception perhaps of electrical phenomena, could be named respecting whose geographical distribution so little is known, whether regard be paid to their amounts, the times of occurrence of the critical phases, or the physical causes on which the differences depend. What, therefore, is required, in the first place, is a sufficient number of facts to show

the geographical distribution of the amounts and times of the oscillations. It is to meet this desideratum that we have collected during the past ten years, as opportunity offered, the following data :—

1*st*. Observations from 335 places in all parts of the globe, showing the mean amplitude of the oscillation from the morning maximum to the afternoon minimum for each month and the year. These are given in Table I., with the hours of observation and the number of years for which the averages have been taken.

2*d*. Mean horary states of the barometer from 81 places, and bi-horary means for 5 places, as indicated in Table I. by "Hourly" in column of "Hours of Obs."

It is believed that these include nearly all the observations at present available for such an inquiry. For much valuable assistance in this inquiry, I have to express my best thanks to Dr HANN, Vienna; Dr BUYS BALLOT, Utrecht; Professor MOHN, Christiania; M. MARIÉ DAVY, Paris; Dr GUSTAVUS HINRICHS, Iowa City; and Mr HARTNUP, Liverpool.

Of the four daily oscillations, the most important, in relation to the daily march of temperature and vapour, and as respects amplitude in nearly all cases, is the oscillation from the morning maximum to the afternoon minimum. Fortunately, this also is the one oscillation regarding which meteorological observations supply the fullest data—mean monthly and annual results having been obtained, as just stated, for 335 places in different parts of the globe. Since sufficient materials are thus available for giving a first approximate representation of the amount of this oscillation over the globe from month to month, it is proposed in the meantime to limit the inquiry exclusively to this oscillation.

Time of occurrence of the A.M. maximum.—From the hourly observations it is seen to occur in *January*, from 9 to 10 in tropical and sub-tropical regions, as far as 50° lat. N.; in higher latitudes the time varies from 7 at Bogoslovsk in Siberia (60° lat.), to 11 at Helder in the Netherlands, and noon at Valentia in the south-west of Ireland, both places being almost surrounded by the sea. In July it occurs from 9 to 10 at stations only so far as about 40° lat. In higher latitudes the times vary, being 8 or even 7 at many continental places, and as late as 11 at some places near the sea, and at noon at Helder and Valentia.

Time of Occurrence of the P.M. Minimum.—In January it occurs generally from 3 to 4; but there are many exceptions north of lat. 40°, where it occurs at 2, and, in one or two cases, as early as 1 o'clock. It is quite different in July, when the time from 3 to 4 is pretty regularly observed as far as about 35° lat.; but in higher latitudes the times are 5 and 6, the latter hour being observed at the more strictly continental stations. During this season at Melbourne and Hobart Town in the southern hemisphere, the hour is 2 P.M., being in this respect similar to the winter of the northern hemisphere.

In the northern hemisphere, during the winter months, the forenoon maximum rises to a greater extent above the mean of the day than the afternoon minimum falls below it at 71 per cent. of all the stations; whereas the afternoon minimum falls to a greater extent below the mean than the forenoon maximum rises above it at only 29 per cent. of the stations. On the other hand, during the summer months, the afternoon minimum falls below the mean of the 24 hours to a greater extent than the forenoon maximum rises above it at 64 per cent. of the stations. These results are confirmed by the observations made in the southern hemisphere, thus suggesting that the influence of the sun and the other causes on which the diurnal oscillations depend, tend during the winter months of both hemispheres generally to raise the morning maximum to a greater extent above the mean than to lower the afternoon minimum below it; and during the summer months the same cause or causes tend to lower the afternoon minimum to a greater extent than to raise the forenoon maximum above it.

Geographical Distribution.—At all places in Table I. for which hourly values have been obtained, it will be observed that the oscillation is stated in three ways, viz.: 1, The difference between the highest and lowest hourly means; 2, the difference between 9 A.M. and 3 P.M. means; and 3, the difference between the means at 10 A.M. and 4 P.M. It will be seen that the amplitude of the oscillation indicated by the 9 A.M.–3 P.M. and the 10 A.M.–4 P.M. means differs but little from that indicated by the highest and lowest hourly means, in cases where three years' observations and upwards are available.

The amounts of the oscillations deduced from the means at 9 A.M.–3 P.M. and 10 A.M.–4 P.M., at places where observations at these hours only were made, were corrected by adding to them the mean of the small differences found to obtain at two, or in some cases three places, between their oscillation for the same hours and the absolute oscillation as shown by the hourly means. Care was taken to make the comparison in every case with stations similarly situated as regards latitude, proximity to the sea, and elevation. The results thus obtained have been used in constructing the charts, the actual oscillations as observed being given in Table I. Table II., on the other hand, does not give actually observed, but only corrected oscillations for 49 places. At these places observations were not available at 9–10 A.M. and 3–4 P.M., but only at 8 A.M. and 2 P.M.; 7 A.M. and 2 P.M., &c. To the mean oscillation obtained from these hours' observations, a correction was added by which an approximation to the true oscillation was obtained. This correction was determined from the hourly means at two or three places similarly situated as regards latitude, proximity to the sea, and elevation, It is, however, to be remarked that less weight has in all cases been given to the means of Table II. than is given to those of Table I.

In no case have the means at the hours observed been reduced to sea-level in calculating the amount of the oscillation, because the present state of meteorology as regards the methods of observation pursued does not warrant this being done; but the oscillations are taken directly from the means as observed at the place. The question of the influence of elevation on the diurnal oscillation will be afterwards dealt with.

In order to show the geographical distribution of the amplitude of this oscillation in different regions of the globe and in different months, the amounts at the 335 places enumerated in Table I. and at the 49 places enumerated in Table II., or at 384 places in all, were entered on 13 charts for the months and the year (Plate XXXIV.)* *Lines* were then drawn representing *equal amplitudes* of 0·100, 0·080, 0·060, 0·040, 0·020, and 0·010 inch. It has not been judged necessary to draw lines representing a larger amplitude than 0·100; all such regions, however, over which in any month a larger amplitude obtains are shaded. The lines representing 0·020 and 0·010 inch oscillation are hatched, for the sake of distinction.

The general results, broadly stated, are these:—This oscillation is greatest in the tropics, and diminishes on advancing into higher latitudes; greater over the land than the sea, and rapidly increases on proceeding inland; nearly always greater with a dry than with a moist atmosphere; and generally, but by no means always, greatest in the month of highest temperature and greatest dryness combined. The regions characterised by largest amplitude of oscillation include the East India Islands, Eastern Peninsula, India, Arabia, tropical Africa, tropical South America, and Central America, over which it either exceeds, or at least closely approaches, 0·100 inch. At Sibsagar, Assam, it amounts to 0·133 inch. It is also large in the interior of continents in equatorial regions. Thus, at Mexico City it is 0·111 inch; at Mañaos, 0·126 inch; and at Gondokoro, if correctly observed, 0·145 inch. In the tropical parts of the ocean, the oscillation is from 0·020 to 0·030 inch less than on land.

In January and July, the two extreme months as regards temperature and humidity, the more marked seasonal changes at a few individual stations are these:—For the two months the oscillations are respectively 0·120 and 0·067 inch at Bombay; 0·133 and 0·059 inch at Poonah; and 0·132 and 0·091 inch at Calcutta, thus showing the influence of the dry and wet seasons. At Madras, where the rain-bringing character of the monsoons is reversed, the numbers are 0·114 and 0·115, being nearly equal, and at Roorkee, in the

* Some of the mean oscillations given in Kaemtz's "Meteorology" have also been used in constructing the charts, the figures, however, being treated merely as rough approximations, seeing that Kaemtz's "mean oscillations" represent only the mean difference of the two maxima and two minima.

Weight has also been given in drawing the lines to what appears to be, in a few cases, an undue sluggishness in the movements of the barometer, which the returns for Seftenberg, Bologna, Berne, Port Said, Ajmere, Jhansie, False Point, Nassau and Caraccas, seem to suggest.

N. W. Provinces, where rain falls all the year round, 0·088 and 0·075 inch. Again, at Aden, Arabia, where the weather of summer is peculiarly hot and dry, while the winter oscillation is 0·109 inch, that of summer rises to 0·142 inch. The point to be insisted on here is that whatever be the cause or causes to which the daily oscillation is due, the absolute amount at particular places is largely dependent on comparatively local causes; numerous illustrations of which, in addition to the above, may be adduced from many other parts of the earth, showing the influence in the same direction of prevailing dry or wet, and hot or cold seasons on the amplitude of the oscillation. But the most striking illustration occurs over Southern Asia, where, on comparing the July with the January chart, the space shaded for 0·100 inch and upwards all but disappears from this extensive region during the season of the summer monsoon.

The lines of 0·080 inch appear to pass round the globe in December, January, March, and April, but not in the other months. The lines of 0·060 inch, and those indicating a smalier oscillation, pass round the globe at all seasons. It will also be seen that the lines taken as a whole attain their greatest degree of parallelism with the lines of latitude in January, and that on the other hand they are most distorted in July, particularly as regards the lines of smallest amplitude of oscillation.

The amount of the oscillation at places immediately bordering on the Mediterranean Sea is greatest during the winter months. In April, when the temperature is rising very rapidly, the amount is diminished, and a small patch to the west and the south of Italy begins to be formed, over which the oscillation does not amount to 0·020 inch. In May this area of small oscillation is greatly extended south and south-eastwards, and in June still further, in which month the maximum extent of this well-marked and annually recurring diminished diurnal range is reached; in July it is much reduced in area, and in August it has almost disappeared, being now apparently limited to Sicily and the extreme south of Italy.

A state of things, the reverse of the above, obtains over the extended peninsula of the south-west of Europe, lying between the Mediterranean Sea on the one hand, and the Atlantic, North, and Baltic Seas on the other, including thus the inland districts of the Spanish Peninsula, France, Switzerland, Germany, and Austria. Of this extended peninsula, the part most completely enveloped by the ocean is Spain and Portugal, and here it will be observed that the amount of the oscillation is greatest, and that it begins markedly to rise in March. On the other hand, in Germany and Austria, where the breadth of the land is greatest, and where the peninsula blends with the Europeo-Asiatic Continent, the amount of the oscillation is least; whereas, in France, the amount is intermediate between that of Spain and that of Germany. This maximum oscillation during the warmer months of the year may be studied on the maps

by noting the course of the line of oscillation of 0·040 inch, which in March begins to overspread a larger portion of the region indicated. It continues to increase and extend steadily from month to month to the annual maximum in August, after which it diminishes in extent from month to month till in December it appears only in the Spanish Peninsula.

Quite analogous to the above are the facts of the oscillation as they are presented by the inland districts of the Italian Peninsula, where, as in Spain and Portugal, numerous observations are available for a closely approximate definition of the geographical distribution of this oscillation. Indeed, the copious data obtained from these countries with respect to this oscillation are of the highest meteorological value.

At Bilbao, Oviedo, Coruña, Oporto, Lisbon, Gibraltar, Barcelona, Marseilles, Genoa, Leghorn, Naples, Reggio, and all places on and near the sea, the winter greatly exceeds the summer oscillation, and the more strictly insular the situation, the greater is the difference. This peculiarity would appear to extend farther inland over the comparatively broad peninsula of Spain than over the narrower peninsula of Italy; and these two countries may be pointed to as likely yet to furnish the key to the explanation of this marked feature of the oscillation under discussion, which doubtless plays an important part in determining the diurnal changes peculiar to sea-side climates as regards wind, temperature, and humidity.

On the contrary at Madrid, Zaragoza, Huesca, Jaen, Campo Maior, Montpellier, Dijon, Metz, Geneva, Milan, Vienna, Prague, Leipsig, Halle, Upsal, and other continental situations, the spring or summer exceeds the winter oscillation.

The influence of the Channel, the North Sea, and the Baltic on the regions adjoining is similar to that of the Mediterranean. A comparison of the coast stations of Holland, Belgium, and North of France with neighbouring inland stations will at once show this; whilst the influence of Great Britain on the other hand resembles that of the Spanish and Italian peninsulas; but in both cases the amount of the disturbance is much less. The few observations we have from the Black and Caspian Seas point in the same direction.

But it is over the Atlantic and regions bordering it, where in certain seasons the greatest amount of disturbance occurs in the deflection of the lines, particularly those representing 0·040, 0·020, and 0·010 inch of oscillation, from what may be regarded as their normal course. In January these lines run tolerably parallel with the latitude, but in March they begin decidedly to curve southwards over this ocean, thus indicating a diminution in the amount of the oscillation as compared with the land adjoining, just as has been seen in the case of the Mediterranean and other sheets of water. The disturbance goes on augmenting till it reaches the maximum in July, after which it begins to diminish month by month, till in the end of October it has all but disappeared.

An examination of the July lines will best show the character of this singular fact in the distribution of the oscillation in the summer of the northern hemisphere. The following figures will also present the point in a striking light:—

	Lat. N.	Long. W.	Oscillation. Inch.
	° ′	° ′	
San Francisco, . .	37 48	122 23	0·068
Fort Churchhill, . .	39 18	119 15	0·091
Washington, . .	38 56	76 58	0·060
Angra da Heroisma, .	38 36	27 15	0·009
Ponta Delgada, . .	37 44	26 55	0·006
Lisbon, . . .	38 43	9 8	0·036
Campo Maior, . .	39 5	6 50	0 049
Zaragoza, . . .	41 39	0 58	0·063
		Long. E.	
Reggio, . . .	38 6	15 38	0·008
Naples, . . .	40 52	14 14	0·027
Corfu, . . .	39 37	19 55	0·020
Tiflis, . . .	41 43	44 47	0·084
Pekin, . . .	39 54	116 26	0·060

At these places, which scarcely differ four degrees in latitude from each other, the differences of the oscillation are enormous,—the extremes being 0·091 inch at Fort Churchhill in California, and 0·006 inch at Ponta Dalgada in the Azores, the one being 15 times greater than the other; or if a correction of 0·007 inch be added to correct the latter approximately to the true oscillation, the oscillation at Fort Churchhill is still 7 times greater than in the Azores.

In lower latitudes the differences, though not so great, are yet considerable, thus—

	Lat. N.	Long. W.	Oscillation. Inch.
	°	°	
City of Mexico, . .	19 25	99 5	0·079
Up Park Camp, Jamaica,	18 0	76 56	0·059
Barbadoes, . .	13 5	59 40	0·046
Cape Verd, . .	14 40	17 25	0·043
		Long. E.	
Kuka Lake Tchad, .	12 52	12 23	0·095
Aden, . . .	12 46	45 5	0·115
Bombay, . . .	18 54	72 51	0·067
Madras, . . .	13 5	80 70	0·115
Fort Blair, . . .	11 41	92 42	0·065

Here the extremes are Aden, 0·115 inch, and Barbadoes, 0·046 inch, the oscillation being less than a third in Barbadoes of what it is at Aden, though both places are very nearly in the same latitude.

Looking a little more closely at the July oscillation with reference to the Atlantic as compared with Europe adjoining, the strictly local nature of its amplitude is very striking, as the following mean oscillation at places situated in lines running in three different directions will show:—

	Inch.		Inch.		Inch.
Valentia,	0·010	Dublin,	0·012	Edinburgh,	0·011
Helston,	0·007	Oxford,	0·023	Christiansand,	0·016
Paris,	0·020	Ostend	0·004	Christiania,	0·032
Geneva,	0·048	Brussels,	0·021	Upsal,	0·024
Turin,	0·051	Vienna,	0·050	St Petersburg,	0·002
Rome,	0·035	Odessa,	0·024		
Reggio,	0·008	Tiflis,	0·084		

No such enormous differences, or anything that could be regarded as an approach to them, occur during the summer of the southern hemisphere, between the oscillations at places in or near the same latitudes. The following illustrations for January will show this :—

	Lat. E.	Long. W.	Oscillation. Inch.
Santiago de Chili,	33° 26′	70° 37′	0·040
		Long. E.	
Capetown,	33 56	18 27	0·035
Graham's Town,	33 18	26 29	0·045
Petermaritzburg,	29 30	30 2	0·056
Freemantle,	33 2	115 45	0·048
Deniliquin,	35 32	145 2	0·087
Sydney,	33 52	151 11	0·065
Auckland,	36 50	174 50	0·035

In this case the extremes are 0·087 inch at Deniliquin, on the Murray River, Australia, and 0·035 inch at Auckland, in the north of New Zealand, these being respectively the most inland and most insular stations of the group.

Whatever be the cause or causes on which the diurnal oscillations of the barometer depend, the influence of the relative distribution of land and water in determining the absolute amount of the oscillation in particular localities, as well as over extended regions, is very great. From the facts detailed above, it will be seen that this influence gives a strong local colouring to the results, particularly along the coasts, and that the same influence is extensively felt over the Channel, the Mediterranean, the Atlantic, and other sheets of water on the one hand, and on the other, over the inland portions of Great Britain, Europe, and the other continents. It will also be observed from the charts, that the lines are as strongly marked as are the lines which show the distribution of the temperature, pressure, &c., of the atmosphere, and that they show equally as great abnormal deflections in particular seasons over particular regions.

The regions more or less extended, to which more special attention has been drawn, have annual maximum and minimum periods, depending very largely, though not nearly altogether, on the position of the sun, the humidity of the air, and the direction of the wind, particularly considered as a sea or a land wind. The general course of the lines over the globe has also a well-marked annual period—the minimum of deflection from the course of the parallels of latitude occurring in January, the amount of the deflection being then small, and the maximum deflection in July.

It follows that the minimum deflection occurs at the season of the year when the earth presents the minimum extent of land, in other words, the most uniform surface, to the perpendicular rays of the sun ; and the maximum deflection occurs at the season when the earth presents the maximum extent of land, in other words, the most varied surface, to the sun. Beyond this broad generalisation of the facts of distribution of this oscillation, we are scarcely warranted to go at this stage of the discussion. While, as has been pointed out, numerous illustrations can be adduced showing a larger oscillation over the same region with a high temperature and a dry atmosphere, than with a low temperature and a moist atmosphere, the small summer oscillation on the coasts of the Mediterranean and those of the Atlantic adjoining, is in direct opposition to the idea that any such conclusion is general. For over these parts of the Mediterranean and Atlantic the temperature is hottest in summer and the air is driest, so dry indeed that no rain or next to none falls, and yet there the amplitude of the oscillation now contracts to its annual minimum. On the western coasts of the Atlantic, from the Bahamas northwards to Newfoundland, the temperature is at the annual maximum, but the air is not dry, being liberally supplied with moisture, and the rainfall is generous. But with these very different meteorological conditions, there occurs equally, as in southern Europe, a diminished oscillation during the summer months in the islands and near the coasts of North America. It is also to be noted that at inland situations both in America and in the south of Europe, the oscillation reaches its annual maximum just at this season when the annual minimum occurs near the sea-coasts, even although the general characteristics of the atmosphere be substantially the same in both cases.

Hence, then, it is not merely latitude and the states of the atmosphere which call for consideration in this inquiry, but it is these, combined with the effects of solar and terrestrial radiation, of currents of air, and possibly also of electro-magnetic conditions, as modified in each locality by the relative distribution of land and water. The development of this question would be most materially furthered by establishing in different parts of the globe strings of stations extending from the sea-shore inland for thirty or forty miles ; and, it may be added, that with observations obtained from stations so planted, the investigation of the important question of sea-side and other local climates would be most satisfactorily carried out, since it would thereby be placed on a strictly scientific basis.

In Part II. it is proposed to discuss by the usual mathematical formulæ, the mean hourly and bi-hourly observations which have been collected from eighty-six stations, with the view of approximately determining the exact time of occurrence and the amount of the two daily maxima and minima of each month, and the time of occurrence of the four daily mean values.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
IRELAND—		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Belfast,	28	54 35	− 5 56	3	9½ 3½	·015	·023	·028	·018	·011	·013	·009	·024	·024	·013	·019	·026	·019
Armagh,	207	54 21	− 6 39	1	Hourly	·014	·021	·014	·031	·018	·016	·016	·006	·013	·004	·030	·024	·017
Do.	207	54 21	− 6 39	1	9 3	·007	·014	·007	·026	·017	·010	·008	+ 004	·007	+ 019	·030	·019	·010
Do.	207	54 21	− 6 39	1	10 4	·005	·017	·012	·030	·015	·011	·012	·000	·011	+ 012	·030	·019	·013
Enniskillen,	246	54 20	− 7 34	3	9½ 3½	·017	·021	·030	·025	·010	·013	·010	·017	·026	·020	·012	·025	·019
Dublin,	154	53 22	− 6 21	22	9 3	·023	·018	·019	·020	·018	·016	·012	·013	·020	·016	·019	·015	·017
Do.	159	53 22	− 6 21	4	9½ 3½	·021	·028	·033	·022	·002	·022	·018	·023	·032	·014	·021	·022	·024
Do.	25	...	...	7	10 4	·018	·021	·022	·023	·025	·021	·012	·010	·023	·011	·013	·015	·018
Limerick,	92	52 40	− 8 38	4	9 3	·019	·029	·024	·020	·019	·017	·008	·014	·013	·028	·021	·014	·019
Valentia,	23	51 55	− 10 18	1	Hourly	·018	·028	·016	·021	·008	·005	·010	·008	·003	·011	·034	·022	·015
Do.	23	51 55	− 10 18	1	9 3	+ 002	·015	·008	·009	+ 001	+ 003	+ 001	+ 012	+ 007	+ 008	·030	·008	·003
Do.	23	51 55	− 10 18	1	10 4	·004	·022	·014	·017	·004	·000	·002	+ 008	+ 004	·001	·028	·005	·007
SCOTLAND—																		
Aberdeen,	88	57 10	− 2 6	1	Hourly	·024	·022	·007	·011	·016	·016	·017	·009	·018	·001	·029	·014	·015
Do.	88	57 10	− 2 6	1	9 3	·022	·021	·003	·008	·009	·014	·008	·006	·014	+ 024	·028	·006	·010
Do.	88	57 10	− 2 6	1	10 4	·020	·022	·005	·010	·013	·013	·014	·008	·018	+ 032	·028	·003	·011
Stirling,	233	56 6	− 3 55	3	9½ 3½	·012	·012	·014	·017	·013	·009	·007	·007	·010	·007	·016	·013	·011
Edinburgh,	307	55 57	− 3 11	9	9½ 3½	·021	·020	·026	·019	·018	·013	·011	·012	·019	·016	·014	·016	·016
Glasgow,	110	55 53	− 4 17	5	9½ 3½	·017	·018	·021	·023	·024	·015	·008	·014	·014	·012	·015	·015	·019
Do.	184	55 53	− 4 18	1	Hourly	·019	·023	·010	·025	·024	·034	·022	·006	·011	·001	·028	·027	·019
Do.	184	55 53	− 4 18	1	9 3	·008	·019	·003	·020	·018	·023	·010	·003	·010	+ 027	·022	·021	·012
Do.	184	55 53	− 4 18	1	10 4	·010	·021	·006	·025	·022	·025	·017	·004	·011	+ 022	·028	·022	·014
Makerston,	213	55 35	− 2 39	4	Hourly	·014	009	·010	·018	·024	·021	·019	·016	·029	·014	·009	·017	·014
Do.	213	55 35	− 2 39	4	9 3	·006	+ 005	·002	·017	·020	·013	·013	·012	·027	·013	·001	·011	·011
Do.	213	55 35	− 2 39	4	10 4	·008	·001	·007	·017	·021	·016	·015	·013	·027	·013	·006	·015	·014
Dumfries,	61	55 4	− 3 36	3	9½ 3½	·021	·027	·030	·027	·016	·009	·007	·015	·019	·018	·020	·025	·020
ENGLAND—																		
Newcastle,	187	54 59	− 1 36	3	9½ 3½	·024	·030	·044	·027	·010	·015	·009	·019	·022	·025	·019	·024	·022
Stonyhurst,	361	53 51	− 2 28	1	Hourly	·019	·028	·015	·030	·022	·031	·020	·012	·025	·002	·031	·032	·022
Do.	361	53 51	− 2 28	1	9 3	·009	·023	·011	·024	·015	·022	·009	·009	·016	+ 011	·028	·022	·016
Do.	361	53 51	− 2 28	1	10 4	·010	·025	·013	·028	·018	·024	·019	·009	·020	+ 011	·029	·028	·018
Liverpool,	0	53 25	− 2 59	3	Hourly	·014	·010	·019	·031	·023	·021	·014	·016	·021	·020	·008	·031	·016
Do.	0	53 25	− 2 59	3	9 3	·006	·010	·018	·026	·014	·010	·005	·009	·018	·020	·005	·029	·014
Do.	0	53 25	− 2 59	3	10 4	·009	·009	·019	·029	·017	·015	·010	·012	·020	·020	·005	·028	·016
Oxford,	210	51 46	− 1 16	18	Hourly	·010	·030	·025	·031	·027	·030	·023	·024	·020	·021	·021	·025	·024
Do.	210	51 46	− 1 16	18	9 3	·005	·030	·021	·028	·025	·028	·019	·022	·018	·020	·020	·025	·022
Do.	210	51 46	− 1 16	18	10 4	·006	·027	·025	·031	·025	·028	·021	·023	·020	·019	·017	·022	·022
Greenwich,	159	51 28	0 0	6	Hourly	·019	·024	·020	·023	·024	·029	·023	·020	·024	·019	·018	·025	·022
Do.	159	51 28	0 0	6	9 3	·012	·014	·013	·020	·013	·019	·015	·013	·019	·014	·015	·020	·015
Do.	159	51 28	0 0	6	10 4	·012	·023	·020	·018	·019	·024	·019	·018	·023	·017	·015	·024	·020
Kew,	34	51 28	− 0 19	1	Hourly	·022	·027	·030	·036	·029	·040	·040	·039	·035	·017	·028	·020	·030
Do.	34	51 28	− 0 19	1	9 3	·012	·024	·029	·030	·023	·029	·026	·030	·032	·015	·023	·008	·023

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Kew,	34	51 28	– 0 19	1½	10 4	·013	·027	·030	·036	·025	·031	·032	·033	·033	·008	·027	·013	·025
Southampton,	78	50 55	– 1 24	4	9½ 3½	·022	·032	·032	·030	·023	·020	·014	·026	·033	·022	·029	·026	·026
Sidmouth,	30	50 41	– 3 14	3	9 3	·012	·015	·021	·025	·017	·008	·009	·010	·015	·019	·023	·020	·016
Falmouth,	211	50 9	– 5 4	1½	Hourly	·022	·030	·022	·033	·016	·013	·012	·020	·014	·003	·033	·022	·020
Do.	211	50 9	– 5 4	1½	9 3	·007	·017	·010	·020	·004	·003	·001	·011	·003	+ 013	·021	·001	·008
Do.	211	50 9	– 5 4	1½	10 4	·011	·023	·017	·027	·013	·007	·007	·016	·010	+ 007	·030	·004	·014
Helston,	106	50 7	– 5 12	19	9 3	·024	·023	·017	·017	·007	·007	·007	·008	·018	·018	·027	·030	·017
Guernsey,	204	49 28	– 2 32	12	9 3	·015	·019	·014	·025	·012	·007	·003	·004	·014	·013	·015	·022	·014
NORWAY—																		
Alten,	...	69 58	23 2	12	9 3	·004	·000	·001	·007	+ 005	·001	·011	·009	·005	·005	+ 004	·003	·003
Christiania,	74	59 55	10 44	27	9 4	·007	·014	·030	·032	·040	·040	·032	·030	·033	·018	·012	·007	·024
Do.	74	59 55	10 44	3	Hourly	·017	·011	·003	·001	·000	·000	·000	·000	·003	·015	·009	·010	·003
Do.	74	59 55	10 44	3	9 3	·008	·006	+ 004	+ 010	+ 019	+ 023	+ 016	+ 007	+ 006	·010	·005	+ 003	+ 005
Do.	74	59 55	10 44	3	10 4	·012	·010	+ 001	+ 007	+ 013	+ 015	+ 010	+ 004	+ 001	·014	·007	+ 001	·000
Bergen,	50	60 24	5 20	2½	Hourly	...	·015	·017	...	·013	·012	·010	·008	·001	·007	·013	·016	·008
Do.	50	60 24	5 20	2½	9 3	+ 004	·008	·011	+ 005	·010	·004	·009	+ 006	+ 003	·006	·012	·014	·004
Do.	50	60 24	5 20	2½	10 4	+ 004	·011	·015	+ 006	·011	·007	·009	+ 001	+ 002	·006	·013	·016	·008
Christiansand,	?	58 8	8 0	2	Hourly	·016	·023	·008	·009	·023	·029	·016	·018	·012	·006	·025	·005	·016
Do.	?	58 8	8 0	2	9 3	·013	·017	·005	·006	·011	·020	·006	·001	·007	+ 001	·017	+ 002	·009
Do.	?	58 8	8 0	2	10 4	·015	·023	·007	·008	·011	·023	·010	·009	·010	·002	·024	+ 001	·012
SWEDEN—																		
Upsal,	77	55 52	17 38	6	Hourly	·006	·014	·022	·018	·017	·018	·024	·014	·013	·011	·011	·011	·015
Do.	77	55 52	17 38	6	9 3	+ 003	·008	·017	·014	·013	·011	·015	·009	·010	·008	·008	·002	·009
Do.	77	55 52	17 38	6	10 4	·001	·012	·022	·017	·015	·014	·020	·012	·012	·010	·009	·009	·013
DENMARK—																		
Copenhagen,	12	55 41	12 35	13	9 4	·011	·003	·010	·018	·014	·014	·008	·006	·007	·012	·004	·003	·010
Apenrade,	36	55 3	9 26	1	9 3	·012	+ 002	·013	·018	·005	·015	·001	+ 004	·011	·012	+ 002	·032	·009
NETHERLANDS																		
Helder,	14	52 57	4 45	8	Hourly	·016	·017	·017	·007	·014	·008	·011	·009	·010	·020	·013	·026	·010
Do.	14	52 57	4 45	8	9 3	·006	·009	·010	·001	·003	+ 008	+ 002	+ 003	·000	·015	·006	·020	·005
Do.	14	52 57	4 45	8	10 4	·010	·013	·014	·006	·010	+ 002	·004	·003	·003	·016	·008	·024	·009
Groningen,	49	53 13	6 34	10	Hourly	·019	·017	·017	·016	·016	·011	·007	·009	·017	·018	·023	·017	·016
Do.	49	53 13	6 34	10	9 3	·011	·009	·011	·011	·012	·006	·002	·002	·010	·012	·019	·009	·009
Do.	49	53 13	6 34	10	10 4	·014	·016	·014	·016	·015	·010	·004	·005	·015	·017	·022	·013	·013
Urecht,	44	52 5	5 7	20	Hourly	·019	·018	·015	·018	·022	·013	·009	·015	·016	·027	·022	·021	·018
Do.	44	52 5	5 7	20	9 3	·016	·018	·013	·017	·019	·009	·007	·013	·016	·027	·021	·017	·016
Do.	44	52 5	5 7	20	10 4	·014	·018	·015	·018	·020	·013	·009	·014	·012	·026	·016	·017	·017
BELGIUM—																		
Ostend,	16	51 14	2 55	9	9 3	·013	·019	·013	·015	·012	·006	·004	·008	·013	·024	·020	·022	·014
Ghent,	35	51 3	3 43	29	9 3	·022	·020	·020	·019	·021	·017	·013	·019	·026	·022	·021	·022	·020
Louvain,	98	50 53	4 42	6	9 3	·024	·019	·019	·019	·029	·022	·018	·018	·024	·023	·023	·017	·021
Brussels,	186	50 51	4 22	10	Hourly	·021	·025	·026	·030	·028	·024	·021	·024	·027	·030	·023	·026	·025

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Brussels,	186	50 51	4 22	10	10 4	·016	·025	·026	·030	·027	·023	·019	·024	·027	·030	·027	·025	024
Do.	186	50 51	4 22	33	9 3	·014	·018	·021	·022	·024	·018	·015	·020	·023	·022	·020	·017	·020
St Trond,	187	50 49	5 11	6	9 3	·013	·021	·023	·015	·026	·014	·021	·015	·028	·014	·015	·020	·018
Bastogne,	1657	50 4	5 36	3	9 3	·022	·014	·020	·026	·017	·011	·016	·016	·022	·035	·017	·014	·019
Namur,	491	50 28	4 51	13	9 3	·021	·021	·028	·027	·024	·025	·017	·026	·024	·026	·022	·022	·023
FRANCE—																		
Cambrai,	197	50 11	3 14	2	9 3	·014	·018	·016	·020	·023	·017	·018	·012	·020	·015	·025	·016	·018
Cherbourg,	23	49 39	− 1 38	2	9 3	·030	·012	·014	·007	·010	·014	·006	·015	·032	·012	·011	·036	·016
Rouen,	128	49 26	1 5	4	9 3	·003	·005	·006	·003	·016	·004	·004	·008	·026	·017	·007	·004	·009
Metz,	594	49 7	6 11	25	9 3	·026	·021	·033	·035	·037	·030	·028	·031	·033	·032	·025	·021	·029
St Lo,	139	49 7	− 1 6	3	9 3	·015	·015	·017	·020	·018	·015	·008	·006	·026	·025	·020	·024	·017
Paris,	216	48 50	2 20	30	9 3	·029	·032	·034	·027	·031	·028	·020	·032	·032	·030	·026	·023	·029
Do.	216	48 50	2 20	1	Hourly	·016	·035	·031	·056	·050	·042	·025	·025	·041	·028	...	·017	...
Do.	216	48 50	2 20	1	10 4	·015	·035	·030	·052	·045	·037	·019	·019	·037	·022	...	·016	...
Versailles,	602	48 48	2 7	6	9 3	·022	·029	·036	·033	·026	·027	·026	·027	·033	·027	·023	·020	·027
Goersdorf,	748	48 57	7 46	2	9 3	·008	·024	·027	·031	·016	·029	·013	·008	·013	·020	·024	·008	·018
Vendome,	281	47 47	1 4	2	Hourly	·034	·026	·048	·052	·035	·036	·038	·034	·043	·035	·037	·034	·038
Do.	281	47 47	1 4	2	9 3	·027	·022	·046	·045	·032	·028	·027	·027	·041	·035	·031	·023	·032
Do.	281	47 47	1 4	2	10 4	·032	·024	·048	·050	·034	·032	·033	·032	·042	·035	·033	·033	·035
Dijon,	806	47 19	5 2	7	9 4	·026	·031	·034	·030	·033	·041	·038	·028	·031	·028	·032	·018	·031
Syam,	1854	46 45	5 54	4½	9 3	·015	·012	·009	·004	·009	·008	·004	·006	·009	·014	·017	·010	·010
Bordeaux,	75	44 50	− 0 35	3	9 3	·030	·027	·024	·030	·024	·034	·018	·028	·036	·028	·022	·022	·027
Orange,	149	44 8	4 48	36	9 3	·033	·033	·031	·032	·035	·031	·018	·026	·026	·033	·019	·026	·029
Toulouse,	650	43 37	1 28	13	9 3	·039	·036	·045	·039	·041	·042	·039	·043	·043	·038	·032	·032	·039
Montpellier,	193	43 36	3 54	10	8 4	·028	·035	·039	·036	·035	·040	·049	·044	·039	·036	·031	·026	·037
Marseilles,	153	43 18	5 22	4	9 3	·035	·035	·041	·028	·021	·019	·022	·028	·032	·029	·040	·031	·030
PORTUGAL—																		
Oporto,	278	41 9	− 8 27	7	9 3	·036	·033	·031	·034	·017	·016	·017	·025	·029	·029	·036	·035	·028
Guarda,	3409	40 32	− 7 16	7	9 3	·014	·012	·008	·007	·004	·004	·000	·006	·007	·007	·014	·012	·008
Campo Maior,	927	39 5	− 6 50	6	9 3	·040	·040	·051	·048	·044	·050	·049	·055	·060	·047	·054	·039	·048
Lisbon,	336	38 43	− 9 8	15	9 3	·044	·044	·041	·036	·026	·026	·030	·038	·044	·043	·043	·044	·037
Do.	312	38 43	− 9 8	2	Hourly	·050	·050	·049	·040	·033	·034	·036	·037	·061	·028	·043	·038	·039
Do.	312	38 43	− 9 8	2	9 3	·043	·042	·040	·037	·028	·027	·028	·033	·056	·026	·037	·037	·036
Do.	312	38 43	− 9 8	2	10 4	·050	·047	·049	·040	·032	·033	·032	·037	·061	·026	·041	·034	·039
Evora,	1027	38 34	− 7 39	1	9 3	·035	·043	·039	·043	·043	·047	·039	·039	·043	·035	·047	·035	·041
Lagos,	41	37 7	− 8 37	6	9 3	·026	·038	·033	·024	·013	·017	·020	·025	·029	·031	·039	·035	·028
SPAIN—																		
Oviedo,	738	43 23	− 5 53	5	9 3	·051	·054	·057	·052	·039	·037	·039	·037	·048	·044	·055	·050	·048
Coruña,	115	43 22	− 8 28	5	9 3	·059	·054	·061	·058	·042	·046	·039	·037	·046	·054	·063	·057	·051
Bilbao,	52	43 15	− 2 59	5	9 3	·056	·063	·058	·055	·052	·042	·042	·042	·056	·058	·052	·056	·053
Vergara,	551	43 7	− 2 21	5	9 3	·067	·074	·067	·065	·058	·046	·046	·052	·065	·064	·061	·061	·061

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Santiago,	896	42 53	− 8 38	5	9 3	·055	·059	·054	·051	·039	·035	·034	·033	·041	·044	·055	·054	·046
Leon,	2789	42 36	− 5 37	$1\frac{2}{5}$	9 3	·048	·052	·061	·058	·055	·047	·042	·051	·045	·055	·046	·043	·050
Búrgos,	2822	42 20	− 3 46	5	9 3	·039	·046	·049	·048	·042	·036	·036	·040	·046	·039	·040	·043	·042
Huesca,	1476	42 7	− 0 20	4	9 3	·048	·057	·058	·055	·050	·053	·054	·050	·055	·046	·049	·045	·052
Balaquer,	755	41 48	1 55	$1\frac{2}{5}$	9 3	·069	·068	·079	·073	·064	·076	·069	·065	·071	·052	·065	·069	·068
Soria,	3504	41 44	− 2 33	5	9 3	·041	·052	·049	·043	·046	·042	·042	·041	·045	·039	·047	·045	·044
Valladolid,	2493	41 39	− 4 47	5	9 3	·052	·062	·062	·067	·060	·056	·056	·053	·060	·054	·065	·054	·058
Zaragoza,	604	41 39	− 0 58	5	9 3	·046	·058	·057	·064	·056	·056	·063	·061	·064	·056	·054	·050	·058
Barcelona,	49	41 22	2 6	5	9 3	·051	·058	·064	·048	·042	·035	·033	·040	·043	·045	·055	·056	·048
Salamanca,	2671	40 58	− 5 4	5	9 3	·044	·056	·055	·054	·045	·046	·037	·035	·038	·044	·051	·050	·046
Madrid,	2149	40 24	− 3 45	5	9 3	·044	·055	·057	·065	·057	·059	·059	·063	·063	·057	·055	·036	·056
Villaviciosa,	2180	40 21	− 3 56	2	9 3	·042	·055	·058	·057	·053	·048	·041	·045	·042	·040	·054	·038	·048
Palma,	*	39 33	2 34	5	9 3	·042	·044	·043	·036	·030	·027	·029	·028	·028	·030	·042	·048	·036
Albacete,	2276	39 0	− 1 55	5	9 3	·043	·055	·056	·057	·047	·056	·054	·054	·055	·056	·050	·053	·053
Ciudad-Real,	2247	38 59	− 3 59	5	9 3	·044	·046	·049	·046	·044	·045	·041	·050	·049	·048	·044	·043	·046
Valencia,	79	39 28	− 0 26	5	9 3	·052	·058	·062	·054	·046	·043	·042	·045	·055	·055	·057	·055	·052
Alicante,	92	38 21	− 0 32	5	9 3	·058	·058	·059	·046	·043	·036	·035	·042	·049	·051	·053	·055	·049
Murcia,	141	37 59	− 1 12	5	9 3	·069	·082	·083	·074	·058	·054	·058	·063	·070	·073	·067	·065	·068
Do.	141	37 59	− 1 12	9	9 3	·071	·083	·083	·067	·060	·052	·060	·067	·071	·071	·067	·067	·067
Jaen,	1930	37 47	− 3 51	5	9 3	·048	·047	·048	·051	·045	·046	·050	·052	·055	·046	·046	·041	·048
Sevilla,	295	37 23	− 6 4	5	9 3	·052	·052	·055	·059	·052	·055	·060	·061	·063	·065	·051	·055	·057
Granada,	2231	37 11	− 3 42	5	9 3	·051	·060	·061	·060	·051	·049	·057	·059	·061	·065	·056	·056	·057
San Fernando,	92	36 28	− 6 16	3	9 3	·047	·047	·043	·039	·035	·043	·043	·051	·047	·051	·051	·043	·044
Gibraltar,	46	36 6	− 5 21	6	$9\frac{1}{2}$ $3\frac{1}{2}$	·048	·041	·047	·036	·029	·024	·033	·047	·045	·033	·040	·039	·039
Tarifa,	49	36 0	− 5 40	5	9 3	·043	·045	·047	·045	·037	·035	·036	·036	·041	·043	·046	·044	·042
SWITZERLAND																		
Bâle,	912	47 33	7 35	36	9 3	·030	·031	·040	·041	·040	·034	·034	·037	·038	·036	·029	·023	·034
Einsiedeln,	2986	47 7	8 45	6	10 3	·026	·019	·018	·019	·023	·015	·015	·016	·019	·023	·017	·030	·020
Berne,	1883	46 57	7 35	8	Hourly	·023	·026	·017	·025	·028	·021	·027	·022	·028	·034	·029	·025	·024
Do,	1883	46 57	7 35	8	9 3	·017	·024	·015	·023	·026	·018	·022	·021	·025	·029	·025	·025	·023
Do.	1883	46 57	7 35	8	9 4	·020	·026	·017	·024	·024	·017	·021	·020	·027	·033	·028	·022	·023
Geneva,	1335	46 12	6 9	4	Hourly	·034	·043	·041	·060	·057	·044	·048	·046	·051	·048	·032	·041	·043
Do.	1335	46 12	6 9	4	9 3	·029	·042	·036	·057	·046	·036	·042	·039	·046	·047	·029	·035	·041
Do,	1335	46 12	6 9	12	10 4	·032	·039	·038	·049	·046	·039	·045	·040	·052	·042	·038	·036	·041
Gt. St Bernard,	8127	45 42	7 7	4	Hourly	·018	·011	·005	·004	·006	001	+ 001	002	·008	·014	·011	·023	·007
Do.	8127	45 42	7 7	4	9 3	·011	·007	·000	+ 001	·000	+ 003	+ 005	+ 002	·005	·011	·009	·016	·004
Do.	8127	45 42	7 7	12	10 4	·010	·011	·004	·000	+ 002	+ 003	·000	·000	·007	·009	·013	·014	·005
ITALY—																		
Aosta,	1969	45 44	7 18	7	9 3	·020	·012	·016	·012	·016	·020	·016	·016	·020	·020	·012	·016	·016
Udine,	381	46 3	13 13	7	9 3	·055	·039	·035	·031	·028	·028	·031	·035	·039	·035	·031	·051	·037
Biella,	1224	45 34	8 2	7	9 3	·012	·012	·024	·020	·020	·016	·016	·012	·020	·016	·024	·012	·017
Milan,	482	45 28	9 9	9	Hourly	·036	·041	·050	·053	·047	·058	·060	·053	·047	·039	·030	·035	·044

* Near sea-level.

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.*,—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Milan,	482	45 28	9 9	9	9 3	·035	·039	·045	·047	·040	·048	·052	·044	·038	·036	·027	·032	·040
Do.	482	45 28	9 9	9	10 4	·035	·041	·050	·051	·045	·054	·053	·049	·047	·039	·030	·035	·044
Do.	482	45 28	9 10	7	9 3	·024	·039	·043	·039	·043	·047	·043	·039	·043	·039	·039	·031	·039
Venice,	66	45 25	12 20	4	9 3	·022	·028	·024	·029	·024	·029	·025	·025	·029	·029	·024	·017	·025
Pavia,	321	45 11	9 8	7	9 3	·031	·039	·043	·051	·051	·047	·047	·047	·043	·043	·043	·039	·043
Turin,	906	45 4	7 41	7	9 3	·031	·035	·059	·051	·039	·039	·051	·047	·051	·043	·039	·035	·043
Moncalieri,	852	44 59	8 40	7	9 3	·031	·043	·047	·051	·047	·047	·055	·047	·047	·043	·043	·035	·045
Guastalla,	98	44 55	10 38	7	9 3	·016	·031	·031	·039	·047	·031	·039	·039	·035	·024	·028	·020	·032
Alessandria,	321	44 54	8 37	7	9 3	·031	·035	·035	·047	·031	·047	·051	·051	·047	·035	·041	·028	·040
Modena,	211	44 39	10 54	7	9 3	·031	·028	·039	·047	·047	·043	·051	·047	·051	·039	·039	·031	·041
Bologna,	277	44 30	11 28	7	9 3	·020	·024	·020	·028	·028	·024	·028	·028	·031	·020	·024	·020	·025
Mondovi,	1824	44 23	7 51	7	9 3	·051	·024	...	·043	·039	·043	·051	·043	·043	·031	·035	·016	...
Genoa,	157	44 25	8 57	7	9 3	·024	·024	·024	·012	·020	·016	·012	·024	·008	·024	·028	·024	·020
San Remo,	67	43 48	7 45	7	9 3	·012	·031	·020	·012	·012	·008	·008	·008	·004	·012	·028	·012	·014
Florence,	238	43 46	11 13	7	9 3	·024	·039	·039	·043	·039	·043	·047	·043	·051	·039	·035	·028	·039
Do.	238	43 46	11 13	10	9 3	·034	·039	·045	·041	·035	·041	·041	·039	·046	·038	·033	·034	·039
Urbino,	1480	43 43	12 37	7	9 3	·024	·028	·024	·024	·020	·020	·020	·016	·028	·024	·028	·020	·023
Leghorn,	78	43 32	10 17	7	9 3	·024	·024	·024	·016	·016	·012	·012	·004	...	·028	·028	·028	...
Jesi,	387	43 30	13 12	7	9 3	·055	·043	·024	·031	·031	·031	·039	·020	·028	·035	·039	·059	·036
Siena,	1143	43 18	11 18	7	9 3	·028	·043	·035	·039	·031	·031	·031	·031	·035	·035	·029	·024	·034
Perugia,	1706	43 8	12 22	7	9 3	·028	·028	·031	·035	·031	·031	·028	·031	·035	·031	·035	·024	·031
Camerino,	2156	43 6	13 3	7	9 3	·031	·028	·024	·024	·016	·020	·012	·028	·008	·012	·028	·016	·021
Rome,	163	41 54	12 28	7	9 3	·028	·047	·035	·031	·031	·028	·035	·039	·035	·039	·039	·035	·035
Naples (S. R.)	489	40 52	14 14	7	9 3	·028	·035	·028	·024	·020	·020	·012	·008	·024	·020	·028	·031	·023
Do. (O. U.)	187	40 52	14 9	7	9 3	·028	·035	·028	·024	·016	·016	·008	·016	·024	·020	·028	·035	·024
Do. (S. R.)	489	40 52	14 14	3	Hourly	·026	·030	·023	·033	·028	·020	·027	·034	·035	·024	·033	·053	·028
Do. (S. R.)	489	40 52	14 14	3	9 3	·018	·024	·017	·027	·020	·009	·019	·025	·029	·016	·026	·039	·022
Do. (S. R.)	489	40 52	14 14	3	10 4	·022	·028	·023	·033	·026	·016	·026	·031	·034	·024	·033	·046	·028
Palermo,	237	38 7	13 21	7	9 3	·024	·024	·024	·024	·016	·004	·004	·012	·024	·024	·039	·047	·022
Reggio,	75	38 6	15 38	7	9 3	·016	·016	·020	·004	·012	·008	·008	·004	·004	·008	·012	·012	·010
Malta,	232	35 54	14 30	4	9½ 3½	·059	·050	·025	·024	·012	·021	·021	·030	·028	·040	·034	·051	·033
AUSTRIA—																		
Prague,	660	50 5	14 23	13	Hourly	·028	·021	·030	·041	·044	·042	·043	·041	·038	·034	·022	·028	·032
Do.	660	50 5	14 23	13	9 3	·024	·016	·025	·034	·036	·032	·031	·034	·036	·031	·018	·021	·030
Do.	660	50 5	14 23	13	10 4	·028	·018	·030	·038	·039	·035	·033	·036	·037	·034	·020	·025	·031
Krakau,	708	50 4	19 55	5	Hourly	·015	·014	·016	·026	·025	·029	·025	·022	·021	·023	·019	·013	·019
Do.	708	50 4	19 55	5	9 3	·010	·011	·010	·021	·020	·023	·020	·019	·015	·017	·016	·007	·016
Do.	708	50 4	19 55	5	10 4	·013	·012	·015	·025	·023	·026	·023	·022	·020	·022	·017	·010	·019
Kremsmünster,	1258	48 3	14 6	10	9 3	·027	·031	·036	·042	·037	·037	·039	·036	·037	·035	·021	·016	·033
Do.	1258	48 3	14 6	10	10 4	·034	·032	·039	·043	·044	·045	·041	·040	·041	·039	·030	·027	·038
Vienna,	638	48 12	16 20	10	Hourly	·038	·031	·038	·048	·051	·045	·050	·050	·048	·035	·027	·031	·040
Do.	638	48 12	16 20	10	9 3	·035	·028	·032	·040	·038	·034	·041	·038	·043	·030	·022	·025	·034

TABLE. I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Vienna,	638	48 12	16 20	10	10 4	·037	·031	·038	·046	·043	·037	·044	·044	·043	·035	·026	·030	·039
Salzburg,	1362	47 48	12 57	6	Hourly	·037	·027	·039	·037	·045	·049	·040	·040	·034	·037	·025	·028	·034
Do.	1362	47 48	12 57	6	9 3	·036	·018	·036	·035	·039	·038	·031	·034	·032	·036	·022	·024	·032
Do.	1362	47 48	12 57	6	10 4	·027	·023	·039	·036	·040	·039	·032	·037	·034	·036	·022	·024	·033
Seftenberg,	1381	50 5	16 25	10	Hourly	·027	·016	·014	·022	·020	·024	·020	·019	·017	·027	·012	·013	·019
Do.	1381	50 5	16 25	10	9 3	·023	·013	·007	·016	·012	·016	·012	·013	·010	·022	·008	·008	·013
Do.	1381	50 5	16 25	10	10 4	·027	·011	·013	·021	·016	·022	·016	·017	·015	·017	·008	·009	·016
GERMANY—																		
Munich,	1708	48 9	11 36	26	Hourly	·025	·022	·025	·033	·035	·032	·031	·029	·029	·025	·022	·025	·025
Do.	1708	48 9	11 36	26	9 3	·019	·017	·023	·029	·029	·026	·024	·023	·024	·025	·020	·019	·022
Do.	1708	48 9	11 36	26	10 3	·021	·019	·025	·033	·031	·029	·027	·026	·028	·024	·020	·020	·024
Halle,	362	51 30	11 57	2	Hourly	·013	·018	·018	·029	·031	·037	·026	·032	·032	·038	·019	·021	·026
Do.	362	51 30	11 57	2	9 3	·009	·016	·015	·023	·024	·030	·022	·024	·029	·032	·015	·014	·021
Do.	362	51 30	11 57	2	10 4	·012	·018	·018	·028	·028	·032	·024	·029	·031	·038	·018	·017	·024
Leipsic,	390	51 20	12 23	2	Hourly	·016	·022	·031	·041	·029	·024	·025	·022	·040	·036	·015	·011	·023
Do.	390	51 20	12 23	2	9 3	·008	·019	·023	·033	·022	·013	·010	·014	·027	·029	·011	·005	·018
Do.	390	51 20	12 23	2	10 4	·012	·020	·030	·038	·024	·018	·018	·020	·035	·036	·013	·005	·023
Crefeld,	140	51 21	6 33	21	B hourly	·020	·025	·024	·029	·033	·027	·024	·027	·031	·026	·023	·017	·025
Do.	140	51 21	6 33	21	9 3	·020	·021	·024	·028	·031	·021	·018	·027	·028	·026	·023	·017	·024
TURKEY—																		
Corfu,	74	39 37	19 55	5	$9\frac{1}{2}$ $3\frac{1}{2}$	·028	·025	·034	·029	·020	·018	·020	·026	·027	·038	·028	·049	·029
RUSSIA—																		
St Petersburg,	15	59 56	30 16	15	Hourly	·011	·014	·009	·003	·003	·004	·002	·002	·007	·015	·013	·012	·007
Do.	15	59 56	30 16	15	$9\frac{1}{3}$ $3\frac{1}{3}$	+ 001	·004	·005	·002	·001	·001	·000	+ 002	·000	·009	·008	·001	·004
Do.	15	59 56	30 16	15	$10\frac{1}{3}$ $4\frac{1}{3}$	·003	·010	·008	·003	·003	·002	·001	+ 001	·002	·012	·011	·006	·006
Dorpat,	224	58 23	26 43	13	Hourly	·007	·005	·008	·011	·016	·020	·017	·018	·010	·007	·008	·008	·009
Do.	224	58 23	26 43	13	9 3	·003	·003	·006	·009	·010	·015	·014	·014	·007	·004	·005	·003	·008
Do.	224	58 23	26 43	13	10 4	·005	·002	·008	·010	·014	·017	·016	·016	·010	·006	·005	·005	·009
Kasan,	233	55 47	49 8	4	10 4	·014	·015	·007	·017	·029	·019	·021	·018	·022	·021	+ 006	+ 005	·014
Zlatouste,	1362	55 10	59 40	20	Hourly	·008	·010	·011	·017	·021	·017	·014	·015	·012	·007	·004	·005	·010
Do.	1362	55 10	59 40	20	9 3	·008	·009	·010	·013	·014	·010	·008	·010	·010	·006	·004	·005	·009
Do.	1362	55 10	59 40	20	10 4	·007	·008	·008	·012	·013	·010	·009	·010	·010	·007	·004	·005	·009
Kursk,	687	51 45	36 8	5	9 3	·012	·015	·003	·014	·021	·023	·019	·022	·010	·019	·014	·009	·015
Lugan,	203	48 35	39 20	20	Hourly	·015	·009	·013	·031	·039	·037	·037	·042	·041	·026	·018	·021	·027
Do.	203	48 35	39 20	20	9 3	·013	·008	·011	·024	·030	·028	·028	·033	·037	·023	·016	·016	·022
Do.	203	48 35	39 20	20	10 4	·011	·005	·010	·023	·030	·028	·028	·034	·033	·024	·016	·016	·021
Nicolaief,	180	46 58	31 58	6	10 4	·027	·022	·030	·028	·020	·027	·026	·028	·022	·022	·014	·016	·023
Odessa,	174	46 29	30 44	6	10 4	·020	·017	·023	·027	·026	·022	·023	·028	·025	·021	·016	·012	·022
Do.	174	46 29	30 44	$7\frac{1}{2}$	9 3	·022	·025	·026	·030	·024	·024	·024	·028	·033	·025	·026	·019	·025
Bogoslovsk,	635	59 45	60 1	17	B hourly	·008	·012	·020	·018	·013	·020	·019	·010	·012	·008	·006	·011	·012
Do.	635	59 45	60 1	17	10 4	+ 001	·004	·011	·013	·009	·017	·015	·007	·005	·000	+ 001	·005	·008

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.*—Continued.

	Height in ft.	Lat.	Long.	No. of Years Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Nijne Taguilsk,	621	57 56	60 5	11	8 3	+ 002	·003	·015	·016	·015	·013	·015	·003	·007	+ 002	·001	·001	·007
Catherinenburg,	931	56 49	60 35	9	Hourly	·018	·013	·014	·024	·027	·021	·023	·023	·012	·013	·003	·007	·012
Do.	931	56 49	60 35	9	9½ 3½	·018	·010	·010	·019	·023	·013	·015	·022	·009	·013	·002	+ ·006	·012
Do.	931	56 49	60 35	9	10½ 4½	·007	·006	·010	·021	·023	·014	017	·021	·011	·010	+ ·004	+ 011	·011
Krasnoyarsk,	150	56 1	92 54	6	10 4	·009	·024	·031	·030	.029	·027	·017	·025	·026	·030	·017	·008	·023
Barnaul,	480	53 20	83 57	9	Hourly	·006	·010	·014	·018	·024	·021	·016	·018	·027	·023	·011	·016	·015
Do.	480	53 20	83 57	9	9 3	+ 003	·008	·011	·014	·015	·016	·012	·011	·020	·013	·008	·010	·011
Do.	480	53 20	83 57	9	10 4	+ 001	·009	·013	·018	·022	·021	·014	·015	·024	·020	·011	·016	·015
Nertchinsk,	1940	51 19	119 36	10	Hourly	·027	·037	·048	·059	·050	·061	·046	·042	·050	·036	·025	·021	·038
Do.	1940	51 19	119 36	10	9½ 3½	·025	·035	·044	·055	·043	·054	·035	·039	·045	·036	·022	·018	·037
Do.	1940	51 19	119 36	10	10½ 4½	·024	·037	·046	·052	·039	·055	·034	·037	·045	·030	·021	·012	·035
Tiflis,	1509	41 43	44 47	10	Hourly	.050	·048	·063	·073	·072	·070	·084	·087	·078	·068	·051	·052	·063
Do.	1509	41 43	44 47	10	9 3	·048	·048	·059	·068	·063	·061	·064	·075	·074	·068	·051	·049	·061
Do.	1509	41 43	44 47	10	10 4	·047	·048	·059	·066	·062	·058	·066	·069	·074	·063	·048	·047	·059
SANDWICH IS.																		
Lahaineluna,	653	20 52	156 40	2	9 3	·106	·091	·071	·067	·087	·071	·051	·051	·059	·059	·079	·091	·074
JAPAN—																		
Jokohama,	10	35 27	139 40	1⅓	10 4	·051	·051	·075	·063	·067	·039	·035	·039	·047	·047	·043	·031	·049
Decima,	26	33 44	129 42	2½	9 3½	·024	·030	·024	·024	·024	·026	·018	·015	·019	·029	·019	·034	·024
CHINA—																		
Pekin,	120	39 54	116 26	7	B hourly	·087	·089	·112	·108	·103	·091	·060	·065	·081	·090	·078	·074	·085
Do.	120	39 54	116 26	7	9 3	·087	·087	·102	·095	·092	·075	·048	·056	·076	·084	·078	·074	·080
Shanghae (quart.)	23	31 30	121 28	...	10 4	·076	...	...	·074	...	..	·048	...	...	·069	...	...	·067
Hong Kong,	35	22 16	114 10	7	9½ 3½	·098	·094	·095	·083	·070	·055	·053	·061	·073	·088	·085	·089	·079
ARABIA—																		
Tor,	18	28 13	33 39	1½	8¾ 3	...	...	...	...	·047	...	...	...	...	...	...	...	...
Djedda,	25	21 28	39 13	⅙	9 3½	...	...	...	...	...	...	·053	·060	...	...	...	...	...
Aden,	199	12 46	45 5	Term	Hourly	·085	·126	·113	·122	·132	·161	·115	·150	·156	·149	·132	·115	·127
Do.	199	12 46	45 5	Term	9 3	·085	·126	·109	·113	·127	·144	·099	·126	·156	·149	·125	·089	·120
Do.	199	12 46	45 5	Term	10 4	·080	·103	·093	·114	·121	·159	·115	·147	·143	·129	·132	·093	·120
INDIA—																		
Simla,	7487	31 6	77 11	3	Hourly	·051	·052	.047	.052	·054	·049	·055	·051	·052	·048	.053	·055	·050
Do.	7487	31 6	77 11	3	9½ 3½	·046	·042	·037	·042	·039	·033	·038	·040	·045	·046	·049	·049	·042
Do.	7487	31 6	77 11	3	10½ 4½	·051	·052	·047	·050	·051	·047	·052	·050	·052	·047	·053	·055	·041
Darjeeling,	6912	27 3	88 18	7	10 4	.055	·050	·050	·053	·047	·046	·041	·043	·048	·048	·049	·051	·048
Sibsagar,	332	26 59	94 40	1	10 4	·130	·129	·144	·135	·117	·128	·118	·127	·145	·137	·141	·146	·133
Goalparah,	386	26 11	90 40	6	10 4	·131	·129	·138	·145	·126	·111	·106	·110	·126	·126	·124	·126	·125
Dehree (A.)	362	25 54	84 13	1½	10 4	·121	·112	·142	·127	·114	...	·084	.098	·110	·105	·110	·108	...
Patna,	179	25 37	85 8	7	10 4	·128	·115	·130	·131	·127	·106	·099	·101	·113	·106	·111	·118	·115
Shillong,	4792	25 33	91 54	4⅔	10 4	·093	·092	·090	·084	·077	·067	·066	·076	·082	·079	·072	·074	·079
Arrah (A.)	195	25 32	84 43	2	10 4	·097	·102	·120	·126	·121	·118	·100	·098	·114	·098	·101	·104	·108

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Monghyr,	160	25 22	86 30	7	10 4	·126	·126	·136	·145	·138	·113	·104	·108	·119	·115	·116	·118	·122
Gya,	347	24 42	85 2	3	10 4	·125	·118	·138	·130	·119	·113	·105	·102	·115	·111	·118	·123	·118
Silchar,	89	24 49	92 50	1½	10 4	·135	·139	·140	·142	·127	·119	·118	·129	·135	·126	...	·125	...
Cachar,	...	24 45	92 50	2⅔	10 4	·143	·138	·152	·139	·128	·122	·121	·127	·133	·134	·134	·127	·133
Berhampore,	65	24 6	88 17	7	10 4	·135	·137	·142	·152	·137	·109	·108	·115	·126	·117	·125	·132	·127
Hazareebagh,	1996	24 0	85 24	7	10 4	·103	·107	·109	·105	·104	·081	·081	·089	·100	·091	·093	·099	·097
Dacca,	35	23 43	90 27	7	10 4	·124	·125	·131	·132	.121	·101	·096	·105	·117	·113	·111	·118	·116
Burdwan,	102	23 14	87 54	1⅓	10 4	·128	·126	·132	·155	·136	·108	·106	·107	·114	·120	·131	·132	·125
Jessore,	20	23 9	89 7	7	10 4	·124	·125	·132	·135	·121	·097	·090	·101	·107	·107	·111	·117	·114
Calcutta,	18	22 33	88 21	6	Hourly	·132	·139	·141	·142	·145	·098	·091	·104	·114	·111	·103	·124	.120
Do.	18	22 33	88 21	6	9 3	·121	·122	·120	·119	·108	·081	·076	·088	·106	·111	·100	·118	·107
Do.	18	22 33	88 21	6	10 4	·132	·139	·137	·140	·143	·098	·091	·104	·113	·109	·100	·124	·120
Do.	18	22 33	88 21	7	10 4	·133	·131	·132	·132	·118	·094	·090	·099	·112	·112	·118	·128	·117
Chittagong,	90	22 21	91 50	7	10 4	·105	·101	·112	·109	·102	·086	·084	·096	·108	·107	·098	·102	·101
Saugor Is.	6	21 39	88 5	7	10 4	·118	·120	·113	·116	·105	·083	·078	·091	·104	·104	·111	·117	·105
Cuttach,	80	20 29	85 54	7	10 4	·133	·138	·141	·138	·124	·098	·087	·094	·102	·103	·107	·122	·116
False Point,	19	20 20	86 47	7	10 4	·103	·104	·103	·110	·103	·084	·076	·083	·085	·091	·095	·098	·095
Akyab,	21	20 8	92 57	5	10 4	·109	·119	·119	·115	·104	·085	·080	·095	·111	·105	·103	·108	·105
Vizagapatam,	31	17 42	83 22	5	10 4	·118	·130	·122	·115	·110	.096	·093	·101	·108	·103	·105	·114	·109
Madras,	27	13 5	80 17	7	10 4	·116	·123	·125	·131	·116	·113	·113	·118	·125	·110	·102	·105	·116
Do.	27	13 5	80 17	5	Hourly	·118	·124	·130	·127	·121	·125	·115	·119	·129	·122	·108	·106	·119
Do.	27	13 5	80 17	5	8 2	·110	·108	·114	·114	·107	·109	·098	·103	·120	·117	·107	·102	·108
Do.	27	13 5	80 17	5	9 3	·118	·124	·130	·127	·120	·123	·111	·118	·127	·122	·107	·106	·119
Nancowry,	134	8 0	93 46	1	10 4	·078	·087	·084	·085	·074	·077	·071	·063	·081	·072	·061	·082	·076
Port Blair,	61	11 41	92 42	5	10 4	·102	·105	·104	·097	·089	·075	·065	·072	·091	·098	·095	·097	·091
Chukrata,	7056	30 40	77 55	2⅓	10 4	·044	·053	·053	·049	·044	·036	·038	·049	·062	·057	·055	·049	·049
Dehra,	2232	30 20	78 8	5	10 4	·087	·087	·084	·089	·090	·086	·070	·077	·092	·091	·093	·089	·086
Roorkee,	880	29 52	77 56	10	10 4	·090	·090	·092	·099	·098	·098	·082	·086	·095	·091	·093	·093	·092
Meerut,	739	29 41	77 41	4	10 4	·076	·065	·073	·087	·090	·089	·082	·097	·082	·075	·076	·071	·080
Raneekhet,	6143	29 38	79 29	2⅓	10 4	·045	·023	·015	·037	·034	·044	·050	·052	·051	·052	·053	·037	·041
Bareilly,	570	28 21	79 27	4	10 4	·094	·102	·108	·118	·122	·124	·093	·098	·107	·104	·106	·101	·106
Futtehghur,	503	27 23	79 48	4	10 4	·097	·104	·108	·108	·106	·100	·094	·102	·106	·097	·098	·100	·102
Agra,	551	27 0	78 0	8	10 4	·103	·098	·105	·119	·102	·090	·081	·083	·097	·096	·103	·108	,099
Lucknow,	364	26 50	81 0	5	10 4	·107	·107	·112	·121	·127	·109	·093	·103	·106	·102	·110	·117	·110
Do.	...	26 51	80 55	1	Hourly	·110	·109	·134	·132	·129	·129	·092	·085	·112	·097	·088	·113	·110
Do.	...	26 51	80 55	1	9 3	.110	·108	·126	·124	·118	·111	·086	·082	·108	·096	·088	·113	·105
Do.	...	26 51	80 55	1	10 4	·098	·108	·129	·127	·124	·126	·087	·084	·108	·090	·071	·109	·106
Goruckpore,	255	26 46	83 18	2½	10 4	·100	·120	·130	·132	·122	·105	·086	·088	·110	·105	·110	·108	·110
Ajmere,	1800	26 27	74 40	6	10 4	·079	·075	·080	·079	·072	·076	·062	·058	·068	·066	·062	·066	·070
Jhansie,	940	25 27	78 37	5⅓	10 4	·092	·091	·101	·095	·095	·092	·080	·080	·084	·081	·088	·096	·090

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

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		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Mozuffespur,	...	26 5	85 24	2½	9¾ 4½	·098	·093	·101	·108	·096	·068	·074	·088	·111	·081	·094	·098	·093
Allahabad,	289	25 26	81 52	3	10 4	·120	·118	·124	·126	·122	·106	·103	·104	·110	·106	·123	·118	·115
Benares,	263	25 20	83 2	9	10 4	·100	·109	·109	·107	·105	·093	·085	·084	·094	·103	·102	·101	·099
Chunar,	280	25 7	83 9	1½	10 4	·106	·102	·130	·130	·110	·110	·104	·110	·109	·106	·110	·111	·112
Bombay,	35	18 54	72 51	6	Hourly	·119	·121	·117	·118	·101	·080	·067	·077	·093	·114	·117	·120	·102
Do.	35	18 54	72 51	6	9 3	·118	·118	·112	·109	·091	·069	·053	·067	·089	·114	·117	·120	·098
Do.	35	18 54	72 51	6	10 4	·119	·121	·116	·115	·097	·080	·067	·077	·093	·110	·110	·118	·101
Poonah,	1823	18 31	74 6	4	9·10 4·5	·133	·131	·121	·114	·092	·092	·064	·076	·085	·116	·126	·130	·107
Secunderabad,	1700	17 25	78 40	1	10 4	·143	·145	·145	·138	·144	·122	·096	·119	·112	·117	·122	·122	·127
Dodabetta,	8640	11 32	76 50	Term	Hourly	·059	·059	·058	·064	·060	·042	·057	·075	·062	·075	·060	·064	·060
Do.	8640	11 32	76 50	Term	9 3	·056	·055	·047	·053	·050	·027	·038	·053	·050	·069	·058	·059	·051
Do.	8640	11 32	76 50	Term	10 4	·059	·058	·058	·064	·059	·042	·057	·066	·062	·075	·060	·062	·060
Trivandrum,	130	8 31	77 0	1	Hourly	·120	·126	·123	...	·107	·088	·092	·103	·111	·119	·120	·115	·111
Do.	130	8 31	77 0	1	9 3	·100	·095	·098	...	·081	·073	·071	·081	·085	·090	·097	·089	·087
Do.	130	8 31	77 0	1	10 4	·120	·126	·123	...	·107	·088	·092	·103	·111	·119	·120	·115	·111
CEYLON—																		
Mannár,	12	9 0	79 50	1	9 3	·118	·143	·140	·134	·107	·133	·107	·112	·122	·129	·120	·147	·126
Puttalam,	11	7 57	79 51	3	9 3	·124	·136	·126	·123	·099	·087	·094	·097	·108	·109	·115	·117	·111
Colombo,	42	6 56	79 50	5¼	9 3	·118	·126	·124	·115	·094	·082	·082	·091	·098	·108	·112	·115	·105
Do.	18	6 56	79 50	6	9½ 3½	·122	·132	·122	·110	·094	·082	·079	·088	·102	·106	·112	·121	·106
Galle,	40	6 1	80 15	3	9 3	·101	·111	·115	·107	·090	·075	·077	·085	·095	·106	·102	·104	·098
Anurádhapura,	312	8 23	80 23	3	9 3	·114	·118	·128	·113	·099	·074	·082	·088	·092	·098	·095	·090	·099
Kandy,	1650	7 18	80 43	3	9 3	·115	·131	·129	·124	·102	·089	·092	·105	·114	·113	·115	·113	·112
Gangaroowa,	1560	7 17	80 37	5	9½ 3½	·117	·123	·125	·117	·098	·084	·086	·095	·108	·117	·111	·110	·108
Badulla,	2220	7 5	81 4	2½	9 3	·094	·108	·091	·080	·075	·082	·088	·090	·105	·120	·099	·101	·094
Nuwara Eliya,	6150	6 57	80 47	3	9 3	·063	·066	·065	·064	·073	·071	·067	·066	·071	·069	·065	·067	·067
Ratnapura,	114	6 43	80 27	3	9 3	·133	·138	·151	·137	·103	·081	·104	·104	·113	·117	·122	·129	·120
Jaffna,	9	9 38	80 3	3	9 3	·108	·121	·129	·122	·105	·096	·097	·105	·115	·113	·107	·106	·110
Trincomalee,	175	8 34	81 12	3	9 3	·103	·109	·121	·127	·104	·090	·102	·109	·112	·115	·105	·100	·108
Batticaloa,	21	7 41	81 41	3	9 3	·083	·098	·098	·106	·096	·076	·087	·093	·093	·103	·096	·095	·094
Hambantota,	40	6 13	81 16	2	9 3	·097	·103	·110	110	·089	·080	·088	·114	·115	·120	·100	·111	·103
EAST INDIA IS.																		
Sincapore,	50	1 17	103 51	5	Hourly	·088	·094	·101	·103	·090	·077	·073	·080	·091	·096	·094	·089	·088
Do.	50	1 17	103 51	5	9 3	·078	·082	·085	·082	·082	·073	·067	·075	·077	·090	·090	·089	·081
Do.	50	1 17	103 51	5	10 4	·088	·094	·099	·098	·090	·076	·070	·080	·091	·094	·092	·088	·088
Do.	50	1 17	103 51	2½	9 3	·100	·101	·112	·116	·100	·110	·094	·120	·104	·110	·105	·101	·106
Padang,	220	− 0 56	100 2	3½	9 3	·098	·102	·091	·083	·083	·087	·071	·094	·091	·094	·094	·091	·090
Batavia,	23	− 6 11	106 50	3	Hourly	·101	·106	·111	·105	·106	·099	·103	·112	·123	·108	·105	·096	·103
Do.	23	− 6 11	106 50	3	9 3	·095	·096	·108	·104	·105	·098	·100	·110	·120	·106	·104	·091	·100
Do.	23	− 6 11	106 50	3	10 4	·098	·104	·110	·103	·100	·098	·097	·106	·114	·105	·100	·092	·102
Buitenzorg,	889	− 6 37	106 49	6½	9 3	·077	·077	·070	069	·073	·063	·072	·080	·092	·091	·082	·075	·077

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
AUSTRALIA, QUEENSLAND																		
Somerset, Cape York,	70	−10 44	142 36	$2\frac{1}{4}$	9 3	·106	·108	·104	·110	·100	·101	·094	·088	·098	·098	·104	·105	·101
Sweer Island,	31	−17 7	139 41	$1\frac{1}{2}$	9 3	·074	·072	·077	·080	·081	·078	·065	·088	·098	·083	·086	·075	·080
Ravenswood, (A.),	600	−20 20	146 50	$1\frac{1}{2}$	9 3	·100	·090	·110	·100	·080	·090	·070	·080	·060	·070	·090	·070	·084
Rockhampton,	68	−23 26	150 43	$1\frac{2}{3}$	9 3	·072	·086	·064	·078	·086	·077	·075	·082	·094	·100	·097	·082	·083
Brisbane,	140	−27 28	153 6	$6\frac{1}{3}$	9 3	·077	·078	·079	·092	·087	·091	·093	·101	·106	·104	·095	·076	·090
Warwick,	1520	−28 12	152 16	$2\frac{1}{4}$	9 3	·075	·082	·080	·084	·064	·068	·083	·085	·095	·091	·093	·094	·083
Casino,	139	−28 53	153 6	$1\frac{1}{2}$	9 3	·066	·083	·066	·072	·074	·062	·076	·074	·097	·098	·085	·078	·078
NEW S. WALES																		
Armidale,	3258	−30 34	151 46	$1\frac{1}{3}$	9 3	·060	·078	·063	·068	·071	?	·059	·076	·075	·082	·083	·059	·070
Maitland,	98	−32 47	151 35	$1\frac{1}{4}$	9 3	·062	·067	·082	·084	·088	·076	·068	·077	·056	·051	·082	·068	·072
Bathurst,	2333	−33 24	149 37	$1\frac{1}{2}$	9 3	·097	·087	·086	·075	·076	·070	·060	·064	·078	·068	·064	·071	·075
Paramatta,	60	−33 49	151 1	$1\frac{1}{3}$	9 3	·065	·064	·073	·078	·075	·066	·085	·064	·081	·083	·075	·061	·073
Sydney,	145	−33 52	151 15	4	9 3	·065	·061	·071	·078	·068	·071	·072	·076	·085	0·84	·073	·072	·073
Goulburn,	2129	−34 45	149 45	$1\frac{1}{3}$	9 3	·070	·060	·066	·069	·086	·069	·060	·064	·081	·075	·060	·060	·068
Deniliquin,	410	−35 32	145 2	$1\frac{1}{2}$	9 3	·087	·066	·070	·074	·057	·062	·055	·070	·063	·062	·095	·063	·069
Albury,	572	−36 6	147 0	$1\frac{1}{4}$	9 3	·048	·050	·045	·072	·039	·041	·046	·029	·032	·056	·037	·039	·044
Cooma,	2637	−36 13	149 9	2	9 3	·053	·046	·063	·067	·050	·060	·050	·059	·077	·056	·056	·049	·057
VICTORIA—																		
Melbourne,	121	−37 50	143 59	5	Hourly	·060	·063	·068	·063	·055	·060	·055	·061	·068	·069	·063	·063	·059
Do.	121	−37 50	143 59	5	9 3	·052	·057	·064	·062	·055	·056	·050	·060	·061	·069	·055	·055	·058
Do.	121	−37 50	143 59	5	10 4	·056	·060	·065	·058	·048	·055	·051	·060	·063	·060	·052	·057	·057
W. AUSTRALIA																		
Freemantle,	16	−32 3	115 45	3	$9\frac{1}{2}$ $3\frac{1}{2}$	·048	·046	·059	·047	·051	·049	·046	·037	·036	·046	·033	·053	·046
TASMANIA—																		
Hobart Town,	37	−42 52	147 21	8	Hourly	·049	·064	·064	·061	·042	·050	·049	·059	·062	·052	·057	·045	·050
Do.	37	−42 52	147 21	8	9 3	·040	·057	·064	·061	·042	·041	·042	·058	·062	·051	·047	·035	·050
Do.	37	−42 52	147 21	8	10 3	·036	·051	·062	·053	·036	·039	·041	·052	·046	·037	·031	·030	·043
NEW ZEALAND																		
Auckland,	140	−36 50	174 51	5	$9\frac{1}{2}$ $3\frac{1}{2}$	·035	·037	·044	·050	·045	·041	·048	·043	·050	·047	·042	·038	·043
MAURITIUS—																		
St Louis,	20	−20 10	57 30	12	$9\frac{1}{2}$ $3\frac{1}{2}$	·061	·062	·065	·068	·070	·068	·072	·077	·078	·074	·068	·060	·069
Do.	20	−20 10	57 30	Term	Hourly	·042	·059	·065	·077	·077	·080	·088	·084	·092	·069	·075	·066	·073
Do.	20	−20 10	57 30	Term	9 3	·042	·059	·065	·077	·076	·080	·088	·084	·091	·069	·072	·066	·073
Do.	20	−20 10	57 30	Term	10 4	·036	·057	·064	·065	·062	·075	·086	·079	·086	·062	·070	·058	·070
MADAGASCAR.																		
Tamatave,	0	−18 3	49 11	$\frac{1}{3}$	10 4	...	...	...	...	...	...	...	·059	·060	·055	·080	...	...
AFRICA—																		
Port Said,	20	31 18	32 18	2	9 3	·027	·021	·026	·015	·017	·030	·028	·023	·015	·019	·014	·015	·021

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Ismäilia,	25	30 38	32 13	2	9 3	·031	·055	·060	·061	·057	·058	·060	·064	·052	·065	·053	·055	·056
Suez,	10	29 57	32 32	2	9 3	·065	·074	·073	·069	·066	·067	·077	·068	·076	·076	·067	·060	·070
Algiers,	66	36 43	3 3	1	10 3	·028	·043	·055	·016	·043	·032	·033	·022	·043	·063	·051	·043	·039
Oran,	164	35 44	− 0 41	12	10 4	·029	·033	·026	·024	·025	·031	·039	·031	·028	·022	·033	·023	·029
Monstagannen,	374	35 55	0 5	3	10 4	·049	·050	·062	·050	·045	·061	·062	·069	·063	·075	·038	·059	·057
Ghadames,	1283	30 10	9 14	$\frac{1}{6}$	9 3	...	...	...	...	...	...	·071	·075	...	...	...	...	...
Murzuk,	1650	25 54	14 12	$\frac{1}{6}$	9 3	·051	·055	·055	...	...	...	...	...	...	...	·035	·047	...
Oasis Kauar,	1624	18 57	12 10	$\frac{1}{6}$	9 3	...	...	...	...	·122	·087	...	...	...	...	...	...	...
Goree,	19	14 40	− 17 25	4	10 4	·067	·063	·047	·039	·028	·031	·043	·043	·051	·047	·035	·038	·044
San Nicolas,	2175	16 40	− 14 15	$\frac{3}{4}$	...	·063	...	...	...	·051	·047	·047	·059	·059	·059	·067	·055	·055
Kuka,	1168	12 52	13 23	$\frac{1}{6}$	9 3	...	...	...	...	...	...	·094	·079	·110	·126	·134	·118	...
Abd es Senga,	1467	8 54	6 48	$\frac{1}{6}$	9 3	...	·181	·193	...	...	...	...	...	...	...	...	...	...
St George d'Elmina,	75	5 5	− 1 20	Term	Hourly	·123	·086	·083	·098	·096	·082	·082	·110	·126	·102	·091	...	...
Do.	75	5 5	− 1 20	Term	9 3	·123	·078	·055	·074	·082	·063	·075	·081	·118	·098	·102	...	...
Do.	75	5 5	− 1 20	Term	10 4	·114	·082	·082	·098	·096	·078	·078	·106	·126	·094	·083	...	...
Christiansborg,	60	5 24	0 10	Term	Hourly	·106	·104	·102	·102	·096	·084	·095	·100	·114	·103	·100	·099	·099
Do.	60	5 24	0 10	Term	9 3	·106	·104	·102	·098	·090	·078	·082	·094	·102	·103	·100	·099	·097
Do.	60	5 24	0 10	Term	10 4	·997	·092	·097	·100	·095	·084	·094	·100	·110	·097	·092	·089	·095
Lagos,	0	6 12	3 25	$\frac{1}{2}$	...	...	...	...	...	...	·102	·134	·130	·106	·138	·113	102	...
Gondar,	7422	12 37	37 32	$1\frac{7}{12}$	9 3	·066	·069	·065	·071	...	...	...	...	...	·071	·078	·064	...
Massuah,	5	15 36	39 21	$\frac{1}{12}$	9 3½	·106	·101	·110	·107	...	...	...	...	·109	·095	·100	·108	...
Gondokoro,	1450	4 30	31 40	1	9 4	·177	·160	·133	·126	·159	·124	·133	·115	·106	·191	·159	·160	·145
Zambesi Delta,	0	− 18 24	35 30	$\frac{1}{12}$	Hourly	...	...	...	...	...	...	·123	...	...	...	...	...	...
Do.	0	− 18 24	35 30	$\frac{1}{12}$	9 3	...	...	...	...	...	...	·045	...	...	...	...	...	...
Do.	0	− 18 24	35 30	$\frac{1}{12}$	10 4	...	...	...	...	...	...	·123	...	...	...	...	...	...
Pietermaritzburg,	2096	− 29 30	30 2	2	9 3	·056	·068	·071	·070	·076	·071	·090	·080	·081	·075	·060	·069	·072
Graham's Town,	1750	− 33 18	26 29	$4\frac{1}{2}$	9½ 3½	·045	·050	·048	·054	·052	·061	·061	·054	·064	·052	·046	·043	·053
Cape Town,	37	− 33 56	18 27	5	Hourly	·035	·044	·052	·045	·059	·054	·056	·051	·051	·046	·037	·039	·045
Do.	37	− 33 56	18 27	5	9 3	·027	·036	·048	·045	·057	·048	·050	·050	·051	·046	·035	·032	·044
Do.	37	− 33 56	18 27	5	10 4	·034	·043	·052	·042	·054	·049	·054	·051	·050	·045	·037	·038	·045
ATLANTIC OCEAN—																		
Angra do Heroisme,	177	38 36	− 27 15	6	9 3	·041	·035	·030	·021	·009	·011	·009	·013	·018	·022	·037	·034	·024
Ponta Delgada,	66	37 44	− 26 55	6	9 3	·038	·039	·022	·017	·006	·004	·006	·011	·016	·026	·033	·036	·021
Funchal,	83	32 28	− 16 55	6	9 3	·059	·047	·034	·024	·016	·013	·011	·017	·030	·037	·049	·044	·033

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Year.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Oratava,	70	28 27	– 16 38	1 1/12	8·10 3·4	·046	·046	·049	·035	·024	·024	·020	·024	·023	·029	·034	·040	·033
Laguna de Tenerife,	...	28 12	– 16 24	3	9 3	·059	·049	·054	·041	·045	·039	·037	·033	·047	·047	·041	·050	·045
Square, 3 N. half	0	5–10	– 20–30	*	Hourly	·083	·069	·079	·079	·070	·063	·059	·062	·067	·066	·074	·081	·070
Do.	0	5–10	– 20–30	*	9 3	·083	·069	·077	·071	·063	·057	·053	·058	·062	·064	·072	·080	·068
Do.	0	5–10	– 20–30	*	10 4	·077	·064	·076	·079	·070	·063	·059	·062	·067	·066	·074	·078	·070
Do. S. half,	0	0– 5	– 20–30	*	Hourly	·075	·076	·076	·080	·074	·062	·069	·074	·072	·067	·075	·074	·072
Do.	0	0– 5	– 20–30	*	9 3	·075	·075	·073	·075	·068	·053	·062	·069	·069	·065	·072	·073	·069
Do.	0	0– 5	– 20–30	*	10 4	·073	·074	·076	·080	·074	·062	·069	·074	·072	·066	·075	·073	·072
St Helena,	1763	– 15 55	– 5 43	5	Hourly	·069	·070	·075	·078	·072	·062	·061	·064	·061	·061	·067	·068	·067
Do.	1763	– 15 55	– 5 43	3½	9 3	·060	·061	·068	·071	·068	·059	·057	·058	·057	·058	·061	·059	·061
Do.	1763	– 15 55	– 5 43	3½	10 4	·069	·070	·075	·078	·072	·062	·059	·063	·061	·060	·067	·067	·067
Do.	40	– 15 55	– 5 43	3½	9½ 3½	·059	·061	·065	·064	·058	·055	·053	·057	·052	·059	·061	·054	·058
ANTARCTIC REGIONS—																		
Lat. 64° to 78° S., Long. 53° to 58° W., and 155° W. to 168° E., 93 days,	...	...	...	*	Hourly	·005	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	*	9 3	·000	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	*	10 4	·002	...	...	...	...	...	...	...	...	...	...	...	...
Lat. 60° to 78° S., Long. 6° to 56° W., and 158° W. to 165° E., 94 days,	...	...	...	*	Hourly	...	·009	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	*	9 3	...	·005	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	*	10 4	...	·007	...	...	...	...	...	...	...	...	...	...	...
BRITISH AMERICA—																		
Port Foulke,	6	78 18	– 73 0	1	B hourly	·010	·017	·015	·009	·019	·019	·004	·000	·011	+ ·003	·007	·002	·007
Do.	6	78 18	– 73 0	1	8 2	·003	·016	·015	·009	·019	·015	+ ·015	+ ·003	·009	+ ·002	+ ·002	·016	·007
Do.	6	78 18	– 73 0	1	10 4	+ ·017	·010	·005	·006	·002	·003	·001	+ ·006	+ ·010	+ ·002	+ ·001	003	·002
Fort Confidence	...	66 54	– 118 49	½	9 3	+ ·009	·002	·007	+ ·006	...	...	...	...	...	+ ·004	·002	+ 009	...
Do.	...	66 54	– 118 49	½	10 4	+ ·001	·000	·004	+ ·008	...	...	...	...	...	+ ·008	+ 001	+ ·010	...
CANADA (DOMINION OF)—																		
St John's (Newfoundland),	130	47 35	– 52 42	6	9½ 3½	·020	·021	·036	·022	·031	·026	·026	·033	·036	·039	·027	·030	·029
Halifax,	8	44 39	– 63 37	4	9½ 3½	·048	·043	·042	·046	·044	·020	·040	·031	·013	·019	·031	·024	·033
Quebec,	230	46 48	– 71 12	2½	9½ 3½	·046	·046	·030	·030	·053	·038	·032	·042	·048	·043	·025	·025	·038

* Calculated from observations taken at sea.

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Kingston,	294	44 14	− 76 30	4	9½ 3½	·040	·039	·051	·052	·056	·043	·038	·032	·058	·036	·032	·032	·042
Toronto,	342	43 39	− 79 23	6	Hourly	·050	·048	·045	·053	·052	·052	·050	·050	·053	·043	·044	·052	·045
Do.	342	43 39	− 79 23	6	9 3	·042	·048	·045	·049	·044	·039	·038	·042	·050	·043	·038	·044	·044
Do.	342	43 39	− 79 23	6	10 4	·039	·047	·044	·052	·050	·045	·043	·047	·051	·041	·040	·041	·045
New Westminister,	54	49 13	− 122 53	2	9½ 3½	·010	·036	·025	·009	·017	·023	·023	·029	·037	·005	·018	·009	·017
UNITED STATES—																		
Sitka,	20	56 50	− 135 0	10	Hourly	·008	·012	·004	·009	·007	·006	·005	·006	·014	·008	·009	·013	·008
Do.	20	56 50	− 135 0	10	9 3	·004	·005	+ 009	+ 002	+ 006	+ 014	+ 007	+ 008	+ 005	+ 001	·000	+ 005	+ 004
Do.	20	56 50	− 135 0	10	10 4	·005	·007	+ 003	·001	+ 002	+ 011	+ 005	+ 005	+ 001	·002	·003	·002	+ 001
Do.	20	56 50	− 135 0	10	1 6	·005	·006	·003	·009	·004	·003	·003	·004	·014	·007	·007	·010	·007
Astoria,	53	46 11	− 123 50	*	Hourly	·057	·034	·038	·059	·036	·038	·040	·037	·052	·037	·036	·057	·041
Do.	53	46 11	− 123 50	*	9 3	·052	·025	·038	·045	·022	·026	·029	·022	·037	·029	·032	·052	·034
Do.	53	46 11	− 123 50	*	10 4	·054	·029	·037	·055	·032	·034	·035	·028	·047	·035	·036	·057	·040
Fort Hall (Idaho),	...	42 15	− 112 29	⅓	10 4	...	...	...	...	...	·061	·077	·066	·058	...	...	...	...
Fort Churchill,	4319	39 18	− 119 15	*	Hourly	·067	·058	·076	·104	·078	·090	·091	·087	·088	·082	·068	·068	·072
Do.	4319	39 18	− 119 15	*	9 3	·050	053	·074	·081	·067	·066	·070	·068	·073	·077	·063	·067	·067
Do.	4319	39 18	− 119 15	*	10 4	·063	·058	·075	·098	·057	·069	·072	·066	·079	·080	·056	·070	·071
Carson City,	4720	39 10	− 119 45	*	Hourly	...	...	...	...	...	...	·078	...	...	...	...	...	...
Do.	4720	39 10	− 119 45	*	9 3	...	...	...	...	...	...	·060	...	...	...	...	...	...
Do.	4720	39 10	− 119 45	*	10 4	...	...	...	...	...	...	·057	...	...	...	...	...	...
Hope Valley,	7072	38 47	− 119 54	*	Hourly	·053	...	...	...	...	...	·031	·031	·045	...	...	...	...
Do.	7072	38 47	− 119 54	*	9 3	·043	...	...	...	...	...	·020	·020	·037	...	...	...	...
Do.	7072	38 47	− 119 54	*	10 4	·044	...	...	...	...	...	·021	·021	·037	...	...	...	...
Strawberry Valley,	5707	38 49	− 120 7	*	Hourly	·051	...	...	...	...	...	·023	·026	...	...	...	...	...
Do.	5707	38 49	− 120 7	*	9 3	·049	...	...	...	...	...	·015	·021	...	...	...	...	...
Do.	5707	38 49	− 120 7	*	10 4	·047	...	...	...	...	...	·018	·016	...	...	...	...	...
Placerville,	1965	38 44	− 120 46	*	Hourly	·056	...	...	...	...	...	·044	·044	...	...	...	...	...
Do.	1965	38 44	− 120 46	*	9 3	·050	...	...	...	...	...	·013	·025	...	...	...	...	...
Do.	1965	38 44	− 120 46	*	10 4	·056	...	...	...	...	...	·034	·035	...	...	...	...	...
Sacramento,	81	38 34	− 121 19	*	Hourly	·070	·068	·082	·076	·066	·064	·068	·081	·073	·079	·067	·068	·070
Do.	81	38 34	− 121 19	*	9 3	·062	·063	·056	·049	·036	·034	·040	·054	·059	·066	·059	·062	·053
Do.	81	38 34	− 121 19	*	10 4	·070	·068	·073	·065	·046	·045	·054	·065	·067	·076	·067	·068	·064
San Francisco,	22	37 48	− 122 23	*	Hourly	·080	·069	·067	·058	·038	·055	·068	·066	·079	·077	·065	·075	·064
Do.	22	37 48	− 122 23	*	9 3	·069	·054	·053	·040	·021	·030	·042	·048	·064	·069	·057	·069	·052
Do.	22	37 48	− 122 23	*	10 4	·080	·063	·061	·050	·027	·044	·059	·060	·072	·075	·061	·075	·060

* From *On the Use of the Barometer*, by Major R. S. Williamson. The observations, generally for short periods, were made under Major Williamson's directions.

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.,*—Continued.

	Height in ft.	Lat.†	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Fort Tejon.	3240	34 53	−118 53	*	Hourly	...	...	...	...	...	...	·040	...	...	...	...	...	...
Do.	3240	34 53	−118 53	*	9 3	...	...	...	...	...	...	·021	...	...	...	...	...	...
Do.	3240	34 53	−118 53	*	10 4	...	...	...	...	...	...	·034	...	...	...	...	...	...
Fort Yuma,	200	32 44	−114 36	*	Hourly	...	...	·120	·122	...	...	...	...	...	...	...	...	...
Do.	200	32 44	−114 36	*	19 3	...	...	·105	·114	...	...	...	...	...	...	...	...	...
Do.	200	32 44	−114 36	*	0 4	...	...	·110	·110	...	...	...	...	...	...	...	...	...
Iowa City,	621	41 40	− 91 20	4	8¾ 3¾	·030	·050	·050	·042	·062	·048	·055	·065	·071	·055	·049	·042	·052
Philadelphia,	112	39 38	− 75 11	4	Hourly	·074	·060	·064	·081	·078	·064	·058	·054	·064	·069	·068	·072	·063
Do.	112	39 38	− 75 11	4	9 3	·070	·060	·064	·073	·064	·048	·048	·048	·061	·067	·068	·064	·061
Do.	112	39 38	− 75 11	4	10 4	·066	·058	·059	·080	·060	·053	·053	·051	·062	·064	·061	·069	·062
Washington,	103	38 56	− 76 58	5	Hourly	·065	·065	·079	·077	·065	·061	·060	·070	·067	·066	·065	·061	·065
Do.	103	38 56	− 76 58	5	9 3	·065	·065	·077	·072	·050	·051	·047	·062	·062	·065	·065	·060	·062
Do.	103	38 56	− 76 58	5	10 4	·062	·065	·076	·073	·056	·058	·057	·065	·063	·063	·057	·050	·062
Utah,	4260	40 45	−111 16	¾	9 3	·034	·042	·043	·065	·058	·058	·060	·064	·047	...	...	...	...
MEXICO—																		
Veta Grande,	...	12 56	−101 26	2	8½ 4½	·055	·071	·075	·075	·075	·063	·051	·063	·079	·067	·067	·055	·066
Cordova,	2880	19 26	− 97 51	3	9 3	·083	·106	·091	·087	·075	·071	·071	·071	·079	·094	·087	·091	·084
Mexico City,	...	19 25	− 99 5	1	9½ 4	·130	·134	·114	·098	·118	·091	·079	·094	·106	·122	·114	·126	·111
CENTRAL AMERICA—																		
Belize,	4	17 30	− 88 18	2	10 4	·087	·087	·091	·083	·083	·071	·071	·079	·091	·091	·083	·087	·084
WEST INDIES.																		
Bermuda,	123	32 23	− 64 40	3½	9½ 3½	·053	·045	·040	·029	·026	·017	·023	·029	·037	·042	·043	·042	·036
Nassau (Bah),	13	25 4	− 77 22	6	9½ 3½	·053	·049	·040	·042	·037	·025	·020	·026	·032	·032	·045	·048	·037
Havannah, Cuba	63	23 8	− 82 23	10	8 hourly	·082	·083	·077	·071	·057	·051	·053	·055	·064	·069	·073	·080	·067
Do.	63	23 8	− 82 23	10	10 4	·082	·083	·077	·071	·057	·051	·053	·055	·064	·069	·073	·080	·067
Up Park Camp (Jam.),	225	18 0	− 76 56	6	9½ 3½	·080	·077	·082	·066	·053	·053	·059	·058	·057	·076	·082	·068	·068
Binfield (Barbs.)	1107	13 5	− 59 40	8	10 4	·065	·065	·059	·051	·058	·048	·046	·046	·053	·053	·066	·068	·057
St Ann's (do.)	5	13 4	− 59 40	6	9½ 3½	·067	·052	·047	·048	·045	·046	·037	·046	·049	·062	·064	·072	·053
S. AMERICA—																		
Caraccas,	3042	10 30	− 66 55	3	10 4	·055	·063	·051	·055	·055	·043	·039	·047	·055	·051	·055	·047	·051
Bogota,	8727	4 35	− 74 14	2	9 3	·090	·089	·100	·097	·098	·085	·083	·088	·102	·111	·108	·091	·095
Paramaribo,	...	6 44	− 44 59	1	10 4	·110	·106	·096	·098	·094	·102	·095	·080	·106	·099	·101	·114	·100
Cayenne,	7	4 56	− 55 39	6½	9 3	·083	·082	·086	·088	·086	·083	·085	·089	·093	·094	·090	·085	·087
Mañaos,	121	− 3 8	− 60 0	⅝	9 3	·126	·118	·126	·130	·134	·110	·110	...	...	·134	·134	·126	·126
Rio de Janeiro,	206	−22 54	− 43 7	5	Hourly	·038	·038	·036	·039	·036	·037	·036	·044	·046	·044	·038	·035	·036
Do.	206	−22 54	− 43 7	5	9 3	·020	·019	·022	·027	·026	·024	·024	·030	·038	·034	·027	·020	·026

* From *On the Use of the Barometer*, by Major R. S. Williamson. The observations, generally for short periods, were made under Major Williamson's directions.

TABLE I.—*Showing the Mean Diurnal Barometric Oscillation, &c.*,—Continued.

	Height in ft.	Lat.	Long.	No. of Years Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Year.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
Rio de Janeiro,	206	− 22 54	− 43 7	5	10 4	·034	·030	·032	·037	·036	·035	·034	·042	·046	·040	·037	·033	·036
Copiapo,	1296	− 27 22	− 70 23	2	9 3	·051	·043	·055	·047	·039	·031	·047	·047	·047	·063	·055	·051	·048
Serena,	59	− 29 55	− 71 17	2	?	·031	·016	·024	·035	·031	·016	·004	·008	·016	·024	·020	·024	·021
Valparaiso,	0	− 33 2	− 71 40	3–5	$9\frac{1}{2}$ $3\frac{1}{2}$	·038	...	...	...	·034	·050	·042	·046	·043	·018	·012	·044	...
Santiago,	1782	− 33 26	− 70 37	3	9 3	·034	·040	·042	·040	·039	·037	·032	·036	·043	·029	·038	·038	·037
Do.	1782	− 33 26	− 70 37	3	9 6	·040	·048	·040	·030	·029	·022	·014	·023	·026	·016	·029	·039	·030
Port Louis (E. F. Is.),	0	− 51 33	− 58 7	$\frac{1}{2}$	Hourly	...	...	...	·037	·030	·054	·020	·017	...	...	·021	...	...
Do.	0	− 51 33	− 58 7	$\frac{1}{2}$	9 3	...	..	...	·034	·030	·009	·013	·014	...	...	·010	...	...
Do.	0	− 51 33	− 58 7	$\frac{1}{2}$	10 4	...	...	...	·032	·022	·005	·006	·006	...	...	·005	...	...
St Martin's Cove	0	− 55 52	− 67 33	$\frac{1}{2}$	Hourly	...	...	...	...	...	...	...	...	·023	...	...	...	...
Do.	0	− 55 22	− 67 33	$1\frac{1}{12}$	9 3	...	...	...	...	...	...	...	...	·017	...	...	...	...
Do.	0	− 55 22	− 67 33	$1\frac{1}{12}$	10 4	...	...	...	...	...	...	...	...	·014	...	...	...	...

TABLE II. (Supplementary to Table I.)—*Showing the Mean Diurnal Barometric Oscillation from the* A.M. *Maximum to the* P.M. *Minimum at different places over the Globe. The amounts have been calculated from Observations made at the Hours specified.*

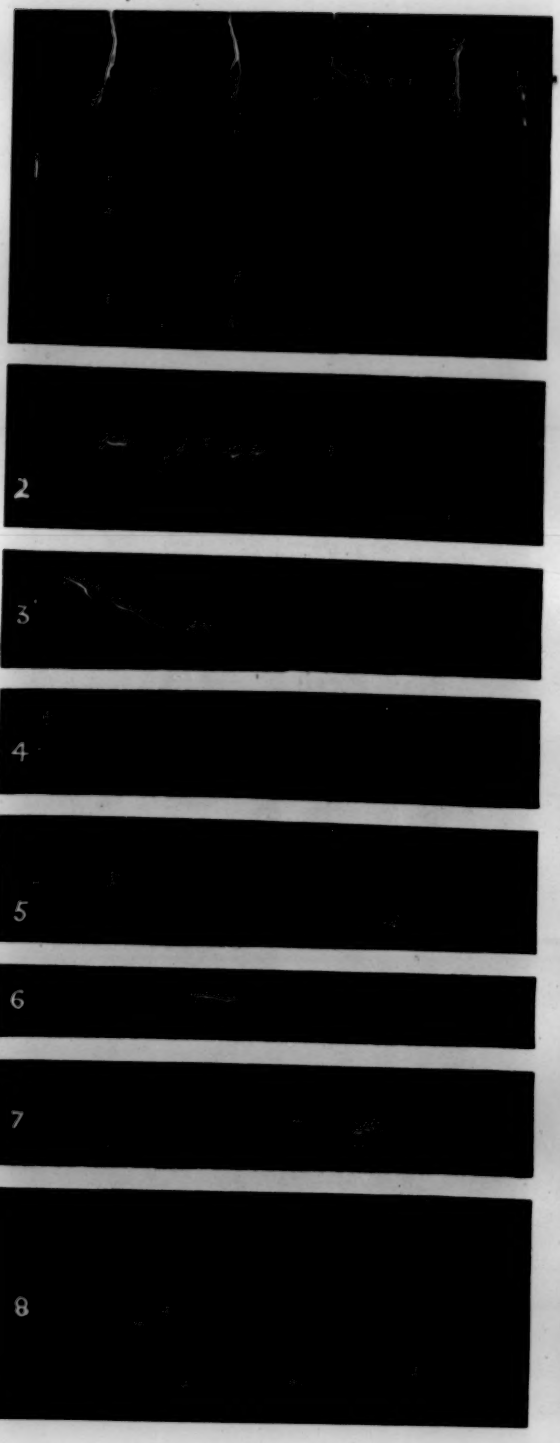
	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
NORWAY—																		
Christiansund,	65	63 7	7 47	8	8 2	·004	·004	·002	·011	·010	·014	·004	·012	·007	·007	+·002	·014	·007
Alesund,	32	62 29	6 9	8	8 2	·006	·008	·002	·009	·003	·012	·003	·003	·001	·007	·002	·012	·006
Bergen,	50	60 24	3 20	8	8 2	·007	·015	·004	·009	·009	·006	·010	·003	·015	·012	·013	·010	·009
Skudesnes,	37	59 9	5 16	8	8 2	·006	·017	·004	·006	·004	·007	·007	·002	·005	·017	·018	·013	·009
Mandal,	54	58 2	7 27	8	8 2	·022	·021	·015	·019	·017	·017	·015	·012	·012	·031	·021	·015	·016
Sandösund,	41	59 5	10 27	8	8 2	·016	+·004	·014	·015	·017	·022	·011	·009	·005	·013	·003	·005	·011
NETHERLANDS																		
Flushing,	15	51 26	3 35	10	8 2	·018	·024	·020	·018	·025	·016	·017	·017	·018	·029	·025	·026	·021
Maestricht,	174	50 52	5 37	10	8 2	·012	·023	·017	·028	·024	·023	·022	·024	·024	·027	·016	·010	·021
BELGIUM—																		
Stavelot,	1022	50 28	5 58	10	9 2	·019	·015	·023	·019	·019	·019	·014	·019	·022	·020	·023	·021	·019
AUSTRIA—																		
Rzeszow,	702	50 3	21 58	4	10 2	·021	·014	·027	·033	·029	·035	·034	·027	·026	·028	·015	·016	·025
Pürglitz,	1010	50 2	13 52	9	7 2	·034	·028	·037	·044	·039	·038	·045	·035	·040	·036	·030	·030	·036
Lemberg,	927	49 50	24 0	5	6 2	·032	·026	·029	·030	·029	·025	·026	031	·028	·031	·024	·029	·028
Linz,	780	48 16	14 12	5½	7 2	·029	·026	·037	·047	·047	·050	·044	034	·037	·033	·018	·025	·036
Alt Aussee,	3095	47 39	13 42	5	8 2	·030	·026	·023	·030	·037	·034	·031	·031	·035	·044	·026	·033	·032
Wallendorf,	1701	47 9	24 36	5	7 2	·028	·033	·034	·039	·036	·036	·035	·036	·035	·038	·036	·020	·034
St Paul,	1235	46 43	14 52	7	7 2	·036	·035	·044	·063	·051	·051	·067	·063	·054	·046	·039	·036	·050
Klagenfurt,	1438	46 37	14 16	9	7 2	·046	·052	·057	·060	·060	·064	·055	·058	·059	·051	·048	·040	·054
Hermanstadt,	1354	45 47	24 7	6	6 2	·028	·034	·029	·038	·034	·034	·035	·038	·038	·032	·027	·016	·032
Triest,	79	45 39	13 44	9	7 2	·024	·039	·038	·041	·033	·041	·036	·032	·035	·036	·025	·031	·034
Ragusa,	64	42 38	18 5	3	7 2	·038	·024	·028	·037	·025	·039	·031	·030	·038	·040	·023	·033	·032
RUSSIA—																		
Kostroma,	348	57 47	40 55	4	7 2	·003	·005	·001	·004	·019	·017	·025	·020	·006	·006	·006	+·002	·010
Orel,	542	52 57	36 5	4	8 2	·017	·006	·002	·021	·028	·029	·024	·015	·025	·020	·008	·016	·018
Orenburg,	340	51 46	55 6	5	10 2	·017	·010	·013	·018	·025	·027	·023	·026	·026	·029	·020	·023	·021
Catherinoslav,	210	48 28	35 5	4	10 2	·020	·033	·021	·015	·024	·026	·019	·026	·030	·032	·020	·032	·025
Astrakhan,	5	46 21	48 2	2	7 2	·031	·032	·026	·030	·024	·027	·026	·024	·032	·030	·019	·027	·027
Sebastopol,	136	44 37	33 31	3	7 2	·030	·025	·024	·021	·017	·015	·029	·023	·029	·027	·027	·030	·025
Tobolsk,	240	58 12	68 16	8	7 3	·015	·006	·008	·016	·015	·014	·014	·007	·012	·006	·012	·011	·011
Nijne Taguilsk,	621	57 56	60 5	11	8 3	·005	·006	·017	·020	·020	·019	·023	·010	·010	·002	·002	·006	·012
Irkutsk,	1350	52 11	104 6	15	7 2	·029	·032	·041	·038	·045	·042	·039	·028	·035	·032	·023	·029	·034
Semipalatinsk,	380	50 24	80 16	6	6 2	·037	·031	·030	·034	·040	·037	·032	·036	·036	·043	·026	·031	·034
Fort Ouralsk,	358	48 37	61 16	5	6 2	·025	·020	·019	·025	·044	·039	·037	·043	·041	·044	·027	·027	·033
Fort No. 1,	170	45 45	64 27	5	6 2	·032	·028	·032	·041	·045	·037	·033	·037	·040	·041	·031	·031	·036
Ajansk,	30	56 27	138 36	2	7 2	·028	·028	·032	·032	·024	·037	·031	·025	·041	·038	·037	·030	·033
Nikolaevsk,	54	53 8	140 45	4	7 2	·040	·028	·030	·041	·038	·028	·030	·031	·032	·035	·032	·033	·033
Isle Douai,	...	50 50	144 27	2	6 2	·038	·036	·037	·040	·030	·029	·026	·027	·030	·033	·034	·035	·033

TABLE II.—*Showing the Mean Diurnal Barometric Oscillation, &c.*—Continued.

TABLE II.—*Showing the Mean Diurnal Barometric Oscillation, &c.*—Continued.

	Height in ft.	Lat.	Long.	No. of Years' Obs.	Hours of Obs.	Jan.	Feb.	March.	April.	May.	June.	July.	August	Sept.	Oct.	Nov.	Dec.	YEAR.
		° ′	° ′			inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.	inch.
CANADA—																		
Sydney,	...	45 35	−61 5	1	*	·035	·054	·023	·071	·031	·024	·052	·048	·063	·041	·026	·066	·044
Charlotte Town,	...	46 14	−63 10	1	*	·041	·046	·032	·057	·036	·032	·057	·051	·047	·047	·035	·076	·046
Father Pt.,	...	48 31	−68 3	1	*	·066	·049	·068	·074	·053	·055	·058	·057	·059	...	...	...	...
Montreal,	...	45 31	−73 35	1	*	...	·074	·073	·058	·066	·076	·061	·064	·060	·061	·031	·024	...
Ottawa,	...	45 20	−75 43	1	*	·043	·080	·074	·062	·067	·073	·068	·067	·079	·053	·048	·046	·063
Brockville,	...	44 37	−75 46	1	*	·047	·074	·050	·057	·055	·059	·048	·060	·059	·044	·048	·040	·053
Kingston,	...	44 15	−78 35	1	*	·050	·065	·067	·053	·054	·053	·065	·052	·052	·037	·040	·033	·052
Toronto,	...	43 39	−79 2	1	*	·041	·050	·060	·075	·051	·060	·058	·057	·063	·041	·038	·043	·053
Port Dover,	..	42 47	−80 18	1	*	·044	·048	·059	·066	·049	·056	·051	·074	·052	·057	·030	·046	·053
Port Stanley,	...	42 41	−81 18	1	*	·043	·051	·052	·058	·040	·044	·041	·053	·049	·042	·029	·050	·046
Saugeen,	...	44 15	−81 0	1	*	·034	·027	·023	·063	·020	·045	·035	·053	·044	·043	·046	·043	·040
Fort Garry,	...	49 56	−92 5	1	*	·032	·046	·053	·042	·059	·046	·064	·056	·050	·060	·050	·018	·048
S. AMERICA—																		
Arica,	...	−18 25	−70 22	$1\frac{1}{6}$	6 5	·070	·073	.068	·057	·058	·061	·064	·051	·055	·050	·044	·064	·060
Buenos Ayres,	60	−34 36	−58 23	$2\frac{1}{2}$	8 2	·071	·056	·072	·058	·063	·073	·073	·052	·060	·056	·064	·075	·064

* Observations made at 7.25 A.M. and 4.25 P.M. Civil Time of Toronto.


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2
3
4
5
6
7
8

XX.—*Photographs of Electric Sparks in Hot and Cold Air.* By Professor Tait. (Plate XXXIV.)

At the meetings of the Society on February 1 and 15, 1875, I exhibited a number of photographs which had been executed for me in my laboratory by Mr A. Matheson, one of my students who has acquired great skill in working with very sensitive films of collodion. My object in advising him to undertake this work was to discover, if possible, the cause of the peculiar zig-zag form which electric sparks, and specially those of a Holtz machine, always show in ordinary air. At the desire of the Council, a selection of a few of the more interesting of these photographs has been printed (by Mr Dallas, F.R.S.E.) for the *Transactions* from the original glass negatives, by what is commonly called the carbon process.

Mr Matheson used a quartz lens which I had procured for experiments on Fluorescence. It was usually placed so as to give an image about half the dimensions of the spark itself; sometimes a little larger. The sparks were furnished by a double Holtz machine made by Ruhmkorff. It was kept in perfect order by placing below it an inverted box of sheet iron, inside of which three small Bunsen lamps were kept burning.

The first eight sets in the plate were taken in ordinary air.

Figures 9–12 inclusive were taken in free air, a foot or two over the flame of a powerful 6-barrelled Bunsen burner.

13–16 inclusive were taken in a wide glass tube, through which air had been passed (under slight increase of pressure) from a long narrow spiral tube of iron, kept at a dull red heat. 13 represents the result just after the current of hot air began to pass; 14 after a few minutes; 15 and 16 after nearly half an hour.

The first thing to be observed is, that in the hot air the sparks form in general much smoother curves than in the cold. A specially excellent instance of this is furnished by the two sparks in 9, taken at a fraction of a second interval, of which the upper passed through the ascending column of very hot air, while the lower (as is easily seen by its being partially out of focus, as well as being much more luminous) passed *beside*, but not *through*, the hot column.

In various sparks of the first eight groups there are cases of sudden change of direction apparently through more than a right angle. To make absolutely

certain of this, however, it would be necessary to be sure of the position of the osculating plane at some one of these points. The simplest mode of doing this would have been to obtain simultaneous photographs of the spark from different points of view.

Another thing to be remarked is, that the zig-zag is sometimes directed nearly towards the observer, thus giving the appearance of a brighter spot at one point of the spark. Specially curious instances of this are to be seen in 2, 3, 6, 11, and 12. Those in 2 and 12, at least, distinctly indicate this particular origin. Some of the others may possibly have a different cause.

Bifurcation is very common. Thus, in a series of seventeen successive sparks (fig. 1)—the camera having been slowly (and, as is clear from the result, rather irregularly) moved (by hand) during their discharge—the first (on the left), the twelfth, and thirteenth are obviously each in part double. The lower spark in 5 presents a magnificent bifurcation. Two very fine instances (in successive sparks) are seen in 13. 6 shows a bifurcation, of which one branch is very feeble compared with the other. Other good examples occur in the upper spark in 7, and the middle one in 8.

The general result of an examination of these photographs is, that the zig-zag appearance depends upon something which heat is capable of removing from the air. This is therefore not aqueous vapour,—nor is it very minute drops of water, for even falling drops of water were found to produce no effect, beyond a mere *interruption* in the photographed spark (see fig. 12),—but is probably organic matter, which, as SCHROEDER and PASTEUR have shown, would be as effectually kept out of our apparatus (if once it were got rid of) by a plug of cotton wool as by actual combustion.

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XXI.—*On the Expiatory and Substitutionary Sacrifices of the Greeks.*

By James Donaldson, LL.D.

(Read 17th May 1875.)

I have to explain at the outset that the title of my paper has been chosen for the sake of brevity. Both the words "expiatory" and "substitutionary" are liable to be misunderstood and abused in argument. I shall, therefore, avoid the word "expiate" as much as possible; but as I cannot do without a frequent use of the words "substitution" and "substitutionary," I merely note at present that there are various modes of substitution and various purposes served by it, and that it is very important in an argument to state accurately both the mode and the purpose.

The object of my paper is to place the two prevalent theories of the origin of sacrifice in juxtaposition with the facts which we have in regard to Greek sacrifices, in order that we may see what light the facts may throw on the theories. For this purpose I first state the two theories.

I.—The Two Theories of the Origin of Sacrifice.

The one theory supposes that man gradually emerged from a low state of intelligence and morality; that in the course of this emergence he formed the idea that there were celestial beings superior to himself and having control over him; that he conceived these beings to possess the same appetites, desires, and passions as he himself had; and that, in order to gain the good-will and avert the anger of these beings, he offered them dainties which he imagined they ate, and he presented them with ornaments which they delighted to look on; he regaled them with sounds which they delighted to hear, and covered in houses for them in which they delighted occasionally to dwell. According to this theory, sacrifices were gifts of food or drink, or of a similar nature, by which, through giving real pleasure to the gods, men hoped to gain from them what they wanted.

The other theory requires to be stated with greater caution, and I shall therefore adopt, for the most part, the words of the scholar who has given the best exposition of the subject,—Lasaulx, in his treatise "Die Sühnopfer der Griechen und Römer und ihr Verhältnis zu dem einen auf Golgotha."* According to him, "the sühnopfer," or expiatory sacrifices, "are the centre of

* It appeared first at Würzburg 1841, and afterwards in "Studien des Classischen Alterthums, Akademische Abhandlungen," von Ernst von Lasaulx. Regensburg, 1854.

all positive religions." He conceives the matter thus: the original man was one with God; his will was identical with God's. He clung to God, as the child to its mother. The very thought of sacrifice in such circumstances was impossible. But when he sinned, at once there arose a chasm between him and the Divine mind. He had lost his living connection with God, his very life, through sin. But man still longed for unity; and in his efforts to regain that unity "he sought to expiate the life which he had lost through sin through the voluntary sacrifice of life. All sacrifices are therefore, as a consequence of sin, essentially expiatory sacrifices; but, according to their form, they are substitutionary, in that, through the presentation of the outer life, they seek to make up for the defective devotion of the inner will" (p. 236). Sacrifices, therefore, according to him, have gone through three stages of deterioration. "Originally," he says, "the sinner voluntarily offered his own life as a sacrifice; after that, instead of the guilty, another who was innocent went to the sacrificial death; and finally, instead of a man, an animal was offered as a substitute" (p. 237).

II.—The Mode of Proof.

The first theory is one that requires no explanation, and if it is true, will admit of easy and ready proof. Did the Greeks think that their gods ate and drank? Did they, when trying to avert the anger of their gods, think of them as they would think of angry men whose wrath could be turned aside by large gifts and a ready obedience to their wishes? Or, was there some other notion besides this in their sacrifices?

The first theory has this also to be said in its favour, that, whether true or not of the origin of sacrifice, it is itself unquestionably a fact of the present day. Among barbarous tribes sacrifices are offerings of food and drink, of which the god is believed to partake, and by which he is believed to be gratified. I may be allowed to quote two decided instances, taken from Mr S. Baring Gould's "Origin and Develcpment of Religious Belief:"—"Thus, among the Iroquois, when an enemy was tortured at the stake, the savage executioners leapt around him crying, 'To thee Areskoui, great spirit, we slay this victim, that thou mayest eat his flesh and be moved thereby to give us henceforth luck and victory over our foes!' The dead were also invoked and bidden join in drinking the blood of the sufferer and eating the flesh of the dead."—*Muller.*

"We bring," says a Chinese authority, "fat cattle and sheep to the sacrifice. Prayer and oblation are made at the gate. The sacrifice is completed, and our ancestor appears. He takes the offering. Pious descendants have luck. The kettle is heated in haste. Some roast, some bake flesh, and offer to the guest, then to the host. The wine is poured out. The patron spirit is present. The

pious offerings smell. The meat and drink gratify the spirit. The spirit has satisfied himself wiith the wine."*

With the second theory the case is quite different. There is no heathen tribe of the present day that entertains the belief that the sinner ought to sacrifice himself in order to restore his unity with the Divine Being. And it may be questioned how far there is a real basis in man's consciousness for the idea at all. Moreover, the proof of the theory in history is equally difficult. We should imagine that to establish the theory it would be necessary to prove that men at one time or another sacrificed themselves to propitiate the Divine Being. Then it is difficult to conceive the next stage. Is it maintained that men, out of a consciousness of sin, murdered other men to please the Divine Being? In a few cases there might be voluntary sacrifices, but in the majority of cases men would not willingly be sacrificed for the sake of others. And then it would be necessary to prove that those who offered up the human sacrifices regarded the sacrifices as innocent, while they looked upon themselves as guilty. It is still more difficult to deal with the third stage. If animal sacrifice is a substitute for human sacrifice, will there not be a consciousness that the animal sacrifice is defective, and that human sacrifices would be more appropriate? And if this idea is lost altogether, how can it be proved that animal sacrifices arose out of and were a substitute for human?

These are only some of the difficulties which beset the proof of this theory, and I mention them because the principal defenders of the theory have not chosen to argue the matter out systematically. They appeal to the very existence of animal sacrifice as if it were absurd in itself, and could have its origin only in a desire to procure a substitute for human; and they appeal to the fact of human sacrifices as if this single fact proved the theory. At the same time, they allow that though a truth underlies sacrifice, that truth was imperfectly apprehended, and that all the sacrifices of men, animal and human, were alike defective, inasmuch as moral innocence was wanting.

TREATMENT OF GREEK SACRIFICES.

In discussing the Greek sacrifices, I divide my authorities into three classes. First, I take Homer and Hesiod, then the writers during the classical period of Greek literature, and finally the writers during the decadence of Greek literature, both heathen and Christian. You will find that the time of the witnesses is a very essential element in this investigation. Greek religion passed through many phases, and we must pay regard to this fact if we are to judge the doctrine of sacrifices aright.

* Many instances are given in TYLER'S "Primitive Culture," vol. ii. p. 341, and Sir JOHN LUBBOCK'S "Origin of Civilization" (sec. ed.), p. 269.

I.—First Period—The Homeric Sacrifices.

1. The great majority of the Homeric sacrifices had nothing to do with sin. According to the Homeric conception, the gods bestowed all good and evil on man.* They were the causes of all things; and, accordingly, the Homeric man was bound to acknowledge his gratitude to the gods by offering the first-fruits of everything to them. They therefore offered to the gods a portion of every animal slain and of all the wine drunk. And they conceived that in this way they ate and drank along with the gods. The Homeric poems contain express statements of this belief. "For my altar," says Zeus, "never was without the equally apportioned banquet, the libation, and the steam of fat; for this is the special honour which fell to our lot, τὸ γὰρ λάχομεν γέρας ἡμεῖς."† Of course, all men did not thus honour the gods; but they who did were loved by them, and the offering up of the sacrifices was a special claim to mercy and pity. One instance will suffice. Æneas comes into conflict with Achilles, and is on the point of being killed by him when Poseidon interferes, and among other things says, "Why, I pray, does this man, though innocent, suffer sorrows without deserving them, because of another man's woes, though he always gives delightful gifts to the gods who inhabit broad heaven? Let us therefore rescue him from death, lest perchance the son of Kronos be angry."‡ And so much was this the case, that a prayer is almost never offered up without a vow to sacrifice, or without an appeal to the sacrifices that had been offered. These sacrifices are accordingly offered on every possible occasion, without the slightest reference to any sin committed. The Homeric Greeks sacrifice before going to bed, before setting sail, before entering on a deliberation, while praying for a return, for obtaining a victory, for coming safely out of danger.

The notices of sacrifice in Hesiod are few; but all that occurs is in harmony with the Homeric practice. A fragment of Hesiod, quoted by Origen, says,—"For at that time there were banquets and there were meetings common to immortal gods and mortal men."§ No doubt, Hesiod referred here to a period anterior to his own; but it proves that he believed that the gods had banquets. In the "Works and Days" (v. 336), the injunction is given "to offer up sacrifices to the immortal gods, chastely and purely, according to one's power, and to burn splendid thighs, and to propitiate them with libations and incense on other occasions, both when you go to bed, and when the sacred light comes, that they may have a heart and soul merciful to you."‖

2. There cannot be a doubt that the Homeric poems represent the gods as

* Nägelsbach, "Homerische Theologie," p. 61, section 34; Professor Blackie, "Horæ Hellenicæ," p. 10.

† Il. iv. 48. Il. xxiv. 69, 70.

‡ Il. xx. 299.

§ "Contra Celsum," iv. p. 216, Spencer.

‖ See also fr. 209, clxxviii. in Goettling.

partaking of the offerings presented to them. The gods are continually described as eating and drinking. Hephæstus urges Here to cease quarrelling with Zeus; for if she does not, "there will," he says, "be no pleasure in the noble banquet."* And a little farther on, the poet says of the gods,—"Thus then they feasted the whole day, till the setting of the sun; nor did their souls fail to enjoy the well-apportioned banquet."† Accordingly, Poseidon is described as having gone to the Ethiopians "to partake of the hecatomb of bulls and rams."‡ Zeus and the other gods spend eleven days with the Ethiopians at a banquet.§ On another occasion Iris goes to the winds, who are feasting together, and being asked to sit down, says she cannot, as she wishes to go to the Ethiopians, to share in a feast that the gods are enjoying there.‖ And in perhaps the most elaborate description of a sacrifice occurring in Homer, the poet, after detailing the preparations, says,—"Athene came to partake of the sacrifices;"¶ and a line after this, we are told that the horns of the ox were gilt, "that the goddess might rejoice, seeing the ornament." So, in the Iliad, Apollo, it is hoped, "will partake of the fat of lambs and of goats,"** and be appeased. In another part it is said that Artemis was exceedingly offended because she did not get her share. "The other gods feasted on hecatombs; but to her alone, the daughter of mighty Zeus, he [Oineus] did not offer sacrifice."†† And accordingly Artemis punished him severely for his neglect of her. So Hermes goes to the island of Calypso, enjoys the repast she sets before him, and tells her that the gods generally do not go to places where there are no cities, and therefore no sacrifices and hecatombs.‡‡

It will be noticed that the gods came personally to receive the sacrifices The smoke of the fat ascended to them. "The fat went swirling up, wrapt in smoke, to heaven," are the words of one passage;§§ and in another, commonly regarded as an interpolation, but allowed to represent the general Homeric sentiment, it is said, "They offered up perfect hecatombs to the immortals, and the winds carried the delightful fat from the plain into heaven; but the blessed gods did not divide it among them, nor did they wish to do so; because sacred Ilium was exceedingly hateful to them."‖‖

It is through fire, then, that the sacrifices went up to the gods; and accordingly the word *θύω*, which afterwards signified to slay for sacrifice, always signifies in Homer to offer up through fire. The slaughter was not essential to the sacrifice: it was the burning of the portions set apart for the gods which is the principal feature. And accordingly, in the Homeric poems, no notice is taken of the blood. The weak and pithless spirits of the departed require blood to give them vigour; but the gods are never said to partake of the blood.

* Il. i. 575. † Il. i. 601. ‡ Od. i. 25. § Il. i. 424.
‖ Il. xxiii. 206. ¶ Od. iii. 435. ** Il. i. 65. †† Il. ix. 535.
‡‡ Od. v. 101. §§ Il. i. 317. ‖‖ Il. viii. 547.

3. The sacrifices in Homer which can be imagined to have any connection with the ideas propounded in the second theory, are the propitiatory. These have to be divided into two classes, the first of which has nothing to do with the idea of sin. The gods, I have said, bestow good and evil on man. Their attitude towards man is therefore neutral; they neither wish well nor ill to him, except for special reasons; and man did not know whether he was to expect good or evil from any particular god. It was therefore natural for him to assume that the god might be planning evil against him, and accordingly he felt a desire to propitiate the god. He knew that if he did wrong, or injured the property or interests of the god, the god would dislike him; but he did not know that if he did well the god would like him and take care of him; and therefore he offered presents to the god, that he might be kind and merciful to him. The man had committed no sin; he was not conscious of any wrong; but still he deemed it the wisest course to try to get the god on his side. One instance of this nature will suffice. Athene has visibly appeared to Nestor and Telemachus, and aided them by her counsel. At length, however, she vanishes like an osprey. Astonishment seizes hold of them, and Nestor prays —"Be merciful to me, O queen, and grant me great glory . . . . . and I will sacrifice to thee a cow."*

Take away propitiatory sacrifices of this kind, and there remain only a few sacrifices which can in any way be supposed to be connected with the idea of sin. In the propitiatory sacrifices of this second class, a sin has been committed, some insult has been offered to the god, or there has been neglect in offering him the proper sacrifices: the god shows his anger by sending evils: and the man or men afflicted try to propitiate him.

To understand the nature of these sacrifices, it is essential that we should understand the ideas of Homeric men as to the nature of punishment. Their only notion of punishment was that of compensation in one shape or another. When a man did an injury to another, the other had a right to exact ample compensation for it; and if the injuring man wished to gain over the injured man to be his friend, he would give very handsome compensation, and flatter and soothe him by gifts and banquets. The Homeric men thought of their gods as they thought of men; and so here we have the entire explanation of the second class of propitiatory sacrifices.

In the first book of the Iliad Apollo is offended because his priest Chryses is treated with insolence. "May the Greeks pay for my tears by thy shafts," prays the priest. The god fulfils the prayer of the priest. The Greeks are at length brought to a right state of mind; and accordingly they send a sacred hecatomb to Apollo; they give back to the priest his daughter without

* Od. iii. 371–382.

ransom, and "all day long they appeased the god with song and dance, celebrating the Far-darter; and he rejoiced in his mind, listening to the song." The priest, moreover, prays to the god to cease afflicting the Greeks. Here the god is pacified by large offerings, by a feast, by singing and dancing, and by satisfaction done to the priest. There is no consciousness that sin, as such, requires a sacrifice; there is no idea that the life of the animal is the substitute for the forfeited life of the sinner; but they are conscious that they have done an injury to the god and his priest; they pay ample damages, and they soothe both in the best way they can.

The real state of the case is still more distinctly brought before us in the ninth book of the Iliad. Achilles is enraged with Agamemnon, exactly in the same way as Apollo was enraged with the Greeks. An embassy is sent to try to reconcile him; and among other arguments addressed to him, it is said, "You ought not to keep a pitiless heart; for the gods themselves can be moved, and men turn away their anger by offers of incense, and by soothing vows, by libations, and steam of fat, praying to them whenever one is insolent towards them or neglectful of them."* Or as Hesiod, quoted by Plato, says, "Gifts persuade the gods; gifts persuade awful kings."†

These sacrifices, therefore, have two objects. They first repair the injury done; and, secondly, they try to bring the god into good humour. The idea of sin never appears in them.

The only idea approximating to that of sin is the idea of pollution. But sacrifices did not take away pollution. The Greeks, in the first book of the Iliad, are polluted by the numerous funeral pyres; and before offering up the sacrifice, they get rid of their pollutions by casting them into the sea. The sea is the great source of purity. In the Homeric poems blood is never used to purify, and no intimation is given that the Homeric Greeks believed that blood could purify. Purification, moreover, was essential to the acceptable offering up of a sacrifice. "I have a religious horror of offering up a libation of sparkling wine with unwashed hands,"‡ says Hector,; "nor is it seemly to pray to dark-clouded Kronion stained with blood and gore." Eurycleia, the nurse, recommends Penelope to wash herself with water, and put on clean robes, and go up to her chamber with attendant women, and pray to Athene, the daughter of Ægis-bearing Zeus, for then she might rescue Telemachus from death.§ Achilles had a cup from which he alone drank, and he made libations with it only to Father Zeus. When he sends Patroclus into the field instead of himself, he takes this cup out of his chest, purifies it with sulphur, then cleans it with pure water, washes his own hands, pours out the wine into it,

* Il. ix. 497.
† The verse is quoted anonymously by Plato: it is ascribed to Hesiod by Suidas and Macarius.
‡ Il. vi. 266. § Od. iv. 753.

prays in the midst of the court, and offers up a libation, looking to heaven His prayer was that Patroclus might be brave, and return safe. Zeus granted the first request, and refused the second.* It is worthy of note, also, that these pollutions arise more from external circumstances than from a consciousness of sin. It has often been remarked, for instance, that in the Homeric age a man received no religious contamination from being in company with a murderer. I may quote here the remarks of Mr GLADSTONE on this point, as they happen to state very clearly what I regard as the true nature of the satisfaction given in a propitiatory sacrifice offered on account of an offence:— "Among the Greeks, to have killed a man was considered in the light of a misfortune, or at most a prudential error, an ἄτη πυκυὴ, when the perpetrator of the act had come among strangers as a fugitive for protection and hospitality. On the spot, therefore, where the crime occurred, it could stand only as in the nature of a private and civil wrong, and the fine payable was regarded, not (which it might have been) as a mode, however defective, of marking any guilt in the culprit, but as, on the whole, an equitable satisfaction to the wounded feelings of the relatives and friends, or as an actual compensation for the lost services of the dead man. The religion of the age takes no notice of the act whatever."†

4. There is no instance in Homer of a human sacrifice being offered up to a god. There are words, indeed, which might have explained the usage had it been found. "If," says Zeus to Here, "you were to enter within the gates and long walls, and eat Priam and Priam's sons and the rest of the Trojans raw, then would you cure your wrath."‡ But these words are not to be taken literally. Similar words, evidently of a proverbial nature, occur in Xenophon, and even stronger language is found in Theognis. Nearer to the idea is the expression "to satiate Ares with blood," where the brutal god is no doubt conceived as enjoying the blood; but it may be doubted if it was supposed that he tasted the blood. The words are applied to men struck down in battle.§ There is also an instance of young men being slain in honour of the dead. Into the funeral pyre of Patroclus four noble steeds and two dogs were thrown; no doubt to keep their master company in the realms of Hades, and to gratify him. Achilles, "enraged at his death,"‖ slew twelve beautiful Trojan youths. "And these all along with thee the fire eats up."¶ These twelve youths are described as ἀποινή, or compensation for the dead Patroclus.

5. There are instances of symbolical acts which have some resemblance to sacrifices, but are only in some points like them. When the Greeks are about to make a treaty with the Trojans, Menelaus urges the Trojans to bring a white lamb for the sun and a black lamb for the earth, and they themselves are

* Il. xvi. 225–250. † "Homeric Studies," vol. ii. p. 436. ‡ Il. iv. 34.
§ Il. v. 289. ‖ Il. xxiii. 23. ¶ Il. xxiii. 171–182.

to bring a lamb to Zeus. On the arrival of Priam and his friends the heralds carry in the victims, and mingle the wine, and pour water on the hands of the kings. Then Agamemnon cuts hairs from the heads of the lambs, and the heralds distribute them among the chiefs. After this Agamemnon offers up a prayer to Zeus, the sun, the rivers, and the earth, and the avenging gods below, that punishment may befall the man who violates the treaty. He then cuts the throats of the lambs, and places the animals quivering on the ground; and they poured out the wine from the goblet, and prayed to the immortal gods, "whoever first breaks this treaty may his brains and his children's flow to the ground like this wine."* Priam refuses to remain, and accordingly mounts his chariot, taking the lambs along with him. In another case of a similar nature a boar is the victim. Agamemnon cut off some of the hairs, and prayed to Zeus. He then killed the boar, and the herald Talthybius threw it into the sea.† In these two cases the only portion of the service corresponding to the sacrifice is the offering up of the hairs. The killing of the animal is merely symbolical of the fate of the man who breaks the treaty. No part of the animal is burned or eaten, but it is taken away and cast into the sea.

6. There is no instance of substitution of any kind, or no consciousness of any kind of substitution, except in one case. The companions of Ulysses, after seizing the cows of the sun, offer some of them up to the gods; but not having barley, they substitute the leaves of the oak, and not having wine to pour over the burning sacrifices, they substitute water.‡

To sum up, most of the Homeric sacrifices cannot be connected with the idea of sin in any way. They are given to the gods to gain favour with them, and they are believed to please their senses. The few that are specially intended to appease the anger of the gods are not dictated by a consciousness of sin, but by the calamities which are brought upon mortals for having acted insolently to some god, or having neglected to honour him; and they are of the nature of compensation for the injuries done, and of gifts for the purpose of soothing and pleasing the god. There is no instance in Homer of a human sacrifice offered up to a god, and there is no instance or consciousness of substitution for it. If the phenomena of the Homeric poems are thus in every point antagonistic to the second theory, it seems to me that it can derive no support from the Greek sacrifices. Homer is certainly nearest to the original man, and therefore ought to give us most exactly the original ideas in regard to sacrifice. If changes are introduced in subsequent periods in the mode of offering up sacrifices, or of thinking of them, it is antecedently probable that these changes are the result of the progress of the nation, or of foreign introductions.

* Il. iii. 103, 104, 245, 268, 310. † Il. xix. 268. ‡ Od. xii. 357, 362.

II.—Second Period.

I now come to the second period, ranging from the time of Hesiod to the conquests of Alexander—the classical period.

1. During the second period, as during the first, we find that sacrifices were offered up on almost every important occurrence, public and private. In very many of these cases there could be no consciousness of sin leading to the sacrifice. Sacrifices were offered up to secure victory, to secure success in any undertaking; sacrifices were offered up as thanksgivings for the attainment of success; "in fact," as Nägelsbach says, "on every occasion on which one could seek or recognise the favour of the gods."*

Nor are we left in any doubt as to how the Greeks viewed these sacrifices. They were viewed as gifts to the gods. This comes prominently out in the writings of Plato. In the "Euthyphro," in which the doctrine of holiness is the subject of discussion, Euthyphro defines holy things as those things that take place "if one knows how to say and to do things pleasing (κεχαρισμένα) to the gods in prayer and sacrifice."† Socrates is so far pleased with the definition, but wishes to go farther. Accordingly, after getting from Euthyphro an assent to the statement that holiness is the knowledge how to sacrifice and pray, he puts the question to him, "Is not sacrificing giving gifts to the gods, and is not praying making requests to the gods?"‡ Socrates introduces the discussion, in order to undermine the common idea that the gods could be influenced by gifts and offerings to abet wickedness. The same idea of sacrifice is given in the "Politicus," where it is said, "There are also priests, who, as the law declares, know how to give the gods gifts from men, in the form of sacrifices which are acceptable to them, and to ask for a return of blessings from them."§ In the "Republic" and "Laws" he attacks the popular notions with great vigour and earnestness. In the "Republic," it is plain that Plato includes among the sacrifices that are gifts, propitiatory sacrifices; for he speaks of the mendicant priests claiming "a power procured from the gods, if there has been any act of injustice done by any one himself or his ancestors, of curing it (ἀκεῖσθαι,—that is, undoing the mischief) by sacrifices and charms, accompanied with pleasures and festivals (μεθ' ἡδονῶν τε καὶ ἑορτῶν)," and he says, a little further on, that these quacks persuade men "that there are releases from and purifications of acts of injustice, through sacrifices and pleasures of amusement (διὰ θυσιῶν καὶ παιδιᾶς ἡδονῶν), for living and dead."‖ He quotes the

* "Nachhomerische Theologie," p. 207; see also K. F. Hermann's "Lehrbuch der gottesdienstlichen Alterthümer der Griechen," which contains a very full notification of the passages in Greek writers relating to sacrifice.

† P. 14, B. ‡ P. 14, C. § P. 290, C, D. Prof. Jowett's Translation.

‖ Rep. ii. p. 364, C, E.

passage from the Ninth Book of the Iliad with reprobation. He alludes to the same passage in the "Laws." "To whom of the above-mentioned classes of guardians would any man gravely compare the gods? Will he say that they are like pilots who are themselves turned away from their duty by draughts of wine and the savour of fat?"*

Now it is impossible, if the masses connected the idea of sacrifice with that of sin, that Plato should have thus argued and spoken. The sacrifices were in the popular mind, as in the Homeric days, gifts to the gods to persuade them to confer benefits and to forgive offences. Plato maintains that this is to think of the gods as if they could be bribed—as if they could be seduced into winking at iniquity for the sake of the presents.

2. The principal element through which the food offered on the altar passed to the gods is still fire. There is in Herodotus a curious instance of the transference of this notion to the conveyance of goods to the dead. Melissa, the wife of Periander, appeared after her death to her husband; but refused to give him information, because, as she said, she was cold and naked, for the garments buried with her were of no use, as they had not been burned. Whereupon Periander brought, by a stratagem, all the Corinthian wives together, stripped them of their clothes, threw the clothes into a pit, and, calling on the name of Melissa, burned the whole heap. The ghost of Melissa was thereby pacified, and granted to her husband the information which he had in vain sought from her before.†

There is no doubt that a considerable change passed over the opinion of the cultivated among the Greeks in regard to the pleasure which the gods derived from sacrifice; and we find a tendency to gratify them more by splendid temples and beautiful works of art, than by appeals to the grosser passions. We also occasionally meet with substitutions, no doubt arising from the belief that the gods did not care particularly for the special dainties usually set before them, and that they regarded the disposition of the worshipper rather than the gift. Accordingly, we hear of pigs of dough, or even of clay, being presented instead of real ones; and, when there are no means of offering up the proper sacrifice, rather than give up the sacrifice altogether, they contrive some figure or representation of it, and offer up that.‡

No stress is anywhere laid on the blood as the essential, or indeed as any part of the sacrifice. On the contrary, many of the sacrifices were bloodless, fruits, and cakes and incense; and so far were the Greeks from regarding blood as essential to a sacrifice, that Aristotle believed that the first sacrifices were sacrifices of the first fruits of the earth. The passage is a remarkable one, and shows the Greek mode of thought. "All these associations," he says,

* Legg, x. p. 906 E. Prof. JOWETT's Translation. † Herod, v. c. 92.
‡ Many examples given by LASAULX, p. 259; see also GERHARD, "Abhandlungen," vol. ii. p. 340.

"seem to rank under the political, (for the political does not aim at a mere transient advantage, but one that is to continue throughout life,) where those who associate together offer up sacrifices and hold gatherings in connection with them, pay honours to the gods, and provide rest with pleasure for themselves. For the ancient sacrifices and gatherings seem to have taken place after the ingathering of the fruits, like first fruits."* This, we shall see afterwards, was also the opinion of his pupil Theophrastus; and if we may trust Porphyry, it was the opinion of Empedocles, who lived a hundred years before Aristotle.†

3. We find still the two classes of propitiatory sacrifices I have mentioned above. The Greeks continued to believe that the gods looked down with jealous eyes on the unbroken happiness or prosperity of mortals; and this sentiment is again and again expressed by Herodotus, and illustrated by several beautiful and well-known tales. The Greek of his time, therefore, felt the same need as the man of the Homeric period to pray and offer sacrifices to a god, that he might be merciful to him, even though he had committed no sin against him.

But it is the propitiatory sacrifices offered up when sins were committed to which we are bound to give special attention, as they are the only sacrifices which can be supposed in any way to support the second theory. Now, these sacrifices have reference only to peculiar sins,—namely, those by which a god was insulted, and those which led to the death of a man. Other sins called for no sacrifice; and, indeed, the first class of sins never seem to call for sacrifices by themselves. The god first shows his anger by a plague, and then the people feel that something must be done.

In this matter considerable difficulty has arisen, from a confusion of three different acts which the sin entailed upon the culprit: purification, compensation, and propitiation. K. Otfried Müller has peculiar merit in having called attention to this matter.‡

By certain acts the Greek deemed himself defiled. He was, for the time, cut off from religious sacrifices. These acts, as is most frequently the case, were often not of a sinful nature, but purely ceremonial. Such, for instance, were touching the dead, touching tombs, sexual intercourse; but they also included homicide.

Now, for all these a purification was necessary. This purification took place sometimes through water, either sea, river, or fountain; sometimes through fire, sometimes sulphur, and sometimes through blood. The blood used was that of a pregnant sow. But in this case, though the rite was no doubt accompanied with sacrifice, the animal was not a sacrifice. In all such cases where

* Eth. Nic. viii. 9.

† Porphyry, "De Abst." ii. 21, who quotes the following line from Empedocles:—

"Ταύρων δ' ἀρρήτοισι φόνοις οὐ δεύετο βωμός."

‡ In his Dissertations on the Eumenides of Æschylus, p. 112 of the English translation.

blood was used the animal was taken away and not used for food, and was therefore equally unfit for being offered to the gods. We know very little about the matter, however, as, though the authors of this period occasionally mention this peculiar method of purification, it is not till the time of Apollonius Rhodius (196 B.C.)* that we get a description of the ceremony.†

Besides the purification, the sinner had to pay for the damage he had done. This compensation often becomes blended with the propitiatory offerings and sacrifices which the offender presents. Thus Crœsus, because he had disbelieved Apollo, and thereby offered an insult to the god, sent to Delphi many gold and silver presents (*ἀναθήματα*), and offered up very many sacrifices.‡

It is especially in cases of homicide, however, that all the peculiarities of propitiatory sacrifices make their appearance; and here it is essential to notice that the Greeks believed that, in the case of certain crimes, no sacrifices were of any avail in propitiating the gods. For instance, if the murder of one of kindred blood were committed voluntarily, the only possible issue was the death of the murderer; and, if he was not killed, the curse fell upon his children, until at length a violent death fulfilled the demand of justice. In this case substitution is impossible. Plato's words§ in regard to this matter are—"For the tale or tradition, whether under this or some other name, has been plainly set forth by priests of old; they have pronounced that the justice which inspects and avenges the blood of kindred, follows the law of retaliation, and ordains that he who has done any murderous act should of necessity suffer that which he has done. He who has slain a father shall himself be slain at some time or other by his children; and, if he have slain his mother, he shall of necessity take a woman's nature, and lose his life at the hands of his offspring in after ages; for where a family is polluted with blood there is no other puri-

* Argon. iv. 702–715.

† We give here a literal translation of the passage in the Argonautica:—"But immediately Circe recognised the doom that entailed flight and the crime of murder.* Wherefore, revering the law of suppliant Zeus, who is mightily angry, but mightily aids the slayers of men, she offered up a sacrifice such as that by which guilty suppliants purify themselves when they come to the hearth. First of all, stretching above as a purification of unalterable murder, the offspring of a sow whose breasts still flowed from the productive womb, she moistened her hands with blood, cutting its neck; and, on the other hand, she soothed the god with other libations, calling on purifying Zeus, the helper of murderous supplications (*i.e.*, supplications made on account of murder). And the attendant Naiads, who procured each thing for her, carried away all the pollutions in a mass out of the house, and she within, beside the hearth, burned cakes and soothing foods, offering up vows of dry sacrifices, in order that she might make the dreadful Erinyes cease from their anger, and he himself might become propitious and gentle to both, whether they come with their hands polluted by a stranger's blood or by that of a kinsman."—iv. 698–717.

The scholiast says, on v. 704, "*λυτήριον* is the purifying portion (*τὸ καθάρσιον*), namely, a little pig, which is sacrificed by those who purify, and then they moisten with its blood the hands of the person who is being purified."

‡ Xen. Cyrop. vii. 2, 19.

§ Legg. ix. 872, D.

* She knew that Jason had committed murder.

fication, nor can the pollution be washed out until the homicidal soul which did the deed has given life for life, and has propitiated and laid to sleep the wrath of the whole family. These are the retributions of heaven, and by such punishments men should be deterred."*

A remarkable instance of this is given by Herodotus.† The Persian king sent heralds to Sparta. The Spartans killed them. The impious act was punished by the wrath (μῆνις) of Talthybius, the herald of Agamemnon. Accordingly, two of the noblest Spartans undertook to pay the penalty (ποινὴν τῖσαι) to Xerxes for the heralds of Darius that had perished in Sparta. They went for this purpose to Xerxes, and offered themselves as the penalty. But the Persian king spared their lives, and they returned to Sparta. For a short time the wrath of Talthybius ceased; but soon it awoke again, and was not finally ended until by a strange, and, as Herodotus calls it, most divine (θειότατον) occurrence, the sons of the men who had volunteered were put to death by the Athenians.

In the case of deliberate murder, then, the Greeks denied, in the most decisive manner, the possibility of substitution; and it seems to me that this single fact completely overturns LASAULX's theory, so far as Greek sacrifices are concerned, and hits it in its most vital part.

Plato lays down minute directions as to what should be done in cases of involuntary homicide.‡ We extract a portion of the passage devoted to these: "If he kill a slave thinking that he is his own, he shall bear the master of the dead man harmless from loss, or shall pay a penalty of twice the value of the dead man, and the judges shall assess the value of the slave; but they must use purifications greater and more than in the case of those who committed homicide at the games,—what they are to be, the interpreters whom the god appoints shall be authorised to declare. And if a man kills his own slave, when he has been purified according to law, he shall be quit of the homicide; and if a man kills a freeman unintentionally, he shall undergo the same purification as he did who killed the slave. But let him not forget also a tale of olden time, which is to this effect: He who has suffered a violent end, if he has had the soul of a freeman in life, is when newly dead angry with the author of his death; and being himself full of fear and panic by reason of his violent death, when he sees his murderer walking about in his own accustomed haunts, he is said to become disordered, which disorder of his, aided by the guilty recollection of the other, is communicated by him with overwhelming force to the murderer and his deeds. Wherefore he must get out of the way of the sufferer for the entire period of a year, and must not be found in any of the places that belong to him in the whole country."§

* (πρὶν φόνον φόνῳ ὁμοίῳ ὅμοιον ἡ δράσασα ψυχὴ τίσῃ).—Prof. JOWETT's translation.
† vii. 133. ‡ Legg. ix. 865. § Prof. JOWETT's translation.

We have here a psychological explanation of what took place. First, the murderer had to purify himself from the ceremonial stain which he had contracted; secondly, he had to repair the damage done; and, thirdly, he had to appease the departed spirit. Often the damage done was repaired by appealing to the departed spirit, or to the deities under whose protection he was.

Now, the compensation and propitiation were not accomplished by blood. The passages quoted to prove that murder could be expiated by murder, mean that the spirit of the murdered man and the Chthonian deities under whose care he was, could be appeased only by the violent death of the murderer, or of those of the same stock. No substitution was possible. But if the murder was not intentional, or if the feeling of vengeance had died away through the lapse of time, a compensation different in kind altogether from the wrong committed, might appease the vengeful Erinyes of the man. Some Phocæans were stoned to death. As the men of Agylla passed the place where their bodies lay, disease attacked them. The Agyllæans consulted the oracle, and, according to its answer, they offered annual libations to the dead, and instituted gymnastic and equestrian games.* The Aegidæ, at one time, lost all their children, whereupon they built a temple to the Erinyes of Laius and Œdipus, and the mortality ceased.† Here there is no possibility of substitution. The means of propitiation are entirely different from the actual injury done. So in all the sacrifices offered to the Chthonian deities. They are always genuine sacrifices; things that would gratify the deities: and there is no instance in which the sacrifice symbolises the *life* of the individual, or in which the shed *blood* of the man is expiated by the blood of an animal.

4. In this period we first hear of human sacrifices. These sacrifices are carefully to be divided into two classes,—those which belong to the mythic times, and those which are said to have been offered up in historical times. We shall take the historical sacrifices first. In regard to them, we have only two passages. The one is in the "Minos," attributed to Plato. "For instance," the writer says, "it is not the custom (*νόμος*) with us to sacrifice human beings, but it is unholy to do so. Yet the Carthaginians do so, in the belief that it is holy and lawful for them; and some of them actually sacrifice their own sons to Kronos, as perhaps you yourself have heard. But not only do barbarous men follow different customs from ours, but what (*οἵας θυσίας*) sacrifices do those well-known inhabitants of Lycæa, and the descendants of Athamas who are Greeks offer up!"‡ The second occurs in the "Republic."§ "Clearly, when the ruler begins to do the same thing as the man in the tale which is told of the Arcadian temple of the Lycæan Zeus. What tale? The tale is, that he who

* Herod. i. 167. † Herod. iv. 149. ‡ Minos, c. v. p. 315, C.
§ Rep. viii. c. xvi. p. 565, D.

tasted the human entrails, when the entrails of one man had been minced up with those of other victims, had to become a wolf." These are all the references that occur in writers of this period to human sacrifice among the genuine Greeks.

Some have adduced a third passage from Plato ("De Legibus," vi. xxii. p. 782 C.) "We see the custom of men sacrificing each other still remaining amongst many; and, on the contrary, we hear that we did not venture to taste* even the ox; and amongst others animals were not used as sacrifices to the gods, but only cakes and fruit steeped in honey, and such like pure sacrifices; but they abstained from flesh, in the belief that it was not holy to eat it, or to pollute the altars of the gods with blood." The first clause does not assert that the custom of offering up human sacrifices prevailed in any of the genuine Greek states; and therefore it does not concern us here.

The statement in the "Minos" is exceedingly indefinite. It does not affirm that either the Lycæans or the descendants of Athamas offered up human sacrifices; though, unquestionably, the inference is to be drawn from its statement that they offered strange sacrifices. The authorship of the "Minos" is a matter of dispute. Most critics are inclined to the opinion that it is not the production of Plato, but they differ as to the date of its composition,—some thinking that it was written in the lifetime of Plato, and some that it is much later. A few critics of some note, such as GROTE, maintain that it is genuine. The text of the work is also in an unsatisfactory condition; and the passage before us gives us a city Lycæa, which is not mentioned by any writer but STEPHANUS BYZANTIUS. It is possible that the reference to the Lycæan sacrifice in the "Minos," as well as in the "Republic," notwithstanding the present tense (*θύουσιν*), is to the story of Lycaon, which we shall find afterwards in Pausanias; and it is likely that if the writer of the "Minos" had stated all that he had heard, he would have added that the sacrificer was always changed into a wolf. The reference to the descendants of Athamas is explained by a passage in Herodotus. Herodotus states (vii. 197), that when "Xerxes reached Alus, in Thessaly, his guides told him a tradition of the country, relating to the temple of Zeus Laphystius. They narrated how Athamas, the son of Æolus, along with his wife Ino, plotted the murder of Phrixus, and how the Achæans, in consequence of an oracular response, impose upon his descendants the following task:—They have to keep the eldest of the race out of the prytaneum, and if he enters, he cannot get out before he is going to be sacrificed. They further related how many persons who were already going to be sacrificed escaped to another country; but in the progress of time they came back, and if they are caught entering the prytaneum, they told how they are

* This clause STALLBAUM deems corrupt. He would read, "We hear, in the case of others, that they did not dare to taste the ox, and," &c.

sacrificed, covered completely with garlands, and led out with a procession. This is what the descendants of Cytissorus, the son of Phrixus, suffer, because when the Achæans, according to an oracle, were making Athamas, the son of Æolus, a purification of the country, and were about to sacrifice him, this Cytissorus, coming from Aia in Colchis, rescued him, and by so doing brought the wrath of the god upon his descendants." The passage in Herodotus is difficult of interpretation in some parts; but I think that we can clearly gather from it that there were no real human sacrifices at Alus. There was a peculiar rite, as STEIN has well pointed out. The eldest of the family had to find his way into the prytaneum, to make good his claim on the community. But the other members watched him as he did this; and if they caught him, and could detain him in the prytaneum, they kept him there till the time of some annual festival, when he would be led forth to sacrifice. They gave him, in the meantime, ample opportunity to escape, and, of course, he would take advantage of it, and travel for some time. The guides speak as if sometimes the young man was actually sacrificed; but the tale bears its purpose on the face of it. They give it to inspire Xerxes with reverential awe and dread. Accordingly they pass from the past to the present when they describe the sacrifice. The tale had the desired effect, and Xerxes kept away from the temple and its enclosure. That a good deal of the narrative of the guides was dictated by a special motive, is to be inferred from the difference which exists between their version of the story and the common one handled by the poets. Sophocles had a play on this subject, a few lines of which are parodied by Aristophanes in the "Clouds." The scholiast on Aristophanes* gives us the version which Sophocles followed, and Apollodorus gives us another, slightly different.† These writers at once place us in the region of myth. The account of Apollodorus is as follows:—Athamas, the son of Æolus [and therefore connected with the winds] was ruler of Bœotia, and had two children by Nephele [Cloud], Phryxos [the Roaster] and Helle [Brightness]. Afterwards he married Ino [earth goddess], and Ino, the stepmother, plotting against Phryxos and Helle, persuaded the women to roast (φρύγειν) the corn-seed, so that when the harvest-time came there were no crops. Athamas sent to Delphi to inquire what ought to be done; but Ino persuaded the messengers to say that the dearth would cease if Phryxos were sacrificed to Zeus [Laphystius]. Athamas resolved to obey, and had already led Phryxos to the altar when Nephele carried her son off, and a golden fleeced ram, supplied by Hermes, conveyed Phryxos and Helle through the sky, until Helle fell into a sea, called after her the Hellespont, and Phryxos reached Colchis. The names in this myth may be explained in different ways;‡

* Nub. 257. † Apoll. Bibl. i. c. ix. 1–5.

‡ The father of Athamas, according to LAUER, Æolus, is the variegated sky (des bunten Himmels) "System der Griech. Mythologie," p. 219.

but no one can fail to see that we are here in the region of cloudland and sunshine, and that *human* sacrifices are here impossible. Sophocles made Nephele demand satisfaction from Athamas for the loss of her children; and Athamas himself was accordingly led to the altar, covered with chaplets, but was rescued by Heracles. There could be really no descendants of Athamas. The Athamantidæ traced their race to a mythical founder; and we need not be surprised if they traced their customs also to mythical sacrifices. But we cannot get historical facts out of such narratives.

We have to notice in this connection the term φαρμακοί. The word occurs in two passages of Aristophanes; in the "Equites," 1405—

"And for this I invite you to the prytaneum and to the seat where that φαρμακός used to be."

And in the Ranæ, 732—

"And now we employ for everything those who have come last, whom the state formerly would not easily have used without due consideration, even as φαρμακοί."

These are all the references to the φαρμακοί in writers of this age. The passages give us no clue to the exact meaning of the word. In the first passage it denotes a low scoundrel; in the second it denotes an office of the meanest description. Our more definite explanations of this word come from writers most of whom are far on in our third period. Perhaps we should add to these two passages a third, which was said to be taken from the speech of Lysias against the impiety of Andocides (53). Harpocration, who quotes the passage, doubts if the speech is genuine; and most critics are inclined to share his doubt. The words of the speech are—"Now we must purify the city, and offer up a propitiatory sacrifice, and send away a φαρμακός, and get rid of the mischief." Here again, even supposing that the speech was genuine, all the information we get is that the φαρμακός was sent away when a propitiation took place.

When we come to our third period, various explanations of the φαρμακοί present themselves. Harpocration (180 P.C. ?) and Helladius (fourth century

Phrixos, written with an *i*, is, according to LAUER, Cloud (Wolke); according to PRELLER, the fertilising rain, from φρίσσω, which is used to express the bristling shiver of rain (von starrenden Schauer des Regens (Gr. Myth. ii. 210).

Helle, according to LAUER, is the gleaming cloud (leuchtende Wolke); according to PRELLER, the light-gleam of the heights inhabited by Zeus (Lichtglanz der von Zeus bewohnten Höhen).

Ino, according to LAUER, is an earth-goddess (Io Erdgottheit); according to PRELLER, a sky and sea-goddess (eine Himmels-und Meeresgöttin).

The ram is, according to PRELLER, the symbol of the fertilising cloud (das Symbol der befruchtenden Wolke).

Zeus Laphystios, according to LAUER, is the heaven that sucks up the clouds (der die Wolken aufsaugende Himmel); according to PRELLER, the dark Zeus of storms and winter (der finstre Zeus der Stürme und des Winters).

B.C.), probably the two earliest, say nothing of a human sacrifice. Harpocration says (*sub voc.* φαρμακός), in explanation of the words attributed to Lysias, "At Athens they led out two men to serve as a purification of the city at the Thargelia, one for the men and one for the women." "Ister (236 B.C.), in his first volume of the 'Appearances of Apollo,' has stated that Pharmakos is a proper name, and that he, having stolen the sacred cups of Apollo, and being captured by Achilles, was stoned, and the custom observed at the Thargelia is an imitation of this." Helladius's account is:—"It was a custom in Athens to lead two φαρμακοί, the one for men and the other for women, being led for purification; and one of the men had black figs around his neck and the other white. They were named συβάκχοι. But this purification was an averting of pestilential diseases, taking its origin from Androgeos, the Cretan, who having been put to death contrary to law in Athens, the Athenians suffered under a pestilential disease, and the custom always prevailed of purifying the city with the φαρμακοί."* In both of these writers the φαρμακοί are led in procession. They are not put to death.

If we turn to the scholiast on Aristophanes (Eq. 1136), we find the following note on the word δημοσίους—"Supply oxen, or bulls, or some such victim. The δημόσιοι are the so-called φαρμακοί, who purify the city by their own murder; or the δημόσιοι are those who are fed by the city; for the Athenians fed some exceedingly base and worthless people, and on the occasion of any calamity coming upon the city, such as a pestilence, they sacrificed these in order to be purified of the pollution, and these, therefore, they also called purifications (καθάρματα), and in the Ranæ, 'one would not have readily used them without due consideration as φαρμακοί.'" DINDORF, in his edition of the Scholia, has pointed out that the Scholia belong to very different ages. In this case, we can have little doubt in assigning the first clause to an Alexandrian grammarian, who gives the right interpretation, that βοῦς is to be supplied to δημοσίους, and that the reference is to cattle fed for the public sacrifices. The rest of the note belongs to a date probably posterior to the time of Helladius, and we cannot be far wrong in supposing that the writer has based his story of the φαρμακοί on his misinterpretation of this passage. In the passage in Aristophanes, the Chorus rebukes Demos for allowing himself to be cajoled and flattered. Demos replies that he is not such a fool. He enjoys the fun of the flatteries. He allows the officials to gorge themselves with plunder; but when once they are full he strikes them down. The Chorus replies—"In this way you are acting well, and there really is, as you say, a very great amount of prudence in your conduct, if you nourish these as public property in the Pnyx for this purpose, and then, when you happen to have no dainty, sacrifice whoever of them happens

* Photius Bibl. Cod. 279, p. 534, A. 2, Bekker.

to be fat, and feast upon him."* Here there is no allusion to the φαρμακοί. The officers of the state are compared to victims fed at the public expense, to be sacrificed whenever the Athenians chose. The scholiast, failing to understand the joke, identifies the δημόσιοι with the φαρμακοί, and infers from the latter part of the verse that these φαρμακοί were really slain, and purified the city by their blood. The subsequent writers who deal with the φαρμακοί present us with a blending of the information given by Harpocration and the scholiast of Aristophanes. Photius (850 P.C.) repeats the statements of Helladius in his Lexicon. SUIDAS (eleventh century) has three articles on φαρμακός. In the one he describes the φαρμακός "as a person slain for the purification of the city, whom they call κάθαρμα." In a second he says "they are persons nourished at the public expense, who purified the cities by their own murder." In the third he repeats the statements of Helladius. TZETZES (twelfth century) gives us additional information—"Placing the victim in a suitable spot, and giving into his hand cheese and cakes and figs, they struck him seven times on the penis with squills and wild figs and other wild plants, and finally burned him with fire on wild wood, and scattered his ashes to the waves and to the winds" (Chil. v. 736).† TZETZES does not confine the custom to Athens, but supposes the sacrifice to take place when a calamity befalls a city. He also quotes passages of Hipponax, in which the meaning of φαρμακός is doubtful, but it is generally taken to be sorcerer or magician. The treatment of the body of the φαρμακός TZETZES seems to have borrowed from the conduct of pagans towards the ashes of Christians, as in the case of the martyrs of Lyons and Vienne.

We have given all the authorities on this subject, and, from our survey, we think that the inference cannot but be drawn that we have no trustworthy information, and that, therefore, there is no good evidence for the human sacrifice.

It is natural, in such circumstances, that those critics who attempt to come to definite conclusions should differ widely from each other. Some think that

* Equit. 1131, VELSEN's text.

† We give the original of TZETZES—specimens of the *versus politici* or accentual verses—

ὁ φαρμακὸς τὸ κάθαρμα τοιοῦτον ἦν τὸ πάλαι.
ἂν συμφορὰ κατέλαβε πόλιν θεομηνίᾳ,
εἴτ' οὖν λιμὸς εἴτε λοιμὸς εἴτε καὶ βλάβος ἄλλο,
τῶν πάντων ἀμορφότερον ἦγον ὡς πρὸς θυσίαν,
εἰς καθαρμὸν καὶ φάρμακον πόλεως τῆς νοσούσης.
εἰς τόπον δὲ τὸν πρόσφορον στήσαντες τὴν θυσίαν,
τυρών τε δόντες τῇ χειρὶ καὶ μάζαν καὶ ἰσχάδας
ἑπτάκις γὰρ ῥαπίσαντες ἐκεῖνον εἰς τὸ πέος
σκίλλαις συκαῖς ἀγρίαις τε καὶ ἄλλοις τῶν ἀγρίων
τέλος πυρὶ κατέκαιον ἐν ξύλοις τοῖς ἀγρίοις,
καὶ τὸν σποδὸν εἰς θάλασσαν ἔρραινον εἰς ἀνέμους
καὶ καθαρμὸν τῆς πόλεως, ὡς ἔφην, τῆς νοσούσης.

the φαρμακοί were offered up every year at the Thargelia; others that they were offered up only when calamities prevailed; others that they were not sacrificed at all. K. O. MÜLLER imagines that they were hurled into the sea, but that means were taken to save their lives, and they were sent out of the country. SCHÖMANN adopts the story of TZETZES, and thinks that the men were really offered as sacrifices at an early period, but that a milder custom was afterwards introduced ("Griechische Alterthümer," ii. p. 435). And AUGUST MOMMSEN thinks that the ceremony consisted in this, that the god Apollo demanded a human sacrifice, that blood was drawn from the victim, but that in most cases the god then showed mercy, and the victim was spared; but when there was a calamity prevailing, the victim may have been really sacrificed ("Heortologie," p. 420).

But even if we assume that the sacrifices actually took place, they could only be explained on the principle that it was believed that the god savagely delighted in the slaughter of men. The first victim, a mythical one indeed, was a sacrilegious thief, and deserved his death. The subsequent victims were worthless individuals whom it would be no sacrifice in a state to lose.

The sacrifices of the mythical era have a general likeness, with the exception of one. This one is recorded by Herodotus (ii. 119). He says that Menelaus sacrificed two Egyptian children to obtain a favourable wind for sailing. The rest have the following characteristics. They are offered to prevent a calamity or procure a victory. No explanation is given why a human sacrifice should be chosen except that the human being is more valuable than a brute. The sacrifice is offered up by the direct injunction of the oracle; and the persons offered up belong to those from whom the calamity is averted or to whom the victory is granted. The victims are therefore offered up, not as expressive of the consciousness of sin, or of the belief that sin can be expiated only by death, nor are the victims, strictly speaking, substitutionary, but representative. The most remarkable of these victims is Iphigenia, the first human sacrifice, as NÄGELSBACH remarks, mentioned in Greek writers; and we have full light thrown on this subject by the circumstance that her fate is partly the subject of tragedies of Æschylus, Sophocles, and Euripides, still extant. The Chorus in Æschylus pronounces the state of mind in which Agamemnon offered up Iphigenia impious, impure, and unholy,* bringing a curse upon the race of the Atridæ, and thinks that Agamemnon yielded to the base ambition of a warrior in offering up the sacrifice, when he should have listened to the promptings of a father's heart. Clytemnestra also justifies her conduct in murdering her husband by saying that she was avenging his polluted deed in butchering her daughter.† Sophocles likewise makes Clytemnestra excuse her murder of her husband by his cruel treatment of her daughter, and then Electra explains how

* Agam. v. 220.

† *Ibid.* v. 1420.

the calm came on because Agamemnon had killed a stag belonging to Artemis, and had acted insolently towards the goddess.* And Euripides doubts the whole story, sends her to the Tauric Chersonese, and, moreover, objects to the idea that the Tauric Artemis accepted human sacrifices.† "But I censure these wiles of the goddess; for if a mortal touch the blood of a murdered man, or woman in childbed, or a corpse with his hand, she drives him away from her altars, regarding him as abominable, and yet she herself delights in human sacrifices. Yet it is impossible that Leto, the spouse of Zeus, could have given birth to such an ignorant being. I, for my part, judge that the banquet given by Tantalus to the gods is a thing incredible, that it is incredible they should have pleasure in eating a child. And I think that the men of this place, being themselves slayers of men (ἀνθρωποκτόνους), attribute the same evil custom to the god: for I think that none of the gods is wicked." These words may mean that the men of the Tauric Chersonese delighted in murdering men, and that they represented their goddess as delighting in seeing the murders perpetrated. But it is more probable that the poet intended to say that the men of the Tauric Chersonese were at one time cannibals, and that they believed that their goddess enjoyed the flesh of men.

No other mythic sacrifice is mentioned in the extant works of Æschylus and Sophocles. In Euripides there are several. First there is the sacrifice of Macaria. Eurystheus and the Argive army are coming to Athens to demand the expulsion of the Heraclidæ. Demophon is prepared to resist the demand, but the oracle declares that, if he is to be successful, he must sacrifice a virgin of noble family to the daughter of Demeter.‡ "But I," says Demophon, "will neither kill my own daughter, nor will I compel any other of my citizens to kill his against his will; and with his will who has such an evil soul as to give up his dearest children out of his hand?"§ The passage shows that the Greek mind, at this period, revolted from human sacrifices. In this case an easy solution is found. Macaria, daughter of Heracles, voluntarily offers herself as a sacrifice, and she thus gives a brilliant example of a course of conduct on which the Greeks delighted to lavish their praise,—the duty of the individual to sacrifice himself for the general welfare. It is noteworthy that she calls her death a cause of pollution (μίασμα), and that this sacrifice is a kind of substitutionary sacrifice. "You see me giving the bloom of my marriage to die instead of these."‖ If she had not died her kindred would have perished, and so she is said to have died instead of them. But there is no idea of substitution as producing the efficacy of the sacrifice. The sacrifice is commanded by the oracle. No reason is given for it.

* El. 566. † Iph. in Tauri, 380.
‡ The reading is doubtful. Some read, "A maiden to Demeter."
§ Heraclidæ, 411. ‖ *Ibid.* 580.

There are two other mythic sacrifices in Euripides, but they are not properly of the nature of sacrifices to gods. The one is that of Menoikeus. Teiresias tells Kreon that he can succeed against Polynices only on condition of slaying his own son. Here no god is mentioned as receiving the sacrifice, and indeed it comes out that it is not a sacrifice, but a payment of blood for blood, such as might have taken place among men. Cadmus had slain the dragon, the offspring of Ares, and Ares now demands vengeance from the race of him that had wrought the evil deed or he will not assist them. "The young man must be slain in recesses where the earth-born dragon dwelt, the guardian of the fountain of Dirce, and pour out his blood as a libation to earth, in consequence of the ancient wrath against Cadmus of Ares, who now avenges the murder of the earth-born dragon."* Kreon utterly refuses to perpetrate such a cruel act, but the young man happened to be present when Teiresias informed Kreon of the necessity of his slaughter; and he himself slays himself at the appointed place, another example of that self-sacrifice for one's country of which Euripides delights to sing.

The other instance of sacrifice is that of Polyxena, mention of which is made both in the Hecuba and the Troades. The ghost of Achilles appeared above his tomb and demanded that Polyxena, the daughter of Hecuba, should be given to him as a prize (γέρας),† detaining the Greek host until his demand should be granted. Here we have an unquestionable sacrifice, and the son of Achilles acts as priest.‡ But the object of the sacrifice is to gratify the desire of a dead hero. The shade of the hero is asked to drink the blood of the maiden. "O son of Peleus, and my father, receive these soothing libations at my hands, that evoke the dead; and come that thou mayest drink the dark pure blood of the maiden which I and the army present to thee!"§ We have here no act of worship, but a shade below desires a drink of blood, as all the shades do, according to the Odyssey, and gets it.

The case of Alcestis is an instance of genuine substitution, but it is not a sacrifice. She died instead of her husband. There is no slaughter, no priest, no altar, and she is offered up to no god. She simply passes away from life, and her husband is spared because she dies.

These are all the sacrifices noticed in the extant works of Euripides; but we know from various sources that several others of a similar nature were taken as themes by the tragic writer. There was a special reason for selecting such tales of self-sacrifice. The patriotic Lycurgus, in his oration against Leocrates (330 B.C.), thinks Euripides deserves great praise for selecting the sacrifice of the daughter of Erechtheus as the subject of a play. "Justly," he says, "would one bestow praise on Euripides, because, while in other respects

* Phœnissæ, 938. † Hec. 41. ‡ *Ibid.* 224, 521. § *Ibid.* 532.

a good poet, he preferred to make this story (μῦθον) the subject of his poetry, believing that the deeds of those persons would be a most beautiful example to the citizens, so that looking to them and contemplating them they might become accustomed in their souls to love their country."* The story of the daughter of Erechtheus is the same as that of Macaria. An army invades Attica; Erechtheus sends to Delphi to inquire what he should do to gain the victory over the enemy. "Sacrifice your daughter," is the reply; and the father and mother are proud to yield up their daughter for the common welfare. Lycurgus quotes from the play, which we know was called Erechtheus, the mother's speech in regard to the sacrifice. She insists on the duty of the individual to sacrifice himself for the state. If she had had sons she would, without fear of death, have sent them into the battle array; now she is glad that her daughter also will gain renown by dying for the state. Athens, indeed, seems to have been famous in mythic narrative for these sacrifices. A plague and famine came on the city, and the Athenians, according to an ancient oracle, sacrificed the daughters of Hyacinthus (Apollod. iii. 15, 5); and the Leocorion in Athens was an enclosure sacred to the daughters of Leos, who were sacrificed to save the city (Ælian Hist. Var. l. xii. c. 28). Other cities, no doubt, had similar legends to stimulate their patriotism. A considerable number of such stories are found in Antoninus Liberalis, Pausanias, and other late writers, who will claim notice in our third section, and who are mentioned here because it is likely that they derived several of these tales from Greek epics and tragedies now lost.

In regard to all these mythic sacrifices, the remark has to be made that they belong to the mythology of the Greeks, and that the sacrifices are no more a real indication of what the Greeks thought and did than are the mutilation of Kronos, the marriage of Zeus with his sister, his innumerable amours with women and beasts, and other wild excesses, which admit of an easy solution. All the persons concerned in the sacrifices have a closer connection with the celestials than with mortals; and, in the case of most, it can be clearly proved that they were immortal,† and therefore could not easily be permanently sacrificed.

5. Occasionally we find symbolical acts, similar to those I have noticed, in Homer. In the Ajax, Sophocles represents Teucros as making Eurysaces, the son of Ajax, stand near his father's corpse, and then, in order that Eurysaces may protect the dead body, he puts into the boy's hand Teucros's own locks, the boy's own locks, and his mother's, and says—

* C. 24.

† Writers on mythology all allow that Iphigenia was the goddess Artemis herself (see Preller, i. 195); Macaria is Eastern, and connected with the Tyrian Heracles (Gerhard, Griech. Myth. sect. 646, 2 b); and Polyxena is a goddess of the dead (Gerhard, sect. 884, 4).

"And should one
In all our army tear thee from the dead,
May he, thus base, unburied, basely die,
An exile from his home, with all his race,
As utterly cut off, as I now cut
This braided lock."*

Herodotus gives an account of an Egyptian sacrifice in which a magical transference takes place.† He says—"The following is their manner of sacrifice: They lead the victim, marked with their signet, to the altar where they are about to offer it, and setting the wood alight, pour a libation of wine upon the altar in front of the victim, and at the same time invoke the god. Then they slay the animal, and cutting off his head, proceed to flay the body. Next they take the head, and heaping imprecations on it, if there is a market-place and a body of Greek traders in the city, they carry it there and sell it instantly; if, however, there are no Greeks among them, they throw the head into the river. The imprecation is to this effect—They pray that if any evil is impending over those who sacrifice, or over universal Egypt, it may be made to fall upon that head."‡ Such magical transference is totally unknown to the Greeks.

There is one passage to which I must allude before I leave this part of my subject. It occurs in the Œdipus Coloneus, vv. 498, 499, and is brought prominently forward by LASAULX. It runs thus:—"For I think that one soul paying§ these offerings is sufficient, instead of ten thousand, if it lend a willing presence." This statement has been deemed something extraordinary; but if taken in its connection it states a recognised sentiment among the Greeks. Œdipus has come into the precincts of the Eumenides, and he is bound to offer them sacrifice. He is, however, personally unable, and he asks one of his daughters to go and give the offering instead of him. In asking her he utters the lines quoted. It merely means that, in presenting an offering to a god, one willing worshipper is as good as ten thousand. The sacrifice was really paid by Œdipus; the daughter is the mere representative minister. There is not a trace of substitution here.

III.—THIRD PERIOD, FROM THE TIME OF ALEXANDER THE GREAT TILL THE FOURTEENTH CENTURY, P.C.

In my third period I take both pagan and Christian writers together. The ideas which during this period prevailed in regard to sacrifice were intimately connected with the singular religious movements of the time, and with the peculiar condition of society. I can only indicate here that at this period

* v. 1179, Prof. PLUMPTRE'S Translation. † Herod. ii. c. 39. ‡ RAWLINSON'S Translation.
§ The reading ἐκτίνουσαν is a questionable emendation; but the point has no bearing on our present subject.

foreign worships became fashionable, Eastern and Western ideas blended in a strange manner, the great pagan writers strove after a pure paganism, and the great masses of the people were devoted to the most abject and contemptible superstitions. I must also notice that it was a period when scandalous stories of every kind received ready credence, and especially that it was a common belief that Christians feasted on the blood and bodies of infants, and indulged in promiscuous intercourse.

1. This period furnishes us with incontestable proof that the mass of pagans believed that the gods devoured the fumes from the victims of sacrifice, and delighted in them. There is a treatise on sacrifices ascribed to Lucian, but which in all probability was written at a later date, and by a Christian. This treatise presents us with the common opinions, and is a satire upon them. The writer expresses his doubts whether "we ought to call these people pious, or, on the contrary, enemies to the gods, who have formed such a low and base conception of the Divine Being as to believe that he stands in need of men,* and delights in being flattered by them, and is vexed at being neglected." Is it really possible, he argues, that all the calamities that happened in connection with Meleager and the Calydonian boar could have occurred because Artemis was left out of a feast alone of all the gods? People who believe these stories, he affirms, represent the gods as selling goods to men, health for a little ox, wealth for four oxen, a kingdom for a hecatomb, and the voyage from Aulis to Ilium for a royal maiden. After banter of this kind, the writer proposes an ascent into heaven, and there he finds the gods looking down to earth with outstretched necks to see "if they can observe a fire being lighted, or the odour of the fat borne aloft rolling up in smoke; and if any one sacrifices, they all feast, gaping over the smoke, and drinking the blood that is poured round the altars like flies; but if they take their food at home, they sup on nectar and ambrosia."

The reference to the blood in this passage is peculiar, and is no doubt connected with the changed ideas which now began to prevail among cultivated heathens as to the nature of the gods. A deep spirit of piety arose in the second century, combined with an earnest moral feeling. The common mythological tales proved in this state of mind a great obstacle, and a way out of the difficulty must be found. It was found in the dæmonic theory. This theory, which Plutarch says was held by Pythagoras, Xenocrates, and the old theologians,† is explained in the Symposium of Plato, and expounded in the Isis and Osiris of Plutarch. According to this theory, the so-called gods of the Greeks were not the true gods, but beings of extraordinary power, not necessarily good

* This mode of speaking is eminently characteristic of the Christian writers. "But we must not bribe," says Tatian, "the ineffable God; for he who needs nothing, must not be misrepresented by us as being needy." The same word ἐνδεής occurs in both. (Tatian, Orat. ad Græcos, c. 4.) "Worshipping God who needs not (ἀνενδεῆ) blood and libations and incense" (Justin Martyr, Apol. i. 13). See also Acts xvii. 25.

† Isis and Osiris, 25.

or bad. Some of them were good, some bad. The bad ones delighted in every mischief, and all the base actions attributed to Zeus and the other immortals were really the acts of these demons. These demons delighted in sensuous pleasure; they gaped after blood; they caught greedily at the sacrifices; they were fond of savage and cruel practices; they had real pleasure in the fastings, and distortions, and bloody rites which accompanied some of the religious services.

The Christian writers adopted this dæmonic theory, but maintained that the demons were the children of fallen angels who had desired the fair daughters of men, and agreed with thinkers like Plutarch and Porphyry in attributing the evil actions of the gods to these beings. Innumerable quotations on this point could be given. I content myself with one from Justin Martyr:—"But the angels transgressed this appointment, and were captivated by love of women, and begat children, who are those who are called demons; and besides, they afterwards subdued the human race to themselves, partly by magical writings, and partly by fears and the punishments they occasioned, and partly by teaching them to offer sacrifices, and incense, and libations, of which things they stood in need after they were enslaved by lustful passions; and among men they sowed murders, wars, adulteries, intemperate deeds, and all wickedness." *

2. During this period we hear on every hand of human sacrifices having taken place in earlier times. Antoninus Liberalis (150 B.C.) and the other mythographers give mythical instances in considerable numbers, and Plutarch (110 B.C.) and Pausanias (160 B.C.) give numerous instances, both mythical and historical. In addition to these, the lexicographers and grammarians speak generally of the same custom in giving explanations of peculiar words. At length it came to be a fashion to gather together the most prominent cases, and, accordingly, we have lists of them in Clemens Alexandrinus, Porphyry, and Eusebius.

We shall look into these various sources. Antoninus Liberalis supplies us with instances similar to those which we have already had from the tragedians and Apollodorus. A plague † arose in Bœotia, and the people died in great numbers. Messengers were sent to the Gortynian Apollo, and the oracle answered that the plague would cease if two maidens voluntarily became sacrifices. The two daughters of Orion resolved to save their land, and striking their collar bone with the shuttle with which they were in the habit of weaving, thus accomplished the sacrifice. Persephone and Hades took pity on their lifeless bodies, and turned them into stars. As the book of Antoninus Liberalis deals with transformations, the sacrifices are seldom accomplished. Iphigenia‡ is carried off, and a calf is slain in her stead. A wild beast called Sybaris dwelt in a cave at the foot of Mount Parnassus.§ It was continually carrying off cattle and men. The Delphic oracle was consulted, and gave for answer that the

* 2 Apol. c. 5, Translation of the Ante-Nicene Library.
† Fab. xxv.
‡ Fab xxvii.
§ Fab. viii.

inhabitants must expose at the entrance of the cave a young man who was one of the citizens. A young man was selected by lot, the sacrificial fillet was put round his head, and he was led forth. Happily he is met by another young man, of powerful build, who, falling deeply in love with the victim, tears the fillet from his head and places it on his own. He is led to the cave, rushes into it, drags the animal from its lair, and hurls it down the rocks. The animal disappeared, but from the spot which its body struck gushed forth a fountain.

Such stories as these throw no real light on human sacrifice.

We get a good deal more light from Pausanias. Some of the tales which he relates, though belonging to a mythical age, show how human sacrifices might have been offered to deities with some show of reason. In mentioning the temple of Artemis Triclaria, he tells us that a virgin acted as priestess till she was married. Once upon a time an exceedingly beautiful virgin of the name of Comaetho discharged this duty. An exceedingly handsome young man, Melanippus, fell in love with her, and she soon came to return the passion with equal ardour. But the course of true love did not run smooth. The parents of both the young people objected to the marriage. But the lovers were not to be baffled in this way, and accordingly they met regularly in the temple of Artemis, and "were going to use the temple as a bedchamber." Whereupon the anger of the goddess became manifest. The crops failed, diseases began to rage, and many died. In distress, the people applied to the Delphic oracle, and the Pythia accused Melanippus and Comaetho, ordered them to be offered up as a sacrifice to Artemis, and enjoined the annual sacrifice of the most beautiful young man and young woman. Pausanias bewails the fate of the young men and women who suffered from no fault of their own; but he thinks that the lovers are not to be pitied: "for," says the old traveller, "to man alone success in love is a full equivalent for life." Pausanias then goes on to relate how the human sacrifice came to an end—a part of the story which we need not relate, but which is of value to us, as it states that the sacrifices took place while Troy still stood. Pausanias, in this latter part of the story, makes the Delphic oracle describe human sacrifice as a foreign sacrifice (*θυσία ξένη*). (Paus. *lib.* vii. *cap.* xix. 2.) Pausanias tells another story of a similar nature. He mentions the existence of a temple of Dionysus Aigobolos in Potniæ in Bœotia. The inhabitants sacrificing to this god proceeded under the influence of drink to such a pitch of insolence as to kill the priest of the god. The god took vengeance immediately and sent a plague, and the plague did not cease until, instructed by the oracle of Delphi, a beautiful boy was sacrificed to Dionysus. Soon after, the god changed the sacrifice, and took a goat instead of a boy (Paus. *lib.* ix. *cap.* viii. 1). In both of these mythical cases a real and serious offence was committed against an individual god; and the sacrifice is a direct punishment of the offence. The

feeling of vengeance is roused in the god, and the original culprits, or some fair victim, must satisfy this thirst for blood. Sometimes the god is animated by purely personal liking in this matter. Pausanias tells us that there was at one time an image of Dionysus in Calydon, and that Coresus was one of the priests of the god. Coresus, as was the custom in these early times, fell in love with a beautiful maiden. Her name was Callirrhoe. But Coresus was unfortunate, for the more he showed his love, the more she hated him. Coresus believed that the god whose priest he was would sympathise with him in his distress, and he made strong supplication to him. Dionysus heard his prayers, and inflicted upon the Calydonians a kind of insanity, as if they were all continually intoxicated. They had still sense enough remaining to send to the oracle at Dodona. The god replied that the madness would not cease until either Coresus sacrificed Callirrhoe to Dionysus, or some one who should have the courage to die instead of her. No one offered to die for her. So with much reluctance she was led to the altar, and Coresus stood ready to offer her up. But he plunged the sacrificial knife into himself and not into the maiden. Whereupon the maiden changed her mind, and pitied Coresus exceedingly, and slew herself, letting her blood flow into a fountain "which, from her," says Pausanias, "subsequent generations call the fountain of Callirrhoe" (*lib.* vii. *cap.* xxi. 1). All these tales are purely mythical, and belong to mythical times. Pausanias gives us other instances, which seem to belong to times that are almost historical. He explains the Spartan custom of scourging boys thus:—The Lacedæmonians had an image of Artemis Orthia which they believed was the very image of the goddess brought by Orestes and Iphigenia from Taurica. The Limnatæ of the Spartans and the Cynosurians, when sacrificing to this Artemis, began to quarrel, and the quarrel ended in slaughter, whereupon a plague attacked the people; and then an oracle came to them that they must imbue the altar with the blood of men. Accordingly a human victim was chosen by lot, and the practice continued until Lycurgus changed it into the scourging of the youths, thinking that the oracle was fully carried out by this course (*lib.* iii. *cap.* xvi. 6, 7). Pausanias thinks that the scourging of women, which took place in Alea in Arcadia in honour of Dionysus, had a similar origin (*lib.* viii. *cap.* xxiii. 1). Another instance of human sacrifice is given by Pausanias when relating the wars between the Messenians and Spartans. The oracle of Delphi declared that a pure virgin must be sacrificed to the gods below (*νερτέροισι δαίμοσι*). Difficulties were raised to the accomplishment of this injunction; but at length Aristodemus was ready to offer up his own daughter. But his daughter had a lover, and the lover affirmed that, as they had been betrothed, the daughter was no longer in the power of the father. The argument failed to make an impression on Aristodemus; but the lover was ready for everything, and affirmed that

he had had intercourse with her, and that she was with child. On this, Aristodemus flew into a rage, slew his child, and, cutting her open, demonstrated to the eye that the lover's assertion was false. The diviner affirmed that this was not a sacrifice, but a murder, and that another must offer up his daughter. The Messenians were not inclined to follow the diviner's suggestion, and so they held to the opinion that Aristodemus had really sacrificed his daughter (*lib.* iv. *cap.* ix.) I think it likely that Lycurgus and Aristodemus were really historical personages; but there can be no doubt that we cannot now separate the mythical details of their history from the real, and that in these particular cases there is not the shadow of historical evidence that the sacrifices took place.

Pausanias throws some light on the human sacrifices said to be offered up to Lycæan Zeus. He asserts (*lib.* viii. *cap.* ii. 1.) that Cecrops, and Lycaon the son of Pelasgus, belonged to the same age, but acted very differently to the gods. Cecrops did not think it right to offer up what possessed life, but only cakes made in the country. Lycaon, on the other hand, offered on the altar of Lycæan Zeus the child of a man, and poured out the blood on the altar, whereupon "they say that he immediately became a wolf instead of a man; and I myself am convinced by this statement." Further on he says (*lib.* viii. *cap.* xxxviii. 5), "There is, on the highest peak of Mount Lycæus, a mound of earth, which serves as an altar of Lycæan Zeus. . . . . On this altar they sacrifice to Lycæan Zeus in secret; but it was not pleasant for me to examine minutely into all that concerns the sacrifice, but let it be as it is, and as it was from the beginning." The secret character of this sacrifice, as in the case of the meetings of the Christians, would give rise to the story that human beings were sacrificed; but the story is as likely to have been true as the many other stories which were told of this wonderful and awful place. We give two instances. In Mount Lycæus there was a portion of ground (τέμενος) set apart to Lycæan Zeus, and no man must enter it; and if any one transgressed the law, he was sure to die within a year. Nothing, whether man or beast or lifeless object, within the precincts ever casts any shadow, and this is the case at all times of the year (*lib.* viii. *cap.* xxxviii. 5).

In Plutarch we have the only instance of a definite human sacrifice which is said to have taken place among genuine Greeks in historical times. In his Life of Themistocles (c. 13) he asserts that as the Greek statesman was sacrificing beside the admiral's ship, three prisoners were brought to him, most beautiful in appearance, and magnificently apparelled and decked with gold. They were said to be the sons of the king's sister. On seeing them, the soothsayer, Euphrantides, urged Themistocles to sacrifice them to Dionysus Omestes. The crowd favoured the suggestion, the omens were propitious, and the youths were sacrificed. "These circumstances," says Plutarch, "have been narrated by Phanias the Lesbian, a man who was a philosopher, and not unacquainted with historical studies." Phanias, a pupil of Aristotle and a friend of Theophrastus,

must have lived at least 150 years after the battle of Salamis, and we can gather from his fragments no clear idea of his trustworthiness or of his bias. There is, therefore, really no sufficient evidence to support the statement, and the silence of Herodotus is strongly against it. Additional doubt is thrown on the statement by the circumstance that Plutarch gives a different account of the event in his Life of Aristides (c. 9). He there represents the sacrifice taking place at the end of the battle of Salamis; while in the Life of Themistocles it is said to have taken place before the battle. The three youths are said to have been taken captive at Psyttaleia and sent from that place to Themistocles. Herodotus, on the contrary, expressly affirms that all the Persians found on the island were slain (viii. 95). Supposing the sacrifice to have been offered, we get no light from it as to the underlying ideas. The sacrifice is offered at the instigation of a soothsayer. It is purely accidental; it is a singular occurrence; and it is enemies that are sacrificed.

We have another remarkable passage on human sacrifices in the Life of Pelopidas (c. 21). Just before the battle of Leuctra, Pelopidas dreamed that he saw the daughters of Scedasus, who had been, in long times past, ravished by the Lacedæmonians, and were buried at Leuctra, come to him in tears demanding vengeance, and their father said to him that, if he wished to be successful, he must sacrifice a yellow-haired maiden to his daughters. On this a consultation was held. The soothsayers urged the sacrifice, and appealed to the cases of Menoikeus and Macaria in antiquity, and in more recent times to the slaughter of Pherecydes, the wise man whose skin was kept by the kings, according to an oracular response, the death of Leonidas at Thermopylæ, and the victims slain by Themistocles. They also reminded Pelopidas that Agis had brought on disaster by refusing to obey a divine warning which he had had to offer a human victim. Others affirmed that "such a barbarous and unlawful sacrifice could be pleasing to none of those who were superior to us and above us; that it was not Typhons and Giants who ruled the world, but the father of all, of gods and of men." "Perhaps," said they, "it is foolish to believe that δαίμονες rejoice in the blood and slaughter of men, but if they do they must be despised as powerless; for these absurd and cruel desires arise and abide only in weak and wicked men." In the end Pelopidas sacrificed a mare of the colour required, which opportunely presented itself.

In the arguments adduced against the sacrifice we can have little doubt that we have a large admixture of the sentiments of Plutarch himself, if the whole narrative be not an expansion of some very simple story.

Plutarch gives us his own opinions fully in his treatise "De Defectu Oraculorum" (p. 417, D. c. 14). He affirms that it is not probable that the gods should either ask or receive the human sacrifices that took place in olden times, nor would kings and generals have endured the sacrifice of their own children; but

they did it to avoid the wrath and indignation of cruel and harsh demons, and to satisfy their vengeful spirit. And he goes on to say, "Just as Heracles besieged Œchalia for the sake of a virgin, so strong and violent demons, eagerly seeking after a human soul enclosed in flesh, and being unable to have bodily intercourse with it, send plagues and barrenness of soil on states, and create wars and seditions until they get and obtain the object of their lust."*

Plutarch, as we have seen above, adduced Phanias as his evidence for the sacrifice of Themistocles. Possibly Phanias was the only historian who recorded such an event. A passage in Athenæus should make us careful how we receive such evidence. He tells us that Neanthes of Cyzicus (241 B.C., Clinton Fast. Hell. iii. 509), affirmed that, when Epimenides purified Attica with human blood, on account of some ancient pollutions, a beautiful youth, of the name of Cratinus, offered himself willingly in behalf of his country, and that his lover Aristodemus killed himself,† and the calamity ended (xiii. 78). Neanthes is again and again praised as a trustworthy historian, and his evidence is as good as that of Phanias,—though we have to take into account that Phanias was much nearer the time of Themistocles than Neanthes the time of Epimenides. Athenæus in the next chapter says, "I am not ignorant that Polemon the traveller (199 B.C., Clinton) in his writings against Neanthes, affirms that the story of Cratinus and Aristodemus is a fiction;" and subsequent critics have regarded Polemon as being in the right.

The opinions of Porphyry, the great antagonist of Christianity at the end of the third century, are given in the second book of his treatise "De Abstinentia." His object is to urge philosophers to abstain from the eating of flesh. He is, therefore, so far under a strong motive to show that neither animals nor human beings were sacrificed to the gods. At the same time, he again and again takes care to caution his readers against thinking that the two things go together,—the eating of animal flesh and the sacrificing it to the gods. The latter may be right when the other is wrong, or at least injudicious for the philosopher. Porphyry agrees with Plutarch in his doctrine of the δαίμονες.

Porphyry thinks that man, in his earliest stage, lived entirely on the fruits of the earth. He is a strong believer in the theory of development. Grass

* Plutarch frequently refers to mythic sacrifices. These are all noticed in the text except the following:—"When a famine prevailed at Lacedæmon, the god gave an oracle to the effect that it would cease if they sacrificed a noble virgin every year. On one occasion the lot fell on Helen, and, as she was led forth arrayed for the sacrifice, an eagle flew down and snatched away the sword, and carrying it to the herds laid it upon a heifer, in consequence of which they refrained from the slaughter of virgins. This is related by Aristodemus in his third collection of myths, ἐν τρίτῃ μυθικῇ συναγωγῇ" (Greek and Roman Parallels, xxxv.; Moralia, p. 314, C). Who this Aristodemus was is uncertain.

† The words of Athenæus are peculiar: ᾧ καὶ ἐπαπέθανεν ὁ ἐραστὴς Ἀριστόδημος. "On whom also his lover Aristodemus died," probably implying that he slew himself on the dead body of his favourite. The word ἐπαποθνήσκω occurs in Plato with the meaning, "to die immediately after."

existed on the earth long before trees; trees before animals. The first men, therefore, sacrificed grass, sending it up to the immortals, and indeed rendering it itself immortal through fire, the element most like to the gods. Then after this, when trees began to grow, men offered up the leaves of the oak, until by degrees they came to offer up the first fruits of the crop,—ground corn, oil, and honey. At last came the lawless sacrifices, "for men slew each other,* and stained the altars with blood, after that they had themselves tasted blood, being tempted by famines and wars" (ii. 7). The whole of these opinions seem to be taken from Theophrastus (ii. 5, 7, 20), and, as they harmonise with the short statement in the "Ethics," we may believe that they were the opinions of Aristotle. Porphyry seems to think that human sacrifices were anterior to animal sacrifices: "Men in time of famine had recourse to human flesh, but before partaking of it offered up a portion to the gods." In speaking of the Bassari in Thrace, he says that they sacrificed human beings, and they conjoined with this the eating of human flesh. "Just," says Porphyry, "as we do now in the case of animals; for offering up the first portions we feast on the rest" (ii. 8). Last of all, therefore, came the sacrifice of animals; and in Attica, for instance, it was by accident that, killing some animals, they tasted them. Most, indeed, think that it was during famine that men tasted animal flesh, and, having once tasted it, they presented the first portion to the god, and thus arose animal sacrifices (ii. 10).

Porphyry thinks that it is the δαίμονες who receive the sacrifices of animals. "Falsehood is natural to them, for they wish to be gods, and the power that presides over them wishes to seem to be the greatest god. It is those that rejoice in the libation and the steam of fat, by means of which their own spiritual and bodily constitution is fattened. For this lives by vapours and incense variously through various means, and is strengthened by the steam from blood and flesh" (c. 42). It is to the demons that all sacrifices of blood,

* The text here presents difficulties. It has simply, *σφαξάντων τῶν ἀνθρώπων καὶ τοὺς βωμοὺς αἱμαξάντων*: "Men having slain and stained the altars with blood." I think that the context suggests *ἀλλήλους*: "Men having slain each other." Eusebius, in his "Demonstratio Evangelica" (i. 10), paraphrases the passage from Porphyry, thus: "*πόρρω δὲ παρανομίας ἐλαύνοντας τοὺς μετὰ ταῦτα ἀνθρώπους αἱμάξαι τοὺς βωμοὺς ζώων σφαγαῖς*," p. 34; and certainly Porphyry may have meant simply animals, whether men or beasts, if he wrote *σφαξάντων* without any object. Perhaps I am too definite in saying that Porphyry places human sacrifices before animal. He certainly seems to do so. But he is not very precise; and he may really have not definitely put the question to himself, regarding human and animal sacrifices under the one category of sacrifices of living beings.

The chapter in the "Demonstratio Evangelica" deserves special study, because sentences taken from it without the context may mislead. Lasaulx, p. 255, and Nägelsbach, p. 194, have quoted from this chapter the words, *ἀντὶ τῆς οἰκείας ψυχῆς τὴν διὰ τῶν ἀλόγων ζώων προσῆγον θυσίαν, πῆς σφῶν ψυχῆς ἀντίψυχα προσκομίζοντες*. But they have not stated that Eusebius explains how the pious Greeks of the olden time objected to animal sacrifices entirely. He affirms that it was the Hebrews who offered up animal sacrifices, and they did this enlightened in their souls by the Divine Spirit. It is of them that the words quoted are spoken, with this reason assigned, *μηδὲν κρεῖττον κα τιμιώτερον τῆς οἰκείας ψυχῆς καθιεροῦν ἔχοντες*.

whether human or animal, are offered. The gods require no such sacrifice; they stand in need of nothing from men. They look to the pious aspirations of the worshipper, and the truest and best sacrifice is a true conception of the character and deeds of the gods.

In regard to human sacrifices, he thinks that man was misled into them through some necessity. He adduces a considerable number of instances to prove that though human sacrifices were offered, yet that is no reason for eating human flesh, in order that he may apply the same arguments to animal sacrifice and animal food. Most of the examples are taken from foreign lands. He relates that human sacrifices were offered up at Heliopolis in Egypt, by the Phœnicians, by the Carthaginians, by the Dumatheni in Arabia, by the Thracians and Scythians, and by the Romans. He mentions one place in Asia Minor, Laodicea, where a virgin was sacrificed. And he mentions a considerable number of the islands of the Ægean and Mediterranean where, he affirms, human sacrifices were offered up (56, 57). There is good reason for affirming that all these islands derived their rites from Phœnicia. They are Cyprus, Crete, Rhodes, Chios, and Tenedos; Laodicea was Persian.*

There are only three statements which he makes in regard to the Greeks proper. He states that Phylarchus (219 B.C.) related that all the Greeks in common killed a man before going forth against the enemy. Phylarchus does not say that the man was offered up as a sacrifice; but whether this was the case or not is not worth inquiring, for the statement is utterly incredible. Phylarchus was given to the fabrication of fictions, and we may apply to this one the words which Plutarch applies to another. It is such, "that not even any ordinary person could be ignorant that it has been fabricated."† The second assertion is this—"Apollodorus says that the Lacedæmonians sacrifice a man to Ares." We have no means of testing this statement of Porphyry. We do not know who the Apollodorus here mentioned was. He may have been the one to whom the authorship of the "Bibliotheca" is falsely ascribed‡ (140 B.C.) Certainly it is very strange if this custom existed, that it should have escaped the notice of all historians. Most probably we have here an exaggerated statement, based on the practice of scourging the youths, and on the legend connected therewith. The third instance is that the Athenians slew the daughter of Erechtheus and Praxithea, one of those mythical sacrifices which we have already discussed. In a different part of the book he notices the human sacrifices on Mount Lycæus. "For from the beginning the fruits formed the sacrifices to the gods; but when, in the lapse of time, men became utterly careless of piety, and there was also a scarcity of fruits, and through want of legitimate nourishment they set about eating the

* See GERHARD'S "Griech. Mythologie," p. 353. † Them. c. 32 (see CLINTON, iii. p. 520).
‡ See DIELS in the Rheinisches Museum, 1876, p. 8.

flesh of each other, then entreating the demonion with many prayers, they first gave a portion of themselves to the gods, not only sacrificing to them what was most beautiful among them, but going beyond the most beautiful, and taking also some of the race; from which time till now, not only do all in common sacrifice human victims at the Lycæan festival in Arcadia, and to Kronos in Carthage, but periodically, in remembrance of the custom, they sprinkle kindred blood on the altars, although the holy rite with them drives away from the sacrifices, by a proclamation at the lustral water, any one who has any share in the guilt of human blood" (ii. 27). This whole passage is one of wild exaggeration.

When we pass to the Christian writers, we find them animated by a different purpose. They denounce sacrifices as entirely wrong, and they denounce human sacrifices as utterly barbarous. "Well, now," says Clemens Alexandrinus (Protrept. c. iii. 42, p. 36 P), "let us say in addition, what inhuman demons, and hostile to the human race, your gods were, not only delighting in the insanity of men, but gloating over human slaughter—now in the armed contests for superiority in the stadia, and now in the numberless contests for renown in the wars, providing for themselves the means of pleasure, that they might be able abundantly to satiate themselves with the murder of human beings, and now, like plagues invading cities and nations, they demanded cruel oblations."*

To prove this savage character of the demons, Clemens adduces various instances of human sacrifice. Most of them are foreign. Sacrifices are mentioned as having been offered in the Tauric Chersonese, in Pella in Thessaly, by the Lyctii in Crete, by the Lesbians, by the Phocæans, by Erechtheus of Attica, and by Marius the Roman. The only genuine Greek sacrifice which he mentions besides that of Erechtheus is one by Aristomenes the Messenian. "Aristomenes," he says, "slew three hundred human beings in honour of Ithometan Zeus, among whom was Theopompus, king of the Lacedæmonians." No one supposes that Clemens had the slightest historical evidence for this sacrifice.

Eusebius is animated by the same spirit. He devotes nearly the whole of the fourth book of his "Præparatio Evangelica" to the subject of sacrifices. He says if the sacrifice through irrational animals was declared by philosophers accursed and an evil sacrifice, polluting, and unjust and unholy, and not without harm to the sacrificer, and for all these reasons unworthy of the gods, what must we think of the slaughter of men? Would not this be the most impious and the most unholy of all? (c. 15.) He thinks that even pagans must see that it is only an evil demon to whom savage and inhuman, and lawless and base deeds are pleasing (c. 16). And he adduces instances of human sacrifices to

* Translation in Ante-Nicene Library.

show "how the plague of polytheistic error ruled the life of man before the evangelical teaching of our Saviour." And he will adduce testimonies from those who are not agreed with him in opinion, to show that before this time there was such wickedness that "the superstitious went even beyond the limits of nature, goaded on by destructive spirits, so as to think that the murderous demons were propitiated by the blood of the dearest, and by ten thousand other human sacrifices" (c. 15). After this introduction he quotes from Porphyry and Clemens Alexandrinus. He also quotes from Dionysius of Halicarnassus, from Philo of Byblos, and from Diodorus, principally in regard to human sacrifices among the Carthaginians, Italians, and Pelasgians.

The lexicographers and scholiasts give us no reliable information. Such words as λαφύστιος, ὠμηστής, ὠμάδιος are applied to gods: λαφύστιος is defined as meaning gluttonous, one who eats with eagerness, tearing; ὠμηστής is one who eats raw flesh. From these definitions it is inferred that reference is made to human sacrifices. "Dionysus ὠμηστής," says Arsenius (thirteenth century P.C.) is the god "to whom the ancients were in the habit of offering up living men."* TZETZES finds in Lycophron the epithet βρεφοκτόνος applied to Palæmon, a sea deity, and he at once explains the epithet by stating that children were sacrificed at Tenedos to Melicerta, the same god as Palæmon, but with another name (ad. Lyc. 229). Such notices do not deserve any serious consideration when we are dealing with matters of fact, for such epithets are applied to the gods to denote some quality that arises out of their character, not to denote actions that are done to them. And they are wrong in principle.

I have now given all the instances of human sacrifice which the Greek writers recorded. I may have omitted one or two cases, which were probably treated by the tragic poets, and which have come down to us in Hyginus or other Latin mythographers. I have purposely neglected to take note of several alleged instances. Many of the best writers on mythology have a firm belief that in early times human sacrifices were common, and they accordingly find survivals of them in customs which can be easily explained otherwise. GERHARD is especially addicted to this habit. He sees in Plato (Leges, xii. p. 945) an allusion to the sacrifice of three men to Apollo and Helios, though the utmost that can be made out of it is, that three of the best men were selected from the community, and specially set apart for the priesthood of Apollo.

KARL OTFRIED MÜLLER† finds in the leap from the Leucadian rock, taken at a festival of Apollo, a remnant of human sacrifice to that god; but this mode of sacrifice would be assuredly very singular. It is much more likely to have been a kind of ordeal by which a person accused of some crime might substantiate his innocence, and every means seems to have been taken to give

* See Paræmiographi Græci, LEUTSCH, vol. ii. p. 735.
† Dorians, vol. i. p. 260, Transl.

him a good chance. Then, again, we find in the "Transformations" of Antoninus Liberalis, that the daughters of Minyas of Orchomenos found fault with the other women of the place for celebrating the orgies of Dionysus. Whereupon Dionysus awoke great terror in the three maidens. The maidens cast lots, and one of them vowed that she would offer up a sacrifice to the god, and in fulfilment of this vow she tore her son Hippasus to pieces, with the aid of her sisters. The three sisters after this duly performed the orgiastic rites, and were ultimately turned into birds. Preller (i. p. 429) sees in this an explanation of the Orchomenian festival of the Agrionia, in which a priest of Dionysus (according to Plutarch, Qu. Gr. 38) followed the female descendants of the daughters of Minyas with a drawn sword with the right to kill any of them that he overtook. And, finally, we have the case of the supposed offering up of two Locrian maidens at the shrine of Athene in Ilium, because Ajax the Locrian violated Cassandra. Mr CLINTON* has collected all the passages relating to this subject. The circumstances are referred to by Aelian, Plutarch, Polybius, Æneas Tacticus, Strabo, Iamblichus, a scholiast on Homer, Jerome, and Tzetzes. They all distinctly mention that the Locrian maidens were sent to Ilium. Not one says a word about killing them until we come to TZETZES. TZETZES says that when they were sent the Trojans who went out to meet them killed them if they could lay hold of them. The story of TZETZES is improbable in itself. It is impossible that all the previous narrators should have known nothing of this hideous practice. And we have here but one of the many instances which TZETZES, in his low opinion of paganism, has contributed to the horrors of the early state of mankind. NAGELSBACH indeed has supposed that Timæus (264 B.C.) is TZETZES's authority for all that he states; but a moment's consideration will lead one to see that TZETZES quotes Timæus as authority only for the date.

I have also excluded from notice such occurrences as the death of Codrus and Leonidas; for these men were not offered up as sacrifices in the strict sense of the term. I have also carefully avoided dealing with the sacrifices of the Romans. The religion of the Romans was widely different from that of the Greeks; and an examination into the ideas which they associated with sacrifices might give us results considerably different from those to which we have now been led in regard to the Greeks.

The conclusions to which this investigation leads may be summed up in the following propositions:—

1. That the sacrifices of the Greeks were offered to the gods with the idea that the food and drink would gratify them, and that the other offerings would

* Fast. Hell. vol. i. p. 134, note v.

in some way or other be pleasing to them; that the common people continued to offer up sacrifices with this belief till the end of Paganism; but that as the more cultivated classes came to believe that the gods did not stand in need of food, drink, or of gifts from them, substitutions became more and more general with them.

2. That certain sacrifices were intended to appease the anger or overcome the dislike of the gods, not because any sin had been committed, but because the Greek worshipper was not sure of the disposition of the special god towards him, and believed that the wisest course was to conciliate him.

3. That no expiatory sacrifices were offered up simply to express repentance for sin in general, but they were always occasioned by some offence against some individual god or gods; that in these cases care must be taken to distinguish between the purification and the sacrifice; that in the case of deliberate murder no expiatory sacrifice was permissible, but the murderer or his descendants must suffer death; and in the case of involuntary murder, the sacrifice was of the nature of a payment of damages.

4. That there is no instance of a human sacrifice in Homeric times. That in the classical times the one or two allusions really refer to mythical times, and that there is only one instance of human sacrifice for which there is the shadow of historical evidence; that the evidence for this human sacrifice breaks down completely on close examination, and thus we have the fact that there is no clear proof that one human sacrifice was ever offered up in Greece during the historical period. We have, on the contrary, abhorrence of such sacrifices frequently expressed. Herodotus denounces human sacrifices as an unholy deed (*πρῆγμα οὐκ ὅσιον*). Æschylus and Euripides* employ language of utmost detestation against it. The Delphic oracle calls it a foreign practice. Pausanias and Porphyry deem it barbarous. And Sextus Empiricus, contrasting the different feelings of mankind in regard to the same acts, says of the Greeks,—"But *we* think that the temples are polluted by human blood."† The same Greek detestation of human sacrifices is embodied in the tradition that Heracles gained renown by doing away with human sacrifice in various parts of the world.‡

5. That there is no satisfactory proof that the Greeks at any time or in any place were in the habit of offering up human sacrifices. Certain rites may find an explanation in the supposition that human sacrifices were at an early period offered up; but there is no historical testimony to show that the practice ever existed. And even in the cases where the practice may by some be regarded as the best explanation of the rite, we have not a genuine Greek race. The

* Welcker thinks that human sacrifices were attacked by Sophocles in his Athamas, by Achæus n his Azanes, and possibly by Xenocles in his Lycaon.—*Die Griechischen Tragödien*, vol. iii. p. 965.

† Hyp. iii. 24, p. 209. ‡ Welcker, "Griechische Götterlehre," ii. p. 769.

ceremonial on Mount Lycæus was Pelasgic. And the Agrionia and the sacrifices of the Athamantidæ are connected with the Minyan Orchomenos, the seat of Pelasgic worship. So that we should have in these three cases the traditions of the worship of the race which preceded the Hellenes, if we were to base any conclusion on the unsatisfactory information which we have in regard to them. And there are really no other decided cases of what can be regarded as survivals.

6. That the writers of the third period, influenced by the belief that the ordinary gods of the Greeks were demons of savage propensities, lent a ready ear to any tale of horror connected with their worship, and that it is in these writers that we hear of the human sacrifices of the Greeks; but if we place the evidence for these sacrifices fairly in the balance, we shall find it not so strong as that which could be adduced to prove that the early Christians killed infants, drank their blood, and indulged in indiscriminate sexual intercourse. And yet no one now believes these accusations against the Christians.

In fact, the Greeks were strangers to the idea of sin until the introduction of Stoicism, as Sir ALEXANDER GRANT has well shown in his Aristotle, and it is likely that the idea was not present to the minds of the earlier Stoics. There is therefore, as it seems to me, no analogy between the sacrifices of the Greeks and the sacrifice of Golgotha. The sacrifice of Christ is, as Dr CRAWFORD has admirably brought out in his "Mysteries of Christianity," p. 230, "exceptional and unique." But in the deeper meaning of sacrifice, the essence of which is self-renunciation, there is a striking parallelism between most of the Greek mythical sacrifices, including also the more or less historical voluntary deaths of Codrus and Leonidas, and the sacrifice of Christ. The oracle decrees that what is noblest, and most beautiful, and most fair must perish. The noblest and the fairest offer themselves up for their country, and present to their country the most beautiful sacrifice that can be offered—a pure human soul. And in like manner the sacrifice of Christ, not indeed devoted, like the Greek sacrifices, to a single land, but offered up for the whole world, is an act of obedience to the will of God, and an infinitely grand exemplification of that self-renunciation which constitutes the essence of all true religion.

XXII.—*New General Formulæ for the Transformation of Infinite Series into Continued Fractions.* By Thomas Muir, M.A., F.R.S.E.

(Read 7th February 1876.)

In Crelle's Journal, vol. x., Stern devotes fifteen pages (pp. 245–259) of his *Theorie der Kettenbrüche* to the examination and elucidation of the following problems:—

Express (I.) $1 + A_1x + A_2x^2 + A_3x^3 + \dots$

(II.) $\dfrac{1 + A_1x + A_2x^2 + A_3x^3 + \dots}{1 + B_1x + B_2x^2 + B_3x^3 + \dots}$

(III.) $\dfrac{1}{1 + B_1x + B_2x^2 + B_3x^3 + \dots}$

as continued fractions of the form

$$1 + \frac{x}{a_1} + \frac{x}{a_2} + \frac{x}{a_3} + \dots$$

where $a_1, a_2, a_3, \dots$ are independent of x.

Express (IV.) $1 + A_1x + A_2x^2 + A_3x^3 + \dots$ as a continued fraction of the form $\dfrac{1}{1} + \dfrac{x}{a_1} + \dfrac{x}{a_2} + \dfrac{x}{a_3} + \dots$.

A mode of solution is shown, and made use of in the case of particular series, but the general problems are left unsolved, Stern having tried in vain, as he himself says,* to discover the law of formation of the partial denominators $a_1, a_2, a_3, \dots$ of the continued fraction from the coefficients $A_1, A_2, A_3, \dots$ of the series.

The main result of the present paper is the discovery of this law. Stern's mode of procedure is followed, but use is made of a more simple and comprehensive notation, to which perhaps the success of the investigation is due.

Problems (III.) and (IV.) being virtually the same, and being like (I.) particular cases of (II.) we start with the second problem, where there is given

$$\frac{1 + A_1x + A_2x^2 + A_3x^3 + \dots}{1 + B_1x + B_2x^2 + B_3x^3 + \dots} = 1 + \frac{x}{a_1} + \frac{x}{a_2} + \frac{x}{a_3} + \dots\dots$$

* "Hier scheint es vielmehr nothwendig zu sein, einen einfachen Ausdruck zu finden, welcher jeden Theilnenner a_n (oder jedes N_n, N_{n+1}) unmittelbar aus den Reihencoefficienten $A_1, A_2, \dots$ finden lehrt. Der Verfasser hat sich vergebens bemüht die Lösung dieser Aufgabe zu finden, vielleicht aber können die mitgetheilten Ausdrücke für N_n, N_{n+1}, welche wohl nirgendwo angegeben sind, darauf führen."—*Crelle's Journal*, vol. x. p. 257.

and we are required to determine $a_1, a_2, a_3, \ldots$ in terms of $A_1, A_2, A_3, \ldots$, $B_1, B_2, B_3, \ldots$

Writing C_1 for $A_1 - B_1$, C_2 for $A_2 - B_2$, &c., the left-hand member becomes

$$1 + \frac{C_1x + C_2x^2 + C_3x^3 + \ldots}{1 + B_1x + B_2x^2 + B_3x^3 + \ldots}$$

whence there results

$$\frac{C_1 + C_2x + C_3x^2 + \ldots}{1 + B_1x + B_2x^2 + B_3x^3 + \ldots} = \frac{1}{a_1 + \dfrac{x}{a_2 + \dfrac{x}{a_3 + \ldots}}}$$

or

$$(C_1 + C_2x + C_3x^2 + \ldots)\left(a_1 + \frac{x}{a_2 + \frac{x}{a_3 + \ldots}}\right) = 1 + B_1x + B_2x^2 + B_3x^3 + \ldots$$

from which, by equating the terms which are free of x, we have

$$C_1a_1 = 1 \quad \text{and} \quad \therefore \quad a_1 = \frac{1}{C_1}.$$

Again, striking these terms out of the equation, and dividing both numbers by x, there results after transformation

$$(a_1C_2 - B_1) + (a_1C_3 - B_2)x + (a_1C_4 - B_3)x^2 + \ldots + (C_1 + C_2x + C_3x^2 + \ldots)\frac{1}{a_2 + \dfrac{x}{a_3 + \ldots}} = 0,$$

and now multiplying both sides by $a_2 + \dfrac{x}{a^3 + \ldots}$ and equating the terms which are free of x, we have

$$(a_1C_2 - B_1)a_2 + C_1 = 0, \text{ and } \therefore\ a_2 = -\frac{C_1^2}{\begin{vmatrix} 1 & B_1 \\ C_1 & C_2 \end{vmatrix}}$$

Proceeding in this way with the equation (which increases rapidly in complexity), we derive equations for the determination of $a_3, a_4, \ldots$. Writing $K(a_1, a_2)$, $K(a_1, a_2, a_3)$, ... for the continuants $a_1a_2 + 1$, $a_1a_2a_3 + a_1 + a_3, \ldots$, that is,* for the successive denominators of the convergents of the continued fraction $\dfrac{1}{a_1} + \dfrac{1}{a_2} + \dfrac{1}{a_3} + \ldots$, then $K(a_1, a_2, a_3 \rbrace C_3, C_2)$ will conveniently denote $K(a_1, a_2, a_3)$ with the annexation of C_3 as a factor to the term of highest degree, and C_2 as a factor to the term of the next degree; and the equations for the determination of $a_3, a_4, a_5 \ldots$ become

$$K(a_1, a_2, a_3 \rbrace C_3, C_2) - K(a_2, a_3 \rbrace B_2, B_1) = 0$$
$$K(a_1, a_2, a_3, a_4 \rbrace C_4, C_3, C_2) - K(a_2, a_3, a_4 \rbrace B_3, B_2) = 0$$
$$K(a_1, a_2, a_3, a_4, a_5 \rbrace C_5, C_4, C_3) - K(a_2, a_3, a_4, a_5 \rbrace B_4, B_3, B_2) = 0$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

* See paper on "Continuants" in the Proc. R. S. E. for 1873–74, and pamphlet on "The Expression of a Quadratic Surd as a Continued Fraction," Glasgow, 1874.

whence we find

$$a_3 = \frac{\begin{vmatrix} 1 & B_1 \\ C_1 & C_2 \end{vmatrix}^2}{C_1 \begin{vmatrix} 1 & B_1 & B_2 \\ 0 & C_1 & C_2 \\ C_1 & C_2 & C_3 \end{vmatrix} - \begin{vmatrix} 1 & B_1 & B_2 \\ 0 & C_1 & C_2 \\ C_1 & C_2 & C_3 \end{vmatrix}^2}$$

$$a_4 = \frac{\begin{vmatrix} 1 & B_1 \\ C_1 & C \end{vmatrix} \begin{vmatrix} 1 & B_1 & B_2 & B_3 \\ 0 & 1 & B_1 & B_2 \\ 0 & C_1 & C_2 & C_3 \\ C_1 & C_2 & C_3 & C_4 \end{vmatrix} - \begin{vmatrix} 1 & B_1 & B_2 & B_3 \\ 0 & 1 & B_1 & B_2 \\ 0 & C_1 & C_2 & C_3 \\ C_1 & C_2 & C_3 & C_4 \end{vmatrix}^2}{\begin{vmatrix} 1 & B_1 & B_2 \\ 0 & C_1 & C_2 \\ C_1 & C_2 & C_3 \end{vmatrix} \begin{vmatrix} 1 & B_1 & B_2 & B_3 & B_4 \\ 0 & 1 & B_1 & B_2 & B_3 \\ 0 & 0 & C_1 & C_2 & C_3 \\ 0 & C_1 & C_2 & C_3 & C_4 \\ C_1 & C_2 & C_3 & C_4 & C_5 \end{vmatrix} - \begin{vmatrix} 1 & B_1 & B_2 & B_3 \\ 0 & 1 & B_1 & B_2 \\ 0 & C_1 & C_2 & C_3 \\ C_1 & C_2 & C_3 & C_4 \end{vmatrix}^2}, \text{ \&c.}$$

Hence,

$$\frac{1 + A_1x + A_2x^2 + A_3x^3 + \dots}{1 + B_1x + B_2x^2 + B_3x^3 + \dots} = 1 + \cfrac{x}{\frac{1}{\gamma_1} + \cfrac{x}{-\frac{\gamma_1^2}{\gamma_2} + \cfrac{x}{\frac{\gamma_2^2}{\gamma_1\gamma_3} + \cfrac{x}{-\frac{\gamma_3^2}{\gamma_2\gamma_4} + \cfrac{x}{\frac{\gamma_4^2}{\gamma_3\gamma_5} + \dots}}}}}$$

$$= 1 + \frac{\gamma_1 x}{1} - \frac{\gamma_2 x}{\gamma_1} - \frac{\gamma_3 x}{\gamma_2} - \frac{\gamma_1\gamma_4 x}{\gamma_3} - \frac{\gamma_2\gamma_5 x}{\gamma_4} - \dots \quad \dots\dots \text{(A)}$$

where γ_1 is written for C_1, γ_2 for $\begin{vmatrix} 1 & B_1 \\ C_1 & C_2 \end{vmatrix}$, γ_3 for the corresponding determinant of the third order, &c., and C_1 stands for $A_1 - B_1$, C_2 for $A_2 - B_2$, &c.

Putting $B_1=B_2=B_3=\ldots=0$, it is easily seen that $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \ldots$ me

$$A_1,\ A_2,\ \begin{vmatrix} A_1 & A_2 \\ A_2 & A_3 \end{vmatrix},\ \begin{vmatrix} A_2 & A_3 \\ A_3 & A_4 \end{vmatrix},\ \begin{vmatrix} A_1 & A_2 & A_3 \\ A_2 & A_3 & A_4 \\ A_3 & A_4 & A_5 \end{vmatrix},\ \ldots.$$

and writing $a_1, a_2, a_3, a_4, a_5, \ldots$ instead of these we have from (A)

$$1 + A_1x + A_2x^2 + A_3x^3 + \ldots = 1 + \cfrac{a_1x}{1 - \cfrac{a_2x}{a_1 - \cfrac{a_3x}{a_2 - \cfrac{a_1a_4x}{a_3 - \cfrac{a_2a_5x}{a_4 - \ldots}}}}} \qquad \ldots\ldots\ldots \text{(B)}$$

Again, putting $A_1=A_2=A_3=\ldots=0$ we find that $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \ldots$ become

$$-\beta_1,\ -\begin{vmatrix} 1 & B_1 \\ B_1 & B_2 \end{vmatrix},\ \begin{vmatrix} B_1 & B_2 \\ B_2 & B_3 \end{vmatrix},\ \begin{vmatrix} 1 & B_1 & B_2 \\ B_1 & B_2 & B_3 \\ B_2 & B_3 & B_4 \end{vmatrix},\ -\begin{vmatrix} B_1 & B_2 & B_3 \\ B_2 & B_3 & B_4 \\ B_3 & B_4 & B_5 \end{vmatrix},\ \ldots.$$

and writing $-\beta_1, -\beta_2, \beta_3, \beta_4, -\beta_5, \ldots$ for these, there results from (A)

$$\frac{1}{1 + B_1x + B_2x^2 + B_3x^3 + \ldots} = 1 + \cfrac{-\beta_1x}{1 - \cfrac{-\beta_2x}{-\beta_1 - \cfrac{-\beta_3x}{-\beta_2 - \cfrac{-\beta_1\beta_4}{\beta_3 - \cfrac{\beta_2\beta_5}{\beta_4 - \ldots}}}}}$$

and $\therefore$ we have

$$1 + B_1x + B_2x^2 + B_3x^3 + \ldots = \cfrac{1}{1 - \cfrac{\beta_1x}{1 - \cfrac{\beta_2x}{\beta_1 - \cfrac{\beta_3x}{\beta_2 - \cfrac{\beta_1\beta_4x}{\beta_3 - \cfrac{\beta_2\beta_5x}{\beta_4 - \ldots}}}}}} \qquad \ldots\ldots \text{(C)}$$

Lastly, putting x^{-} for x, then multiplying both sides by x, and simplifying, we find

$$\frac{1 + A_1x^{-2} + A_2x^{-4} + A_3x^{-6} + \ldots}{x^{-1} + B_1x^{-3} + B_2x^{-5} + B_3x^{-7} + \ldots} = x + \cfrac{\gamma_1}{x - \cfrac{\gamma_2}{\gamma_1x - \cfrac{\gamma_3}{\gamma_2x - \cfrac{\gamma_1\gamma_4}{\gamma_3x - \cfrac{\gamma_2\gamma_5}{\gamma_4x - \ldots}}}}} \qquad \ldots\ldots \text{(D)}$$

$\gamma_1, \gamma_2, \ldots$ bearing the same signification as before.

GAUSS in the treatment of his general series,* $F(\alpha_1 \beta_1 \gamma_1 x)$, viz.

$$1 + \frac{\alpha\beta}{\gamma}x + \frac{\alpha(\alpha+1)}{1.2}\frac{\beta(\beta+1)}{\gamma(\gamma+1)}x^2 + \frac{\alpha(\alpha+1)(\alpha+2)}{1.2.3}\frac{\beta(\beta+1)(\beta+2)}{\gamma(\gamma+1)(\gamma+2)}x^3 + \ldots.$$

† *Disquisitio circa seriem infinitam* in the Abhandl. der Götting. Gesellsch. d. Wissensch. Bd. ii. 1812.

developes readily a special property, from which he succeeds in showing that

$$\frac{F(a,\beta+1,\gamma+1,x)}{F(a,\beta,\gamma,x)} = \cfrac{1}{1-\cfrac{\frac{a(\gamma-\beta)x}{\gamma(\gamma+1)}}{1-\cfrac{\frac{(\beta+1)(\gamma+1-a)x}{(\gamma+1)(\gamma+2)}}{1-\cfrac{\frac{(a+1)(\gamma+1-\beta)x}{(\gamma+2)(\gamma+3)}}{1-\cfrac{\frac{(\beta+2)(\gamma+2-a)x}{(\gamma+3)(\gamma+4)}}{1-\ddots}}}}}$$

This is but a particular case of (A) when the reciprocal of both sides is taken, and so also are one or two other previously discovered theorems of less generality and importance.

The only perfectly general method hitherto known for the transformation of series into continued fractions is that discovered by EULER,* and it is important to notice the radical distinction between the continued fractions given by the two methods for any particular series. Taking, for example, the series for $\tan^{-1}x$, viz., $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \ldots$, the two continued fractions are found to be

$$\cfrac{x}{1+\cfrac{x^2}{3-x^2}+\cfrac{3^2x^2}{5-3x^2}+\cfrac{5^2x^2}{7-5x^2}+\ddots} \qquad \text{and} \qquad \cfrac{x}{1+\cfrac{x^2}{3}+\cfrac{2^2x^2}{5}+\cfrac{3^2x^2}{7}+\ddots}$$

the former being that given by EULER's method. Now, if in the case of the left-hand fraction we stop at any particular partial denominator, the portion taken represents *exactly a corresponding number of terms of the series* for $\tan^{-1}x$: whereas, if we stop at a particular partial denominator of the right-hand fraction, we find that the portion taken represents *an infinite series which for a certain number of terms agrees with the given series* for $\tan^{-1}x$.

The results of the present paper are thus seen to refer only to *infinite* series, the continued fractions coinciding, it may be, in the limit with these series; and it is a problem of some interest to determine the correction required on stopping at any particular partial denominator, or, as is said in the case of series, to determine "the remainder after n terms." STERN's method is unsuited for this; but now that the theorems have been found, other methods of proof will in all likelihood appear, to which there may not be the same objection.†

* *Opuscula Analytica*, t. ii. p. 138.

† One such method has since been discovered by the author, and communicated to the Mathematical Society of London, in a paper read February 10th, 1876.

XXIII.—*On the Stresses due to Compound Strains.* By Professor C. Niven. Communicated by Professor Tait.

(Received 22d January 1874.—Read 16th February 1874.)

§ 1. The general problem in elasticity, as usually presented for solution, supposes the elastic substance to pass initially from a state without strain. But important cases exist where it must be conceived to start from a state already under considerable stress. When this is the case, the constitution of the solid undergoes great change, as is shown by the fact that strained glass loses its isotropic property, and becomes doubly refractive. This subject was long ago attacked by Cauchy, who, by means of the theory of molecular actions, deduced the existence, in the expressions for the stresses due to the secondary strains, of terms proportional to the initial or primary stresses. The problem has been since discussed by MM. De St Venant and Boussinesq, who have applied to it Green's expression for the energy stored up during the strain. But the question, in their hands, still retains traces of Cauchy's hypothetical element, inasmuch as their expression for the potential energy was deduced by means of the molecular theory. M. De St Venant even considers it a strong argument for the truth of the latter, that it is indispensable in the discussion of this problem. These authors have also failed to see in what way the remaining part of the potential depends on the original strain.

In the present paper the treatment of the matter is grounded solely on the laws according to which one set of distortions may be superposed on another. The resultant strains, it is shown, differ from the primary by linear functions of the secondary ones, whether the latter be small or large. The general expression for the potential energy is thus found independently of any hypothesis, and so far coincides with the result of M. Boussinesq. In the further development of the subject, I have confined myself to the case where the potential due to the primary strain is a quadratic function of the corresponding strains, and also where the substance was originally isotropic.* In this case, it appears that the increase of the potential energy, so far as it involves the primary strain, depends only on six quantities called *quasi-strains*. The primary stresses, each multiplied by the dilated unit-volume, also depend only on these six functions.

* See, however, the note added 16th February 1876.

From the formulæ which determine the small motions of such a substance, when strained homogeneously at first, it follows that the vibrations in a plane wave are not generally in its front, and that for each position of the latter there are three real and different velocities of transmission. If the primary stress be symmetrical with respect to an axis, the wave surface breaks up into an ellipsoid of revolution and a surface of the fourth class.*

In these investigations it was necessary to determine expressions for the stresses due to distortions of any magnitude. My results for these, though not their symbolical form, have, as I find, been already published in the "Comptes Rendus," t. lxxi. p. 400, by M. Boussinesq. In the present paper two demonstrations are offered, one derived from the rules for compounding two strains.

The present paper contains also a general theory of the laws according to which strains and certain other physical magnitudes are transformed with respect to different sets of rectangular axes.

§ 2. Let the three intersecting edges of a rectangular element-parallelopiped PH, PK, PL, be called h, k, l, and let them be strained into P'H', P'K', P'L', the displacements of P being u, v, w. The co-ordinates of H', K', L', relative to P' are h, k, l, multiplied respectively by the members of the successive columns

of the determinant $\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \nabla,$

where $u_1 = \frac{d}{dx}(x+u) = 1 + \frac{du}{dx}$, $u_2 = \frac{d}{dy}(x+u) = \frac{du}{dy}$, . . . $w_3 = 1 + \frac{dw}{dz}$.

The minors of its constituents $u_1 \ldots w_3$ we denote by $U_1 \ldots W_3$; and the geometrical elements of the strain are expressed by these laws:—

(1) Any infinitesimal volume, $d\mathrm{V}$, strains into $d\mathrm{V}'$, where $d\mathrm{V}' = \nabla . d\mathrm{V}$.

(2) If any infinitesimal surface-element $d\Sigma$, whose projections on the co-ordinate planes are A_x, A_y, A_z, strains into $d\Sigma'$ having corresponding projections A'_x, A'_y, A'_z, then

$$\left.\begin{aligned} \mathrm{A}'_x &= \mathrm{A}_x \mathrm{U}_1 + \mathrm{A}_y \mathrm{U}_2 + \mathrm{A}_z \mathrm{U}_3 \\ \mathrm{A}'_y &= \mathrm{A}_x \mathrm{V}_1 + \mathrm{A}_y \mathrm{V}_2 + \mathrm{A}_z \mathrm{V}_3 \\ \mathrm{A}'_z &= \mathrm{A}_x \mathrm{W}_1 + \mathrm{A}_y \mathrm{W}_2 + \mathrm{A}_z \mathrm{W}_3 \end{aligned}\right\} \quad . \quad . \quad . \quad (2).$$

(3). The six *strains* are given by half the sums of the squares of the elements in the 3 columns of ∇ less 1, and by the sums of the products of the elements

* If the solid be "incompressible" for small strains, and if the primary strains be small, the equation giving the velocity of transmission of a plane wave is similar in form to that given by Fresnel in his Theory of Double Refraction.

in each pair of columns. They are denoted by s_{xx}, s_{yy}, s_{zz}, s_{yz}, s_{zx}, s_{xy}. Somewhat further on, in dealing with long formulæ, I shall write them simply $\frac{1}{2}$A, $\frac{1}{2}$B, $\frac{1}{2}$C, D, E, F. They are readily expressible in terms of the edges of the tetrahedron P'H'K'L'.

The infinitesimal strained sphere $x_1'^2+y_1'^2+z_1'^2=K^2$ was produced by straining the ellipsoid

$$x_1^2 + y_1^2 + z_1^2 + 2(s_{xx}.x_1^2 + s_{yy}.y_1^2 + \ldots + s_{xy}.x_1y_1)=K^2. \quad \ldots \quad (3).$$

[$(x_1\ y_1\ z_1)$ are the relative co-ordinates of a point very near to P].

This result will be afterwards useful in determining the laws of resolution of a strain-system from one set of axes to another.

Equation of Energy.

§ 3. The energy required to strain any element $dx\ dy\ dz$ into its final form is W $dx\ dy\ dz$, where W is a function of the strains which, as will be found, cannot be of a lower degree than the second.

Let S_{xx}, S_{yy}, ... S_{xy} be the six stresses at any point considered as tractions. The work on any triangular element, and so on any element $d\Sigma'$ of the strained surface, due to an infinitesimal displacement δu, δv, δw is the sum of the quantities of work done in its projections on the three co-ordinate planes. Its value is therefore

$$A_x'(S_{xx}\delta u + S_{xy}\delta v + S_{xz}\delta w) + A_y'(S_{yx}\delta u + S_{yy}\delta v + S_{yz}\delta w) + A_z'(S_{zx}\delta u + S_{zy}\delta v + S_{zz}\delta w).$$

The work done over the surface of a strained body is therefore

$$\begin{aligned} &\iint dy\,dz\,\{(S_{xx}.U_1 + S_{xy}V_1 + S_{xz}.W_1)\,\delta u + \text{two other similar terms}\} \\ &+ \iint dz\,dx\,\{\text{three similar terms}\} + \iint dx\,dy\,\{\text{three similar terms}\} \quad . \quad (4). \end{aligned}$$

Now the work done by internal stresses is of course $\iiint \delta W\,.\,dx\,dy\,dz$.

$$\left.\begin{aligned} \text{And} \quad & \delta W = \frac{dW}{ds_{xx}}\delta s_{xx} + \frac{dW}{ds_{xy}}\delta s_{xy} + \ldots \\ \text{and} \quad & \delta s_{xx} = u_1\delta u_1 + v_1\delta v_1 + w_1\delta w_1 \\ & \delta s_{yz} = u_2\delta u_3 + v_2\delta v_3 + w_2\delta w_3 + u_3\delta u_2 + v_3\delta v_2 + w_3\delta w_2 \end{aligned}\right\} \quad . \quad (5).$$

The work done by the impressed forces $= \rho_0 \iiint (X\delta u + Y\delta v + Z\delta w)dx\,dy\,dz.$

That consumed in producing motion $= \rho_0 \iiint \left(\frac{d^2u}{dt^2}\delta u + \frac{d^2v}{dt^2}\delta v + \frac{d^2w}{dt^2}\delta w\right)dx\,dy\,dz.$

On substituting these values in the equation which expresses the conservation of energy, and equating according to LAGRANGE's principles the coefficients of δu ... to zero, both throughout the interior and over the surface, we obtain

the equation of equilibrium, and expressions for the elastic forces. It is of course to be noticed that the expression for the work due to the potential energy of strain must be first integrated by parts throughout the volume of the solid under consideration. This volume may be any portion whatever of the whole solid, or the whole body itself.

Determination of Stresses and Equations of Equilibrium.

§ 4. On equating separately to zero the co-efficients of δu . . . taken over the three separate boundary integrals, we obtain nine equations to find the six stresses. But this difficulty is explained by the symmetry of the expressions for the tangential stresses which indicate that they may be derived from different sets of equations by different routes.

The general type of these fundamental boundary conditions is given by

$$\left.\begin{aligned} S_{xx}\cdot U_1 + S_{xy}\cdot V_1 + S_{xz}\cdot W_1 &= \frac{dW}{ds_{xx}}\cdot u_1 + \frac{dW}{ds_{xy}}u_2 + \frac{dW}{ds_{xz}}\cdot u_3 \\ S_{xx}\cdot U_2 + S_{xy}\cdot V_2 + S_{xz}\cdot W_2 &= \frac{dW}{ds_{yx}}\cdot u_1 + \frac{dW}{ds_{yy}}u_2 + \frac{dW}{ds_{yz}}\cdot u_3 \end{aligned}\right\} \quad . \quad . \quad (6).$$

From these we may derive either the stresses in terms of the rate of change of W or conversely. The former are given by the general formulæ—

$$\left.\begin{aligned} S_{xx}\cdot\nabla &= \frac{dW}{ds_{xx}}\cdot u_1^2 + \frac{dW}{ds_{yy}}u_2^2 + \ldots + 2\frac{dW}{ds_{xy}}\cdot u_1u_2 \\ &\vdots \\ S_{xy}\cdot\nabla &= \frac{dW}{ds_{xx}}u_1v_1 + \frac{dW}{ds_{yy}}u_2v_2 + \ldots + \frac{dW}{ds_{xy}}(u_1v_2 + u_2v_1) \end{aligned}\right\} \quad . \quad . \quad (7).$$

These results are manifestly capable of expression in a symbolical form, which can be best explained by studying any one form and comparing it with its more complete development. The form is

$$S_{\lambda\mu}\cdot\nabla = \frac{dW}{ds}(\delta_{1,x} + \delta_{2,y} + \delta_{3,z})^{\lambda}\cdot(\delta_{1,x} + \delta_{2,y} + \delta_{3,z})^{\mu} \quad . \quad . \quad (\alpha),$$

where $u = \delta^x$, $v = \delta^y$, $w = \delta^z$.

A similar form exists for the expression of $\frac{dW}{ds_{\lambda\mu}}$ in terms of the stress. It is

$$\frac{dW}{ds_{\lambda\mu}}\cdot\nabla = S\cdot(D^x_{i,x} + D^y_{i,y} + D^z_{i,z})\cdot(D^x_{j,x} + D^y_{j,y} + D^z_{j,z}) \quad . \quad (\beta),$$

where $U = D^x$, $V = D^y$, $W = D^z$, and the suffixes i, j mean 1, 2, 3 according as λ, μ mean x, y, z.

The typical form of the equation of motion or equilibrium is

$$\rho_0 F_{\lambda} + \left(\frac{d}{dx}\cdot\frac{dW}{ds_x} + \frac{d}{dy}\cdot\frac{dW}{ds_y} + \frac{d}{dz}\cdot\frac{dW}{ds_z}\right)\cdot(\delta_{1,x} + \delta_{2,y} + \delta_{3,z})^{\lambda} = \rho_0\frac{d^2\delta^{\lambda}}{dt^2} \quad . \quad (\gamma).$$

§ 5. In cases where the displacements are exceedingly small, it will be sufficient to neglect terms in $S_{\lambda\mu}$ of the order $\frac{d(u, v, w)}{d(x, y, z)}$ compared to unity whenever they enter. Thus, in this case,

$$S_{\lambda\mu} = \frac{dW}{ds_{\lambda\mu}} \quad . \quad . \quad . \quad . \quad . \quad (8).$$

In investigating strains to this order of magnitude, it will be sufficient to use the unstrained area $d\Sigma$ instead of the strained $d\Sigma'$ in the foregoing process. It is also obvious that W must be at least a quadratic function of the strains, for otherwise S might be finite while $(s_{xx} \ldots)$ were zero. This can only happen in the case of

§ 6. *Compound Strains.*—Let the displacements, strains, and stresses due to the first strains be $(\alpha\ \beta\ \gamma)$, $(s^0_{xx} \ldots)$, $(S^0_{xx} \ldots)$; and let the new position of $(x\ y\ z)$ be $(\xi\ \eta\ \zeta)$, where $\xi = x + \alpha$, $\eta = y + \beta$, $\zeta = z + \gamma$.

Let displacements and strains in second strain be $(u\ v\ w)$, $(\sigma_{xx} \ldots)$; then in the state of stress produced by superposing the latter strain on the former, the displacements are $\alpha + u$, $\beta + v$, $\gamma + w$, while the strain and stress systems will be denoted by s and S. The values of $s_{xx} \ldots$ may be found in terms of $\sigma_{xx} \ldots$ and $\alpha_1 \ldots \gamma_3$ either by direct differentiation or by the general method employed by THOMSON and TAIT for compounding strain-displacements (§ 185).

Let the co-ordinates of any unstrained point very near P be, relatively to P, $p\ q\ r$, and let them become, after the first and second strains $(p'\ q'\ r')$ $(p_1\ q_1\ r_1)$. then

$$\left.\begin{aligned} p' &= \alpha_1 p + \alpha_2 q + \alpha_3 r \\ q' &= \beta_1 p + \ldots \\ r' &= \ldots + \gamma_3 r \end{aligned}\right\} \qquad \left.\begin{aligned} p_1 &= u_1 p' + u_2 q' + u_3 r' \\ q_1 &= v_1 p' + v_2 q' + v_3 r' \\ r_1 &= w_1 p' + w_2 q' + w_3 r' \end{aligned}\right\};$$

whence finally,

$$\left.\begin{aligned} p_1 &= (\alpha_1 u_1 + \beta_1 u_2 + \gamma_1 u_3)\, p + (\alpha_2 u_1 + \beta_2 u_2 + \gamma_2 u_3) q + \ldots \\ q_1 &= (\alpha_1 v_1 + \beta_1 v_2 + \gamma_1 v_3)\, p + (\alpha_2 v_1 + \beta_2 v_2 + \gamma_2 v_3) q + \ldots \\ r_1 &= (\alpha_1 w_1 + \beta_1 w_2 + \gamma_1 w_3)\, p + (\alpha_2 w_1 + \beta_2 w_2 + \gamma_2 w_3) q + \ldots \end{aligned}\right\}.$$

We thus find

$$\left.\begin{aligned} s_{xx} &= s^0_{xx} + \alpha_1^2 \sigma_{xx} + \beta_1^2 \sigma_{yy} + \gamma_1^2 \sigma_{zz} + \beta_1\gamma_1 \sigma_{yz} + \gamma_1\alpha_1 \sigma_{zx} + \alpha_1\beta_1 \sigma_{xy} \\ s_{yy} &= s^0_{yy} + \ldots \\ s_{xy} &= s^0_{xy} + 2\alpha_1\alpha_2\sigma_{xx} + 2\beta_1\beta_2\sigma_{yy} + 2\gamma_1\gamma_2\sigma_{zz} + (\beta_1\gamma_2 + \gamma_1\beta_2)\sigma_{yz} + (\gamma_1\alpha_2 + \gamma_2\alpha_1)\sigma_{zx} + (\alpha_1\beta_2 + \alpha_2\beta_1)\sigma_{xy} \end{aligned}\right\} (9).$$

The values of $s_{xx} - s^0_{xx}$, $\ldots s_{xy} - s^0_{xy}$ we shall denote by $D_{xx} \ldots D_{xy}$.

It may be noted that these expressions, which are rigorous, show that the order in which the strains take place is not indifferent, and that $D_{\lambda\mu}$ is a linear function of the superposed strains.

The energy due a compound strain may be readily found, for let W^0 correspond to the initial strain, and W to the final,

$$W^0 = \phi(s^0), \quad W = \phi(s^0 + D).$$

Whence by TAYLOR'S theorem,

$$W - W^0 = D_{xx}\frac{dW^0}{ds^0_{xx}} + \ldots + \frac{1}{2}\left\{\frac{d^2W^0}{ds^{0\,2}_{xx}}D^2_{xx} + 2\frac{d^2W^0}{ds^0_{xx}ds^0_{xy}}D_{xx}D_{xy} + \ldots\right\}.$$

If ∇^0 correspond to the first strain, $dx\,dy\,dz\ \nabla^0 = d\xi\,d\eta\,d\zeta$; substituting which, and remembering the expressions given in equations (7) for $S_{\lambda\mu}$, it follows that

$$W dx\,dy\,dz = W^0 dx\,dy\,dz + (S^0_{xx}\sigma_{xx} + \ldots + S^0_{xy}\sigma_{xy})d\xi\,d\eta\,d\zeta + \frac{1}{2\nabla^0}\left(\frac{d^2W^0}{ds^{0\,2}_{xx}}D^2_{xx} + \ldots\right)d\xi\,d\eta\,d\zeta \quad (10).$$

The two last parts of this expression I shall usually write $W'd\xi\,d\eta\,d\zeta$ and $w'd\xi\,d\eta\,d\zeta$ respectively.

[§ 7. Without assuming a knowledge of (7), the second member of (10) may be written

$$W' = T^0_{xx}\sigma_{xx} + \ldots + T^0_{xy}\sigma_{xy}$$

where the forms of $T^0_{\lambda\mu}$ are known. If we now suppose the second strains to be very small, and after differentiation, according to the formula (8), which are well known, put $u = v = w = 0$, we obtain

$$T^0_{\lambda\mu} = S^0_{\lambda\mu},$$

which furnishes a new proof of the forms for S_{xy}.]

§ 8. The system of additional stresses $(S - S^0)$ introduced by the second strain, which we shall always suppose to be *small* and of the first order, can now be found. The part of it which arises from w' may be found from equation (8).

We have in general, on putting $\nabla = \nabla^0 . \nabla'$,

$$\left.\begin{aligned} &\nabla' . S_{\lambda\mu} = S^0(\delta_{1,x} + \delta_{2,y} + \delta_{3,z})^\lambda . (\delta_{1,x} + \delta_{2,y} + \delta_{3,z})^\mu + \frac{dw'}{d\sigma_{\lambda\mu}} \\ &\nabla' = 1 + \frac{du}{d\xi} + \frac{dv}{d\eta} + \frac{dw}{d\zeta}. \end{aligned}\right\} \quad (11).$$

In particular,

$$\left.\begin{aligned} S_{xx} - S^0_{xx} &= -S^0_{xx}\left(\frac{du}{d\xi} + \frac{dv}{d\eta} + \frac{dw}{d\zeta}\right) + 2\left(\frac{du}{d\xi}S^0_{xx} + \frac{du}{d\eta}S^0_{xy} + \frac{du}{d\zeta}S^0_{xz}\right) + \frac{dw'}{d\sigma_{xx}} \\ S_{xy} - S^0_{xy} &= -S^0_{xy}\cdot\frac{dw}{d\zeta} + \frac{dv}{d\xi}\cdot S^0_{xx} + \frac{du}{d\eta}S^0_{yy} + \frac{dv}{d\zeta}S^0_{xz} + \frac{du}{d\zeta}S^0_{yz} + \frac{dw'}{d\sigma_{xy}} \end{aligned}\right\} \quad (12).$$

The equations of motion of the solid, as has been remarked by ST VENANT, assume a very simple form, even in the case where (S^0) are not constant throughout the substance. If they were produced solely by external force acting on the surface of the body, the new equations have for their type

$$\rho X' + \left(\frac{d}{d\xi}\cdot\frac{dw'}{d\sigma_{xx}} + \frac{d}{d\eta}\cdot\frac{dw'}{d\sigma_{xy}} + \frac{d}{d\zeta}\cdot\frac{dw'}{d\sigma_{xz}}\right) + \left(S^0_{xx}\frac{d^2}{d\xi^2} + \ldots + 2S^0_{xy}\frac{d^2}{d\xi d\eta} - \rho\frac{d^2}{dt^2}\right)u = 0 \quad (13).$$

If the strains (S^0) were produced under the action of forces X_0, Y_0, Z_0, we must add to the left hand member of this equation the expression

$$-\rho\left(X_0 + X_0\frac{du}{d\xi} + Y_0\frac{du}{d\eta} + Z_0\frac{du}{d\zeta}\right).$$

§ 9. *Resolution of Strains.*—In the foregoing results nothing has been assumed respecting the constitution of the solid, and they all apply equally well whatever that may be. But in the reduction to the case of isotropism, it is important to know the laws according to which strains are resolved in different directions; or, to put more concisely a particular case of the general problem, to know what conditions must be satisfied that two systems of stresses, defined with reference to different sets of rectangular axes, may be equivalent. When the strains are of the first order of small quantities, the system $(2s_{xx}, 2s_{yy} \ldots s_{xy})$ follow the same laws of resolution as the stress system $(S_{xx} \ldots S_{xy})$. For in the case of isotropic media we have

$$S_{xx} = A\theta + 2B\,.\,s_{xx}, \; S_{xy} = Bs_{xy},$$

when $\theta = s_{xx} + s_{yy} + s_{zz}$ and is an invariant, and A, B are constants.

But the same law holds good when the strains are not *small*, as I first found by actual transformation of co-ordinates. A simpler proof, however, is furnished by considering the strain ellipsoid, which as already shown may be written

$$x_1^2 + y_1^2 + z_1^2 = x^2 + y^2 + z^2 + 2\,(s_{xx}x^2 + \ldots + s_{xy}\,.\,xy) = K^2.$$

Expressed with regard to new axes this becomes

$$x_1'^2 + y_1'^2 + z_1'^2 = x'^2 + y'^2 + z'^2 + 2\,(s_{x'x'}x'^2 + \ldots + s_{x'y'}\,.\,x'y') = K^2,$$

and the new strains (s') are expressed *linearly* in terms of the old (s). But since the degree of any power of the distortions $\frac{d(u\,v\,w)}{d(x\,y\,z)}$ is unaltered by transformation, it follows that the parts of the first and second degrees separately, and consequently the whole (s), follow the same law of resolution, which is that of strains of the first order already given.

The sum of the squares of the members of the several rows of ∇^0, and the sum of the products of the corresponding members of each pair of rows, will be of great use to us in what follows. They are denoted by a, b, c, d, e, f. These quantities also follow the same laws of resolution as stresses, a property arising from the fact that $\frac{d}{dx}\frac{d}{dy}\frac{d}{dz}$ follow the ordinary parallelepiped law of resolution, just as $u\ v\ w$ do, when the axes are rectangular.

§ 10. *Reduction to Isotropic Media.*—The theory of the ellipsoid indicates directly that there are three invariant functions of the strains (s), of the first, second, and third degrees respectively. Calling these J_1, J_2, J_3 it follows that all possible invariants of these three degrees are J_1, $mJ_1^2 + nJ_2$, $m'J_1^3 + n'J_1J_2 + p'J_3$.

Also,

$$\left.\begin{aligned} J_1 &= s_{xx} + s_{yy} + s_{zz} \\ J_2 &= -4s_{yy}s_{zz} - 4s_{zz}s_{xx} - 4s_{xx}s_{yy} + s_{yz}^2 + s_{zx}^2 + s_{xy}^2 \\ J_3 &= 4s_{xx}s_{yy}s_{zz} + s_{yz}s_{zx}s_{xy} - s_{xx}s_{yz}^2 - s_{yy}s_{zx}^2 - s_{zz}s_{xy}^2 \end{aligned}\right\} \quad . \quad (14).$$

If we confine ourselves in W to terms of the second degree, we may take

$$2W = mJ_1^2 + nJ_2,$$

where $m = k + \frac{4n}{3}$ (See THOMSON and TAIT, art. 682).

The expressions for the stresses become very simple; for writing $2W = (m - 2n)J_1^2 + nJ_2^1$, where

$$J_2^1 = s_{yz}^2 + \ldots + 2s_{zz}^2.$$

$$\left.\begin{aligned} \nabla.S_{xx} &= (m-2n)J_1.a + n(2s_{xx}.u_1^2 + 2s_{yy}u_2^2 + \ldots + 2s_{xy}u_1u_2) \\ &\vdots \\ \Delta.S_{xy} &= (m-2n)J_1.f + n(2s_{xx}.u_1v_1 + 2s_{yy}u_2v_2 + \ldots + s_{xy}(u_1v_2 + u_2v_1)) \end{aligned}\right\} \quad (15).$$

in which $a, b \ldots f$, have the values in the last paragraph of § 9, but now refer to ∇.

These expressions admit also of the following transformations.

$$\left.\begin{aligned} \nabla.S_{xx} &= (\overline{m - 2n}.J_1 - n)a + n(a^2 + e^2 + f^2) \\ &\vdots \\ \nabla.S_{xy} &= (\overline{m - 2n}J_1 - n).f + n(\overline{a + b}.f + de) \end{aligned}\right\} \quad . \quad (15a),$$

where also

$$2J_1 + 3 = a + b + c. \quad . \quad . \quad . \quad (15b);$$

or, with reference to the developments which follow in the next article, into these forms

$$\left.\begin{aligned} \nabla.S_{xx} &= (mJ_1 + 2n)a + n(\overline{e^2 - ac} + \overline{f^2 - ab}) \\ &\vdots \\ \nabla.S_{xy} &= (mJ_1 + 2n).f + n(de - cf) \end{aligned}\right\} \quad . \quad (15c).$$

There is a remarkable corollary from these equations. There exists, as we shall see in § 13, or as we may deduce from the laws of transformation of $a \ldots f$, one set of rectangular axes at each point for which $d = e = f = 0$. If the solid be homogeneous after the strain, these have the same direction at every point. It follows from the above equations that for these axes there is no tangential stress, and the normal stresses are the sums of two parts of which one is directly as the corresponding a, b, c, and the other part as its square. And, generally, at every point of an isotropic solid the stresses (each multiplied by ∇) are functions of $a \ldots f$, which we may call the *quasi-strains*.

§ 11. The calculation of w' will usually be a very long and troublesome matter; for it contains 21 terms of the form $D_{\lambda\mu} \cdot D_{\lambda'\mu'}$, each of which terms $D_{\lambda\mu}$ contains 21 terms of the form $\sigma_{\lambda\mu}$. We should thus have, in general, to take account of 441 terms. But if we narrow the problem, as in last article, by taking W to consist of terms of the second degree only in (s), and by supposing it isotropic with regard to these terms, w' appears reduced to a form of remarkable and rather unexpected symmetry.

When $W = \phi(s)$ is homogeneous and of the second degree in (s), then also

$$\left.\begin{aligned} \nabla^0 . w' &= \phi(D_{xx} \ldots) \\ &= \tfrac{1}{2}(mI_1^2 + nI_2) \\ &\text{where } I_1 \text{ and } I_2 \text{ are the invariants of (D)} \end{aligned}\right\} \quad . \quad (16).$$

If for $2\sigma_{xx}$, $2\sigma_{yy} \ldots \sigma_{xy}$ we write A, B ... F, we find

$$I_1 = \frac{Aa + Bb + Cc}{2} + Dd + Ee + Ff \quad . \quad . \quad (17).$$

The calculation of I_2 is most easily performed by find finding $I_2 + 2I_1^2$; when this work, which naturally is not devoid of symmetry, is gone through, we arrive finally at

$$I_2 = -\left\{\begin{aligned} &(BC - D^2)(bc - d^2) + (CA - E^2)(ca - e^2) + (AB - F^2)(ab - f^2) \\ &+ 2(EF - AD)(ef - ad) + 2(FD - BE)(fd - be) + 2(DE - CF)(de - ef) \end{aligned}\right\} \quad (18).$$

In these results $a \ldots f$ are the initial *quasi-strains*.

[§ 12. The strain ellipsoid corresponding to the second strain may be written

$$Ax^2 + By^2 + Cz^2 + 2Dyz + 2Ezx + 2Fxy = 1 \quad . \quad (\delta),$$

while the quasi-strain ellipsoid corresponding to the initial strain is

$$ax^2 + by^2 + cz^2 + 2dyz + 2ezx + 2fxy = 1 \quad . \quad (\delta').$$

Let the discriminants of these surfaces be $S = 2J_2^0$ and s, and let them be

reciprocated with regard to the spheres whose radii are $\frac{1}{\sqrt{S}}$ and $\frac{1}{\sqrt{\sigma}}$; thence we obtain

$$\left.\begin{aligned}(\mathrm{BC}-\mathrm{D}^2)x^2+\ldots+2(\mathrm{DE}-\mathrm{CF})xy&=1\\(bc-d^2)x^2+\ldots+2(de-cf)xy&=1\end{aligned}\right\}\quad\ldots\quad(\epsilon).$$

$2\mathrm{I}_1$ and $-\mathrm{I}_2$ are the invariants of the first order of the first and second pairs of surfaces].

§ 13. If the body after the first strain be homogeneous, and we choose the co-ordinate axes parallel to these of the quasi-strain ellipsoid, we may put $d=e=f=0$, and a, b, c will be constant throughout.

Thus,

$$\left.\begin{aligned}\mathrm{I}_1&=a\sigma_{xx}+b\sigma_{yy}+c\sigma_{zz}\\\mathrm{I}_2&=bc(\sigma_{yz}^2-4\sigma_{yy}\sigma_{zz})+ca(\sigma_{zx}^2-4\sigma_{zz}\sigma_{xx})+ab(\sigma_{xy}^2-4\sigma_{xx}\sigma_{yy})\end{aligned}\right\}\quad(19.)$$

We are now in a position to determine the plane waves which can be propagated unchanged through such a solid.

[*Inserted* 16*th February* 1876, *and following the notation of* §§ 16, 18.

The equations of motion are as follows:—

Putting

$$\nabla_1=a\frac{d^2}{dx^2}+b\frac{d^2}{dy^2}+c\frac{d^2}{dz^2}$$

$$\nabla_2=\mathrm{P}^2\frac{d^2}{dx^2}+\mathrm{Q}^2\frac{d^2}{dy^2}+\mathrm{R}^2\frac{d^2}{dz^2}$$

we find, after some reduction, that the equations take the following forms:

$$\left.\begin{aligned}\rho\frac{d^2u}{dt^2}&=n(\nabla_2+a\nabla_1)u+\tfrac{1}{2}\mathfrak{m}\frac{d\Theta}{dx}\\\rho\frac{d^2v}{dt^2}&=n(\nabla_2+b\nabla_1)v+\tfrac{1}{2}\mathfrak{m}\frac{d\Theta}{dy}\\\rho\frac{d^2w}{dt^2}&=n(\nabla_2+c\nabla_1)w+\tfrac{1}{2}\mathfrak{m}\frac{d\Theta}{dz}\end{aligned}\right\}\quad\ldots\quad(20.)$$

where $\mathfrak{m}=4m-n$.

To solve these, put

$$u, v, w=(u_0, v_0, w_0)\sin\frac{2\pi}{\lambda}(px+qy+rz-\mathrm{V}t),$$

where

$$p^2+q^2+r^2=1,$$

and let

$$\left.\begin{aligned}\mathrm{D}_1&=ap^2+bq^2+cr^2\\\mathrm{D}_2&=\mathrm{P}^2p^2+\mathrm{Q}^2q^2+\mathrm{R}^2r^2\\\rho\frac{\mathrm{V}^2}{n}-\mathrm{D}_2&=\psi\\\theta&=apu_0+bqv_0+crw_0\end{aligned}\right\}$$

we arrive at the following:—

$$\left.\begin{aligned} n(\psi - aD_1)u_0 &= \mathfrak{m} \cdot ap\theta \\ n(\psi - bD_1)v_0 &= \mathfrak{m} \cdot bq\theta \\ n(\psi - cD_1)w_0 &= \mathfrak{m} \cdot cr\theta , \end{aligned}\right\} \quad . \quad . \quad . \quad (21.)$$

and on the elimination of $u_0\ v_0\ w_0$, the following equation for ψ

$$\frac{a^2p^2}{\psi - aD_1} + \frac{b^2q^2}{\psi - bD_1} + \frac{c^2r^2}{\psi - cD_1} = \frac{n}{\mathfrak{m}} . \quad . \quad . \quad . \quad (22.)$$

It appears from this result that there are three values of V^2 for each plane wave, and the wave which spreads out from a centre will, therefore, possess three sheets.

But if the solid be incompressible, and the primary stress be finite, we shall have $\mathfrak{m} = \infty$, $m = \infty$, and also $J_0 = 0$; and in this case the equation which gives V reduces to

$$\frac{a^2p^2}{\psi - aD_1} + \frac{b^2q^2}{\psi - bD_1} + \frac{c^2r^2}{\psi - cD_1} = 0 .$$

and the equations which give $u_0\ v_0\ w_0$ require us to suppose $\theta = 0$, $\mathfrak{m}\theta =$ finite.

Now $\theta = 0$ is equivalent to $a\frac{du}{dx} + b\frac{dv}{dy} + c\frac{dw}{dz} = 0$; and at first sight it seems difficult to understand the significance of this condition. We must remember, however, that the word "incompressible," as here used, is a relative term, and denotes that when the strains are indefinitely small, the stresses in such a solid required to produce a very small cubic compression, are indefinitely great compared to those necessary to produce a shear of the same magnitude. This implies that m is indefinitely great compared to n; and the preceding investigation shows that if such a solid, having previously undergone considerable strain, be still further subjected to stresses of small magnitude, the distortions which they produce are such as to make $a\frac{du}{dx} + b\frac{dv}{dy} + c\frac{dw}{dz} = 0$.

With these explanations we now proceed to consider the case where $a-1$, $b-1$, $c-1$ are such that their squares and products may be neglected. I shall also suppose that $mJ_0 = 0$. Let therefore $a = 1 + \alpha$, then $P^2 = a(a-1) = \alpha$, and similarly $Q^2 = b - 1 = \beta$, $R^2 = c - 1 = \gamma$; thus $D_2 = \alpha p^2 + \beta q^2 + \gamma r^2$, $D_2 = 1 + D_1$ and $aD_1 = (1+\alpha)(1+D_2) = 1 + \alpha + D_2$, . . .

The equation for V becomes $\frac{(1+\alpha)^2p^2}{\frac{\rho V^2}{n} - 2D_2 - a} + \frac{(1+\beta)^2q^2}{\frac{\rho V^2}{n} - 2D_2 - b} + \frac{(1=\gamma)^2r^2}{\frac{\rho V^2}{n} - 2D_2 - c} = 0 .$

It appears therefore that $\frac{\rho V^2}{n} - 1$ is of the order α, β, γ, and we may write

therefore 1 for $(1+\alpha)^2\ldots$, whence we obtain

$$\frac{p^2}{\frac{\rho V^2}{n}-2D_2-a}+\frac{q^2}{\frac{\rho V^2}{n}-2D_2-b}+\frac{r^2}{\frac{\rho V^2}{n}-2D_2-c}=0,$$

an equation not unlike that occurring in the theory of double refraction.]

It cannot be said that our investigations, so far at least, throw much light on the true cause of double refraction in strained glass beyond the fact that such glass is no longer isotropic. Nor, indeed, was it to be much expected, in our ignorance of the true nature of the luminiferous medium and the mode of its connection with terrestrial bodies.

§ 14. I add the calculation of the constants for the case of a substance *under longitudinal stress F.*

Here
$$\alpha=(\epsilon-1)x,\ \beta=(\lambda-1)y,\ \gamma=(\lambda-1)z,\ \nabla^0=\begin{vmatrix}\epsilon & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{vmatrix}.$$

We have
$$0=(m-2n)J_1-n+n\lambda^2,$$
$$\epsilon\lambda^2F=[(m-2n)J_1-n]\epsilon^2+n\epsilon^4,$$

whence
$$\left.\begin{aligned}&(3k-2n)\epsilon^2+2(3k+n)\lambda^2=9k\\ &F=\frac{n\epsilon(\epsilon^3-\lambda^3)}{\lambda^2}\end{aligned}\right\}.$$

In the case of glass, for which we have, very nearly, $3k=8n$, we find

$$\epsilon^2+3\lambda^2=4,\ \epsilon^3+\frac{F}{n}\epsilon^2-\epsilon-\frac{4F}{n}=0,$$

which latter equation has only one real root, as might have been expected.

On the Laws of Resolution of Forces and Stresses.

§ 15. These investigations may be fitly terminated by some general considerations on the law of resolution of stresses, which has been proved in § 9 to apply to strains, and to what I have called quasi-strains. It would seem that physical magnitudes possessing direction divide themselves naturally into two classes, in one of which the law of resolution along rectangular axes is that of ordinary forces, and in the other that of elastic stresses. To each class pertains a series of divariants and invariants, and each possess groups of derived directed magnitudes in some of which the law of resolution is that of their primitives, and in others the opposite law. We may term these shortly the law of forces and the law of stresses, and the corresponding groups force-groups and stress-groups respectively.

Def. 1. Let x, y, z be rectangular co-ordinates, and (u, v, w) a ternary group such that for rectangular axes,

$$ux + vy + wz = \text{an invariant, J not} = 0 \quad . \quad . \quad (23),$$

then $(u\ v\ w)$ constitute a force group.

Def. 2. Let (a, b, c, d, e, f) be a sextic group such that

$$ax^2 + by^2 + cz^2 + 2dyz + 2ezx + 2fxy = \text{an invariant, J}' \quad (24),$$

where J′ differs from zero, then $(a, \ldots, f)$ from a stress group.

The known properties of planes and quadries permit us at once to enunciate.

Theorem (*a*). For an infinite number of sets of rectangular axes, one axis of which is fixed, the ternary group (u, v, w) becomes $(u', 0, 0)$, and the axis of x' is the axis of the group; so for *one* set of rectangular axes the sextic group becomes $(a', b', c', 0, 0, 0)$, and the axes of x', y', z' may be called the axes of the group.

Since in the above definitions the quantities $uv \ldots a \ldots$ are involved linearly, we derive the following

Theorem (*b*). From two force- or two stress-groups we may derive new force- or stress-groups by taking the sums or differences of the corresponding constituents of the groups for constituents of the new group.

Since $x.x + y.y + z.z = r^2$, an invariant, it follows that the characteristic law of resolution of (x, y, z) is itself that of forces; and since $x^2.x^2 + y^2.y^2 + z^2.z^2 + 2yz.yz + 2zx.zx + 2xy.xy = r^4$ an invariant, it follows the group $(x^2, y^2 \ldots xy)$ is itself a stress-group, and has the corresponding characteristic law of resolution. Hence follows,

Theorem (*c*). If (u, v, w), (U, V, W) be two force-groups, then $uU + v.V + w.W$ is an invariant J; and conversely if $uU + vV + wW$ be an invariant, and one of the groups be a force-group, so also is the other: Also if $(a \ldots . f)$ $(A \ldots . F)$ be stress-groups, then is $Aa + Bb + Cc + 2Dd + 2Ee + 2Ff$ an invariant; and conversely, if this expression be an invariant, and one of these groups be a stress-group, so must the other.

The theory of planes and quadries allows at once to write down the fundamental invariants of each group, as we may do in

Theorem (*d*). The invariant of $(u\ v\ w)$ is $I = u^2 + v^2 + w^2$; and the invariants of the stress-group $(A \ldots . F)$ are

$$J_1 = A+B+C, J_2 = BC+CA+AB-D^2-E^2-F^2, J_3 = ABC+2DEF-AD^2-BE^2-CF^2.$$

The following are a few of the applications of this theory in various depart-

ments of mathematics: in most cases the results run parallel in the two groups. We may denote force-groups in general by the symbols $(\alpha\ \beta\ \gamma)$, (u, v, w), &c., while stress-groups will be expressed by $(A \ldots F)$, $(a \ldots f)$, &c. Invariant functions will be represented by the characteristic symbols J, J^1.

Theorem (*e*).

Force-Groups.	*Stress-Groups.*
Lines (x, y, z)	$(x^2, y^2, z^2, yz, zx, xy)$
	Since $J \cdot x^2 + Jy^2 + J \cdot z^2 = Jr^2$ we have the groups $(J, J, J, 0\ 0\ 0)$, $(r^2, r^2, r^2, 0\ 0\ 0)$.
Moments of a particle with regard to three planes.	$(y^2+z^2, z^2+x^2, x^2+y^2, -yz, -zx, -xy)$.
Moments of a solid with regard to three planes.	Moments and negative products of inertia of a solid.
Forces.	*Stresses.*
Displacements.	Strains and quasi-strains.

Theorem (*f*). Since $dJ = \frac{dJ}{dx}dx + \frac{dJ}{dy}dy + \frac{dJ}{dz}dz$, and $dJ^1 = \frac{dJ^1}{dA}dA + \ldots + \frac{dJ^1}{dF}dF$, it follows that $\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right) J$ form a force-group and $\left(\frac{d}{dA}, \ldots \frac{1}{2}\frac{d}{dF}\right) J^1$ a stress-group. Moreover, as these characteristic laws of resolution of $\frac{d}{dx} \ldots \frac{1}{2}\frac{d}{dF}$ depend on the relative directions only of the new and old fixed axes, and not on the subject of operation, we may remove the restriction that the latter is to be an invariant, and extend them to the case where it may be any directed quantity. In this way we establish the invariancy of such expressions as $\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}$, $\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$, and $\frac{d^2A}{dx^2} + \frac{d^2B}{dy^2} + \ldots + 2\frac{d^2F}{dx\,dy}$.

Theorem (*g*). If we choose $J' = \frac{J_1^2}{2} - J_2$ we reproduce the group $(A \ldots F)$, while by choosing $J' = J_3$ we see that $(BC - D^2, \ldots, DE - CF)$ form a stress-group. This result enables us also to deduce the new invariants

$$J^1_{-1} = a(BC - D^2) + \ldots + 2f(DE - CF)$$
$$J^1_{-2} = (bc - d^2)\ (BC - D^2) + \ldots + 2(de - cf)\ (DE - CF).$$

The application of (f) to J^1_{-1} produces the stress-group

$$(B\beta + C\gamma - Dd, \ldots\ldots, De + dE - C\alpha - Fc).$$

Theorem (h). If we choose $J = \nabla = \begin{vmatrix} x & y & z \\ u & v & w \\ \alpha & \beta & \gamma \end{vmatrix}$ we obtain the force-group $(\gamma v - \beta w, \ldots \beta u - \alpha v)$; while if we choose $J^1 = \nabla^2$ and put $\alpha^2 = a$, $u^2 = A \ldots$, $\alpha\beta = f$, $uv = F$, we reproduce the stress-group last found.

By putting $u = \frac{d}{dx}$, $v = \frac{d}{dy}$, $w = \frac{d}{dz}$ we arrive at the force-group $\left(\frac{dv}{dz} - \frac{dw}{dy}, \ldots.\right)$ and the stress-group $\left(\frac{d^2B}{dy^2} + \frac{d^2C}{dz^2} - \frac{d^2D}{dy\,dz}, \ldots\ldots\right)$.

Theorem (i). $J = (u\alpha + v\beta + w\gamma)(\alpha x + \beta y + \gamma z)$ and $J^1 = (ux + vy + wz)(\alpha x + \beta y + \gamma z)$ generate respectively the force- and stress-groups $(uA + vF + wE, \ldots)$, $(u\alpha, \ldots., \frac{1}{2}\overline{u\beta + v\alpha})$.

These include as particular cases the groups $\left(\frac{dA}{dx} + \frac{dF}{dy} + \frac{dE}{dz}, \ldots\right)$ and $\left(\frac{d\alpha}{dz}, \ldots, \frac{1}{2}\left(\frac{d\beta}{dx} + \frac{d\alpha}{dy}\right)\right)$.

Theorem (j). $J = \nabla . (u\alpha + v\beta + w\gamma)$ generates the force-group $(\overline{b - cD} - d\overline{B - C} + fE - eF, \ldots)$; while $J^1 = \nabla . (\alpha x + \beta y + \gamma z)$ generates the stress-group $(vE - wF, \ldots \frac{1}{2} . \overline{wA - B - uE - vF})$.

These include as particular cases those in which $\alpha = \frac{d}{dx}$, $a = \frac{d^2}{dx^2} \ldots f = \frac{d^2}{dx\,dy}$.

[*Added* 16*th February* 1876.]

§ 16. The reduction of the work necessary to produce a compound strain in an isotropic solid has been effected, in the foregoing paper, on the hypothesis that the work necessary to produce a state of strain from perfect freedom, is a quadratic function of the component strains. But it may be useful to show how the same problem may be solved when the work is a function of any degree in these strains.

Calling xyz the relative co-ordinates of two particles of the unstrained solid, $x_1y_1z_1$ those of the same particles in the state of strain, it has been shown that

$$x_1^2+y_1^2+z_1^2-(x^2+y^2+z^2)=\mathrm{A}x^2+\mathrm{B}y^2+\ldots+2\mathrm{F}xy, \quad . \quad (25).$$

where $\mathrm{A}=2s_{xx}$, $\mathrm{B}=2s_{yy}$, $\ldots$ $\mathrm{F}=s_{xy}$.

From this result the following invariants result—

$$\mathrm{J}=\mathrm{A}+\mathrm{B}+\mathrm{C}$$
$$\mathrm{H}=\mathrm{D}^2+\mathrm{E}^2+\mathrm{F}^2-\mathrm{BC}-\mathrm{CA}-\mathrm{AB}$$
$$\mathrm{K}=\mathrm{ABC}+2\mathrm{DEF}-\mathrm{AD}^2-\mathrm{BE}^2-\mathrm{CF}^2.$$

And there can be no more invariant functions of the strains; for these equations are sufficient to determine the three principal strains in terms of J, H, K; and if there were a fourth invariant, it must be a function of these principal strains, and consequently of J, H, K.

The work done in producing from freedom any state of strain must, therefore, be a function of J, H, K of the form—

$$\mathrm{W}=\tfrac{1}{2}(m\mathrm{J}^2+n\mathrm{H})+p\mathrm{J}^3+q\mathrm{JH}+r\mathrm{K}+\ldots, \quad . \quad . \quad (26).$$

[The n here used is the same as that used by Thomson and Tait, p. 710, but the m is different. If the m there used be written $\mathfrak{m}$, we shall have $\mathfrak{m}=4m-n$].

We have now to find how J, H, K for a compound system depend on its component elements; and to do so, I use capital letters with suffix $_0$ to define the primary strains, letters without suffix to denote the secondary, and those with suffix $_1$ to indicate the final state; while a, b, c, d, e, f denote, as formerly, the primary quasi-strains. The corresponding invariants will also be similarly distinguished. We have, therefore—

$$\mathrm{J}=\mathrm{A}+\mathrm{B}+\mathrm{C},\ \mathrm{J}_0=\mathrm{A}_0+\mathrm{B}_0+\mathrm{C}_0,\ \mathrm{J}_1=\mathrm{A}_1+\mathrm{B}_1+\mathrm{C}_1,\ j=a+b+c,$$

and the remaining invariants $\mathrm{H}, \mathrm{H}_0, \ldots . k$ may be similarly written down.

The following "mixed concomitants," which make their appearance, are thus denoted :—

$$\left.\begin{aligned} \mathfrak{J} &= Aa+Bb+Cc+2Dd+2Ee+2Ef \\ \mathfrak{J}_1 &= A(bc-d^2)+B(ca-e^2)+\ldots.+2F(de-cf) \\ -\mathfrak{J}_2 &= (BC-D^2)(bc-d^2)+\ldots.+2(DE-CF)(de-cf). \end{aligned}\right\} \quad (27).$$

§ 17. We now proceed to find J_1, H_1, K_1. If we turn to equations (9), we find they may be written—

$$1+A_1 = \alpha_1^2(1+A)+\beta_1^2(1+B)+\ldots+2\alpha_1\beta_1 F$$
$$\vdots$$
$$F_1 = \alpha_1\alpha_2(1+A)+\beta_1\beta_2(1+B)+\ldots+(\alpha_1\beta_2+\alpha_2\beta_1)F.$$

I. To find J_1 we have

$$3+J_1 = a(1+A)+b(1+B)+\ldots+2fF,$$

whence

$$\left.\begin{aligned} J_1 &= J_0+\mathfrak{J} \\ J_0 &= j-3 \end{aligned}\right\} \quad \ldots\ldots \quad (28).$$

II. To find H_1, it is most convenient to calculate, in the first place, $(1+A_1)^2+(1+B_1)^2+(1+C_1)^2+2D_1^2+2E_1^2+2F_1^2$, which is found, after a little reduction, to be equal to

$$\Sigma a^2(1+A)^2+2\Sigma(bc+d^2)D^2+2\Sigma d^2(1+B)(1+C)+4\Sigma af(1+A)F$$
$$+4\Sigma ef(1+A)D+4\Sigma(ad+ef)EF,$$

where Σ indicates summation extended to all terms of the same type. On expanding and substituting for $A_1+B_1+C_1$ its value already found, we obtain

$$\left.\begin{aligned} H_1 &= 2j+h-3+hJ+2\mathfrak{J}+\mathfrak{J}_1+\mathfrak{J}_2, \\ \text{and on putting } A=B=\ldots=0 \text{ we find also} \\ H_0 &= 2j+h-3 \end{aligned}\right\} \quad \ldots \quad (29).$$

III. To find K_1 we observe that K is the discriminant of the covariant (25), and that the problem of finding $1+A_1, \ldots.$ is the ordinary one of linear transformation, the modulus being ∇_0; hence

$$(1+A_1)(1+B_1)(1+C_1)+2D_1E_1F_1-\ldots = \nabla_0^2\left((1+A)(1+B)(1+C)+2DEF-\ldots\right)$$

Now, it is clear that, if ∇_0^2 be expressed as a determinant, we shall have $\nabla_0^2 = k$,

* It is perhaps worth noting that there is still a fourth invariant, which depends on the systems $(AB\ldots F)$, $(a,b\ldots f)$, namely, $\mathfrak{J}^1 = a(BC-D^2)+b(CA-E^2)+\ldots+2f(DE-CF)$, but it does not present itself in these investigations. It may be observed, however, that it is not really independent of the nine magnitudes $J_1\, H_1\, K_1\, j, h, k, \mathfrak{J}, \mathfrak{J}_1, \mathfrak{J}_2$. For every invariant relating to two systems of strains, or to a system of strains and one of quasi-strains, can depend on only nine elements,—the six principal strains, and the three magnitudes which determine one set of principal strain-axes with regard to the other. In fact, referring one set of strains to their principal axes, we see that the ten invariants involve only nine independent quantities.

This is a special case of the more general theorem that n sets of magnitudes cogredient with strains give rise to $n(2n+1)$ invariants apparently independent, but of which only $6n-3$ are actually independent, the remaining $2n^2-5n+3$ being functions of these.

Hence the above result furnishes the following

$$\left.\begin{aligned} &K_1-H_1+J_1+1=k(K-H+J+1)\\ \text{and by putting } A=B=\ldots=0\text{, this also,}\quad &\\ &K_0=j+h+k-1 \end{aligned}\right\} \quad . \quad . \quad (30.)$$

§ 18. We are now in a position to find the value of the work W_1, which corresponds to the compound strain, in terms of its components; to do this write $W_1=W_0+w$, and analyse w into the terms of the first, second ... degrees in the secondary strains; thus—

$$w=w_1+w_2+w_3+\ldots$$

On substituting for J_1, H_1, K_1 their values, we can readily find w_1, w_2, ... If we confine ourselves to the case where

$$W_1=\tfrac{1}{2}(mJ_1^2+nH_1)$$

we shall have

$$\left.\begin{aligned} w_1&=\tfrac{1}{2}\{2mJ_0\mathfrak{J}+n(hJ+2\mathfrak{J}+\mathfrak{J}_1)\}\\ w_2&=\tfrac{1}{2}(m\mathfrak{J}^2+n\mathfrak{J}_2) \end{aligned}\right\} \quad . \quad . \quad (31.)$$

Now w_1 may be written in the form

$$\left.\begin{aligned} &\tfrac{n}{2}\{(a)A+(b)B+(c)C+2(d)D+2(e)E+2(f)F\}\\ \text{where}\quad &n(a)=2(mJ_0+n)\,a+n(e^2-ca+f^2-ab)\\ &\vdots\\ &n(f)=2(mJ_0+n)f+n(de-cf)\,, \end{aligned}\right\} \quad . \quad (32.)$$

These expressions may be simplified by supposing the axes of co-ordinates to be those of the primary quasi-strain ellipsoid; and if the primary stress be homogeneous throughout the solid, these axes will have the same direction throughout. In this case we shall write

$$\left.\begin{aligned} &(a)=P^2,(b)=Q^2,(c)=R^2,(d)=(e)=(f)=0\\ &P^2=2(\tfrac{m}{n}J_0+1)a-(ab+ac)\,,\ldots\\ &w_1=\tfrac{n}{2}(P^2A+Q^2B+R^2C)\,. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (33.)$$

The simplified value of w_2 has been already found.

The expressions found above (32) may be used to furnish the expressions for the stresses given in my paper at art. 10, by observing that

$$W_1dV=W_0dV+\frac{w}{\nabla_0}\delta V$$

where dV and δV are corresponding elements of the solid in its free state, and after the primary strain has been produced.

We may also apply the method given in art. 7 to find expressions for the stresses, whatever may be the nature of the function which expresses W_1 in

terms of J_1 H_1 and K_1. To do this we must first arrange K_1 according to the terms which are of the first, second, third degrees, in terms of the secondary strains, as follows:—

$$K_1 = K_0 + (h+k)J + \mathfrak{J} + \mathfrak{J}_1 + \mathfrak{J}_2 - kH + kK \quad . \quad . \quad . \quad (34.)$$

The stress S^0_{xx} is given by $\nabla_0 S^0_{xx} = 2\frac{dW_1}{dA}$ where, after differentiation, the secondary strains are to be put equal to zero. When, therefore, we write

$$\frac{dW_1}{dA} = \left(\frac{dW}{dJ}\right)_0 \frac{dJ_1}{dA} + \left(\frac{dW}{dH}\right)_0 \frac{dH_1}{dA} + \left(\frac{dW}{dK}\right)_0 \frac{dK_1}{dA},$$

we need only attend to those terms in H_1 and K_1, which are linear in A, B, C....

Substituting for J_1, H_1 and K_1 we find

$$\nabla_0 S^0_{xx} = 2\left[\left(\frac{dW}{dH}\right)_0 h + \left(\frac{dW}{dK}\right)_0 \overline{h+k}\right] + \left[\left(\frac{dW}{dJ}\right)_0 + 2\left(\frac{dW}{dH}\right)_0 + \left(\frac{dW}{dK}\right)_0\right] a + 2\left[\left(\frac{dW}{dH}\right)_0 + \left(\frac{dW}{dK}\right)_0\right](bc - d^2).$$

If we write this result,

$$S_{xx} = L + Ma + N(bc - d^2),$$

the type of the tangential stresses is

$$S_{xy} = Mf + N(de - cf).$$

XXIV.—*Chapters on the Mineralogy of Scotland. Chapter First.—The Rhombohedral Carbonates.* PART I. By Professor HEDDLE.

(Read April 3, 1876.)

In the series of chapters of which the present is the first, I purpose to submit the results of an analytical examination of all the minerals of Scotland whose composition appeared doubtful; of such as had not previously been examined; or of such as appeared in any way to be of special geologic interest.

In every case, where not otherwise stated, the specimens were gathered by the hands of the writer himself; while the purity of every particle examined was as far as possible secured by an examination under the lens, conducted with the most scrupulous care. All portions selected for the determination of the specific gravity were afterwards divided into small fragments (in the event of any impurity being observed, the determination was discarded); the fragments thus obtained were those employed for the analyses.

Minerals belonging to the same group were as far as possible examined in succession.

The substances first examined are comprehended in this first chapter.

ANKERITE.

Found by DUDGEON and myself in 1873, in a vein of somewhat decomposed yellowish crystalline talc, on the west side of the Ting of Norwick, in Unst, Shetland, at the junction of serpentine with mica slate.

This is an old and well-known locality, and is that referred to by GREG and LETTSOM in their "Manual of Mineralogy," as the locality for *Breunnerite*, when they say—"Hitherto it has been met with in the United Kingdom only at the head of Norwick Bay, in Unst."

So well, apparently, had the locality been searched, that it was with difficulty that the mineral could be found, and then only one piece of some ounces in weight was obtained. This was a readily cleavable mass, composed of mutually penetrating crystals, of about one inch in size. Its colour was bluish grey; its specific gravity, 2·91; its cleavage angle, 106° 6′.

The analysis was executed on 25·025 grains, and afforded—

Carbonate of lime,	51·804
„ of magnesia,	37·998
„ of iron,	7·82
„ of manganese,	2·314
Silica (quartz),	·02
	99·956

This is not the composition of Breunnerite; which is a magnesite, having about 10 per cent. of the carbonate of magnesia replaced almost solely by carbonate of iron.

Moreover, neither the specific gravity nor the angular inclination agree with Breunnerite—the specific gravity of which is from 3· to 3·2, and its angle from 107° 22′ to 107° 32′.

As Ankerite is a Dolomite in which the magnesian carbonate is more or less completely replaced by carbonates of iron and magnesia, the Norwick mineral falls to rank under that name; though the amount of replacement hardly comes up to the limit separating it from Dolomite, as assigned by DANA.

The equivalentic ratios of its constituents are nearly—

$\dot{C}a\ddot{C}$	$\dot{M}g\ddot{C}$	$\dot{F}e\ddot{C}$	$\dot{M}n\ddot{C}$
52	43	7	2

Or more generally—

$$(\dot{C}a\dot{M}n)\ddot{C} \quad + \quad (Mg\dot{F}e)\ddot{C}$$

The lime being to the manganese as 26 to 1, and the magnesia to the iron as 24 to 4.

In gravity and cleavage angle, the mineral agrees fairly with Ankerite; the gravity of which is given as low as 2·95; and its angle by MOHS at 106° 12′,—by ETTLING at 106° 6′.

The theoretical specific gravity of a substance formed with its constituents united in the above ratios is 2·94; and its theoretical cleavage angle, 106° 13′.

Breunnerite, therefore, has to be excluded from the list of British minerals; the second analysis repones it, however.

BREUNNERITE.

On a hill slope overlooking from the north-west the bay of Haroldswick, in the same island of the Shetland group, some exposed portions of weathered talc were pointed out; these, on examination, proved to be from a vein running through serpentine towards the east coast; and appearing in an indentation thereof, called North Cross Geo. After two days' blasting, unweathered specimens were obtained of the following minerals:—Apple-green talc in broad

foliated plates, and, rarely, fibrous in structure; Brucite very rarely; magnetite in octohedra; pearly white Dolomite; and Breunnerite.

This Breunnerite occurs in pale brown rhombohedral crystals, imbedded either in the talc or in the Dolomite. Its specific gravity is 3·095; its cleavage angle, 106° 50′.

The analysis executed on 25 grains afforded—

Insoluble (talc),	·096
Carbonate of magnesia,	91·394
„ of iron,	6·784
„ of manganese,	·780
Silica,	·60
Alumina,	·136
	99·76

Here the magnesian carbonate is to the conjoined carbonates of iron and manganese in the ratio of 16 to 1; and the mineral is within, though barely within, the margin of the ferriferous magnesites, to which HAIDENGER assigned the name of Breunnerite.

The theoretical specific gravity of a carbonate compounded in the above ratio is 3·087, and its theoretical angle, 107° 27′. There was not in the mineral even a trace of lime.

The most frequent order of position of the associated minerals here is—talc, Breunnerite, Dolomite; both talc and Breunnerite are, however, sometimes inclosed in Dolomite.

DOLOMITES.

The Dolomite which has been mentioned as occurring at North Cross Geo is found in numerous ramifying veins; its colour is a pure watery white; its lustre is pearly; it is semi-transparent; though not showing itself in free crystals, it is in large crystalline masses, and is altogether finer than at any other Scottish locality. Its specific gravity is 2·865; its cleavage angle, 106° 17.

Portions from two veins were examined, with a view to ascertain if the replacement of the carbonates was constant,—the first analysis being that of the specimen whose gravity and angle were taken.

	1	2	3	4
	On 25 grs.	On 25·17 grs.		
Insoluble (talc),	·1			
Carbonate of lime,	52·548	55·344		
„ of magnesia,	43·772	41·911		
„ of iron,	1·972	2·193	1·304	1·36
„ of manganese,	1·368	·6	·84	1·108
	99·76	100·048		

The only difference that could be seen between the specimens analysed was that the first was more distinctly crystallised than the others; and it is a fact calling for remark, that in the presence of this free and almost indefinite replacement, the Breunnerite, which is imbedded in this highly calcareous matrix, and which is a mineral in itself obeying the law of homeomorphous replacement, and evidently of contemporaneous origin, should so absolutely in crystallising expel every particle of lime, whilst it incorporated iron and manganese—the former, at least, to a greater extent than had been accomplished by that matrix.

The specific gravity and angle of No. 1 were so near the usual, and the mineral altogether so typical, as to call for no remark.

Dolomite from Scalpa, Harris.—This was analysed on account of its having been mentioned by MACCULLOCH and others as "the great vein of Dolomite."

It forms a vein (? bed), associated with penninite on the one side and steatite on the other, running nearly east and west from a spot a few yards to the south-east of the Lighthouse pier at Scalpa. The enclosing rock is serpentine, distinctly bedded in hornblendic gneiss.

The appearance of this Dolomite is unusual—it is confusedly crystalline, somewhat foliaceous in structure, with its cleavage faces so curved as to be incapable of measurement. It is of unusual hardness and toughness, is milk white, perfectly opaque, and with little lustre. Its specific gravity is 2·87.

Its analysis of 25 grains afforded—

Insoluble (steatite),	·16
Carbonate of lime,	50·244
„ of magnesia,	43·028
„ of iron,	2·504
„ of manganese,	2·896
Silica,	·64
Alumina,	·112
	99·644

The four carbonates are here in nearly the ratios

$\dot{C}a\ddot{C}$	$\dot{M}g\ddot{C}$	$\dot{F}e\ddot{C}$	$\dot{M}n\ddot{C}$
50	49	2	2½

and the theoretical specific gravity of such a compound is 2·913.

Dolomite from the old lead and copper mine on the roadside a little to the south-east of Newton Stewart, Galloway.—It occurs here associated with chalcopyrite and galena in large pale brown simple and twin crystals; also rarely in fine crystals of a beautiful pink colour;—the brown crystals were analysed. Their specific gravity was 2·906; their cleavage angle, 106° 10′.

25 grains afforded—

Insoluble (quartz),	·204
Carbonate of lime,	55·08
„ of magnesia,	37·092
„ of iron,	5·716
„ of manganese,	1·368
Alumina,	·116
	99·576

The ratios here are—

$\dot{C}a\ddot{C}$	$\dot{M}g\ddot{C}$	$\dot{F}e\ddot{C}$	$\dot{M}n\ddot{C}$
55	42	5	1

The theoretical specific gravity is 2·912; the theoretical angle, 106° 10′.

Dolomite from Largybaun, Cantyre.—The mineral here was found in the cliffs a little to the south of the Largybaun caves, which are situated about half way between the Mull of Cantyre lighthouse and the house of Ballygroggan. It is associated with limonite; occurs of a fawn-yellow colour; and is in large cleavable masses; these are somewhat decomposed in spots into a bright red ochre. From its colour it was taken for chalybite. Its specific gravity is 2·824; its cleavage angle, 106° 30′.

25 grains afforded—

Insoluble (quartz),	·048
Carbonate of lime,	55·80
„ of magnesia,	36·296
„ of iron,	6·612
„ of manganese,	1·264
	100·02

The four carbonates are here in the ratios—

$\dot{C}a\ddot{C}$	$\dot{M}g\ddot{C}$	$\dot{F}e\ddot{C}$	$\dot{M}n\ddot{C}$
56	41	6	1

The theoretical specific gravity of such a compound is 2·92; and its theoretic angle, 106° 9′.

Dolomites from igneous rocks.

"Pearl spar" from the round sea stack north of the "Rock and Spindle," near St Andrews. This occurs filling druses in a thick vein which cuts the tuffa; the vein has occasional cavities lined with crystals, which are sometimes studded with "nail-head" (rhombohedron *g*) calcite. This pearl spar contains large white lamellar crystals of baryte imbedded in it. Analcime is found closely adjacent, imbedded in a vein of brown calcite. Colour, pink; specific gravity, 2·782; crystals and cleavages too curved for measurement.

Carbonate of lime,	50·026
„ of magnesia,	39·108
„ of iron,	6·7
„ of manganese,	3·736
Silica,	·22
Insoluble,	·62
	100·41

The four carbonates are here in the ratios—

$\dot{C}a\ddot{C}$	$\dot{M}g\ddot{C}$	$\dot{F}e\ddot{C}$	$\dot{M}n\ddot{C}$
50	44	6	3

The theoretical specific gravity of such a compound is 2·946.

*Pearl spar from Walls, Orkney.**—Occurs in a dark purple amygdaloid to the west of Sandsgio, filling druses which are lined with green earth. It frequently contains large white lamellar crystals of baryte imbedded in its substance; sometimes crystals of translucent analcime line the sides of the druses, and very rarely brown "Babel quartz." Its cavities sometimes contain curved crystals; its colour is pink; its specific gravity is 2·780; its crystals and cleavage too curved for measurement.

Its analysis afforded—

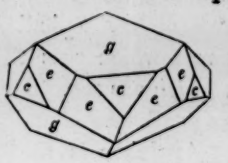
Insoluble (quartz),	·16
Carbonate of lime,	62·4
„ of magnesia,	32·056
„ of iron,	1·74
„ of manganese,	4·276
	100·632

The four carbonates are here present in the ratios—

$\dot{C}a\ddot{C}$	$\dot{M}g\ddot{C}$	$\dot{F}e\ddot{C}$	$\dot{M}n\ddot{C}$
42	24	1	$2\frac{1}{2}$

The theoretical specific gravity, 2·88. The association of baryte and analcime with both of these pearl spars is worthy of remark.

Dolomite pseudomorphous after calcite.—Occurs in druses in trap tuff north of the Rock and Spindle, at Kinkell, St Andrews. Besides the mineral, the druses contain salmon-coloured rock crystal, blue baryte, and nail-head calcite. The Dolomite is pseudo after the scalenohedron *r* ("dog-tooth" scalenohedron) in general; the portion analysed was however pseudo after the more complex form depicted.



* Incorrectly entered as Ankerite in Greg and Lettsom's "British Minerals," on the writer's authority.

The colour of these pseudomorphs is dull ochre yellow. Their specific gravity, taken on perhaps a somewhat porous bit, = 2·742.

The analysis on 25 grains afforded—

Insoluble,	1·780
Carbonate of lime,	51·48
„ of magnesia,	37·42
„ of iron,	7·016
„ of manganese,	1·928
	99·624

The four carbonates are here present in the ratios—

$\dot{C}a\ddot{C}$	$\dot{M}g\ddot{C}$	$\dot{F}e\ddot{C}$	$\dot{M}n\ddot{C}$
51	42	6	2

Its theoretical specific gravity would be 2·93.

These "pearl spars" somewhat approach Ankerite in composition; having, however, low gravity, curved faces, and pearly lustre, they are true pearl spars.

COLOURED CALCITES.

Pomegranate coloured calcite.—From the old workings at Tomnadashin, Loch Tay. Occurs in pale pomegranate red crystals, with cleavage angle 105° 14′, associated with a ferruginous calcite, sheafy baryte, fahlerz, quartz, and pyrite. Specific gravity, 2·687.

Insoluble,	·004
Carbonate of lime,	97·763
„ of iron,	·765
„ of manganese,	1·119
„ of magnesia,	·076
	99·727

The very peculiar colour was possibly due to a suboxide of copper; derived from the adjacent fahlerz; no copper was, however, actually found.

Pink calcite occurs in the porphyry south of the town of Gourock, associated in druses with white baryte; purple, yellow, green, and colourless fluor; and minute needles of a substance much resembling Göthite, but which are magnetic.

Colour of the calcite nearly as bright a pink as that of manganspath. Specific gravity, 2·732; cleavage angle, 105° 43′.

Carbonate of lime,	93·16
„ of magnesia,	·472
„ of iron,	1·984
„ of manganese,	4·276
Alumina,	·044
	99·936

The colour was, doubtless, due to the large quantity of the manganesian carbonate.

Brown calcite.—Pointed out to me by GRIEVE as filling small druses in a very friable amygdaloid a little to the west of Kinghorn, Fifeshire, on the sea-shore.

The specimens are of a radiating structure, but not fibrous;—they at first sight resemble brown stilbite. Specific gravity, 2·736.

When dissolved in dilute acid, numberless minute globules of a light brown oily matter separated, and were seen suspended in the liquid; these had a pleasant odour, resembling that of hawthorn blossoms. Upon passing the solution through a filter, these globules were retained, but rapidly evaporated from its surface, leaving the paper somewhat brown tinted.

Insoluble (siliceous),	·056
Carbonate of lime,	94·2
„ of magnesia,	1·276
„ of iron,	1·628
„ of manganese,	1·868
Alumina,	·042
	99·07

The colour probably was derived from the bituminous oil.

Green calcite occurs with a skin of Delessite in the amygdaloid vein which traverses the tuff of the Rock and Spindle, near St Andrews; and which vein is connected with the line of volcanic "necks" to the eastward. Specific gravity, 2·704.

Left upon solution 4·376 per cent. of siliceous matter mixed with Delessite. This, upon continued digestion in strong hydrochloric acid, left 3·94 per cent. of silica;—·436 is therefore green earth (Delessite).

Carbonate of lime,	88·08
„ of magnesia,	4·996
„ of iron,	2·028
„ of manganese,	·48
Silica,	3·94
Alumina,	·036
Green earth,	·436
	99·996

Anthraconite.—From a hill two miles north of Campbeltown, Cantyre. The specimen, which was given to me by my late colleague Dr MACDONALD, consisted of a saccharoid mass of a dark blue or greyish-black colour; the crystalline granules were of the size of peas, and were united by a much paler cement,

which was still more siliceous than the dark foliaceous and cleavable granules. The specific gravity of the cleavable portions was 2·89; their cleavage angle, 104° 45′.

25 grains, not absolutely free from adhering paler cement, yielded—

Silica,	19·24	
Alumina,	·236	
Carbon,	·92	
Carbonate of lime,	71·44	89·39
„ of magnesia,	5·112	6·396
„ of iron,	1·744	2·182
„ of manganese,	1·624	2·032
	100·316	100·000

Anthraconite.—From the north-west side of Loch Earn, Perthshire. The specimen was also given me by Dr MACDONALD. Occurs as a vein in clayslate in large foliated imbedded crystals; colour, grey black, with granular cement of the same colour. Specific gravity of general mass, 2·759; cleavage angle, 104° 58′.

29·6 grains yielded—

Insoluble,	12·205
Carbon,	·016
Carbonate of lime,	86·741
„ of iron,	·598
	99·55

Other specimens equally dark in colour yielded no carbon whatever.

The large quantity of foreign matter reduces the cleavage angle of these anthraconites.

To the Dolomite above-mentioned, as occurring pseudo after scalenohedra of calcite in the trap tuff at Kinkell, near St Andrews, very unusual interest attaches. Its mode of occurrence and appearance are as follows :—

The extremely friable tuff of this locality, which from its being wave-washed has probably to a considerable extent been exhausted of all constituents soluble in water, is rifted and reticulated with shrinkage cracks, which by the process of exfiltration are now filled with the above Dolomite. On these "exfiltration veins," amplifications or "bunches" rarely occur; these being still more rarely drusy or cavernous.

The contents of the druses, arranged in the order of their deposition or nearness to the sides of the cavity, are the following :—

The scalenohedra of Dolomite after calcite of a dull yellow colour; semi-transparent quartz of a pinkish salmon colour, in doubly terminated prisms; "nail-head" corded aggregations of rhombohedron *g* crystals of calcite, devoid of colour; double six-sided pyramids of colourless quartz.

The scalenohedra of Dolomite are considered to be pseudomorphs upon the following grounds :—

1*st*, Fully developed crystals of the form *r* do not occur of Dolomite, though that form occurs as a hemihedral modification of some of its dominant rhombohedral forms; while the scalenohedron *r* is one of the commonest forms of calcite.

2*d*, The cleavage of the crystals, instead of being accordant with a single crystal of *r*, and continuous across the crystal, is confused, interrupted, and evidently in several directions; leading to the belief that the fracture meets a number of crystals lying in every direction.

3*d*, The crystals under the lens present every appearance of being pseudomorphic. Upon being broken, the great mass of some is seen to consist of unaltered or rather unabstracted colourless calcite; there being merely an outside layer of the yellow Dolomite: in others a core of calcite, and that frequently loose, and which rattles when shaken, alone remains within a thick casing of Dolomite; while in a considerable number no trace of calcite is to be seen.

In these last, namely, in those wholly consisting of Dolomite, a central cavity invariably occurs; in the specimens which contain a core of calcite, an annular vacuity between it and the Dolomite is as unvariably to be seen; while slight pressure with the nail will crack off and remove the thin layer mentioned in the first of these cases; there being a want of absolute attachment between this layer and the invested calcite.

4*th*, The surfaces of the faces of the scalenohedra are commonly rent and depressed; in other words, they do not form flat bounding planes, but show curved lines, or blunt re-entering angles.

5th, While no structure whatever can with the lens be detected on the outer surfaces of these hollow crystals, their inner surfaces, whether a central core of calcite be present or not, are invariably studded with crystals of Dolomite, which are larger the greater the thickness of the walls, and largest when the calcite is altogether absent.

In a word, the scalenohedron of Dolomite is not an individual crystal, built up through the action of crystalli-polar force depositing material upon a central nucleus, and so growing or incrementing *from within outwards*; but is produced by the juxtaposition of a multitude of individual crystals, which have cast or concreted themselves upon a mould, through replacement of its substance; and as that mould was removed by some solvent, have, by the aggregation of an ever-increasing quantity of their own material, continuously augmented in size, and so grown *from without inwards*.

All specimens which have been through marine denudation dislodged, and are rolling on the shore, and also all specimens the walls of which have been crushed in, have the displacement complete—that is, the crystals contain no calcite; while druses which are found and opened by quarrying into the rock still have the *central* calcitic core; and if they be deep-seated the Dolomite presents itself only as a thin layer, for it cannot be called a coating.

Here then we unquestionably have pseudomorphism through true chemical replacement going on continuously almost under our eyes; and the question of deepest interest connected therewith is this,—How comes it that the resulting pseudomorphs are invariably wanting in solidity?—the incompleteness of their filling up being greater in the ratio of the completeness of the chemical change.

Hollow pseudomorphs are far from uncommon; Blum and Bischof mention many. The latter,—who hoped to solve the mystery of metamorphism solely by the application of Neptunian principles,—after the detail of a number of purely chemical experiments, at once opens his great work with pseudomorphic changes; and repeatedly and persistently returns to speak of hollow pseudomorphs—founding upon them perhaps more than on any other bit of nature's evidence.

And yet his explanation of the mode of formation of these hollow pseudomorphs—or, more precisely, of the cause of the vacuity thereof—does not satisfy.

We find it first given at page 39 of the English edition; and it would appear that, read in the light of his law as afterwards enunciated, there is —with all reverence be it spoken—some confusion of ideas in the way it is launched:—"If during the metamorphism the specific gravity of the mineral is increased, this tends to give the mineral a porous structure." Is this not confounding effect with cause? In many cases of true chemical interchange the

specific gravity is increased as an effect, as will be afterwards shown; but certainly a porous pseudomorph can only result from the replacing substance being deposited in a *bulk* which is smaller than that displaced.

There follows—"The twenty-eight minerals which occur in forms of carbonate of lime, are all less soluble than it is; the less soluble mineral displaces therefore the more soluble. *It appears that the displacing mineral is always less soluble than that which is displaced,* consequently a mineral which never appears in the form of another, would have a greater affinity for water than those minerals which occur in its form. The greater solubility of the displaced in comparison with the replacing mineral will cause the porosity of the former to increase during the pseudomorphic change."

Here then we have it clearly enunciated that relative solubility is the determining agent both of the interchange and of the alteration in bulk—that *adhesion,* in fact, is the acting force.

At the very outset, however, exceptions to the *facts* above stated are near at hand. On the very opposite page BISCHOF writes—"We see in some belemnites not only the whole sheath, but also the alveole filled by sulphate of baryta." The latter has twice the specific gravity of the fibrous calcite of belemnites, but, instead of producing porosity, it is said to have *filled* the whole thing.

Again, according to KREMERS, the solubility of carbonate of lime in water is one part in 12,860;* that of carbonate of magnesia, one in 5070,† or twice and a half as great; yet in our Kinkell pseudomorph the very soluble substance is that which has replaced the less soluble—has been deposited in the solid form, while the less soluble has been swept away; but this is in direct opposition to BISCHOF'S statement; according to him, it should "never appear."

Turning, however, to page 175 of his third volume, it will be seen that it has appeared even to him; and appeared, moreover, so very important that he has devoted nearly fifty pages, directly or indirectly, to its consideration.

We read—"Carbonate of magnesia is decidedly more soluble in carbonic acid water than carbonate of lime; consequently it might be expected that it would be displaced not only by the minerals that displace carbonate of lime, but also by carbonate of lime itself;—but no pseudomorphs after magnesite are known as yet. The conversion of calc spar into bitter spar, which is actually proved by the occurrence of pseudomorphs, seems to be an exception to the generally greater solubility of the displaced mineral than that by which it is displaced. The pseudomorphs are more or less hollow, which shows that the displaced mineral was more copiously dissolved than that by which it was replaced."

"Seems to be an exception." This is nothing but skimming over a fact

* BINEAU says it is soluble in 62,500 parts of water;—PELIGOT in 50,000 parts.

† FOURCRAY says in 2504 parts.

irreconcilable with his previously enunciated law. Nor certainly is this better—"At the same time, however, it must not be forgotten that the substitution of carbonate of magnesia for carbonate of lime (in the *Dolomitic* pseudomorph) is *only partial*."

It is any such substitution whatever, in opposition to his own law, that has to be explained; and this cannot be regarded in any way as the "proving exception;" nor is it so regarded by BISCHOF himself, though he so trippingly here passes it by.

His fifty page chapter commences—"No rock has attracted greater attention than Dolomite." In no part of his book does he, who tries to prove that water can do anything and everything, more firmly hold to its action than while showing that Dolomite has been formed from limestone, through the action of percolating waters, which carry magnesia in solution as a sesquicarbonate, and which interchange that magnesia for lime; riddling and rendering porous, in the so doing, the whole stratum from top to bottom; and all this in direct opposition to his law—"the displacing mineral is always less soluble than that which is displaced."

It is singular how closely BISCHOF has, as it were, grazed the true explanation without hitting on it;—a consideration of the precise extent of the "only partial" replacement of carbonate of lime by the magnesia salt in Dolomite would have led to it. He has, perhaps, been even nearer when he writes—"V. MORLOT mentions a calculation made by ELIE DE BEAUMONT, according to which, on the assumption that if of two equivalents of carbonate of lime, one equivalent was removed, and one equivalent of carbonate of magnesia deposited in its place, in the production of bitter spar, limestone would, in its conversion into Dolomite, have been reduced in bulk 12·1 per cent. He was induced by this calculation to estimate the actual proportion of hollow spaces in Dolomite to the entire mass. For this purpose he took Dolomite of average porosity, and found that the hollow spaces amounted to 12·9 per cent., or very nearly the calculated value." He was, perhaps, nearer still when he says—"Dolomite presents in its rent and fissured condition so great a resemblance to some hollow pseudomorphs of bitter spar after calc spar, that this character alone seems almost enough to induce one to regard them as having been produced in the same manner. These pseudomorphs prove positively that crystallised carbonate of lime may, by combination with carbonate of magnesia, be converted into a double carbonate. If this is the case with crystallised carbonate of lime, it would be so likewise with amorphous carbonate of lime. Consequently, Dolomite would be produced wherever carbonate of lime in any state is brought into such conditions as are requisite for conversion into double carbonate of lime and magnesia."

Throughout this it must be evident that *equivalentic* replacement was mistily seen; though the force which produces and governs equivalentic interchange

was overlooked;—water (here the mere menstruum of the change), filling the vision to the obscuration of all other agents, and of all forces not directly concerned therewith.

But if it be once admitted that the interchange is equivalentic chemically speaking, that is *gravimetrically*, this of necessity brings with it that it is equivalentic *volumetrically*; every gravimetric equivalent having its own volumetric equivalent. It only remains to find the special volume of these equivalents.

It has been shown years ago by Otto that this is most simple—"We have only to divide the equivalent weights by the specific weights, in order to obtain a quotient expressing the relations of their volumes."

Now the equivalent weight of carbonate of lime is 100; and its specific gravity is 2·72 ;—$\frac{100}{2\cdot72}=368$.

And the equivalent weight of Dolomite is 94; and its specific gravity is 2·88 ;—$\frac{94}{2\cdot88}=326$.

When therefore a crystal of carbonate of lime is through the action of chemical replacement pseudomorphosed into Dolomite, 100 parts by weight of calcite are replaced by 94 parts by weight of dolomite; but this primal gravimetric quantity of calcite has a volume of 368, while the replacing quantity of Dolomite is only 326 ;—there is therefore, numerically stated, a vacuity of 42 volumes = 11·41 per cent. or about one-ninth of the original volume.

Whenever, then, calcite in crystals is replaced by Dolomite, or limestone by the percolation of magnesian waters is converted into it, there must be, in the case of the first, a central vacuity; of the second, a more or less general porosity.*

And in lieu of Bischof's law, which confessedly failed in the most familiar, and out of all comparison the most extended instance, the following presents itself:—*Whenever, through true chemical interchange, the replacing material has an equivalent volume smaller than that of the mineral replaced, the resulting solid must be more or less cavernous.*

The general applicability of this law has now to be tested.

The following table presents in six columns the data as regards the carbonates occurring in nature.

The first column gives the formulæ; the second the gravimetric equivalent; the third the specific gravity; the fourth the equivalent volume, as obtained by dividing the gravimetric equivalent by the specific gravity; the fifth the atomic volume, obtained by dividing the last by the number of atoms in the compound; and the last gives the angular inclination of the terminal faces of the rhomb,—the unequal axis being made vertical ;—showing, as has been before done, that

* The geologic bearings of so great a shrinkage, or *cariousness* of a previously solid stratum, being the direct consequence of dolomitization, must be apparent.

the acuteness of the angle, or, in other words, the relative length of the unequal axis, increases with the increase of the equivalent volume.

	Equivalent.	S.G.	Equiv. Vol.	Atomic Vol.	Angle.
$\dot{C}a\ddot{C}$	100	2·72	36·8	7·36	105° 5′
$2\dot{F}e\ddot{C} + 3\dot{M}g\ddot{C} + 5\dot{C}a\ddot{C}$	99·6	3·02	32·9	6·58	106° 12′
$\dot{C}a\ddot{C} + \dot{M}g\ddot{C}$	94	2·88	32·6	6·52	106° 15′
$\dot{M}n\ddot{C}$	115	3·59	32·0	6·40	106° 51′
$\dot{F}e\ddot{C}$	116	3·83	30·3	6·06	107°
$\dot{F}e\ddot{C} + \dot{M}g\ddot{C}$	102	3·42	29·5	5·90	107° 14′
$\dot{M}g\ddot{C}$	88	3·04	28·9	5·78	107° 29′
$\dot{Z}n\ddot{C}$	125	4·44	28·1	5·62	107° 40′

The above law requires that if any one of the minerals in this table * be displaced by—*i.e.* pseudomorphosed into—any member standing below it, the resulting crystals must be vacuous, or have what has been called "fallen in" sides.

Of about two hundred and twenty pseudomorphs in the author's cabinet, there are only twenty-nine which throw any light on this particular point.

From two localities there are specimens of bitter spar replacing calcite, and both are hollow; there is one specimen of calamine after calcite, and one of siderite replacing calcite, which are both also vacuous. These bear out the law enunciated.†

There is in the author's cabinet one other case (occurring, however, in numerous localities), which from its being a replacement in equal number of atoms is a direct case in point; this is hematite replacing calcite, the resulting crystals being always vacuous. The subjoined formulation shows the accordance with the law.

Hematite.	Calcite.
Fe_2O_3	CaO,CO_2
$160 \div 4·9 = 326$	$100 \div 2·72 = 368.$

"True chemical interchange," however, does not always demand equality in the *number* of interchanging atoms. Professor Fuchs long since noticed that

* From which the calculated angles and specific gravities before given have been derived.

† Being desirous to test the law more fully, the writer bethought him of a collection procured by the late Professor Jameson, and which is thus referred to at page 42 of the English edition of Bischof's "Chemical Geology." "This petrifaction is in the Edinburgh Museum of Natural History. Professor Jameson commissioned Dr Krantz of Bonn to collect the most important minerals, pseudomorphs, &c., which are mentioned in the German edition of this work. I have closely examined this collection, which consists of 664 specimens, and have found many which illustrate the phenomena described much more clearly than the minerals which I used. I shall, therefore, frequently take occasion to refer to especially characteristic specimens in this collection." Upon application being made to the Director of the Industrial Museum, all that could at first be found of this collection was *the catalogue;* though a certain number of specimens supposed to belong to it were afterwards seen:—it would appear that one of the results of the transference of the University collection has been the incorporation of this specially geologic collection with the mineralogic suite; where it is practically useless, and positively an eyesore. It is so far satisfactory to know that an attempt may be made by the Director to separate it.

soda with a certain amount of water is capable of replacing lime; other such cases are known.

Moreover, complicated difficulty lies in the fact that the equivalent volume of compounds is far from invariably the sum of the equivalent volumes of their constituents. The equivalent volume of sulphur is 16·1, that of copper is 7; the equivalent volume of the protosulphide is 23, which is the sum of the above equivalents; but the equivalent volume of the subsulphide is only 26·5, while the sum of the equivalent volumes of two of copper and one of sulphur is 30. Again, the equivalent volume of the bisulphide of iron is only 23·6, while the sum of the equivalent volumes of two of sulphur and one of iron is 39·47. Many other examples might be adduced of a similar condensation.

Neither are we in possession of a sufficient amount of evidence as to how far the law of quantivalence dominates among mineral bodies; the evidence which we do possess of nature's workings is not always consonant with deductions elaborated at the desk.

In this partial state of our knowledge, all probably that we can at present do is, in the cases where minerals with an unequal number of atoms, as expressed in their rational formulæ, replace each other, to consider them as so doing in equal number of atoms; that is, that of a mineral formulated with three atoms, we must employ two parts to replace one formulated with six.

It will therefore be necessary in all such cases to compare the common atomic volume each with each.

The atomic volume is arrived at by dividing the equivalent volume, as before obtained, by the number of atoms in the combination.

Of cavernous pseudomorphs, which have to be considered under this second heading, there are (in the writer's cabinet) the following :—

Three after Pyrite—

Hematite. Fe_2O_3	Pyrite. FeS_2
$160 \div 4{\cdot}9 - \frac{32{\cdot}65}{5} = 6{\cdot}53$	$120 \div 5{\cdot}08 - \frac{23{\cdot}6}{3} = 7{\cdot}4$

Limonite. $2Fe_2O_3 + 3H_2O$	Pyrite.
$374 \div 3{\cdot}8 - \frac{98{\cdot}42}{19} = 5{\cdot}18$	7·4

Magnetite. FeO,Fe_2O_3	Pyrite.
$232 \div 5{\cdot}05 - \frac{46}{7} = 6{\cdot}57$	7·4

Two after Calcite—

Limonite.	Calcite.
As above, 5·18	CaO,CO_2
	$100 \div 2{\cdot}72 - \frac{36{\cdot}8}{5} = 7{\cdot}36$

Pyrolusite.	
MnO_2	
$87 \div 4{\cdot}82 - \frac{18{\cdot}3}{3} = 6$	7·36

Malachite after two—

Malachite.	Azurite.
$CuO, CO_2 + CuO, H_2O$	$2CuO, CO_2 + CuO, H_2O$
$221 \div 3{\cdot}85 - \frac{57{\cdot}3}{10} = 5{\cdot}7$	$344 \div 3{\cdot}615 - \frac{92{\cdot}4}{15} = 6{\cdot}2$

Malachite.	Cuprite.
	Cu_2O
5·7	$142{\cdot}8 \div 6 - \frac{23{\cdot}8}{3} = 7{\cdot}9$

The following is very cavernous:—

Cervantite	after	Antimonite.
SbO_4		Sb_2S_3
$188 \div 4 - \frac{47}{5} = 9{\cdot}4$		$340 \div 4{\cdot}56 - \frac{74{\cdot}6}{5} = 14{\cdot}9$

Vesuvian	after	Andradite.
$9CaO^2SiO_2 + 2(Fe_2O_3, 3Al_2O_3)^2SiO_2{}^3$		$3CaO^2, SiO_2 + Fe_2O_3{}^2, SiO_2{}^3$
$2142 \div 3{\cdot}4 - \frac{630}{91} = 6{\cdot}923$		$1016 \div 3{\cdot}64 - \frac{279}{40} = 6{\cdot}975$

Epidote	after	Andradite.
$3CaO^2, SiO_2 + 2(Fe_2O_3, 3Al_2O_3)^2SiO_2{}^3$		
$1345 \div 3{\cdot}27 - \frac{411}{59} = 6{\cdot}966$		6·975

Epidote	after	Scapolite.
		$3CaO^2, SiO_2 + 2Al_2O_3{}^2, SiO_2{}^3$
6·966		$1288 \div 2{\cdot}7 - \frac{477}{59} = 8{\cdot}08$

Steatite	after	Gehlenite.
$6MgO, 5SiO_2 + 2H_2O$		$CaO^3, SiO_2 + Al_2O_3, SiO_2$
$576 \div 2{\cdot}68 - \frac{215}{33} = 6{\cdot}51$		$391 \div 2{\cdot}95 - \frac{133}{17} = 7{\cdot}82$

Steatite	after	Fassaite.
		$(MgO, CaO, FeO)SiO_2$
6·51		$116 \div 3·3 - \frac{35}{5} = 7$

Serpentine	after	Biotite.
$(\frac{3}{4}MgO + \frac{1}{4}H_2O)SiO_2 + \frac{1}{2}H_2O$		$(\frac{1}{2}MgO, \frac{1}{2}Al_2O_3)^2, SiO_2^3$
$138 \div 2·57 - \frac{54}{6\frac{3}{4}} = 8·$		$320 + 2·9 - \frac{111}{13} = 8·6$

Glauconite	after	Orthoclase.
$(\frac{1}{2}FeO, \frac{1}{2}K_2O)SiO_2 + (\frac{1}{2}Fe_2O_3, \frac{1}{2}Al_2O_3)\ SiO_2^2 + 3H_2O$		$K_2O, SiO_2^3 + Al_2O_3, SiO_2^3$
$466 \div 2·3 - \frac{201}{25·5} = 7·88$		$557 \div 2·55 - \frac{218}{26} = 8·4$

Prehnite	after	Andesine.
$(2CaO, SiO_2 + Al_2O_3, SiO_2) + H_2O$		$(CaO + Na_2O)\ SiO_2 + Al_2O_3, SiO_2^3$
$413 \div 2·87 - \frac{144}{21}\ 6·86$		$402 \div 2·67 - \frac{151}{19} = 7·95$

Prehnite	after	Analcime.
		$Na_2O, SiO_2 + Al_2O_3, SiO_2^3 + 2H_2O$
6·86		$441 \div 2·25 - \frac{196}{25} = 7·84$

In all the above cases the requirements of the law are fulfilled.

It now behoves us to look for exceptions.

The writer's cabinet contains the following *apparent* exceptions:—

Orthoclase	after	Laumonite.
		$CaO,SiO_2 + Al_2O_3SiO_2^3 + 4H_2O$
8·4		$471 \div 2·3 - \frac{205}{31} = 6·61$
or		
Albite.		Heulandite.
$Na_2O, SiO_2^3 + Al_2O_3, SiO_2^3$		$CaO, SiO_2^3, + Al_2O_3, SiO_2^3 + 5H_2O$
$525 + 2·62 - \frac{200}{26} = 7·7$		$609 \div 2·19 - \frac{278}{40} = 6·95$
		Stilbite.
		$CaO, SiO_2^3 + Al_2O_3, SiO_2^3 + 6H_2O$
		$6·27 \div 2·2 - \frac{285}{43} - 6·63.$

These three pseudomorphs occur along with others in the Kilpatrick hills; they are not hollow in the sense of having a central vacuity; they even have

bulging sides, as if there had been some expansion; but they are, in the case of the two latter at least, markedly porous.

I have called the replacing substance orthoclase or albite—it has not been sufficiently determined. The substance of some of the red pseudomorphs of stilbite was many years ago analysed by myself, and found to be albite; the analysis appeared in the "British Minerals" of GREG and LETTSOM, under the name of weissigite. The erythrite of THOMSON (?oligoclase) would appear to be the material of others. The material of Laumonite pseudos was found by BISCHOF to be orthoclase; and the cluthalite of THOMSON, physically ill-defined, may be but a badly selected or badly picked specimen of one or other of these.

Prehnite — after — Laumonite.

6·86 — 6·61

or — Thomsonite.

$$2\left(\left(\tfrac{3}{4}CaO, \tfrac{1}{4}Na_2O\right)SiO_2 + Al_2O_3,SiO_2\right) + 5H_2O$$

$$651 \div 2{\cdot}35 - \frac{277}{41\frac{1}{2}} = 6{\cdot}67$$

Possibly both of those pseudomorphs occur; they have no terminations to their crystals, and are altogether so much of the nature of skeletons that it is not easy to determine the mineral replaced. No weight either way can be attached to any specimen I have seen of these.

Still one marked exception remains to be noted—

Chalcedony — after — Datholite (Haytorite).

SiO_2 — $(CaO, B_2O_3 + CaO, 2SiO_2) + H_2O$

$60 \div 2{\cdot}65 - \frac{23}{3} = 7{\cdot}66$ — $320 \div 2{\cdot}9 - \frac{110{\cdot}3}{18} = 6{\cdot}13.$

The crystals of Haytorite are usually very hollow; the freely radiating crystals of quartz—for they are more of the nature of quartz than chalcedony—of which they are formed, have so much the appearance of having grown in a thoroughly empty cavity, that probably this does not come into the category of chemical replacements.

We are, therefore, in our exceptions, reduced to the cases of the two felspars replacing the zeolites; and, as has been shown, doubt attaches also to them.

So far then as the specimens in the writer's cabinet throw light on the question, no well-marked or indubitable exception to the law previously propounded exists.

Trans. Royal Soc. Edin.

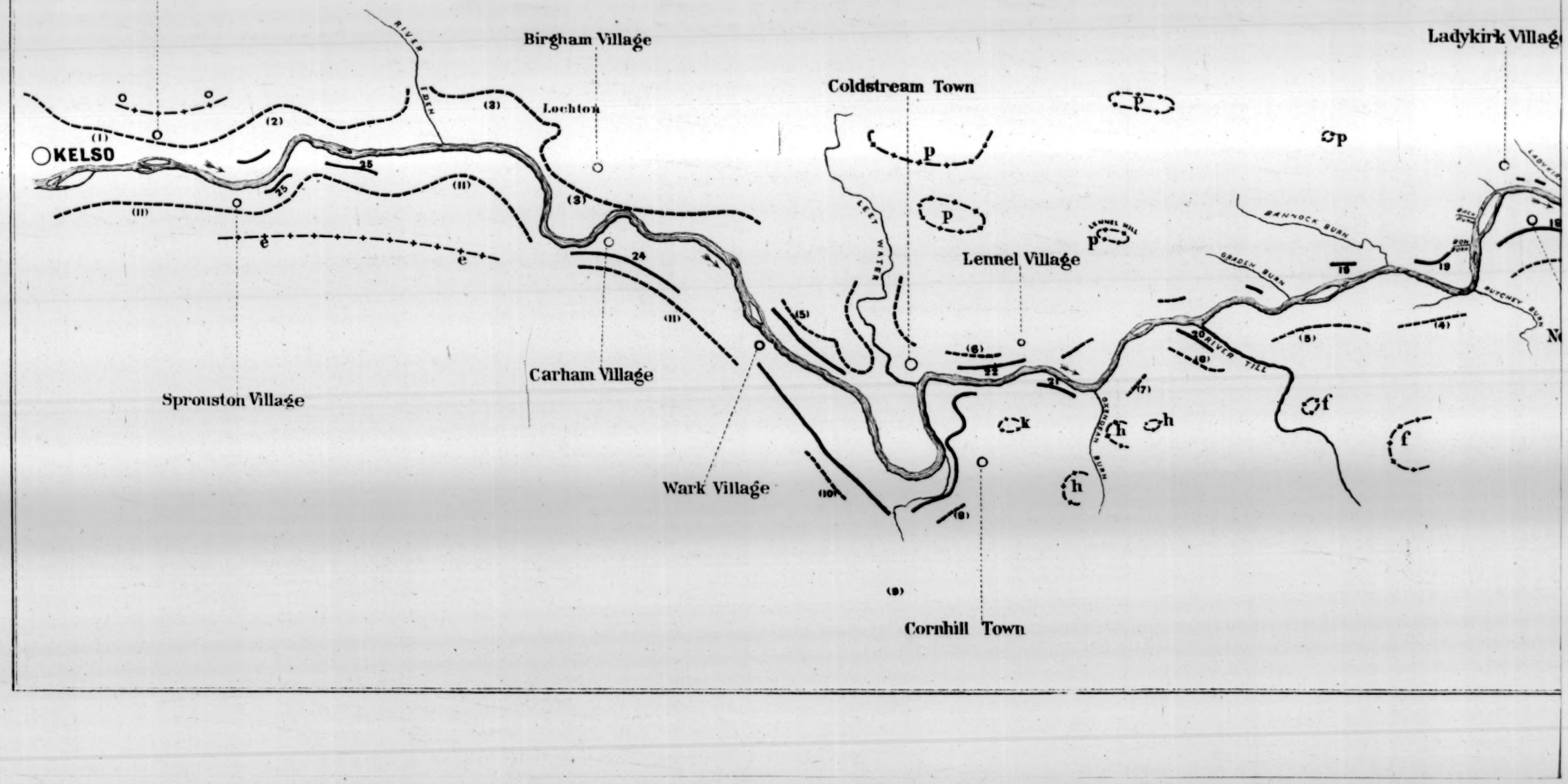
REFERRED TO ON PAGES 517 AND 554 OF TEXT.

PLAN OF TWEED VALLEY FROM BERWICK TO KELSO

SHOWING LINES OF TERRACES

SCALE 1/12 OF 6 INCH ORDNANCE MAP

The Figures on the Lines refer to the Text where the same Figures occur

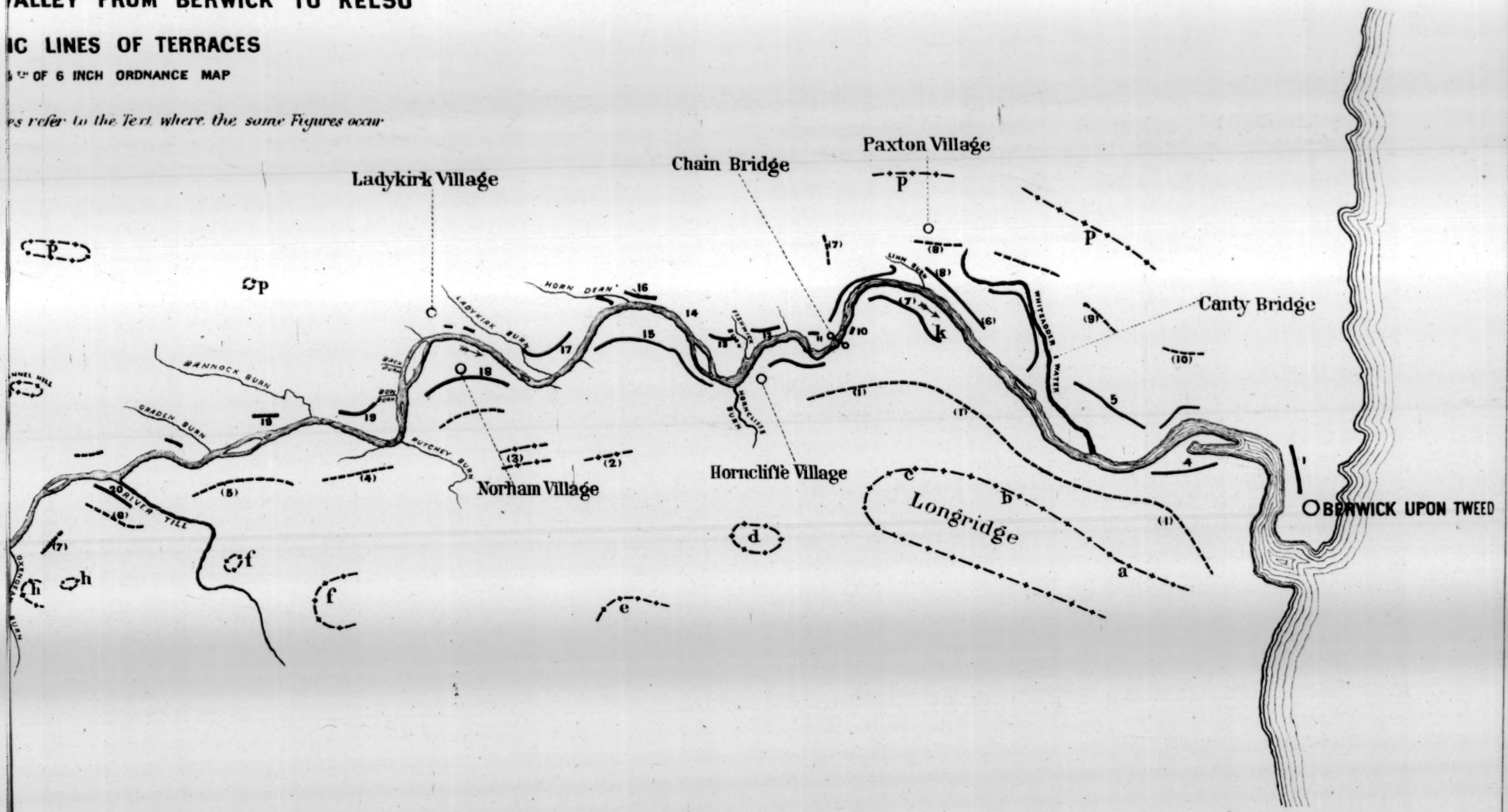


VALLEY FROM BERWICK TO KELSO

IC LINES OF TERRACES

½ OF 6 INCH ORDNANCE MAP

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W. & A. K. Johnston Edinburgh

Fig 1. (See page 526 of text)

Preston Haugh.

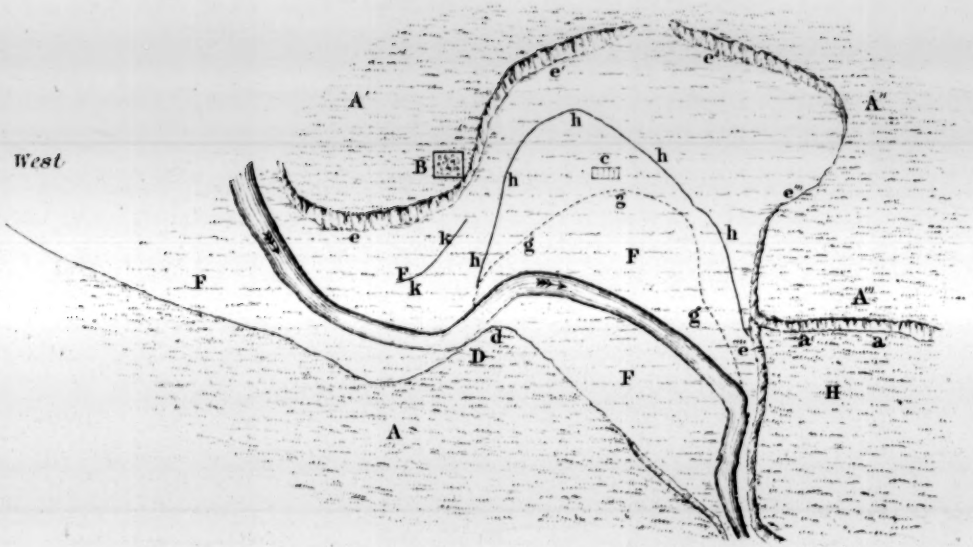
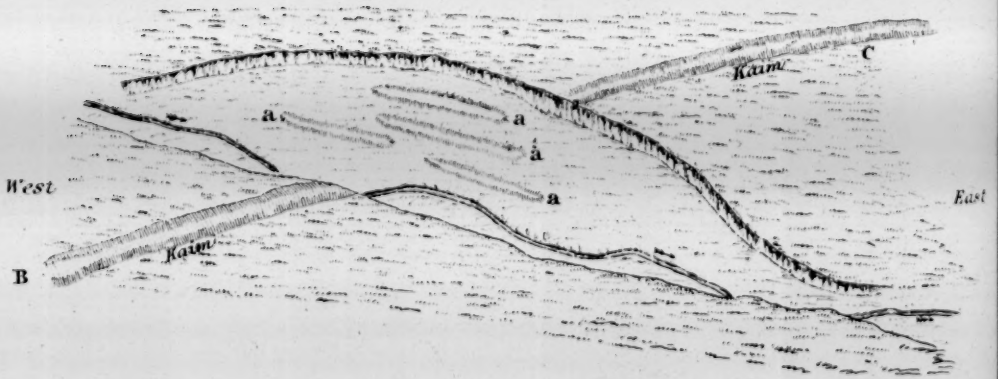
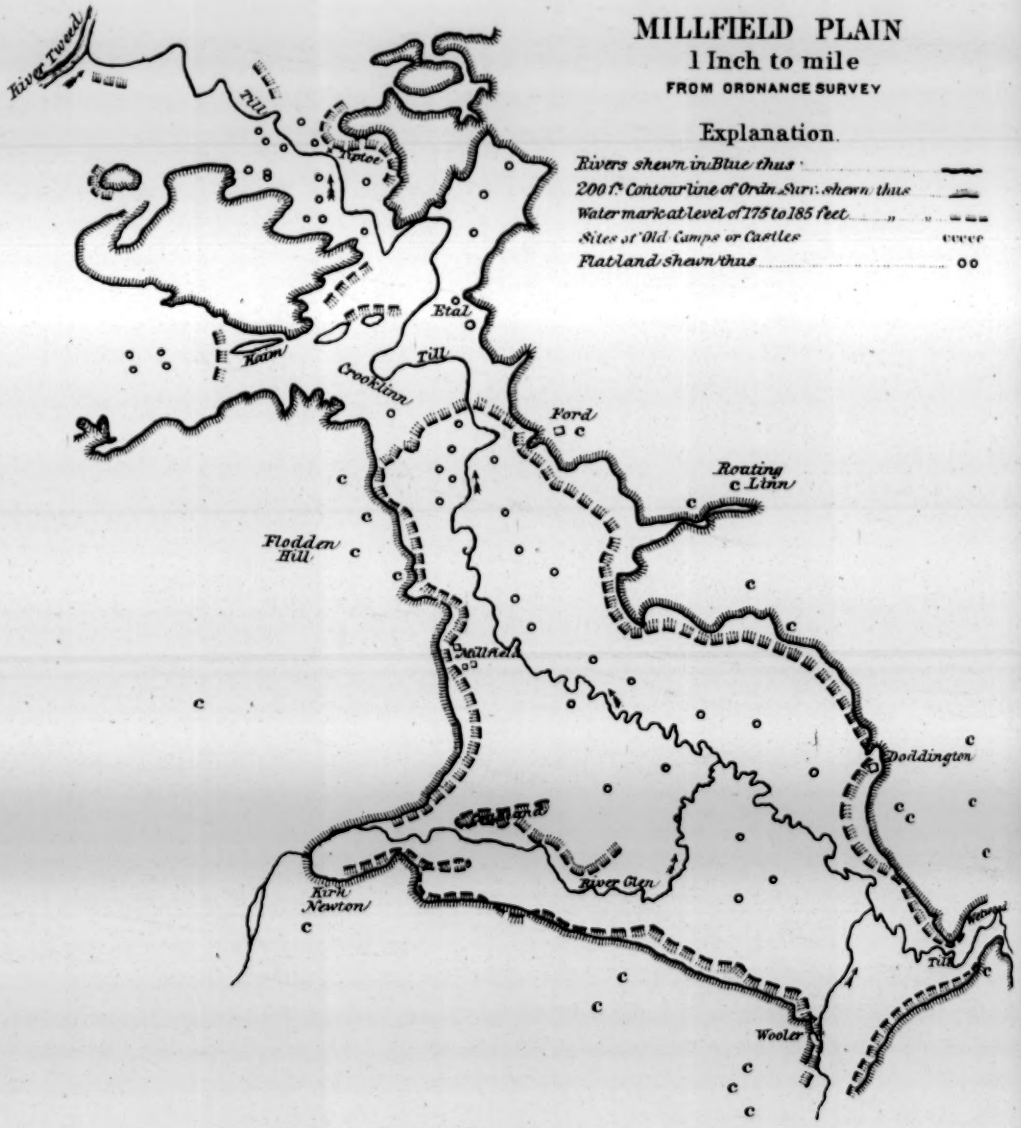


Fig 2. (See page 943 of text)

Oxendean Kaim.



(See page 528 of text)



BRITISH
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VALLEY OF TWEED FROM SEA TO SUMMIT LEVEL AT MOSS PAUL AND ST. MARYS LOCH TO SHOW DIRECTION OF DRIFT RIDGES.

Heights in feet above the level of the Sea, thus 573



XXV.—*Notice of High-Water Marks on the Banks of the River Tweed and some of its Tributaries; and also of Drift Deposits in the Valley of the Tweed.* By DAVID MILNE HOME of Wedderburn, LL.D. (Plates XXXV.–XXXVIII.)

(Read June 7, 1875.)

A few years ago, a memoir on high-water marks on the banks of the Rivers Earn and Teith, in Perthshire, by the Rev. THOMAS BROWN, was read in our Society, and published in our Transactions.

The only other Scotch geologist, so far as I know, who has alluded to the existence of river terraces, much above the level of existing floods, is the late Dr ROBERT CHAMBERS. In his work, entitled "Ancient Sea Margins," Dr CHAMBERS specifies many Scotch rivers, in the valleys occupied by which, he had seen terraces, at considerable heights above the rivers and above the sea.

The explanations of these high river terraces given by the Rev. Mr BROWN and by Dr CHAMBERS respectively, are different. I venture to entertain doubts respecting the soundness of both explanations; and as the subject is of some interest, it appears to me that farther inquiry is desirable.

Dr CHAMBERS was under the belief that almost all the high-level terraces examined by him on the Tweed, Tay, Clyde, and Spey, were *horizontal*, and therefore not formed by rivers. He did not suggest, that they had been formed by lakes. He considered them sea beaches.

The Rev. Mr BROWN, on the other hand, states that all the high-level terraces which he examined on the Rivers Earn, Teith, and their tributaries the Turrit, Keltie, and Ruchil, *slope with the streams;* and he ascribes their formation to river action. He however came to this rather remarkable conclusion, that the highest of these terraces on the Earn, which is about 57 feet above the channel of the river, and in like manner the highest terraces in the other four rivers, had been formed by floods in these rivers, *whilst flowing in their present channels;* to account for which floods, he supposes the existence of what is called "a pluvial period" in Western Europe. The floods in the Earn now, seldom rise higher than 10 or 12 feet. A rise of the river in its existing valley, of 57 feet, certainly would imply a climate and state of things very different from the present.

Startling as this conclusion appeared, I found that it had been also suggested by another geologist, Mr ALFRED TYLOR, in explanation of the high terraces of

the River Somme in France, a river apparently about the same size as the Earn and Teith. In the Somme valley, there are beds of fluviatile gravel, at the height of about 80 feet above the present channel of the river; and these beds, Mr TYLOR maintains, were deposited by the river in its present channel rising to that height when in flood.

It occurred to me, that some test of the correctness of these views might be obtained by an examination of similar high terraces visible in the valley of the Tweed. I had a good opportunity of making this survey, having long had my residence on the north bank of the river, and being therefore well acquainted with the locality.

I shall also in this memoir describe some other things in districts adjoining the Tweed, which seem to be more or less connected with the formation of the terraces.

I divide my paper as follows:—

1*st*, Notice of the River Tweed, between Melrose and Berwick.

2*d*, Notice of districts adjoining the River Tweed.

3*d*, Theoretical explanations.

4*th*, Views held by other persons.

I.—*River Tweed and its Banks.*

1. The channel of the Tweed at Melrose, is about 270 feet above the sea, and about 30 miles distant from the sea.

The towns of Kelso and Coldstream divide the course of the river into three parts, each part being about 10 miles long. The levels of Kelso and Coldstream, above the sea, show that between Melrose and Kelso, the gradient or fall of the river is at the rate on an average of 17 feet per mile; between Kelso and Coldstream, of 7 feet per mile; between Coldstream and Berwick, of 3 feet per mile. The diagram fig. 1 represents these several gradients.

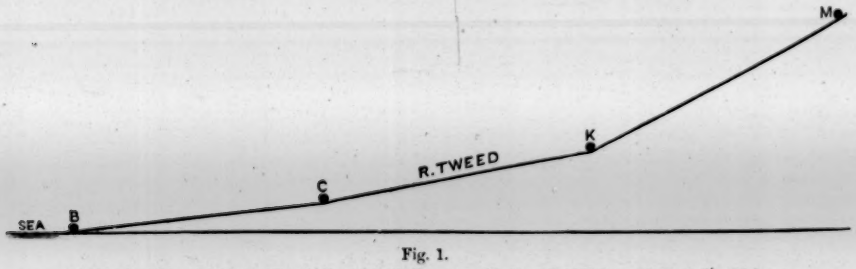


Fig. 1.

B, Mouth of Tweed at Berwick; C, Tweed at Coldstream; K, Tweed at Kelso; M, Tweed at Melrose.

That rate, however, is on the assumption that the river runs from one point to another in a straight line. If in consequence of its windings, 60 miles instead of 30 be assumed as the length of its actual channel, the rate of fall per mile would be very much less.

The neap tide comes up the river to the Chain Bridge, about 5 miles from Berwick, and the spring tide about 2 miles higher up.

The banks of the river along its course differ in height and in distance from one another, as in most rivers.

Where the river passes between hard rocks, the banks are high, the width between them small, and the stream generally deep; when not deep, it is rapid.

Where the river passes through soft strata,—such as marl, clay, gravel, or sand,—the banks are low, with a greater width of stream.

The terraces or flats noticeable along the course of the river, exist chiefly on the kind of banks last described. The stream wears away and undermines detrital materials more easily than rocks; so that marks are most frequently on banks of detritus.

2. The first flat or terrace with its bounding cliff which I notice, is the lowest in level, and therefore the most recent.

The base of this cliff was reached by a flood which occurred on 9th February 1831. It has not, since that date, been again reached.

There had previously been two floods in the memory of persons still alive—one called the Roller Flood,* and the other, the Berwick Fair Flood,—neither, so remarkable as that in 1831.

The fact of this flood of 1831, having risen higher than any previously known, accounts for the circumstance that, at about twenty different places between Melrose and Berwick, individual proprietors, apparently without mutual concert, thought it right to mark, some by stones, others by brass plates, the precise height to which the flood reached on their lands or houses.

The following table indicates the height above the present ordinary summer level of the river, to which the flood of 1831 rose.

Places.	Height.	Remarks.
	Feet.	
Berwick, . . .	15	Pointed out by harbour master.
New Water Haugh, . .	13	Given by gardener, being a mark made on a tree.
Whitadder Mouth,† . .	12 or 13	Base of lowest visible cliff.
Whitadder, Canty's Bridge,	12	Do. do.
Gainslaw, . . .	19	Water rose up two steps of lobby in farm-house.

* I have not learnt in what years, these two previous floods occurred. The Roller Flood takes its name from the circumstance that a large wooden roller for farm purposes, lying on Leeshaugh, near Coldstream, was lifted by the flood, carried down to the mouth of the river, and stranded on Spital beach. Having been recovered, it was sent back to Lees. Probably this flood occurred in October 1797, the year in which Kelso Bridge was swept away.

The Berwick Fair Flood is supposed to have occurred in the month of May,—that being the month of the principal fair at Berwick. Mr Jeffrey, in his "History of Roxburghshire," mentions that a flood occurred in the Teviot in the year 1846, which was greater than that in 1831. In the Tweed, this flood did not rise so high as that in the year 1831.

† Near the junction of the Tweed and Whitadder, a considerable change has visibly taken place in the course and levels of both rivers. The Ordnance Survey Map indicates that the Tweed below

Places.	Height.	Remarks.
	Feet.	
Yard Ford, . . .	15	Base of lowest visible cliff.
Start Fishery, . . .	18	Do. do.
Tweedhill, . . .	21	Marked by a stone with inscription.
Chain Bridge, . . .	20½	Marked by an iron bolt put into north pier.
Field, west of do., . .	23	Marked by a stone with inscription.
Horncliff (east of), . .	22	Base of lowest cliff.
Horncliff (west of), . .	20	Do. do.
Norham Castle (opposite to),	23	Base of lowest visible cliff.
Upsetlington Boathouse, .	23	Mark on window of boat-house.
Milne Graden, . . .	23	Marked by a stone still standing below garden.
Mouth of Till, . . .	15	Base of lowest cliff. Till valley allowed Tweed flood waters to flow into it.
Coldstream Bridge, . .	22	Height pointed out by Jas. Cunningham of Coldstream, who watched the flood.
Carham Hall, . . .	18	Height pointed out by Mr Huntley.
Wark (opposite to), . .	19	Base of lowest cliff.
Sprouston (east of), . .	16	Do. do.
Hendersyde, . . .	15	Do. do.
Broomlands, . . .	16	Do. do.
Kelso (Rosebank), . .	14½	Marked by a stone with inscription.
Kelso (below Bridge), .	14	Marked by brass plate on house with inscription.
Kelso Bridge, . . .	22	Letter from Mr Rutherford, Kelso.
Kelso Mill, . . .	20	Information by the miller, Kelso.
Kelso Teviot-Foot Mill, .	18	Do. Mr Rutherford.
Floors Policy, . . .	14	Marked by a stone 300 yards from river. Stone bears that flood reached the spot at 2 P.M.
Makerstoun, . . .	13	Marked by a stone fixed by the late Sir Thomas Macdougal Brisbane.
Rutherford Mill, . .	14	Marked on mill door.
Mertoun, . . .	14	Marked on a tree pointed out by Lord Polwarth.

On the Tweed, below Drygrange Bridge, two lines are visible,—one on each side of the river at 12 feet; and the second on the left bank, at a height of about 16 feet.

On a general study of the foregoing table, one or two remarks occur.

It will be observed that the flood line is higher at some places than at others.

The difference may be accounted for, in most cases, by the circumstance, that where a river runs in a narrow channel between banks nearly vertical, the stream in a flood must rise more in those parts of its course, than where the banks slope upwards at a low angle. Thus, whilst the Tweed at Coldstream, Kelso, and Milne Graden rose 22 and 23 feet, at the mouth of the Till, situated

Gainslaw farm has left traces of an ancient bed, at least 50 yards north of its present channel; and at that place the south bank shows a high cliff, indicating the action of the stream against it, probably after the river left the north bank. But there are more ancient records of change than those noted by the Ordnance surveyors. About ¼ of a mile to the north there is a steepish bank, the base of which is now 21 feet above the surface of the Tweed; and in the Whitadder valley there is a corresponding bank at the same height, showing that the two rivers had once united, at or near a point about a mile from where the Whitadder now joins the Tweed. The flood mark of the Whitadder now is only about 12 or 13 feet above the stream in its present channel.

between those places it rose only 15 feet. The flooded waters would expand up the valley of the Till, instead of rising vertically in the Tweed.

But notwithstanding this circumstance, it rather appears that in several places, there is a line of cliff a foot or two above the level to which the flood in 1831 rose, implying that at some former period, the water did rise a little higher than in 1831.

This may be explained in two ways:—It may have been, that really on some former occasion beyond tradition, there occurred a greater flood, of which the traces were obliterated, except at a few spots; or it may have been, that more than a 100 years ago, the river, at these parts of its course, flowed in a slightly higher channel.

On the other hand, it is rather a remarkable circumstance, that all the fords across the Tweed, which existed several hundred years ago, remain still shallow and passable. These fords were always guarded on both sides by encampments or fortlets, traces of which yet remain.

There has therefore, during probably more than 1000 years, been little or no change in the actual channel of the river, notwithstanding that the river may now have heavier floods, owing to agricultural drainage.

3. The following is an enumeration of places where old water lines, above the flood marks of the existing river, are observable.

These lines are indicated on the plan of Tweed valley forming Plate XXXV. They are shown by thick black lines, with numbers which correspond to those in the following table.

Name of Place.	Above River.	Above Med. Sea Level.	Remarks.
1. Berwick Town (north side of river),	30	22	1. Bridge Street forms a flat, along the base of a steep bank, up which bank several streets have been formed,—as for example Hyde-hill Street. Flat or terrace here.
2. Tweedmouth (south side of river),	34 or 35	22	2. There is a sloping bank visible, bounding the flat, at this level.
3. New Water haugh (north side of river),	35	22	3. Sloping bank visible for several hundred yards.
4. Yarrow haugh (south side of river),	34		4. Considerable flat, bounded by sloping bank. Height ascertained by aneroid and levelling.
5. Lowhaughs, near junction of Whitadder with Tweed (north side of river),			5. There are three water lines here, as shown in the following section, viz., A, C, and E.

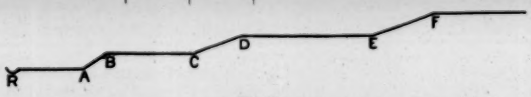
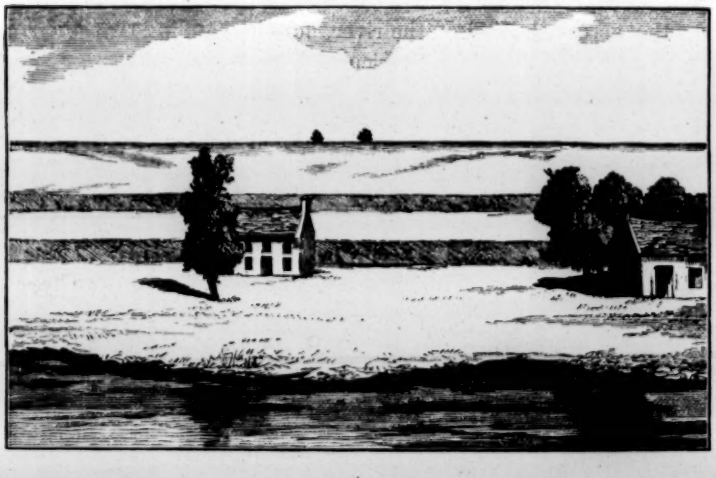


Fig. 2.

R River Whitadder, with haugh land to A covered in ordinary floods, about 6 feet above river.
A B a sloping bank of 19 paces made by these ordinary floods, rising to a level of about 16 feet above river.
B C a flat of about 60 paces reaching to C, about 17 feet above river, where a bank occurs undermined by highest existing floods.
C D sloping bank of about 73 paces, reaching up to a flat D E, having a width of about 120 paces, bounded by another sloping bank E F, the base of which, E, is about 40 feet above river.
Above F there is a general flat about 76 feet above river.

Name of Place.	Above River.	Above Med. Sea Level.	Remarks.
			The above section and measurements were taken along a line of hedge on Ordnance Map running south to north from the river. The diagram is not made to any scale. On both sides of the Whitadder here, there is a very extensive flat. On the west side of the river, the bank bounding this flat has its base line about 50 feet above the river. This flat land and the high bank bounding it, before being cut through by the river Whitadder, have sloped very gently towards the east. The area is so extensive, that only a lake or an arm of the sea will explain it.

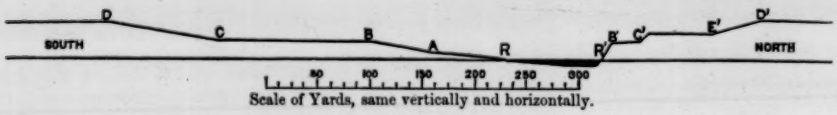
Field between Paxton Toll and Gainslaw House (North Side of River).



6. Field between Paxton Toll and Gainslaw House (north side of river). See sketch given above.	36	49	6. There are two flats and water lines here. The lowest is bounded by a bank, the base of which is about 19 feet above the Tweed, at low water. The upper flat is bounded by a higher and steeper bank, the base of which is a little *below* the 50 feet O.S. contour line, and about 36 feet above the river. The distance between the two water lines here is about 72 paces. The distance of the lower line from the river is about 150 paces. These banks extend the whole length of two Gainslaw fields;—the upper bank stops on the east side of Gainslaw House. Its continuation eastward has been obliterated by the action of the Tweed on the bank, which is here unusually steep. Both banks cross the "*Bound road*" into Paxton Field, of Easter Finchey.

Name of Place.	Above River.	Above Med. Sea Level.	Remarks.
7. Start Fishery (south side of river),	52	56	7. The bank, and the flat stretching from it, are striking objects. The bank here is *above* the O.S. 50 feet contour. Height above river, ascertained by aneroid. The bank is continuous here for nearly half a mile. On examining this bank by a telescopic spirit-level, along a distance of about 600 yards, the base of the bank is found to slope down eastwards several feet;—it is 52 feet above river at westermost point. The bank at its east end has a remarkable curve towards the S.E., as if the river formerly, or a branch of it, had flowed in that direction round the knoll K on map (*see* Plate XXXV.). In the field S.W. of Yard Ford houses, which is about 38 feet above the sea-level, there is an indication of an old water course having flowed towards the east. When the telescope level, planted at the base of the bank, is directed to the bank on the north side of the river, the following is the result. The bank at Gainslaw towards the east, is a few feet lower than the ground on which the telescope stands. There are banks and flats in Paxton Policy on the north bank immediately opposite, which correspond in level. There is a bank at Tweedhill (*i.e.*, up the river) slightly above the level.
8. Paxton Policy. In the Tweed braes old grass field, and in the plantation to the west, there are flats bounded by a steep bank (north side),	53	54	8. The terrace in the field is interrupted by a mass of rock, standing about 20 feet above it, and on which there is no river marking. The banks east and west of this rock, consisting of softer materials, being capable of erosion, show a cliff. The river, when it formed these banks on both sides in this part of its course, must have inclined to run as at present, more towards the north than towards the south. That is to say, the current must have been most rapid on the north side, so that sediment brought down by floods would be deposited chiefly in the stiller water on the south side. Hence on the north side, in this part, the bank is much steeper than that on the south side, and the flats bounded by the flood lines at both heights (viz., the old one at 43 feet, and the existing one at 19 feet above the river) are much narrower on the north than on the south side. The inclination of the stream towards the north may be accounted for by the circumstance that about half a mile up the river, there are high rocks on the south side, causing the river to slant off in a N.E. direction, even from an early period, when the river occupied a channel from 40 to 50 feet above the present channel.
9. Tweedhill Policy (north side),	54	57	9. There is a bank (east of the house) facing the river, the upper part of which is covered with trees, the lower part reaching down into an old grass field where it meets a terrace. The spirit-level on this terrace coincides with the base of the bank on the south side of the river. The flat and its bank are continued in the field west of Tweedhill House; and will be found to be on exactly the same level as the base of the projecting rock No. 10 on the opposite side of the river.

Section of River Banks at Paxton. Hor. and vert. scale the same. Referred to in 7 and 8 of foregoing list.



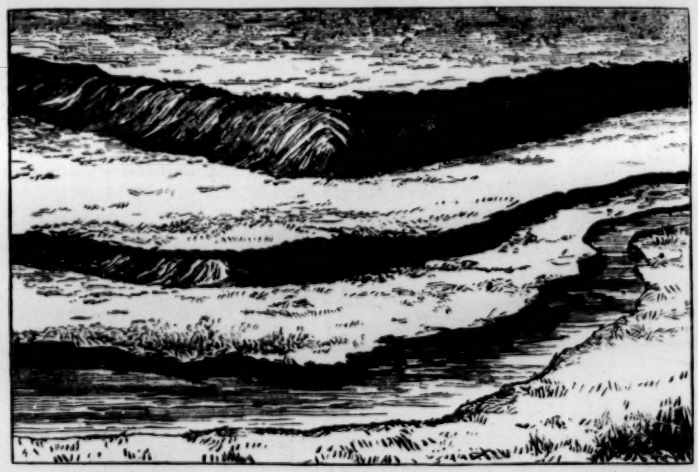
Scale of Yards, same vertically and horizontally.

R R′, the River Tweed, here about 98 yards wide, and about 10 feet above med. sea level.
A, the point to which river reaches in high floods, about 18 feet above river, and 70 yards from river.
A B, a sloping bank of about 62 paces.
B, about 46 feet above river.
B C, an extensive flat, about 143 paces wide here, and running with river for half a mile or more.
C, about 52 feet above river.
C D, a sloping bank of about 110 paces in width.
D, about 102 feet above river, from which point an extensive flat runs south as well as east and west.
B′ C′, a flat about 23 yards wide about 52 feet above river.
A sloping bank of about 13 yards rises to north, when a flat C′ E′ occurs about 62 feet above river for 56 yards.
At E′ this flat is bounded by a steepish bank of 40 yards, reaching to the general flat of district D, which is here about 98 feet above river.

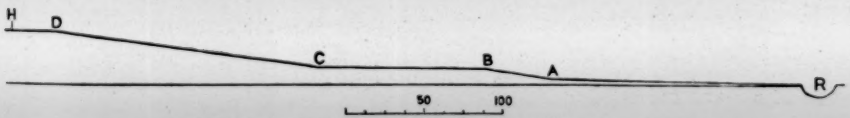
Name of Place.	Above River.	Above Med. Sea Level.	Remarks.
10. Projecting rock, 30 feet long and 14 feet high, about 100 yards on east side of Chain Bridge (south side of river).	55	58	10. This rock has all the appearance of having been hollowed out and smoothed by water. The hollows in the rock are filled with mud and small water-worn pebbles. When this deposit was taken out, the surface of the rock filled with it was found smoothed, as frequently seen in rivers. This rock, when viewed with the telescopic spirit-level from the north bank of the river, is apparently a little higher than the base of the cliffs Nos. 7 and 8.
11. Bit of flat land a quarter of a mile west of Chain Bridge (north side).	54	58	11. This shelf is bounded by a steep bank on the north; but being in a corn-field, its original features have been much effaced.
12. Fishwick Church-yard field (north side),	54	61	12. The flat here, bounded by a steepish bank, is of considerable extent. It occurs in two fields. On the south side of the river, opposite to this place, there is a steep bank facing the river, on several parts of which, at a height of from 50 to 60 feet above the river, there are traces of an old water-line.
13. West of Fishwick Church-yard, opposite to Horncliff village (north side),			13. The indication of this bank is very slight, in consequence of being in fields under the plough. Its height above the river is apparently about the same as at Fishwick Churchyard.
14. Rocks below Old Fishwick House (north side),			14. These rocks reach from the river channel to the top of the bank, which is about 80 feet above the river. There are portions of rock facing the west nearly to the top, much worn and rounded, as if by water.

Name of Place.	Above River.	Above Med. Sea Level.	Remarks.
15. Bank two miles west of Horncliff village (south side),	30 to 40	70	15. For almost half a mile, the south side of the river here shows two flats, with corresponding banks. The relative heights and distances will be understood from the following sketch and section :—

Sketch of Banks on South Side of River Tweed, 2 miles West of Horndean Village, referred to above, No. 15.



15. *Bank west of Horncliff, on south side of River looking from east.*

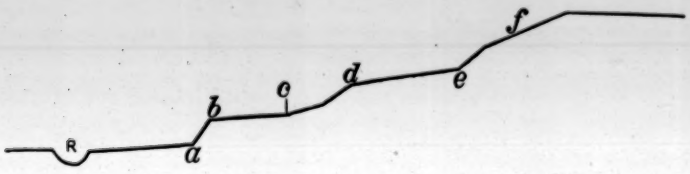


Scale same for Vertical Height and Horizontal Distances, in yards.

R, the River Tweed.
A, 14 feet above R, and 170 paces distant from river.
AB, a slope of 41 paces up to a flat, 29½ feet above river.
C, base of steepish bank, 36 feet above river, and 116 paces from B.
D, slope up from C, 190 yards, and 89 feet above C, and 125 feet above river, angle of slope about 22°.
The above section from A to D was taken along a hedge marked on Ordnance Survey map.
At a place about half a mile farther west, where the two opposite banks are nearer one another,—A the lowest bank is about 20 feet above the river, and C is 40 feet.

16. Horndean Burn, junction of, with Tweed (north side),			16. On each side of the burn, and more particularly on its west or right bank, there is a terrace bounded by a steep bank, the base of which is about 40 feet above the river. The north bank of the Tweed is here about 117 feet high ; and it is through this bank, consisting entirely of sand and fine gravel, that a small stream has excavated a valley, leaving a terrace on each side at the height above mentioned.

Name of Place.	Above River.	Above Med. Sea Level.	Remarks.
17. Bank opposite to Norham Castle (north side),	49	68	17. The different flats and slopes here will be best understood by the following outline :—



R being the river, for about 100 yards there is a flat which rises slowly to a bank at *a*, the base of which is 21 feet above the river,—probably the highest flood-line of the present river.

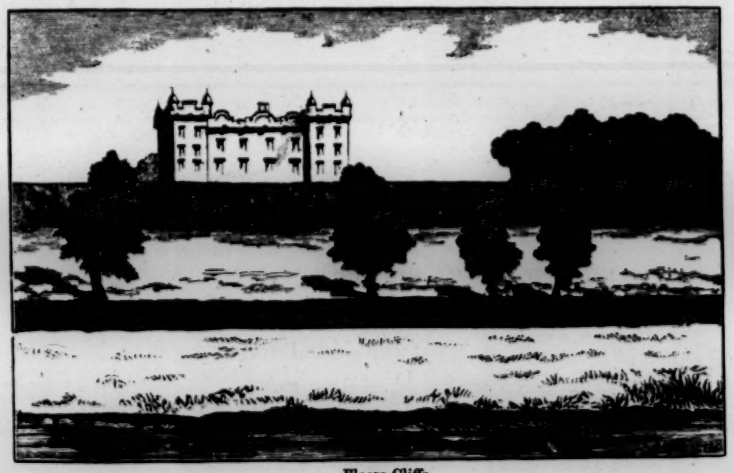
At *c* the 50 feet O.S. contour crosses about 31 feet above the river, and about 147 yards from it.

At *d* the height above the river is about 40 feet, and at *e* 49 feet, at a distance of 252 yards from the river. This cliff is also visible at Frockham, about half a mile to eastward.

The bank, *e f*, is about 45 feet in height, making the highest flat 94 feet above the river, and 115 feet above the sea.

18. Norham village (south side),	40		18. The church and oldest part of the village have been built on a kaim of gravel and sand, lying E. and W., nearly parallel with the river. The ridge of this kaim is about 28 feet above the river. The base of the old river bank is from 35 to 45 feet above the river, and half a mile distant from the river.
19. Ladykirk and Milne Graden (north side),			19. There are faint traces of a bank, with a flat from it, in the policies of these places, about 40 feet above the river, running considerably above the 50 feet O.S. contour line.
20. River Till, at its junction with Tweed on west bank,	37½	70	20. On the west side, the lowest water-line is 15 feet above the stream; but there is on the east side a flat haugh 20 feet above the stream. Between Twizel bridge and Twizel mill, the haugh land is 18 feet above the stream. At the mouth of the river, on the west side, there is a higher flat 37½ feet above the river, bounded by a sloping bank which goes up to another flat 67 feet above the river. A water-line at this last level is also faintly indicated on the north bank of the Tweed opposite.
21. Haugh land opposite to Lennel Churchyard (south side),	34		21. The haugh land here is bounded by a bank, the base of which is about 34 feet above river.

Name of Place.	Above River.	Above Med. Sea Level.	Remarks.
22. Haugh land east of Coldstream Bridge,	30		22. The haughs here on both sides of the river, are bounded by banks, the base of which is about 30 feet above the river. On the north side there is an interesting boulder of chert limestone at this level. On the bank above this boulder (about 30 feet) another boulder of the same nature shows its top above the surface. Large deposits here of sand and gravel form the old river bank. A sand pit shows a depth of 20 feet.
23. Flat between Coldstream Bridge and Cornhill (south) side),	44	72	23. The turnpike road passes over this flat. On the east side of the road a steepish bank rises from the flat. The line of bank is more deeply curved than is usual when formed by a river. It rather resembles the margin of a lake.
24. Carham—base of old bank (south side),	40 to 45	93	24. The old bank, mostly covered by a plantation, rises very steeply from the flat land to a height of about 70 feet. It is continuous and parallel with the river for about two miles between Wark village and Carham church.
25. Bamnf Mill, Sprouston and Wooden (south side),	34 to 44	100	25. The old cliff is distinctly traceable for three or four miles at the places here indicated. At Wooden Mill the water-line coincides with the base of a trap rock which has a vertical side facing the river. The 100 feet contour line of the Ordnance Survey coincides with the base of this rock.
26. Floors Policy (north side).—*See Sketch.*	35	138	26. The bank is very distinct. A similar line is visible on the south side of the river.



Floors Cliffs.

27. Duke's Fishing Cottage, west of Roxburghe Castle (south side),	25 to 30		27. The terrace above the river traceable for ¼ of a mile. At Merton (Lord Polwarth's), on the Tweed, there is an extensive flat, on which the house stands, about 46 feet above present level of river.

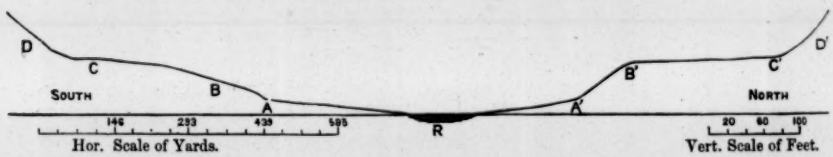
In glancing over the foregoing table, it will be observed that there are at some places high banks, the base of which is not exactly at the same height above the river as at other places. It is not difficult to account for this. In the first place, the author not having had the assistance of a professional surveyor, the measurements stated may be a foot or two wrong. In the next place, the flood-line of a stream may not be everywhere equally high above the channel, because of the water having at some parts of its course risen vertically, and at others spread laterally.

In No. 5 of the foregoing list, the section makes reference to a flat bounded by a sloping bank, the base of which is about 50 feet above the river, and about 68 feet above the medium sea-level. This flat is very extensive. It has apparently extended originally to the west side of the Whitadder, and can be traced for nearly a mile up the right bank. In the Paxton Policy grounds (Tweed Braes) there is a small flat at about the same level. In that case, the flat must have originally formed an area of about 2 miles in a S.E. and N.W. direction, and of half a mile in width. It is so extensive as to suggest rather the bottom of a lake or of an estuary, than of a river. Moreover, the sloping banks which rise from it, are less steep than the banks produced by a river current.

4. In a higher part of the Tweed—viz., at Melrose—thereis a valley with two well-marked *water-lines* on its sides.

The following figure indicates the relative heights of these lines :—

Section of Banks at Melrose, across the Valley in a N. and S. direction.



R, the Tweed. A A′, Flood-line of River. A B C, Sloping ground occupied by town up to railway. D, Eildon hills. A′ B′, a steep bank. B′ C′, Flat land of Gattonside and Allerly.

The above section is on a line drawn across the 6-inch Ordnance Map between the railway station on the south side of the valley and Allerly on the north side. Along the south side of the valley there is an extensive flat about 55 feet above the river, and 325 feet above the sea, occupied by the following places :—viz., Priorbank, Railway Station, Darnick, Chiefs-wood, Hydropathic Establishment, and Bleachfield. Along the north side of the valley, this flat, at the same height, is recognisable in the Pavilion grounds, Gattonside village, and Allerly.

The flat is bounded by the Eildon hills on the south side of the valley, and by the Gattonside hills on the north.

At a little distance from the hills, the flat begins to slope towards the river, and near the river on both sides a steep bank, B B′, reaches down to the haugh land between A A′ and the river. The base of this lower bank is from 12 to 14 feet above the river, and was nearly reached during the great flood of 1831.

It is very evident to any one who walks along the river at Melrose, or who even examines the Ordnance Map, that the river here has not always kept its present course. The Ordnance Map shows a channel on the north side of the main stream, bearing the name of "Little Tweed." A part of the "Ana," or flat land between the river and the south cliff A, bears the name of Gattonside Haugh, implying that at one time the river did not run between that haugh and the village, as it now does.

Mr CURLE, who is proprietor of a farm in this haugh land, informs me that about fifty years ago, a deserted river channel existed, about 100 yards south of the present channel, which his father filled up.*

That the steep bank at A A′ on the foregoing figure was made by water flowing by it, and undermining it, no one can doubt. Some persons have suggested a lake; and there is no doubt that a barrier at the place called the Red Heugh, where the high ground on each side now shows a gorge narrow and high, might have existed, and dammed back the waters. But the base of these banks *slopes down towards the east* with the river, and so excludes the idea of a lake.

The origin of the higher flats between B and C is more problematical. In the first place, as the banks bounding these flats are in most places separated by a space of three-fourths of a mile, it is not likely that the river could have meandered so much, as to produce these banks. In the second place, the base bounding these flats next the hills, seems horizontal. Judging by the 300 feet contour line of the map, the flat, wherever it can be seen, forms a line pretty uniformly about 25 feet above that contour. These considerations suggest the existence of either a lake or an arm of the sea; but more probably a lake, inasmuch as the existence of the higher land which exists towards the east, must have excluded the sea. A barrier of about 60 feet in height above the present level

* As notice has been taken of a considerable change in the course of the river at Melrose, by which, property once on the north side became transferred to the south, a similar case may be mentioned as having occurred between Carham and Cornhill. At this place there is an extensive haugh on the south side of the Tweed, now under cultivation. It is bounded on the south by a bank about 50 feet high, and exceedingly steep. It is plain that this bank has been formed by the river when it ran about 25 feet above its present channel. A hollow along a part of that bank at its base, has from time immemorial gone by the name of "Dry Tweed." There is here a portion of land possessed by Sir JOHN MARJORIBANKS, Bart., as part of his estate of Lees, which estate is situated on the north side of the river, and in Scotland. The river is here the boundary between England and Scotland. This bit of land extends to 12 or 13 acres. Sir JOHN's right having been questioned, he established it by old plans in a court of law some years ago.

of the river, would fill Melrose valley up to the line C C′ in the above diagram, fig. 9. The slopes from C to B, and from C′ to B′, would of course be portions of the lake bottom, the central part between B and B′ having been scoured out by the river after the Red Heugh barrier gave way.

It is not probable that the entire barrier would disappear at once. It would be gradually worn down, and the surface of the lake would subside slowly, whilst the stream in the central parts would also be removing the detritus. A small arm of the lake appears to have covered what is called the Duke's Meadow, where, in making excavations for a new gasometer lately, beds of fine clay (blue, yellow, and red in colour) were discovered, probably deposited in still water, the surface of which must have been at least 25 or 30 feet above the present bed of the river. A small stream comes here from the Eildon hills, which may have brought down sediment.

There are traces in Melrose valley, of a flat even at a higher level than that last mentioned. Along the north side of the valley there are patches of one at about 400 feet above the sea; and it is traceable also on the south side of the valley between Abbotsford and Broomlees.*

5. When the *tributaries of the Tweed* are examined, indications of high water lines are found on their banks, similar to those on the Tweed itself. Three of these tributaries—viz., the Whitadder, the Till, and the Leader—may be noticed.

(1.) The *River Whitadder*, in the higher part of its course, near Cockburn Law, presents on its left back two old haughs—one apparently about 30, and the other 60 feet above the present channel. The highest floods now have, in this part of the river, never been known to rise so much as 7 feet.

At Preston, about four miles lower down, there have been important changes, both in the level and in the course of the river, which deserve description. On Plate XXXVI. fig. 1, a diagram is given to explain the changes.

The present flat haugh land is indicated by the letters F. The scale is 6 inches to the mile. The river in its present course is indicated by the arrows.

Bounding this haugh land on the north and east, there is a very steep bank, about 42 feet above the haugh, which is indicated by the letters e, e', e'', e'''. The base of this bank slopes down eastward as the channel of the river does. This can be proved by studying the Ordnance Survey contour line of 300 feet,—it being below the base of the bank at e on the west, and above the top of the bank at e''' on the east. The materials composing the bank are entirely

* Robert Chambers, in his "Ancient Sea Margins" (p. 180), refers to two terraces noticed by him near Abbotsford and Galashiels—one 346, and the other 395 feet above the sea.

detrital, *i.e.*, boulder clay and coarse gravel, except at e'''', on the east side of the diagram. There, the old red sandstone rocks have been cut through to the depth of about 12 feet, with 8 feet of gravel over them.

The base of the steep bank *e* on the west side of this diagram, is about 23½ feet above the river near it, and at e'''' on the east side of the diagram from 18 to 20 feet above the river at that place.

The highest flood remembered was in the year 1846. The river then rose about 6 or 7 feet. It overflowed the haugh up to the line *h*, *h*, *h*, where there is a hedge and sloping bank, forming from time immemorial the boundary between the parishes of Dunse and Buncle.

C is a woollen factory, the water-wheel of which is supplied by the lade *g*, *g*, *g*. This lade about fifty years ago was the channel of the river.

The present channel, shown by the arrows, was a new cut made by the late Mr Wilson of Cumledge, who erected the factory.

The cliff *a' a'*, on the east side of the diagram, is about 30 feet high, and runs along and above the flat ground H. The base of the cliff *a' a'* is about 22 feet above the river nearest to it. It has all the appearance of a river bank, made at some former period when the river ran in a nearly straight course from west to east.

The line *k k*, on the west side of the diagram, indicates an intermediate bank between the steep bank *e e'* and the lower bank *h h*.

These facts indicate the following changes in the level and direction of the river in this part of its course :—

1*st*, The river, after emerging from the rocky fissure between Cockburn Law and the Stanesheill, ran originally over the flat district, A, A', A'', A''', which is from 70 to 80 feet above the present channel.

2*d*, The river then began to cut for itself a groove through the coarse detritus lying over the rocks, the groove being for a time stopped from getting deeper, by a whinstone rock, which occurs at D. The river then probably ran in a tolerably straight course, traces of which are the bank at *e* and the bank at *a' a'*, about 22 or 23 feet above the present channel.

3*d*, The river, in continuing to cut its way to a lower level, was diverted by the rock at D more towards the north, and then it began to form the bank *e*, *e'*, *e''*, *e'''*.

4*th*, The obstruction at D having been partially overcome, the river sank to a somewhat lower level, and formed a bank, of which a trace exists at *k k*.

5*th*, It next occupied a still lower channel at *g*, *g*, *g*.

The River Whitadder, therefore, in this part of its course, has formerly run in a channel, which was at least 15 feet above the level reached now by its greatest floods.

(2.) The *River Till*, except where it joins the Tweed, presents, in the lower

parts of its course, no old terraces. Its banks are composed chiefly of high rocks. It is only when river banks are composed of detrital matter, that old flats or cliffs need be looked for.

But in the upper parts of the Till, where it meanders through the extensive flat district called Millfield Plain, there are on the rising grounds surrounding the plain, lines of water action, which deserve attention.

The annexed diagram (on Plate XXXVII., reduced from the Ordnance Survey) shows, by the blue line, the course of the River Till from its junction with the Tweed, upwards to the south, and that of its two tributaries—the Glen and the Wooler Water—both of which come from the Cheviot hills situated to the S.W.

The interrupted line surrounding Millfield Plain indicates a bank or slope of land up from the plain, which, along the North, West, and South, and part of the East margin of the plain, is very noticeable.

The continuous black line more or less parallel with the interrupted shaded line, and almost everywhere outside of it, is the Ordnance Survey contour line of 200 feet above the sea.

The interrupted line indicating the base of the sloping bank, has an average height of from 175 to 185 feet above the sea. From the base of this bank, flat ground extends everywhere towards the central parts of the plain. To the east of Kirknewton, this flat ground is occupied, near the base of the bank, by a number of large boulders, apparently of the same rock as the rocky hill to the south, and from which they may have fallen.

It has been mentioned, that the sloping bank is least noticeable on the east side of the plain. No rivers flow into the plain from the eastward, or along the east side of the plain. Therefore no current existed along the east side of the plain, to form a cliff or bank. The general surface of the country on that side, owing to dip of the strata towards the east, slopes *away from* the plain; so that the rain which falls, flows eastward. The Cheviot hills bound the plain on the west, and from these hills several small rivers and streamlets flow down upon the plain on that side.

But even on the east side there are occasional traces of a slope in the surface, down towards the plain, as at Doddington and Fenton. The slopes on that side, are at a very small angle. Therefore the old beach line on that side is at a greater distance from the 200 feet contour line, than on the sides of the plain where the banks are steeper.

Millfield Plain is about 7 miles long, and at one place about 3 miles wide.

The question may now be asked, What has formed this a water-line, which is more or less horizontal, round this extensive area?

It seems very unlikely to have been formed by a river or rivers; for the banks

formed by them would not have been horizontal, especially on so great a distance as 7 miles.

These banks suggest rather the presence of a large body of water, which covered the plain, and into which streams flowed from the adjoining hills.

There are several circumstances which confirm this inference as to the prevalence of a large body of water, over the plain.

Thus, towards the central parts, there is a great depth of fine clay, and beds of gravel, which are generally above the clay.

On Ewart estate, the proprietor, Sir HORACE ST PAUL, informs me that the following borings were made by him :—

On the low haugh land opposite to Humbledon buildings, where the clay is at the surface, he bored down 70 feet, and did not get through it. He "went through a few thin seams of gravel."

At another place, in boring for water, he went first through 25 feet of dry gravel and sand; at that point, having reached the level of the River Glen not far off, he went through more gravel and sand, heavily charged with water, for about 20 feet, a thick bed of clay was then reached. This clay bed was bored to a depth of 100 feet when the rods broke. There was nothing in the clay but a few thin seams of gravel.

Sir HORACE mentions that below these beds of clay, there is sandstone rock. At one place he reached the rock under about 50 feet of sandy clay. But the rock was shattered and extremely hard,—owing, as he supposes, to disturbances by the Cheviot porphyry.

At Flodden Brickwork, situated on the north-west side of Millfield Plain, the clay is mud and sand in combination. It is known to be at least 30 feet deep. A bed of river gravel lies over it about 10 or 12 feet in thickness.

Near Humbledon buildings, having heard that shell marl and peat had been found, I took an opportunity of examining both deposits. The bed was found at the base of a bank at a level of about 180 feet above the sea.

Many beds of peat occur in different parts of the plain.

These facts indicate that a large body of water had prevailed over this district, in which sediment had been deposited to the depth of more than 100 feet.

At the south-east corner of the plain, near Weetwood House, there are several heaps of detritus, at the level of the 180 feet line. As it was there that the River Till joined the body of water before referred to, these detritus heaps would be *deltas* of the river.

On the diagram a small letter C will be observed at a number of places near the 200 feet contour line. The letter indicates the position of old camps or

castles which are noted on the Ordnance Map, and traces of which are still visible on the land. It was a common practice in ancient times to form camps near large bodies of water, as they afforded protection on at least one side.* Like protection, however, would have been afforded if, instead of water, there had been an impassable morass, which is very likely to have been the state of matters more recently on Millfield Plain.† On each side of the great morass west of Stirling, there had been in like manner ancient fortresses, the Broch of Coldoch being one.

There is evidence that since the 185 beach line was formed, the River Glen has greatly changed its course. Near Kirknewton, the river had joined the body of water;—but since that body of water disappeared, the glen has cut out for itself the channel in which it now runs. On the diagram there will be seen an elongated ridge with the word Coupland on it. This ridge is composed of gravel near the west end and sand towards the east. It had formed originally part of the bottom of Millfield lake or estuary. At first the Glen seems to have run on the north side of Coupland; ultimately it diverged into its present course, running for about a mile along the base of a nearly vertical bank of gravel and sand, which it is still undermining. This mass of gravel and sand (182 feet above the sea) gradually slopes down towards the middle of the plain, being, at the entrance-lodge to Ewart House, 166 feet above the sea. Through this mass, the river has cut its way 40 or 50 feet. Most probably the large amount of detrital matter here, was brought down by the River Glen and other streams, when water prevailed to the height of 185 feet.

To the east of Kirknewton, it may be observed that the interrupted line rises a little above the 200 feet contour line. It is difficult to account for the circumstance. It may, however, be, that the interrupted line here indicates, not a water margin, as it does elsewhere, but only an accumulation of detrital matter washed down the sides of the hills, which at this place are close to the contour line, and unusually steep.

As to the question whether this body of water which prevailed over Millfield Plain was a lake or an arm of the sea, there are but few data on which a satisfactory opinion can be formed.

If it was lake, there must have been a barrier of considerable height and extent, at or near New Etal or Crookham, which is the only place where the

* In the "Berwickshire Nat. Club," vol. ii. p. 345, there will be found a list of old camps, castles, and towers situated in this part of the Borderland. In this list several are mentioned as situated near Millfield Plain, which are not marked in the Ordnance Map.

† The names of "Howmyre" and "Floaty," which refer to places or fields on the lower grounds, probably originate from this cause.

lake could have discharged. In that case, the flat land now existing there, was probably the bottom of the lake ; and as it is 125 feet above the sea, the barrier must have been at least 60 feet high, and about a mile in length, judging by the present configuration of the district. Such a barrier might have existed. Bounding Millfield Plain, at its north end, there are now numerous hills of detritus, which rise up to a level of upwards of 200 feet above the sea. Similar detrital hills probably existed near New Etal ; and these could easily have been cut through, by the action of the united rivers which form the Till at this place.

On the other hand, there are some facts which suggest the possibility of an arm of the sea having stretched up here. It will afterwards be shown, that in the valley through which the River *Tweed* flows, there are traces of extensive flats, at various heights, from 175 to 212 feet above the present sea level.

So also along the valley of the River *Till*, at Tiptoe and Old Heiton, there are several flats at a level of from 175 to 185 feet above the sea, some of them bounded by sloping banks. If these flats and banks are due to aqueous action, the lake theory is hardly admissible, because it must have been a body of water which extended over the whole valley of the Tweed ; and even if the sea stood there, no higher than 200 feet above the present level, there would be a passage into Milfield Plain, not only by the valley of the Till, but by Branxton Valley to the north-west.

There are traces of sea action on the hills round Milfield Plain, at greater heights.

One of the proofs of this fact, is the occurrence of great beds of stratified sand. On the road to the east of Kirknewton there is such a knoll, now used as a quarry, at a height of 220 feet above the sea. On the road from Wooler, by Weetwood House, and leading up towards Weetwood Moor, there are masses of stratified sand at heights from 300 to 400 feet above the sea.

On the side of Weetwood Hill, facing the north, at a height of about 400 feet above the sea, there is a large bit of projecting sandstone rock,* noticeable even from Wooler. On inspection, this rock was found much broken or fissured, and, moreover, rubbed and hollowed out, as by water. The fissures and hollows are filled with water-borne, hard pebbles, several of them of Cheviot porphyry. The following woodcut (fig. 10) gives an idea of the spot.

Then there are several traces of a terrace or shelf on the sides of the hills elsewhere, at almost exactly the same height of 400 feet above the sea. For example, Middleton Hall, the residence of —— HUGHES, Esq., situated

* The rock was pointed out to me by Mr HARDIE, Secretary of the Berwickshire Naturalists' Club.

about 3 miles south-west from Wooler, is on a terrace at this height, which is also traceable along the same (the west) side of the valley for some miles, both north and south. Middleton farm cottages, Earle village, and Homildon churchyard are situated on it, and a shelf on Horsden Hill corresponds. On Doddington Hill, which is on the east side of Milfield Plain, a bench on its side occurs at 400 feet.

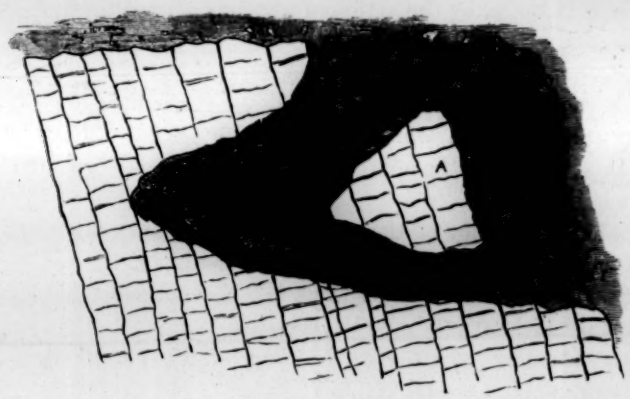


Fig. 10.

A, Sandstone strata.
B, Great Hollow in the Sandstone Rock, filled with water-borne pebbles and sand.

According to that view, the whole valley had originally been filled with water up to the height of 400 feet, and the terraces above named were the beaches formed along the margin of an inland sea.

In the upper part of the Cayle Water (a tributary of the Teviot) in Roxburghshire, there are extensive flats, at a height of about 360 feet above the sea, being the upper surface of huge knolls or mounds of sand,* through which the Cayle has formed nearly vertical cliffs, in some places about 100 feet in height. These large accumulations of sand, of course, imply that sea once stood here in comparatively recent (geological) times, and therefore corroborate the evidences existing elsewhere of a sea which reached to 400 feet, and stood at that level long enough to form indentations on our hills.

With these remarks, I leave the question, as to the nature of the great body of water which covered Millfield Plain. That the sea has left traces of its action on the hills adjoining, at a height exceeding 300 feet above its present level, I think there can be no doubt. But, when the sea subsided to a height of 175

* The Parish Churches of Morebattle and Linton are on the top of sandy knolls, adjoining the Cayle. Their singular position has given rise to legends, notice of which will be found in JEFFREY's "Roxburghshire," vol. i. pp. 41 and 43.

feet, a lake *may* have been formed at a height of 180 or 185 feet above the sea-level, having its overflow at the north end, near Etal and Crookham, kept up by a blockage there, which was in time cut through.

(3.) The *River Leader*, at the town of Lauder, runs through a valley about 3 miles long and a mile wide. Thirlstane Castle stands on an elongated kaim of gravel, whose direction is parallel with the sides of the valley, and whose ridge is about 40 feet above the river. The position of Thirlstane Castle suggests, that the castle had been erected on the kaim for the sake of the protection afforded by the water which originally surrounded it. On the east side of the kaim, there was the river; and on the west side, a deep morass, across which a causeway afforded access to the castle. Between the town of Lauder and Whitslaid Castle (situated about 2 miles lower down the river), traces are visible of what had been the bottom of a lake from 35 to 45 feet above the present bed of the river. After the lake had disappeared, the river cut its way through the beds of sand and gravel which formed the lake bottom—leaving sloping terraces on each side of the present valley, at different levels, still visible at several places.

(4.) Many other places in the Border Counties might be named, where horizontal terraces occur, indicative of lakes long since drained or dried up. The small valley near Grant's House, Berwickshire, through which the North British Railway passes, is an example.

Thornton Loch, in Northumberland, was once a sheet of water a mile long; it is now a green meadow, with banks on each side, betokening its lacustrine origin. Several old camps* and peels had been erected near it, for the protection it afforded.

Primside Loch, in Roxburghshire, is the only one which remains of a series of lakes which had occupied the valleys between Kirknewton and Morebattle.

The former existence of lakes and marshes in Berwickshire is also indicated by such names as "Billie-myre," "Drake-myre," "Dunse Common Myre," and by the general name of "Merse," applicable to the county.

In Roxburghshire, such names as "Mer-ton," "Mer-wich," "Black-myre," "Myre-dyke," "More-battle" (originally "Mere-bottel"), afford like evidence.

6. In the foregoing part of this paper, a description has been given of terraces and cliffs, connected with the River Tweed and its tributaries. But in other parts of the general valley of the Tweed, there are terraces and cliffs, in some respects similar, but probably having a different origin.

* In Eccles parish, at Hardacres, there is an old camp, on the west end of a kaim or high gravel ridge, which had been surrounded by water on three sides. Swinton *Loch* is also spoken of, in "Boston's Memoirs." There is not even a marsh there now.

A. Those to be first noticed, are at a height above the sea of from 105 to 125 feet.

Places on the *South side of the Valley*, beginning near the sea.

This line of cliffs is indicated on the Plan of the Tweed (Plate XXXV.) by a broken line, ------, having figures within brackets, thus — (1), (2), (3), &c. These figures on the map correspond with the figures in this part of the text.

(1.) A steepish bank can be seen from the railway, facing north, running westward from Ord Bone Mill, passing Mount Pleasant on the north, Loanend on the south, and disappearing near Horncliff.

(2.) A trace on Mount Carmel Farm.

(3.) Short lines of a bank at Norham Railway Station, on both the south and the north of the railway.

(4.) A line of bank on north and west of Riffington Farm offices.

(5.) A piece of flat land, with trace of a bank bounding it, south of Twizel Railway Station.

(6.) West side of mouth of River Till, under St Cuthbert's Farm offices, a flat, bounded by steep bank.

(7.) Oxendean Burn, on east side of, south of railway, a flat, bounded by steep bank.

(8.) Campfield, west of Cornhill Railway Station, flats bounded by cliffs.

(9.) Barelees Farm and Toll-bar, several flats bounded by cliffs.

(10.) Between Cornhill and Wark, trace of, in Lamb Knowe field.

(11.) Line more or less continuous from near Wark village, west by Wallace Croft, Redden, Sprouston, and Wooden Mill, where, being only about 30 feet above the river, it is not easy to distinguish between it and the old river bank at about that height. Numerous whinstone boulders show themselves on the part of the line between Wark and Wallace Croft.

Places on the *North side of the Valley*, beginning near Kelso.

(1.) On the bank below Broomlands House there is a terrace, apparently horizontal, in an east and west direction, and bounded by a steep bank.

(2.) The line is traceable through Hendersyde Policy eastwards, till it reaches Edenmouth, where it is interrupted by the River Eden. It seems to go a little way up both sides of the river.

(3.) The line passes through High Ridge Hall and Lochton, at which last place it forms a deep curve or loop—suggestive more of still water than a flowing stream.

(4.) The line crosses the Coldstream road, west of Springhill, and forms a very noticeable steep bank below Birgham village, facing the River Tweed.

5.) Visible again on Haigsfield Farm, where there is a great knoll of sand and gravel, steep on the west and south sides.

(6.) Visible also north-east of Fireburn Mill Toll-bar, and running eastwards as far as Lees, first crossing the road near Coldstream.

(7.) Faint traces west and east of Lennel Churchyard; also on bank opposite to the mouth of River Till.

(8.) No line or bank at or about this level has been noticed till Tweedhill (in Hutton parish) is reached, where a great extent of flat land occurs, bounded by a line of bank on the south, near Tweedhill Lodge gate.

(9.) The village of Paxton occupies a bank sloping to the south. An extensive piece of flat land, 126 feet above the river, lies between the village and the Tweed, and corresponding with a similar flat on the south side of the river.

(10.) A steepish bank, running east-south-east, is seen at this level, on the east side of the River Whitadder, at Grangeburn Mill.

(11.) There is a similar bank at Low Letham, about 2 miles from Berwick.

It is hardly necessary to observe, that the lines of bank mentioned in the foregoing list are by no means distinct. If they were less obscure, their antiquity might be doubted. Of course, it requires some experience before they can be distinguished; but the contour lines of the Ordnance Survey afford great assistance. Indeed, it would be hardly possible to trace them or ascertain their height above the sea, without the aid of these contour lines.

It will at once be perceived, that there are two important differences between the system of terraces last referred to and those first mentioned. Those specified in the Table on pages from page 517 to page 523, slope with the river; whilst the others are horizontal, in an east and west direction. The first set cling to the river banks; the last set appear to have no connection with the river. Looking at the spaces between the set of lines on the south side of the valley and the corresponding set on the north side, amounting in somes place to more than a mile, these last mentioned banks seem *due to a lake or an arm of the sea*, and *not to the river*.

In regard to the terraces on the banks of the Tweed, which I have said slope down eastward, I make a reservation regarding those between Coldstream Bridge and Wark. I am not sure that *they* may not be horizontal; in which case, they could not have been formed by the river. The reasons for my uncertainty are these:—1*st*, The level of these terraces seems to be very nearly equidistant from the Ordnance horizontal contour lines at Wark and at Cornhill. 2*d*, Between Cornhill House and Coldstream Bridge, the line of cliff makes a deep bay or loop, which is indicative more of lake than river. 3*d*, At two places on the north bank of the river (viz., above Lees and above Fireburn Mill), there are beds of small gravel, horizontally stratified, which seem

lacustrine. It was near this last spot that a spear of flint was found, probably used in primitive times for the spearing of salmon.

If these suggestions are correct, there must have been some blockage near Coldstream Bridge. The banks there are high, and approach so near each other, that there is no physical improbability against the lake theory.

B. There is still another line of cliffs and flats in the district, which deserve notice. They are at a level of from 170 to 185 feet above the sea.

Beginning as before on the south side of the valley and near the sea, this system appears at the following places, as shown on the Map (Plate XXXV.) by this line —+—+—+ having on it the same letters as in the text :—

(*a.*) Sunnyside Hill and East Ord, where there is a steepish bank facing the north, having a base line running north-west and south-east, at a level of about 170 feet above the sea.

(*b.*) Middle Ord, a similar bank, running in the same direction.

(*c.*) Between Loanend and Longridge, a bank facing the north, and traceable nearly to Horncliff village.

(*d.*) Thornton school and cottage, a round grass knoll, the base of which is about 170 feet above sea.

(*e.*) Shoreswood Rocks, facing north-west, the base of which is about 170 feet above sea.

(*f.*) Old Heiton and Tiptoe, on River Till, where there is flat land, 180 feet above the sea, bounded by a bank.

(*g.*) Palinsburn. The base of the long ridge or kaim in front of house is 185 feet above sea ; an extensive expanse of flat land to the west, bounded by a steep bank, is about the same level.

(*h.*) At Melkington Hill—another small hill to east—and Cramond Hill, there are steep banks facing north and west, 180 feet above sea.

(*k.*) Hill plantation, north of Cornhill, base of which is 180 feet, above the sea,—steep towards west and north.

(*l.*) Between Carham Railway Station and Kelso Railway Station, a steep bank, traceable south of and nearly parallel with the railway, at about 180 feet above the sea. On Kerchesters Farm, the cliff is from 40 to 50 feet high, and very steep towards the north.

(*m.*) Windywalls Farm, south of Millenden, a similar bank.

(*n.*) At Springwood Park, Old Roxburghe Castle, and Floors, the line at about 180 feet *crosses* the valley, as shown partly by the extent of flat land at these places, but more so by the banks bounding the flat land.

At Floors, the flat is that on which the modern castle stands, bounded by a bank about 180 feet above the sea, fronting south-east. (See page 523.)

(*o.*) The same flat is traceable near Kelso Race-Course, and also to the

north and west of Hendersyde, at all of which places, it is bounded more or less by a bank.

(*p.*) The same bank is noticeable below Hendersyde House, fronting the river.

(*q.*) At the following places along the north side of the valley towards Berwick, a bank, at from 170 to 180 feet, is in like manner distinguishable, viz.—Ednam Hill; Eccles Newtown; Hirsel-law; Lennel Hill; Felloe-hills; Sunwick Farm; Paxton North Mains; High Cock-law; Baldersberry; * and High Letham. A bank, facing the sea, at the height of 170 feet, occurs also at the cemetery north of Berwick.

With reference to the foregoing list, there are two or three points deserving remark.

On the south side of Longridge Hill (Plate XXXV.) there is a remarkable valley, called Murton Dean, the sides of which are from 70 to 80 feet high, and with a bottom about 170 feet above the sea. A mere driblet of water runs there at present,—flowing towards the west, which is inconsistent with the general gradient of the district. The direction of this valley is east and west, and is about a mile in length. Sandstone rocks occur along the south bank, portions of which exhibit rounded surfaces and the circular "pot-holes," known to be often formed by water. That this valley has not been formed by a river or stream, is indubitable. If it was caused by a slip or fracture, what has carried off all the fragments of rock which resulted from the fracture?

With reference to this level of 180 feet, it deserves to be noted that a large number of the boulders in the district are found on spots at exactly this level; as at Roxburghe Castle, Broomlands, Palinsburn, Kirknewton, Carham, High Cocklaw, and Baldersberry.

The terraces and banks in the neighbourhood of Kelso deserve more study than they have yet received. There are apparently two sets—1*st*, Those which slope with the Rivers Tweed and Teviot; and 2*d*, Those which seem horizontal.

Those which slope with the Tweed are partially indicated on the two lists first given in the paper,—viz., 1*st*, A cliff formed by the floods of the existing rivers, the base of which is about 14 feet above the stream (pp. 515, 516); and 2*d*, A cliff whose base is from 20 to 25 feet higher (pp. 517–523).

This higher set, forms the steep and striking bank, from 70 to 80 feet high, which runs through Floors and Springwood Park Policies.

At first I was inclined to look on this bank as having been formed by a

* The original name of this farm was "Boulders Broch"—supposed to mean a broch or fortress formed of boulders. Boulders were formerly there in great numbers, the foundations of strong walls.

lake or an arm of the sea; but being now satisfied by a study of the Ordnance contour lines, that they slope down eastward, I allow that they have been formed by river action.

The flat land, in the Kelso district, about 80 feet above the river, and stretching towards the sea coast horizontally, of course suggests either a lake or an estuary. If due to lake, there must have been a barrier at Berwick, a supposition so unlikely, that the other alternative seems a necessity. The cliffs, which mostly face one another on opposite sides of the valley of the Tweed, are in some places 4 or 5 miles apart; another feature suggestive of an estuary.

A point deserving notice, is the question of exact horizontality of the lines last referred to. Near the sea at Berwick, the line of bank seems scarcely to be higher than 170 feet above the present sea-level; whereas, towards Kelso, the line seems to be between 180 and 185 feet. This circumstance, however, if on a more minute survey confirmed, is not incompatible with the conditions of an elongated estuary.

II.—*Districts adjoining the Valley of the Tweed.*

1. The whole surface of the country *North* of the Tweed, is covered with drift deposits. No rocks are visible, except on the river banks or channels, till the Lammermuir range of hills is reached.

South of the Tweed, there are, at several places, conspicuous outcrops of sandstone and limestone, and one or two whinstone dykes.

The drift deposits are as usual, clay, gravel, sand, and boulders.

The clay occupies generally the lowest position. It is in the lower districts only that brickworks occur. There are none so high as 300 feet above the sea. The clay is generally tough and stoney, almost always containing pebbles, and frequently boulders. From a brickwork at Paxton (160 feet above the sea), there was lately extracted a blue whinstone boulder 12½ tons in weight. The boulder was removed to Paxton Policy, where it now stands.

Beds of gravel and sand abound everywhere, up to at least 1000 feet above the sea.

The beds of sand are extensive. Many near Cornhill, and in lower parts of the River Tweed, show a depth exceeding 40 feet. At Mount Pleasant farm, near Berwick, about a mile south of the Tweed, the tenant bored down 30 feet without reaching the bottom of the deposit.

The following figure represents a section of the rocks, covered by beds of stratified sand and gravel, for a distance of 2 miles along the north bank of the Tweed, between the Chain Bridge and the mouth of the River Whitadder. The bank here reaches to a height of about 90 feet above the river.

The Berwick and Kelso Railway passes through great knolls of sand and gravel at Riffington, and west of Cornhill.

Section of about 2 miles along North Bank of River Tweed between Chain Bridge and Mouth of River Whitadder.



Fig. 11.

The black line at the bottom represents the River Tweed.
The unshaded parts, marked R R, represent Ravines cut by burns through drift and rocks.
The faint broken horizontal lines show Water Marks formed on the Drift.
The Stratified Rocks are indicated by the parallel lines dipping eastward.
The dark shaded parts are Sand; the lighter shaded parts, at top, are Gravel.

In the parishes of Gordon and Greenlaw, about 300 feet above the sea, there are extensive hills of sand and fine gravel. There is sand also along the base of the Lammermuir hills, at heights of from 800 to 900 feet above the sea.

In like manner, on the opposite side of the valley, near the Cheviot hills, there are enormous masses of sand and fine gravel, up to heights of 400 feet above the sea and more, which having been cut through by the small rivers flowing down from the hills, present cliffs nearly perpendicular and occasionally 100 feet high.

In the valley of the Cayle, between Eckford and Kirkbank, there is a kaim or eskar of fine sand, from 400 to 500 yards in length, very round backed. Its width at the base may be about 80 yards. It stands up above the adjoining land to the height of about 30 feet at its west end, and about 50 feet at its east end. Its direction is (by compass) S.W. by W. Its height above the sea is about 200 feet.

The sand and gravel beds are all more or less stratified.

Generally speaking, the gravel lies above the sand.

Occasionally, pebbles occur in the *sand*. Those well rounded consist almost always of hard rocks, not belonging to the adjoining district. Sometimes there are lumps of coal and shale—also so rounded on the edges, as to show that they too have undergone much friction in transport. Near Old Heiton, on the River Till, there are pebbles of buff felspar and coal shale. The former may have come from the Cheviots to the south-west, or Dirrington Hill to the north. The shale may have come from the coal beds near Jedburgh. Lumps of true coal are found in sand pits at Milne Graden and Paxton. As shale and coal is comparatively light, they may have been transported by currents of water, and lodged in the sand beds or banks then forming.

Pieces of coal and shale have been found also in the *clay* of brick works, as at Eyemouth and Broomdykes. Marine limestone pebbles occur in the

Eyemouth brick clay. The nearest strata of coal and lime are in Northumberland, about 20 miles to the south, and in East Lothian about 25 miles to the north-west.

2. The *forms of the sand and gravel hills* in Berwickshire, deserve attention. They are of three kinds—

(1). The commonest are round-backed knolls, the crest of which is from 30 to 50 feet above the adjoining general surface of the land.

One point worthy of notice about them is, that they are generally steepest on the west fronts.

When situated on the south side of the valley, the north-west front is steepest; on the north side of the valley, the south-west front is steepest.

(2). Another form taken by these drift hills, is that approaching an ellipse. Some are as much as a mile in length, and several hundred yards in width.

Kaims in Berwickshire, as represented on the one-inch shaded Map of the Ordnance Survey.

A B

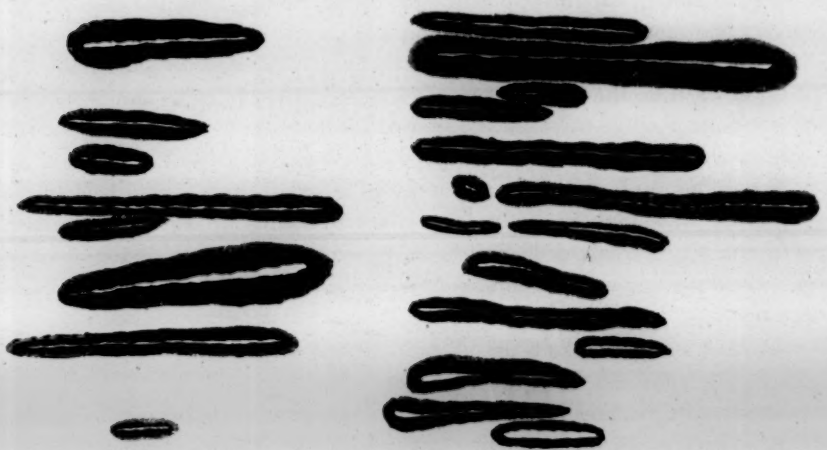


Fig. 12.

The above set of ridges marked A, cross at right angles a line 2½ miles in length, drawn between Dove-Cot Mains and Lennel. The direction of these ridges is about W.S.W. (magnetic).

The above set marked B, cross at right angles a line 2½ miles in length, drawn between Hilton and Ladykirk. The direction of these ridges is about W. by S. (magnetic).

Another interesting feature about these long ridges, is their general parallelism to one another.

As this is a point to which little attention has hitherto been given, the above fig., No. 12, is offered in illustration. The ridges have been copied as exactly as possible from the 1-inch shaded map of the Ordnance Survey. The Ordnance

map itself, if consulted, will show that these ridges prevail through all the lower parts both of Berwickshire and Roxburghshire.*

(3). Whilst in any one district, these ridges when near one another are approximately parallel, it is interesting to observe, that when the whole valley of the Tweed, from the sea upwards is examined, there will be found a considerable change in the direction of the ridges. This is a point which suggests important inferences; and, therefore, some details are excusable.

a. In the first place, an examination of the shaded Ordnance Map of Berwickshire and Roxburghshire reveals this change. When parallel rulers are laid along the ridges in Roxburghshire, it will be seen that whilst the general direction there is N.N.E. *true*, the direction in the lower parts of the valley gradually changes to due east and west, or even a few degrees south of east.

b. An examination of well-formed eskars or kaims in different parts of the valley, gives the following, beginning with those in the higher parts of the valley and going eastward :—

1.	Long Ridge of Sand near Eckford, on River Cayle,	E.N.E.
2.	A Ridge of Gravel and Sand, Castleton Parish,	N.E. by E.
3.	Long Ridge of Gravel and Sand at Riddleton Hill, near St Boswells,	N.E.
4.	Another do., half a mile east,	E.N.E.
5.	Several Ridges of Gravel at North Park Wall of Floors Policy,	N.E. by N.
6.	Ridges, Kaims, and Lines of Bank at Heriot Bank, near Kelso,	E. by N.
7.	Ridge at Kaimflat and Kelso Race-Course,†	E. by N.
8.	Long Ridge of Gravel and Sand at Palinsburn, near Cornhill,	E. by N.
9.	Faddon-hill a Long Ridge near Tiptoe, on the River Till,	E. by S.
10.	Skaithmuir, N.W. of Coldstream,	E. 25° N.
11.	Swinton Parish Ridges,	E. 20° N.
13.	Eccles Parish Ridges,	N.E.
13.	Whitsom Parish Ridges,	E. 15° N.
14.	Horndean Ridges,	E. and W.
15.	Hutton Parish Ridges,	E. 5° or 6° S.
16.	Baldersberry Hill (Berwick bounds),	E. 23° S.
17.	Cocklaw Hill (do.)	E. 24° S.

The ridges in fig. 12 indicate a similar change in direction. Those marked B are about 4 miles lower down the valley than A. The direction of the B ridges differs about 10° from the A ridges.

Representations of these ridges are on the diagram (Plate XXXVIII.) which embraces (of course on a very reduced scale) the whole valley of the Tweed up to the summit level near St Mary's Loch and Moss Paul, about 800 feet above the sea.

On this diagram there is indicated, the general line of the hills bounding

* These ridges of gravel have so influenced the popular mind as to be made use of for identifying places on the Borders. The terms "Long-ridge," "Kaims," "Kaim-know," "Kaim-flat," "Cambridge," "Campton," all have reference to these ridges. The estate of Kaims, in Eccles parish, when its proprietor HENRY HOME, a historian and philosopher, was made a Judge in the Supreme Court of Scotland, supplied him with a title—Lord KAMES.

† The direction of the striæ on the Kerchester Boulders (page 546) and the Carham Limestone rock (page 548) agrees with the direction of the gravel ridges near Kaimflat and Kelso.

the valley on the north and on the south, with several of their heights, as also the general trend of the parallel ridges.

(3). There is a third form taken by these gravel and sand beds.

On Greenlaw Moor there is a ridge of sand and gravel, about 2½ miles in length, having a curved form, the concavity being towards Dirrington Hill, situated to the north. This ridge is from 30 to 50 feet in height, with sides sometimes as steep as 40°. The sand and gravel in this remarkable ridge form separate beds or layers. The following figure 13 is reduced from the Ordnance Survey.

At Oxendean, in the parish of Dunse, and distant from Greenlaw Moor about 4 miles, a similar ridge of gravel and sand runs for 1½ mile. (See fig. 14).

Both of these kaims are on nearly the same level above the sea, viz., about 700 feet. Both are now cut across by small rivers. But an inspection conveys a strong impression, that when originally formed, both of these kaims had been continuous or uninterrupted in their course; in other words, that they were formed before rivers existed, *i.e.*, that they had been submarine banks, when the sea stood 1000 feet at least higher than now.

The interruption of the Oxendean Kaim, where a stream crosses it, is so instructive, that I have attempted to explain it in the diagram (fig. 2, Plate XXXVI.) The kaim itself, running about E.N.E., lies on a mass of gravel. It is interrupted, however, by a deep glen or gully, at the bottom of which runs a small but rapid stream. The cliffs forming the sides of this valley show, that they are composed of gravel and sand for some feet from the surface, and covering old red sandstone rocks, which are not so hard or solid as to have resisted the cutting and undermining action of the stream. The cliffs forming the sides of the gully are about 80 to 120 yards apart. The east cliff is about 40 to 50 feet high, the west cliff (concealed in the diagram from view) is about 60 to 70 feet high, and rocky in its lower part. The haugh ground is higher on the east side of the gully, than on the west. The haugh is occupied by four or five ridges of gravel, almost parallel to one another and to the axis of the valley. These are shown on the diagram by the letters *a*, *a*, *a*, *a*. They are from 60 to 70 yards in length, and from 15 to 25 feet in height. The sequence of changes appears to have been as follows :—1*st*, By some means, the district has had spread over it a sheet of gravel and sand, covering the rocks. How this could have been effected, except by means of sea currents, it is difficult to imagine. 2*d*, The Kaim or Ridge, represented on the diagram, has been formed as a submarine bank, continuous in its whole length of a mile or more. 3*d*, The land rose, so that this part of the district became dry land. 4*th*, A stream was formed from the drainage of the adjoining hill slopes. 5*th*, The progress of this stream to a lower part of the country, was for a time obstructed by this kaim, through which, however, at some low point on its line, the stream at length

cut for itself a channel. 6*th*, The surface of the land at B and at C being higher

North.

GREENLAW MOOR KAIM.

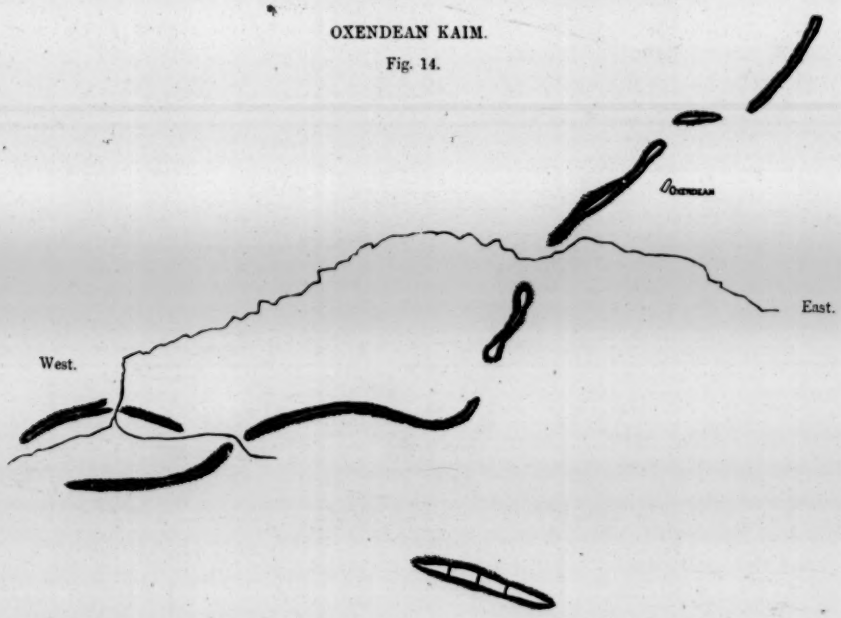
Fig. 13.



than the intervening surface, the kaim as it originally existed, must have been

OXENDEAN KAIM.

Fig. 14.



on a level lower than at B and C. 7*th*, The stream, after cutting through

the kaim, worked its way into the gravelly detritus on which the kaim had rested, to form the gully now occupied by the stream. *8th*, As the haugh land is higher on its east side than on its west, it may be inferred that the stream from some cause gradually changed its course towards the west, obtaining a new channel in the detritus, and leaving a ridge of gravel between its several channels, as represented by *a*, *a*, *a*, *a*, on the diagram.

These kaims differ in form with the nature of the materials composing them. When of sand, they are more round-backed, and the steepness of their sides is less than when chiefly of gravel. For example, the kaim already referred to in Eckford parish (page 539), as composed almost entirely of sand, has a width at the top on an average about 20 yards, being double the width of those composed of gravel.

In Castleton parish, there is a kaim about ½ a mile in length, and 50 to 60 feet high, running N.E. by E. Coarse gravel is in its upper part, fine gravel and sand in its lower part; pebbles of granite,* both red and gray, occur in it.

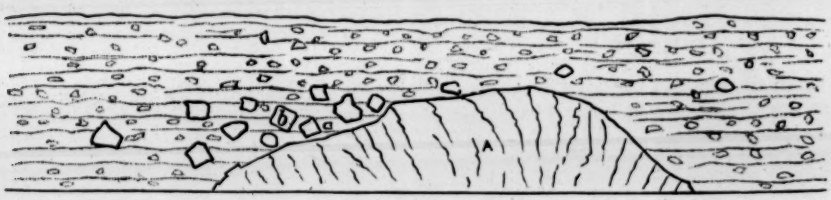
3. The *Boulders* of the district may be classed under three heads.

There are blocks, consisting of rocks belonging to the district, *i.e.*, to the valley of the Tweed.

There are blocks, consisting of rocks belonging, not to Tweed valley, but to the hills skirting the valley on the west, north, and south.

There are blocks which, judging from the nature of the rock, must have come from regions more distant.

(1). The blocks formed of rocks belonging to Tweed valley, are sometimes rounded, and occasionally angular.



Carham.

Near Carham, on the south side of the Tweed, there is a rock, of white chert limestone, which stands well up above the general surface, and bears marks of great grinding and striation on its western fronts. The height is from 180 to 190 feet above the sea.

Large masses of this rock have been carried to the east; none to the west.

The above diagram shows a section from a railway cutting about a mile to the east of Carham Station. A, is porphyry rock, about 20 feet above the

* These granite pebbles must have come from the Dumfriesshire hills.

rails, and 100 feet long, well rounded on west side, and covered with gravelly detritus.

The blocks marked *b*, *b*, *b*, &c., are of chert limestone, up to 6 feet (cubical) in size, mostly lying on the east side of the rock. The parent rock is a mile to the west.

About 2 miles west of Carham, there are several boulders of whinstone, in apparently their original natural position, with striæ on one of them bearing E. ½ S., which is also the direction of the longer axis of the boulder. Their height above the sea is 330 feet.

Two blocks of the Carham chert, each several tons in weight, occur below Coldstream Bridge, about 5 miles E. by N. from Carham, about 40 to 50 feet above the river. (See No. 22, p. 523). These are tolerably rounded, having probably been rolled or pushed from the parent rock by river floods.

Another mass of this peculiar rock, not quite so large, but extremely angular, and singular in shape, was lately excavated out of a bank of gravel, on the lands of Hirsel, about ¾ mile to the north of the river, and distant from the Carham rock about 3 miles. Its height above the river is about 60 feet, and above the sea 100 feet. The rock near Carham is about 70 feet above the river. This angular mass must have been rafted across the valley, in a direction E. by N.—a direction which coincides also with striations on the parent rock.

Two blocks of this Carham limestone are at Palinsburn, about 5 miles due east from Carham. The largest is about 10 feet long and 2 feet square, angular in shape. It has a legend attached to it, of marking the spot where JAMES, king of Scotland, died, after having been wounded on Flodden Field. It has long been known as the King's Stone. Assuming that this block came from the Carham chert rock, it must have been *carried* to its present position, not rolled or pushed.

It is more probable that it was carried on a mass of floating ice than on a glacier. The level of the ground where the boulder now stands, is at the same height above the sea as the Carham rock, viz., 182 feet.*

These chert limestone blocks become more numerous and of larger size in proportion to their nearness to Carham.

In examining the parent rock itself, there is evidence that large portions were broken off by some powerful agent moving over it from the westward. The fragments lying on the east side of the rock are severally many tons in weight.

* A somewhat similar case has been mentioned to me by Mr CURLE of Melrose. He states that, when a cutting was made for the railway between St Boswells and Earlston, a well-rounded boulder was extracted from boulder clay, about 4½ feet by 3 feet in size. It was a porphyry of exactly the same nature as that now quarried on the north side of the Eildon hills, distant to the west of the boulder about 3 miles, and at *about the same level above the sea*. It was lying with its longer axis about east and west, and there were striæ on its surface in the same direction.

The interstices between the fragments are filled with sand. Amongst the sand there is a well-rounded whinstone boulder, which could have come from no other quarter than the west.

Near the Chain Bridge across the Tweed above Paxton House, there is a bed of sand, covered by confused gravel. Partly buried in the sand, there is a sandstone block of angular shape, which probably came from the west, as rocks of that character are chiefly in that direction.

(2.) Of the next class of boulders, being of rocks differing from those in the valley, and occurring in hills adjoining, there are Silurian, Greenstone, dark Basalt, buff compact Felstones, various Porphyries, a black hard metamorphosed Graywacke rock, and red and gray Granites.

The ordinary graywacke, the greenstone, and the basalt, may have come from any of multitudes of places situated to the north or the west. On Kerchesters Farm, in the Redden Burn, there are several whinstone boulders at a height of 330 feet above the sea, sticking in boulder clay. The direction of the longer axis of most of these boulders, is E. $\frac{1}{2}$ S. One of them has striæ on the upper surface bearing E. $\frac{1}{2}$ S. At Marchmont, 539 feet above sea, there is a magnificent blue whinstone boulder, $9\frac{1}{2} \times 5 \times 4$ feet, with striæ on it, parallel with the longer axis. In all the above cases, the only known rocks of whinstone *in situ* are situated to the westward of the boulders.

There are other blocks which must have come from one or two hills, whose position is known, such as the buff felspar of Dirrington Law, the porphyry of Kyles Hill, the granites of Cockburn Law, and the black metamorphosed graywacke from a place near Cockburn Law. All these hills are to the west of the blocks.

The Cockburn Law blocks occupy a fan-shaped track towards the S.E. The *north* side reaches to the sea coast at or near Eyemouth, and the *south* side crosses the Tweed at or near Ladykirk. Two kinds of rock, occurring in Cockburn Law, are found along this track, viz., Granites, grey and flesh coloured; and a black Silurian rock with singular looking iron nodules in it. In this track the size and number of the blocks, is greater in proportion to their nearness to Cockburn Law.

Reference has been made to a block of blue whinstone, $12\frac{1}{2}$ tons in weight, found in Paxton Brickwork. The nearest rock of the same kind is at the Hardens, 2 miles west from Dunse, and bearing from Paxton about W.N.W. (magnetic.) Its longer axis, and sharpest point, were towards that quarter.*

There are boulders from the Cheviot Hills, deserving of notice. Thus, to the S.E. of Wooler, in Northumberland, and about 3 miles distant, there is a hill (Fowberry Moor) which (according to the Ordnance Survey) is

* In the channel of the River Tweed, at Carham, I observed a number of boulders in the stream. Those which were of an oblong shape, lay with their points up stream.

30 feet above the sea. The top of the hill presents a large flat area of sandstone rocks, rising gently to the N.W. Portions of these rocks bear deep striæ, the direction of which is N. ½ W. by compass. On many parts of this hill surface, there are boulders of the various Cheviot porphyries. My companions (Mr HUGHES of Middleton Hall, and Mr HARDIE, Secretary to the Berwickshire Naturalists' Club) who guided me to the moor, are well acquainted with the Cheviot rocks. They pointed out to me Hartsheugh and Watchlaw as the hills from which these boulders had come. The distance between these hills and the present site of the boulders, is about 2½ miles "as the crow flies." But Wooler valley intervenes, the bottom of which is about 320 feet beneath the position of the boulders. The width of the valley is about one mile, with hills on each side, so steep, that it is impossible that any boulder could have been pushed or rolled up. The boulders must therefore have somehow been carried across this valley, from the parent hills, to their present positions.

I understand that in Chillingham Park, situated about 5 miles to the E.S.E. of Wooler, there is a large boulder of red porphyry containing mica, which Mr HARDIE says must have come from "High Cheviot," as that hill alone, he believes, has rock in it of this description. The valley of the Till lies west of Chillingham, so that this boulder must have crossed both Till valley and Wooler valley, to reach its present position.

The direction in which these Cheviot boulders have come was from west, or W. ½ S. (magnetic.)

Near Doddington, there lie blocks of a peculiar porphyry, which Mr HARDIE considers to have come from near Yetholm, about 6 miles to the west.

It will be observed, that this direction does not agree with that of the striæ on the sandstone rocks, of Fowberry Moor, above mentioned, viz., N. ½ W. To account for the direction of these striæ, it may be mentioned, that there is a wide opening towards the north, between the Cheviot hills on the west and Doddington hills on the east. A current could therefore have flowed from the N. ½ W. point, and have passed over Fowberry Moor, producing on its rocks the striations before referred to. On the surface of the hill, there are small boulders of graywacke and blue whinstone, apparently from Scotland. In a direction towards S. ½ E., the country is low enough, to have admitted a current to flow on in that direction from the north.

There is no improbability in the supposition, that if the sea prevailed over these hills, there may have been different currents at different periods.

(3.) The other class of boulders, viz., those from the Scottish Highlands are, so far as known, few in number. There is a block of Mica Schist, weighing about half a ton, about 3 miles north of Dunse, first pointed out by Mr STEVENSON of that town. Several small blocks of a similar rock were found lately in boulder

clay near Berwick. A block of pink Granite was picked up by me at a railway cutting near Burnmouth Station, 8 miles north of Berwick. I sent a bit of it to Mr M'Donald, granite-polisher, Aberdeen, and he reported that it is a kind of Granite rare in Scotland, and known to him only in the form of boulders in Kemnay parish, situated about 10 miles N.W. of Aberdeen, though he believes that it must also occur in rocks not yet worked. Boulders of this pink Granite, he added, have been strewed along the shore south of Aberdeen. Mr Gordon of Cluny informs me, that there are *rocks* of this pink Granite on his estate N.W. of Aberdeen.

Mr James Geikie, in his "Great Ice Age," page 225, states that he had seen "bits of Mica Schist in the Till at Reston, in Berwickshire."

4. *Striated Rocks* occur at the following places:—

(1.) In the valley of the Tweed near Carham Railway Station, at the Lime Quarry, about 180 feet above the sea. This rock has been already referred to as the parent of many boulders lying to the eastward. It is a hard white limestone. The striated surfaces are generally horizontal. The direction of the striæ varies between E.N.E. and E. by N. (magnetic).

(2.) The next locality in height above the sea is at St Abb's Head, where, at about 250 feet above the sea (on the igneous rock), there are striæ, caused by an agent passing from N. by W.

On the east side of Coldingham Loch, about one mile inland from St Abb's Head, there are several well striated rock surfaces facing the N.W. There the direction of striæ, and height above the sea, are the same as at St Abb's Head.

(3.) On Smailholm Craggs (situated in the west of Berwickshire), at a height of 570 feet above the sea, there is an extensive surface of igneous rock, beautifully smoothed and striated. The surface of the rock dips W.N.W. at an angle of 35° or 40°. The direction of striæ is S.W. by W.; some striæ are 18 inches in length. An agent to produce the striæ, taking into account the slope of the rock, probably came from W.S.W. A general examination of these craggs exhibited ten or eleven spots, where the rocks had been rubbed and smoothed on their sides facing the W.S.W., by some agent or body passing over them.

(4.) On a part of the hill, occupied by Hume Castle, there is a surface of igneous rock about 742 feet above the sea, bearing striæ running E. and W.

(5.) The locality next to be mentioned is beyond the district, but it is not far, and it is rather important, being at the sea-level, viz., on Farne Island, situated about 15 miles south of Berwick, and about 8 miles from the coast. The direction of the striæ is N. by W.—agreeing, therefore, with the St Abb's Head striæ.

5. As not unconnected with this subject, it may be mentioned that there are several localities where the rocks bear evidence of disturbance and derangement by the passage over them of some agent or force of considerable power.

(1.) Thus below Preston, on the River Whitadder, there is a high bank which gives a good section of marl strata, covered by boulder clay. The strata have been evidently pushed out of their position by some force pressing against them from the westward. Boulder clay lies over these disturbed strata. It is therefore not unreasonable to suppose, that this boulder clay may have been the means of dislocating the strata. The locality is about 300 feet above the sea.

(2.) At Langton Quarry, about 250 feet above the sea, near Dunse, the sandstone strata have been broken up, and the fragments lie in a confused manner. Over the fragments, there are beds of sand and gravel, evidently deposited by water. (Pointed out by Mr STEVENSON of Dunse.)

(3.) At Letham, not far from Berwick, about 120 feet above the sea, there is a quarry where the sandstone strata have been passed over, and planed down to a horizontal surface; and over them is a bed of clay full of pebbles and small boulders.

(4.) A whinstone dyke runs E. by N. and W. by S. through the districts south of the Tweed, called the *Mattilees* Dyke. It stands up above the general surface of the land in many places. Mr CARR of Felkington pointed out to me when I lately examined this dyke, how in several parts of its course, large blocks of the dyke lie on the *south* side of the dyke, and very seldom on the *north* side; showing that some agent of great power had passed over the dyke from the north, pushing masses of it over to the south.

III.—*Theoretical Views.*

1. Under this head, I may first raise the question, Has the River Tweed formed or excavated the valley in which it flows, or does the river run in a valley previously formed?

Probably several causes have contributed to form the valley.

In the first place, there has been considerable dislocation and rupture of the earth's crust in the district now occupied by the river.

In this part of Great Britain, it will be remembered, that all the principal rivers run east and west. The formation of the Forth river and its estuary, is owing to dislocations, which occurred along the valley now occupied by the estuary and river—dislocations* forming lines whose average direction is nearly east and west.

With regard to the estuary of the Tay, though not so well acquainted with that district, I know that there are eight or ten whinstone dykes which run through Forfarshire, each in a direction nearly east and west. Many years ago, I traced their course and mapped them.

Professor RAMSAY, in a recent paper, explains that the River Eden, which

* These dislocations are explained in a little treatise, entituled "The Estuary of the Forth." Blackwood: 1871.

forms the boundary between England and Scotland in the west, running for 30 miles in an east and west direction, follows the line of a great fault. He ascribes to the existence of similar dislocations, the course taken by a number of other rivers in the west of England. ("Lond. Geol. Journal" for 1872, p. 156).

The Tweed is no exception to the rule. There are three whinstone dykes to the south and one to the north of the river, each running nearly east and west.

In examining the strata on the banks and in the channel of the Tweed, there are many proofs of disturbance, as at Tweedmouth, Horncliff, Norham, Coldstream, and Carham.

The mining operations of the Berwick coal field, extending along the south bank of the river for about 20 miles, show down-casts amounting to as much as 500 feet. Between Norham and Horncliff, in the bed of the river, there may be seen a great down-cast of the red sandstone rocks. The vacuity thus formed, is now occupied by enormous deposits of sand and gravel.

On this subject, a book has lately been published by Mr KINAHAN of the Irish Geological Survey, entituled "Valleys and their Relation to Fissures, Fractures, and Faults." Mr KINAHAN shows that most of the Irish rivers run along lines of rock dislocations; and oddly enough, these fractures of the earth's crust run in Ireland as in Scotland, east and west.

On these grounds, I think it was a true remark of Sir CHARLES LYELL, in his "Principles of Geology" (vol. i. p. 335): "Few great valleys in any part of the world have been excavated by rain and running water alone. During some part of their formation, subterranean movements have lent their aid, in accelerating the process of erosion."

2. If then, the River Tweed now runs along lines of great fractures of the strata, the question suggests itself, What has become of the millions of fragments from the dislocated rocks, many of these fragments hundreds, perhaps thousands, of tons in weight?

The river now runs over a level channel which externally manifests no rents or fractures; nor is there any external vestige of them in adjoining districts. Yet when these fractures occurred, there must have been many vertical cliffs of strata, several hundred feet in height; all of which have disappeared.

The lines of fracture are entirely concealed under extensive drift deposits; and even when these deposits are artificially removed, so as to expose the rocks beneath, the rocks are found in most cases forming a tolerably even floor, levelled, no doubt, by agents which passed over them and planed them down.

I am aware that the prevailing disposition of geologists, is to explain almost everything by the action of glaciers and land ice; and I admit, that there are places in the Scottish Highlands, where that explanation is sound.

But the facts observable in Berwickshire, seem not to admit of this explanation. They suggest the supposition that a sea prevailed over this district, exceeding 1000 feet above the present sea-level, and probably much more.

The facts alluded to are these—

(1.) The extensive beds of sand and gravel, in many cases stratified, which abound in the district, not merely in the lower parts, but in higher parts also, up to the ranges of hills on each side of Tweed valley.*

The relative position of these deposits favours that view; inasmuch as clay or mud occupies the lowest parts of the district, and gravel generally lies above the sand.

(2.) The transport of boulders from the Highlands of Scotland across the estuary of the Forth, and also across two or three ranges of hills, necessitates the existence of a sea, with ice floating on it, to account for this transport.

Even the boulders which have come merely across the Lammermuir and Cheviot hills, respectively require such an explanation.

(3.) A new feature in this question is manifested by the form of the drift ridges, and their remarkable parallelism.

My attention to it was first drawn by Sir HENRY JAMES, the Director of the Ordnance Survey, who, on seeing the shaded maps of the district made by his surveyors, sent to me a copy, accompanied by a note expressing his surprise at the immense number of "lateral moraines" existing in the valley of the Tweed.

I am satisfied however, that these ridges are in no right sense moraines, seeing that they consist chiefly of stratified beds of sand and gravel.

To one unaware of the composition of these ridges, and attending only to the circumstance of their longer axis being parallel with the ranges of hills forming the valley, the theory of lateral moraines was not unnatural. But when lateral moraines occur, they occur, as their name implies, only at the *sides* of valleys, and not over a great portion of the central parts of the valley, as is the case in our Border Counties.

These ridges appear very similar to the submarine banks, composed of sand, gravel, or mud, in existing estuaries formed by the action of tidal currents.

The sea charts, which give soundings and show the forms of submarine banks in estuaries, indicate many features similar to those which these kaims present. In the "Estuary of the Forth" (page 99), there is a diagram indicat-

* Mr LESLIE, C.E., Edinburgh, tells me (18th May 1875), that in the parish of Temple (on the north side of the Lammermuirs) a bore was lately put down under his orders, which went through a solid bed of pure sand to the depth of 130 feet, without reaching rock. The spot is 800 feet above the sea.

ing several submarine banks off the coast of Norfolk. In the annexed diagrams some of the submarine banks off the Belgian coast are shown, in plan and in section.

Plan of Submarine Banks off the Belgian Coast.

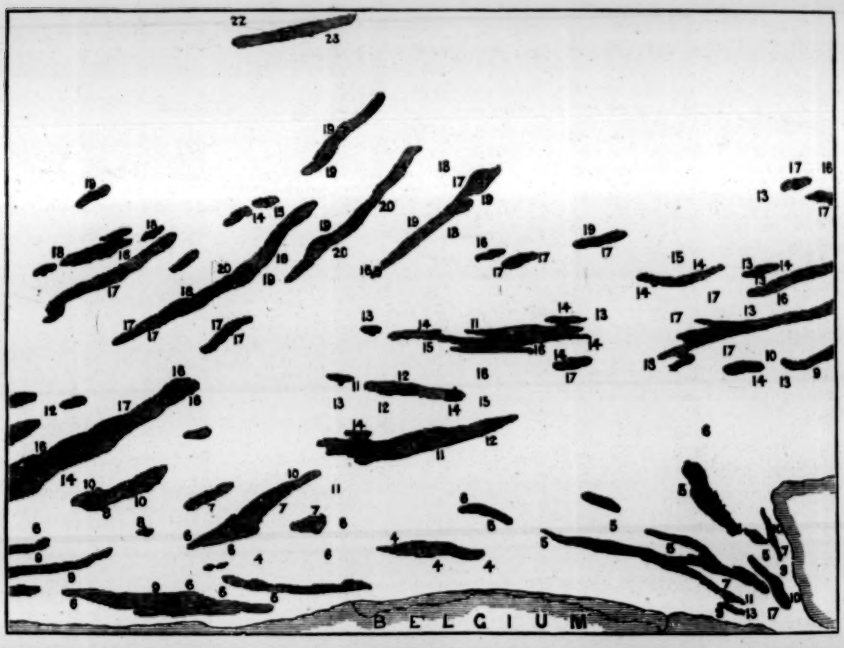
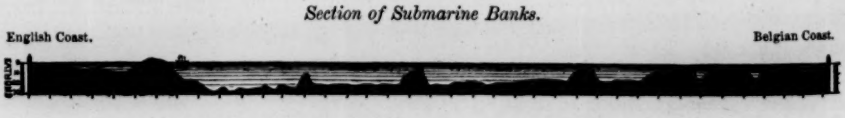


Fig. 16.

Submarine Banks lying between the Belgian and English coasts; reduced from the Admiralty Chart. The figures represent fathoms at low tide. The figures on the shaded parts show the depth of water over the Banks. The figures on the unshaded parts show the depth between the Banks.—Scale, 13 miles to one inch.

The submarine banks, when near one another, are approximately parallel, formed by one set of forces;—those at a distance often exhibit a different direction, due to the action of other forces, or the same forces modified.

Section of Submarine Banks.



Heights of banks shown by the scale of fathoms at each end.
The distance between the two coasts is about 30 miles.

The formation of these submarine banks is no doubt due mainly to the set of the tides; and the tides are influenced, as regards direction and strength, by the adjoining coast lines.

Were the English Channel to be raised out of the sea, would it not present a series of kaims or eskars very similar to those occurring in the Border Counties?

The general depth of the English Channel in this part, is from 150 to 200 feet. The height of the submarine banks of course varies, but sometimes reaches 60 feet above the general level of the adjoining sea bottom.

How the banks are formed, is a question of no moment in this discussion. The sediment moved by the currents may be *heaped up* into ridges; or the sediment forming the ocean bed may be *scoured out*, leaving ridges standing up.

In now turning to the map of the district (Plate XXXVIII.), it will be noticed that the kaims or ridges situated near one another, are approximately parallel, and that they observe a general parallelism to the range of hills on each side of the Border Counties.

With regard to the depth of sea which prevailed when these kaims were formed, one thing is clear, viz., that the beds of sand and gravel, existing in several places as high as 700 or 800 feet, imply a sea which stood greatly above that level. Mr Austin, after studying the features of the English Channel ("Geol. Soc. Journ." for 1849, vol. vi. p. 83), says that "the moving power of the sea at 60 fathoms is limited to fine sand." A somewhat similar opinion had been previously expressed by Lord Anson, as the result of his nautical experience, when he says, that he generally found at the sea bottom, fine sands, mud, and ooze, at from 80 to 60 fathoms; sand with broken shells, at from 60 to 40 fathoms; coarse sands, pebbles, and small stones, at from 40 to 12 fathoms.

According to these rules, if large accumulations of sand and fine gravel exist in Berwickshire and Roxburghshire up to a height of 700 or 800 feet, the depth of the sea in which these beds were deposited, must have been at least 400 feet more, *i.e.*, from 1100 to 1200 feet. But we know that there are parts of Scotland, in which thick beds of sand occur, at heights exceeding 1500 feet above the present sea-level.

The contour lines of the Border Counties show that when sea prevailed over them, a depression of the sea bottom existed along what is now the valley of the Tweed, and that the highest part of this depression was at St Mary's Loch and Moss Paul. The tides would of course be stronger towards the head of the valley, where the hills on each side were closer, and the depth of water less. Hence, it is that, judging by the shaded maps of the Ordnance Survey, the kaims appear to be of larger dimensions in the higher parts of the valley.

The great probability is, that the whole of the central parts of the Tweed valley were at one time occupied with detrital matter, at least 300 feet in thickness, most of which was subsequently removed by currents of water. Hence we

find in some places, kaims near the river, parallel with its course, which are merely portions of a pre-existing sea bottom, the detritus on each side having been scoured out and removed. To this class may belong the very interesting kaim at Wark, on the north side of which the Tweed now runs. That the Tweed had previously run on the south side of this kaim, is plain from the configuration of the ground. So also at Norham, there is a kaim, on which the church stands. There is a hollow along the south side, through which the river ran, before it passed into the lower channel on the north side.

6. If the theory of a sea prevailing at a height of 1000 feet or more, be adopted, it enables us to account for many other facts.

I do not discuss the question how the relative levels of sea and land altered, till they reached the present condition of things; whether it was by the land rising or by the sea falling. Whichever way it was, the change seems not to have been accomplished at once. There have been successive periods at which those ancient sea margins were produced.

I admit that those sea margins, particularly at levels above 100 feet, are, in this part of Scotland, faint and seldom continuous. But if beach lines at *low* levels are well marked, and now acknowledged by experienced and cautious geological observers to be beach lines, *their* existence lends probability to others having in like manner been produced at *higher* levels. Let us see, then, what reliable evidence there is of comparatively low sea margins.

A sea margin 9 to 12 feet above the present high water mark, on both sides of the English Channel, is avouched by Mr Godwin Austin, and on the south coast of Ireland by Mr Kinahan of the Irish Survey.

A horizontal line at that height has been long recognised on many parts of the coasts of Scotland ("Estuary of the Forth," p. 105).

Messrs Brady, Crosskey, and Robertson, in a recent paper in the Transactions of the Palæontological Society, avouch two lines on the Scotch coasts, one at 25 feet, the other at 40 feet. Mr Kinahan refers to one on the Irish coast at 35 feet.

Mr Pengelly ("Lond. Phil. Trans." for 1873, p. 182), refers to a raised beach all along the south coast of England, at a height of 30 feet, containing sea shells of the existing species.*

Sir Henry de la Beche ("Geology of Cornwall," p. 425), describes a raised sea beach on the coast of Cornwall, at a height of 50 feet above the present sea-level.

* The slight discrepancy as to the height of these old sea margins, as given by different authors, may arise from the measurements being made, in some cases, from high-water mark; in others, from the supposed medium sea-level. All the Ordnance Survey measurements are from the medium sea-level. But as it is almost impossible for geologists, in their excursions, to ascertain the medium sea-level, they invariably measure from the apparent line of the last high water, produced by a spring or a neap tide.

Mr AUSTIN describes a sea margin on the Devonshire coast from 60 to 70 feet above the sea (also "Geolog. Journ." vol. vii. p. 128.)

Sir CHARLES LYELL, in his "Antiquity of Man" (p. 112), speaks of "marine shells of recent species in the drift on the banks of the Severn, 100 feet or more above the level of that river."

Mr DANBY describes a raised beach in Jersey at the height of 100 feet above the sea ("Geol. Mag." vol. ii. No. 3, New Series.)

Mr CUMMING of London describes a terrace at from 100 to 115 feet as existing through the great glen west of Inverness, and as traceable along the coasts of Ross-shire, Sutherland, and Caithness, and also in Nairn and Moray shires. Having described at that height a great gravel platform in the Isle of Man, he states that he has no doubt that it belongs to the same period, and is due to the same cause as the terrace in the north-eastern counties of Scotland.

I have visited some of the places mentioned by Mr CUMMING at and near Inverness. I can vouch for an extensive flat there about 90 feet, and a lower flat about 25 feet above high-water mark (spring tides). At Kessock Ferry, I found terraces at respectively 86 and 25 feet above the sea-level, both of which I traced eastward along the coast for some miles, viz., to Rosemarkie. Westward of Inverness there is a horizontal terrace at Dunain, and another in Loch Ness, both of them about 90 feet above the sea, visible near Dochfour House, at Urquhart, as also on Loch Oich. If these flats were, as is probable, part of a *sea bottom*, it is not to be expected that they should all be precisely on one horizontal level. The portions remote from the ancient shore would be at a lower level.

Captain BEDFORD describes an old sea margin, as seen by him at Loch Tarbert at the height of 105 feet above the sea. ("Geol. Soc. Journ." for 1855, vol. xi. p. 549.)

The Rev. Mr BROWN, in the paper published in our Transactions, to which I have already alluded, speaks of what he calls a "high level terrace" seen by him near the River Earn, more than 100 feet above the sea. ("R. S. E. Trans.," vol. vi. p. 14.)

These statements seem to put almost beyond doubt, the fact that in recent (geological) times the sea has stood, and stood for long periods, at the levels above specified. But if further proof were needful, it would be afforded by the discovery of sea shells of existing species in the drift deposits of Aberdeenshire, Dumbartonshire, and Lanarkshire, at numerous places up to a height of 526 feet above the sea; and in England, at more than double that height.

I have dwelt at length on the evidence that the sea has stood at these considerable heights on the land, for two reasons. It will immediately be pointed

out, how this fact throws light on the origin of the high terraces on river banks, as well as on the formation of the kames in our Border Counties.

It may, however, in the first place, be convenient to notice the

IV.—*Views of other Geologists.*

1. In ascribing the formation of the parallel ridges and kaims of Berwickshire and Roxburghshire to tidal action, it is only right to acknowledge, that a different explanation has been suggested. I have already mentioned the opinion of Sir HENRY JAMES, that these drift ridges are *lateral moraines*, and my reason for not acquiescing in that view.

Another geological friend, for whose opinion and experience I have a high regard—Mr JAMESON of Ellon—in his last publication ("Quart. Journ. Geol. Soc." Aug. 1874, p. 329), refers to the great Greenlaw Kaim (before referred to, p. 543), and founding on the circumstance that it is disposed like a horse-shoe with the hollow towards the hills, supposes it must be a *terminal moraine*. If Mr JAMESON had personally visited the district, he would have seen, there was no valley, across the mouth of which this kaim lies. It is situated on the south-east side of Dirrington Law, and about 1½ mile from it. The outline of the kaim is approximately parallel with the contour of the hill, at a level of 800 feet above the sea; so that as the direction of submarine banks is more or less parallel with that of an adjoining coast, this kaim may have been formed by tidal action affected by Dirrington Law. I might further say, that there is no valley or mountain in this district sufficient to have produced a glacier with a moraine at this height above the sea, and forming a rampart of stratified gravel and sand 2 miles long.

Another geologist, the author of a recently published book, called "The Great Ice Age," has specially referred to the parallel ridges and kaims of Berwickshire, with explanations which require notice.

At page 236, Mr JAMES GEIKIE states, that there is a "considerable assemblage of mounds, hillocks, banks, and undulating flats of sand and gravel in the valley of the Kale Water, *between the base of the Cheviot hills and the River Teviot, near Eckford.*"

"We find similar appearances characteristic of the *Lammermuir* districts. The Whitadder water, for example, after leaving the Lammermuir hills, enters upon a low-lying undulating country, which is thickly strewn with sand and gravel over an area many miles in extent; and the great bulk of these is strictly confined to the drainage area of the water." (P. 237.)

The same deposits are again alluded to, accompanied this time, by theoretical views, in the following paragraphs:—

"In the lowlands, the effect produced by the varying direction and unequal pressure of the ice-sheet, is visible in the peculiar outline assumed by the till. Sometimes it forms a confused aggregate of softly swelling mounds and hummocks. In other places it gives rise to

a series of *long smoothly-rounded banks*, or '*Drums*' and '*Sowbacks*,' which run parallel in the direction taken by the *ice*. This peculiar conformation of the till, although doubtless modified to some extent by rain and streams, yet was no doubt assumed *under the ice-sheet;* the 'sowbacks' being the glacial counterparts of those broad banks of silt and sand, that form, here and there, upon the banks of rivers. Perhaps the most admirable example in Scotland of this peculiar arrangement or configuration of the till, *recurs in the valley of the Tweed, between the Cheviot hills and the Lammermuirs.* In this wide district, all the ridges of till *run parallel to one another*, and in a direction approximately east and west."

The foregoing passages ascribe the formation of these parallel ridges somehow to the action of *land ice.*

But in another part of his book (page 243) the author apparently ascribes the formation to *river action.*

After again (pages 241–2) referring to the "gravel beds," "the well-marked ridges," and the "typical kaims of the *Tweed* and *some of its tributaries*," he says, that—

"Putting these various considerations together, the conclusion seems forced upon us, that all those accumulations of water-worn materials owe their origin to currents, that once flowed down the valleys. Not only so, but one must also admit, that those currents were proportionate in size, to the extent of each particular valley-system, in which such accumulations were found. In short, we can only, as I think, account for the appearances described, by attributing the deposition of the greater areas of gravel and sand to *river action.* But if so, then the rivers must have greatly surpassed in volume and breadth their present puny representatives." (Page 243.)

"The explanation appears to be simply this:—The great ice-sheet, underneath which the till accumulated, had, after depositing the boulder clay, continued to retire, until it was reduced to a system of gigantic local glaciers. In summer time, such streams and rivers as flowed in glacier valleys, would be vastly swollen by the water derived from melting snow and ice. Great currents would sweep down the valleys, carrying with them the angular debris derived from terminal moraines, and from freshets running down the slopes of the hills. As this debris was hurried along, it would be *gradually rounded by attrition*, and eventually *pass into good gravel.* At the sametime, the till and ancient moraine debris over which the rivers rushed, would be denuded and washed away from exposed positions." (Page 245.)

In the foregoing passages, the author calls in, for the formation of the till, the agency of an ice-sheet; and for the formation of beds of gravel and sand, the agency of rivers swollen in summer by the melting of the snow and ice.

In the following passage, however (page 246), he not only admits but adopts a very different explanation:—

"But whilst thus admitting, that many mound-shaped hillocks of gravel and sand are only the denuded remains of what were once continuous flats of *fluviatile* origin, still there are appearances connected with the more typical assemblages of kaims, cones, and mounds, which *can hardly be explained by what we know of rain and river action.* To account for some of the phenomena, we are apparently compelled to call in the *agency* of the *sea.* The deep circular depressions, surrounded on all sides by smoothly-rounded cones and banks, and often occupied by lakelets or peat mosses, *cannot possibly be due to the action of rivers.*"

"When we note that strings of gravel ridges and mounds may sometimes be followed up one valley across the dividing col into a totally different drainage system, we cannot but conclude, that ordinary river action is out of the question as an explanation of the phenomena. In the present state of our knowledge, we appear to have no alternative, but in such cases to admit the *marine origin* of such kaims." (P. 248.)

"The same assumption is necessary, to explain the occurrence of those *elevated shelves or terraces* which here and there fringe the slopes of the hills. The shelves of gravel at Eaglesham, for example, appear to be ancient *sea beaches*." "The highest of these terraces does not reach beyond 800 feet above the level of the sea. Similar terraces, however, have been met with at greater elevations. I have traced them on the Moorfoots, up to a height of 1050 or 1100 feet; and these, like the Eaglesham beds, seem equally to require the agency of the *sea*." (P. 248.)

"There are yet other considerations which seem to render it extremely probable, that many of our kaims have been shaped out by the action of the *sea*." (P. 249.)

I have thought it right to refer to these views of Mr JAMES GEIKIE, for two reasons—*First*, as one of the surveyors employed on the Geological Survey of Scotland, he has had opportunities of extensive observation; *second*, he has, if I mistake not, hitherto ascribed the drift deposits in Scotland to the action of land ice.

I am therefore gratified to find that my own opinions on this point are now accepted by a geologist so experienced and acute.

Assuming, then, that to account for the long parallel ridges of drift materials, for the transportation of boulders, and above all, for the accumulation of beds of stratified sand at high levels, it is necessary to suppose that the sea covered the whole of this district of Scotland, overtopping even the Lammermuir hills,—an explanation is afforded of several other matters noticed in this paper.

In such a sea, the currents would vary in direction according to the configuration of the sea bottom. Undoubtedly, there seems to have been everywhere, in Scotland, a very prevailing current from the N.W. But in some places we find a deflection:—and for which, it is not difficult to account. If, as I have supposed, there was a kyle or strait across the south of Scotland—the shallowest part of which is now the watershed between Roxburgh and Dumfries shires—it is natural, looking to the direction of the hill ranges on each side of the valley, that the current there should be not from the N.W. but from W.S.W.—W. by S., and due W. Now, it will be observed, that these are also the directions of the striæ, of the transport of local boulders, and of the parallel ridges.

Then, where it is now sea, off the coast of Berwickshire, the normal N.W. current might naturally have been deflected, so as to produce the striæ on the rocks at St Abb's and on the Ferne Islands, which all run N. $\frac{1}{2}$ W.

These differences in the direction of the striating and transporting agent seem to me much more explicable on the theory of a deep sea carrying ice than on any other. It also accounts for another set of phenomena—the terraces and flats which exist, both in Berwickshire and elsewhere, at high levels.

The bearing of these facts on the river terraces will at once be seen.

If the sea stood, as suggested in the early part of this paper, at 180 feet above its present level, the Rivers Tweed and Teviot must have reached what was then the sea, at points respectively some miles to the west of Kelso.

When the sea sank to the level of 120 feet, the junction of the above rivers with the sea, would be close to where Kelso is situated.

When another subsidence of the sea took place, say to the level of 60 or 70 feet, afterwards to 30 or 40 feet, and ultimately to 12 or 15, the rivers, being on each occasion made to flow down steeper gradients, would acquire greater speed and more cutting power. Their channels being deepened, their banks would also be undermined, and any flood marks previously made on their banks, in the upper parts of their course, would run great risk of being obliterated.

There is thus a relationship between the old sea margins and the high river terraces. As the sea fell from one level to another, so also must the rivers have fallen from one channel to another.

There is, however, this difference between the two. If the successive subsidences of the sea were sudden, the subsidence of the rivers could not have been equally sudden, as time would be required before their streams could cut out deeper channels. How much deeper the new river channel would be than the old, would depend on many circumstances. It would not be equal in every river. It would not be equal even in every part of the same river. Therefore the old flood marks on river banks, after the rivers sank in consequence of a subsidence of the sea, might not be all at an equal height above the deeper channel when acquired. But still, there would be in all rivers, traces of an old flood line above that of the existing stream, when the materials composing the bank were such as to have been susceptible of erosion, and when the banks were not so undermined as to destroy the old flood lines.

The examination of these high river terraces has obtained attention from only a few geologists. The cases of which I have found notices, may be mentioned.

Our esteemed colleague Mr Brown, in his paper to which reference has already been made, vouches from personal observation, for the existence of three distinct terraces on two rivers in Perthshire with their tributaries, viz., the Earn and the Teith.

On both rivers, he states that there are three terraces. In the Earn, these are at 9 feet, 22 feet, and 57 feet above the present channel of the river. On the Turrit and Ruchil, tributaries of the Earn, the same three-fold system of terraces exists.

On the River Teith, though the same terrace system prevails, and if possible, more strikingly (p. 154); and also on the Keltie, a tributary of the Teith, the

terraces are not quite at the same height as on the banks of the Earn. On the Teith, Mr BROWN states the height to be 6, 12 or 14 feet, and 24 feet above the river.

This last height I can myself vouch for. In the "Estuary of the Forth" (p. 3), notice is taken of a river margin in the Teith 25 feet above the present level of the river.

Mr WHITAKER, F.G.S., in his "Guide to the Geology of London and its Neighbourhood," recently published, refers to the gravels deposited by the old Thames, on the flanks of its valley. He adds that—

"From the occurrence of these river drifts, at successive stages or terraces on the sides of the valley, we are led to infer that, after the deposit of the first or highest gravel, *the river deepened its bed*, cutting through that gravel, and depositing another mass at a lower level; in its turn to be cut through, as the channel was *farther* deepened. Naturally the highest of these terraces, of which there are often three in the valley of the Thames, has suffered more from denudation than the others." (P. 66).

Mr TYLOR, referring to the Aire, in Yorkshire, says that it has a flood line about 10 feet above the present level of the river, and that there are escarpments 50 feet above the river, which show (he says), the line, at which the river formerly flowed. ("London Geol. Jour." for 1869, p. 63).

The River Somme, in Brittany, has been very carefully examined by PRESTWICK, TYLOR, and others, on account of the flint implements and animal remains found in its banks. All agree, that there are escarpments of gravel at a height of about 84 feet above the river and 133 feet above the sea, indicating, as these gentlemen think, that the river ran formerly at that higher level.

A similar escarpment is visible on the Rhine. Professor RAMSAY ("Lond. Geol. Journ." for 1874, p. 88), says of the Rhine, that—

"The traces of its temporary levels, as the river *cut its way down*, may still be seen on the cliffs high above the present surface of the river. Thus on the hill behind Bingen, there are the relics of a plain 341 feet above the river. This plateau also, in a fragmentary state, is continued farther down the Rhine.

"As the gorge (near Bingen) was being gradually cut out and deepened, in consequence of this, the Rhine, wandering through the plain beyond Bingen, by degrees lowered *its* surface; so, just in proportion, the Maine, the Neckar, the Kinzig, and other tributary rivers, also lowered *their* channels. The reasoning now applied to the Rhine, is equally applicable to the Danube and other European rivers of equal importance.

"Similar terraces (adds Professor Ramsay), occur on the Mozelle." (P. 95.)

It appears also, that in the valley of the Nile, there are terraces at 30 feet, 100 feet, and even at greater heights, which Sir CHARLES LYELL considers to indicate the levels of the river, before the land was elevated. ("Elements," p. 116).

I mentioned at the commencement of this paper, that there are some

geologists who consider that these high-water marks on the banks of rivers indicate the levels to which the rivers rose, in former times, when in flood,—assuming that the rivers *then flowed in the same channels which they at present occupy.*

Mr TYLOR was, I believe, the first to suggest this view in reference to the River Somme and Oise in France, and the Rivers Aire and Waveney in England.

Mr BROWN also, in his paper on the terraces of the Earn and Teith, adopts that theory; and Mr JAMES GEIKIE, in his book, to which I have already referred, seems to adopt it, when speaking of the immense deposits of sand and gravel formed by existing rivers, by reason of the far greater floods which occurred in them formerly.

Mr BROWN is of opinion, if I have read his paper correctly, that in the River Earn, where floods now reach no higher than 6 feet, floods in former times reached to a height of 57 feet, making the river-flood 57 feet deep. So also it is contended that in the River Teith, the floods, which reach now no higher than 6 feet, in former times reached to a height of 24 feet, forming a stream more than 24 feet deep.

Mr TYLOR, applying that view of the matter to the Somme, a river apparently about the size of the Earn or the Teith, maintains, that when the high level gravel bed was laid down by the river,—the river occupied its present channel 84 feet below that gravel bed; and, therefore, when that gravel bed was deposited, it must have been by a flood 84 feet deep at least.

These views, assumed to be correct, are adduced as evidence in favour of glaciers and other forms of land ice, the melting of which in summer, it is said, would give rise to these enormous floods. In support of these views, suggestions have been made regarding larger amounts of rain which may have fallen in the country, owing to a more rapid condensation of vapour on the ice or snow-capped hills.

I cannot discover good grounds for these views. Mr PRESTWICH, who has carefully studied the physical features of the River Somme with reference to this theory, and for many years devoted himself to what are called the Drift deposits, says, that if Mr TYLOR's views are correct, the quantity of water required to raise the Somme to the height of the highest river gravel beds would be 100 times more than any flood ever known to have occurred in the river, and he cannot see where such a supply of water could be obtained.

No doubt, it may be said, that we are not entitled to reject any geological theory, if it may *possibly* be correct; but if a more probable explanation can be suggested—that explanation surely should be preferred.

The view which I have submitted in this paper, seems to rest on stronger grounds, than there being merely in its favour a larger body of evidence. I think that, if the existence of old sea margins, up to even the smallest heights

above the present level of the *sea* be admitted, then all the rivers *must necessarily have had their channels that much higher than now;* in which case flood-marks must have been made on their banks at heights suited to the higher channels so occupied by them.

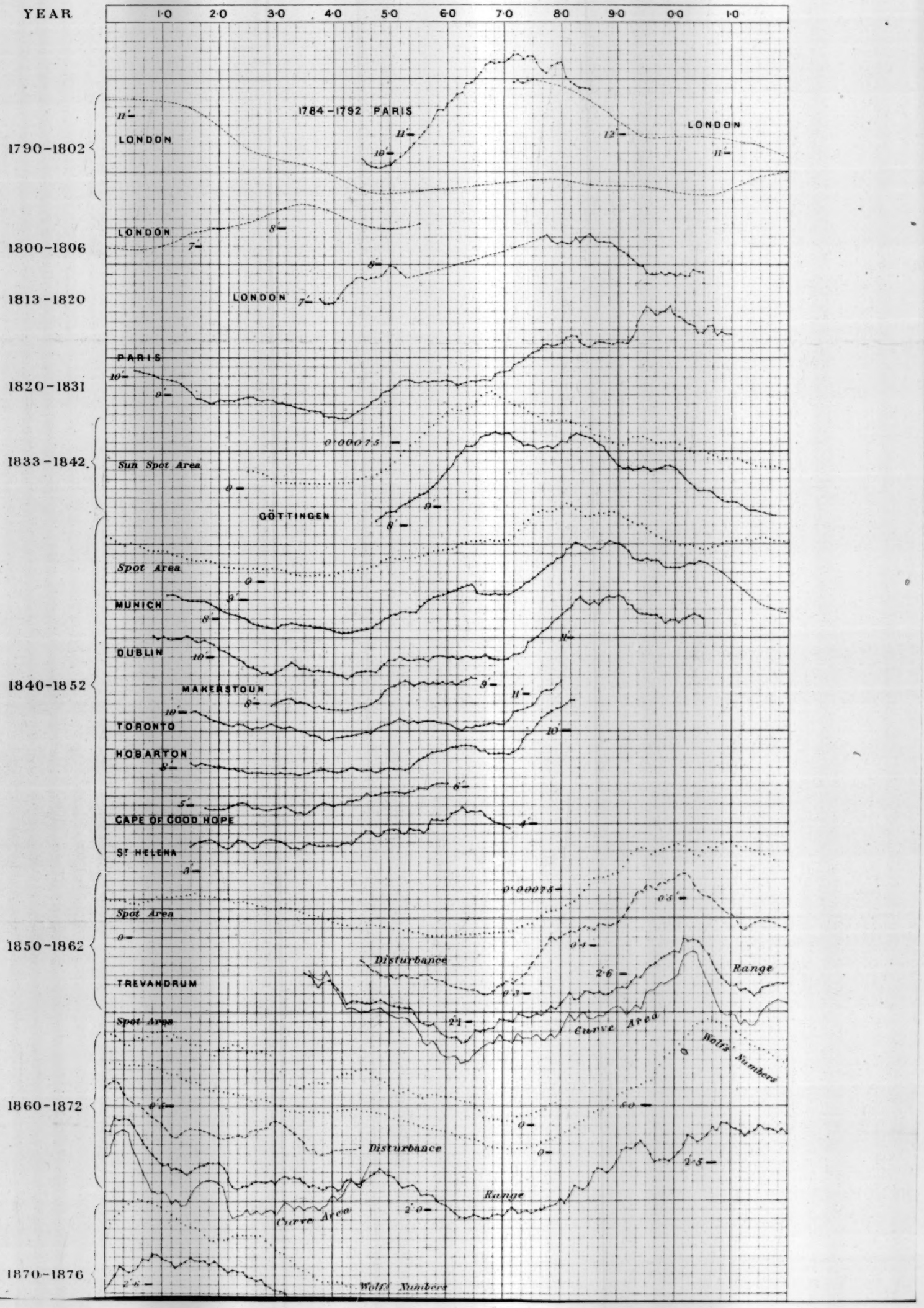
On the other hand, it may be said, that these high-water marks prove rivers to have been of larger size, if these marks indicate the width of the stream in former times. Now, it is true, that in those parts of the river, where there are high water-marks on banks immediately opposite to one another, the width between *those* water-marks is greater than the width between the lines made by *existing* floods. But in examining the map of the Tweed (Plate XXXV.), it will be seen that there are few spots where there are high-water marks on banks immediately opposite to one another; and where such occur, it must not be inferred that these lines are on exactly the same level, as they would be, if made by the river at the same time. On the contrary, they are always on levels which differ a few feet, indicating that they were made by the river when it flowed at different levels, and of course at different periods. A river after having continued to press on one bank more than another, cuts for itself a deeper channel, and then meeting with some obstruction, it changes its course, and begins to press against the opposite bank. Hence, the banks will show flood-marks at levels not exactly the same.

If it be alleged that the water marks, at the height of from 40 to 55 feet, on the banks of the Tweed were produced by floods in the existing channel, will it be also contended that the extensive flat, near the junction of the Tweed and Whitadder, which is about 66 feet above the channels of these rivers, was produced in the same way? The great extent of this flat precludes such a supposition. That flat must have been the bottom of a lake or an estuary; but if so, the Rivers Tweed and Whitadder must have flowed into it, in which case *their channels must have been at this place more than* 50 *feet higher than at present.* Through that extensive flat, they have cut out for themselves the channels in which they now respectively flow; and in the course of this operation, have left the water marks at different levels on their banks.

Before concluding this paper, I must in candour admit, that the problem of these high terraces would have been more satisfactorily discussed, had the levels of the terraces been ascertained with more precision than I have been able to accomplish. That the terraces on the immediate banks of the River Tweed and its tributaries slope downwards with the rivers at most parts of their course, there can be no doubt. On the other hand, in some places, the terrace and base of the bounding cliff appears to be absolutely horizontal,– as between Wark and Coldstream;—and the distance between these cliffs is so great as to suggest a lake. But to verify this view, the horizontality of the base line of the cliffs would have to be ascertained with the greatest precision.

DECENNIAL PERIOD OF THE DIURNAL RANGE OF MAGNETIC DECLINATION AND SUN-SPOT AREA
1784–1876.

PLATE XXIX Royal Soc Trans. VOL. XVII



YEAR

1·0 2·0 3·0 4·0 5·0 6·0 7·0 8·0 9·0 0·0 1·0

1790–1802

1784–1792 PARIS

LONDON

LONDON

1800–1806

LONDON

1813–1820

LONDON

1820–1831

PARIS

1833–1842

Sun Spot Area

GÖTTINGEN

1840–1852

Spot Area

MUNICH

DUBLIN

MAKERSTOUN

TORONTO

HOBARTON

CAPE OF GOOD HOPE

St HELENA

1850–1862

Spot Area

TREVANDRUM

Disturbance

Range

Curve Areas

Wolfs Numbers

1860–1872

Spot Area

Disturbance

Curve Areas

Range

Wolfs Numbers

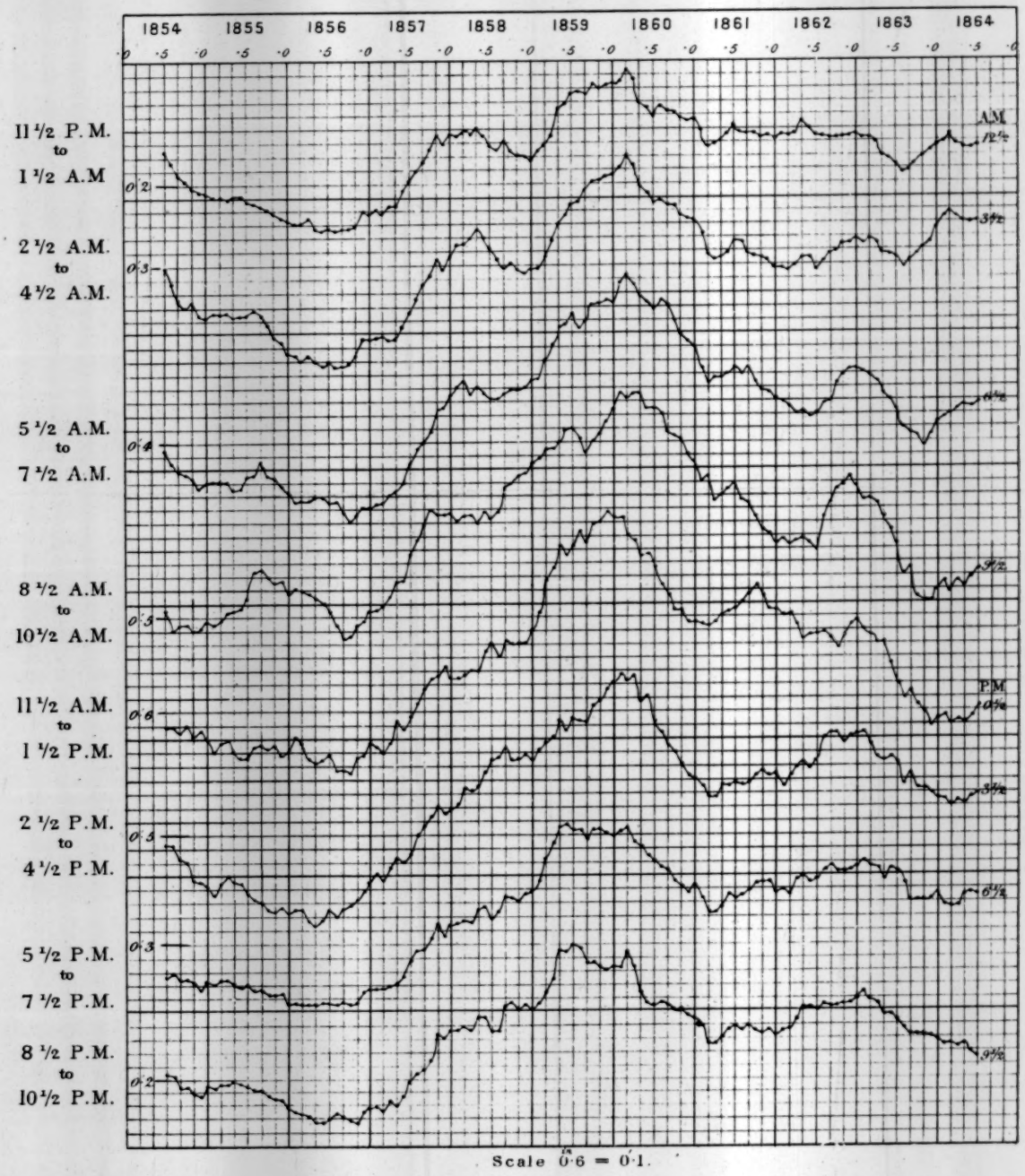
1870–1876

TREVANDRUM

DECENNIAL PERIOD OF DISTURBANCE OF MAGNETIC DECLINATION FOR EACH THREE HOURS AT TREVANDRUM, 1854 – 1864.

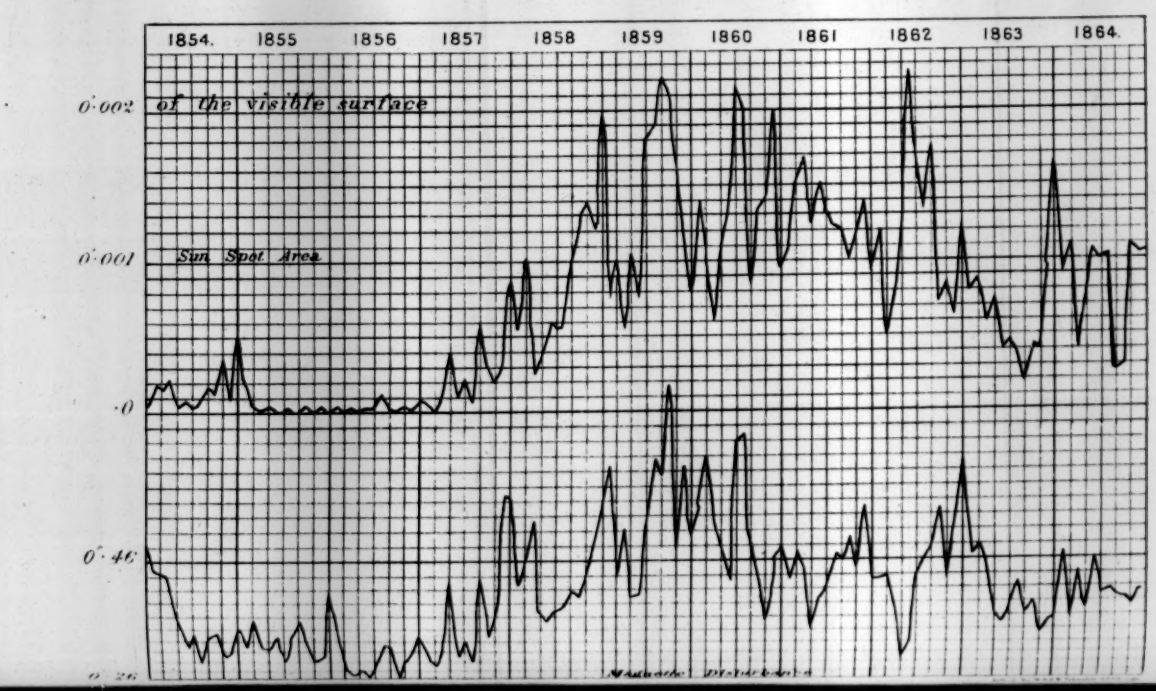
by J. A. Broun, F. R. S.

PLATE XL. Royal Soc. Trans. VOL. XXVII.



MONTHLY MEAN SUNSPOT AREA AND DISTURBANCE OF MAGNETIC DECLINATION AT TREVANDRUM

1854 – 1864.



XXVI. *On the Decennial Period in the Range and Disturbance of the Diurnal Oscillations of the Magnetic Needle, and in the Sun-spot Area.* By J. A. Broun, F.R.S. (Plates XXXIX., XL.)

(Read 15th May 1876.)

1. The presentation to the Royal Society of Edinburgh of results relating to the decennial period, derived from observations of magnetic declination made during nearly a quarter of a century at Trevandrum, has seemed to me a favourable occasion for a determination of the mean duration of this period. Upon the explanation of the decennial variations depends the solution of several important problems in solar and terrestrial physics, and the first step towards this result is to ascertain the true mean duration of the period. Two markedly different results have been obtained, each of which has been accepted by men of the highest scientific reputation.

2. *Discovery of the Decennial Period of Magnetic Variations.*— A century ago the varying positions of the magnetic needle were followed with much perseverance by several men of science, but by none with more ardour than Van Swinden. Unfortunately, the needles employed were supported on steel points, and, owing probably to weak magnetisation and the mechanical difficulties of construction, they gave widely different results at different places, and even at the same place; so that valuable series of observations made with similar instruments have lost, to a great extent, the weight which they would otherwise have merited. Van Swinden, however, had needles from England, where much care had been bestowed on their construction; and he observed every hour from morning till evening during thirteen years, in eight of which three needles were observed simultaneously. The differences of movement of the three needles during magnetic disturbances show that we cannot consider the absolute amounts accurate; the results, however, to which I am about to refer are independent of these differences.

Van Swinden followed with great interest every disturbance (*affolement*) of the needle, and he has given the number of days in each year from 1771 to 1781 on which these *affolements* occurred. As he had not apparently any exact measure of what should be considered a day of disturbance, these numbers cannot be considered quite comparable. They show, however, fewest days in 1771 and 1780–81, while the greatest number of disturbance days occurred in 1773, 1774, 1775, and 1777.* Van Swinden, however, obtained another result with specific limits. The north end of the needle is generally farthest west between

* Analogie de l'Électricité et du Magnétisme, t. iii. p. 85. This volume contains the "Dissertation sur les mouvements irreguliers de l'Aiguille aimantée, par J. H. Van Swinden, La Haye, 1785."

1 and 2 P.M. VAN SWINDEN remarked that on many days without irregular movements the mean law was not followed, that the needle was farthest west before noon or after 4 P.M.; this irregularity is also due, however, to disturbances; he then sought the numbers of irregular days in each year on which the needle attained the maximum before noon or after 4 P.M. From the table which he has given, he concludes—"On voit par là combien le nombre de ces jours a cru de 1774 à 1776, et décru de 1776 à 1780. Y auroit-il quelque période de quatre ans?"* That is of eight years from minimum to minimum. An inspection of VAN SWINDEN'S numbers given afterwards (Art. 30), and a consideration of other facts, will show, I think, that VAN SWINDEN had here discovered one of the results of the decennial period; he does not appear, however, to have examined the amounts of the diurnal range of the needle in each year.

3. Neither CASSINI nor GILPIN seem to have noticed the variation of the range of the diurnal oscillation from year to year; and ARAGO'S observations from 1820 to 1830 were not reduced in such a manner as to show the decennial period till 1854, and even then it was not remarked.†

4. The first who appears to have observed the fact, or thought it worth noticing, that the diurnal range of magnetic declination varied from year to year, was GAUSS. In his discussion of the Göttingen observations for the three years April 1834 to March 1837, he remarked that the range of the diurnal oscillation of the magnetic needle between 8 A.M. and 1 P.M. was greater in each month of the second year than in the corresponding months of the first; and again, greater in the third than in the second year; adding, "But these differences are much too great for us to conclude that they are due to a secular increase, and it is much rather to be expected that by continuing the observations during several years an oscillation (*Hinundherschwanken*), cannot fail to present itself."‡

5. In 1846 Dr LAMONT added to the Göttingen results from 1834 to 1842 his own, derived from the Munich observations from 1842 to 1845, and pointed out the very regular change of the diurnal oscillation during the ten years. The ranges showed the maximum in 1837–38, and Dr LAMONT concluded there was reason to believe that the minimum was then attained (1846)§. The observations of ten years did not prove, however, that the fluctuation predicted by GAUSS was periodic; and it was only in 1851, after the passage of a second maximum in 1848, that Dr LAMONT concluded, with the aid of preceding series of observations, the existence of a period occupying on the average $10\frac{1}{3}$ years.‖

6. Early in 1852 General Sir E. SABINE communicated to the Royal Society

* Analogie de l'Élect. et du Mag. t. iii. p. 129. This result of VAN SWINDEN'S long and persevering labours seems to have been lost sight of.

† Œuvres de F. ARAGO, t. iv. p. 501, Paris, 1854. ‡ Resultate des Mag. Vereins, 1836, S. 54.

§ Resultate des Mag. Obs. in München, 1846, S. 31.

‖ POGGENDORFF'S Annalen, B. 84, S. 572, Dec. 1851.

of London the coincidence between the epochs of minimum and maximum magnetic disturbance and diurnal range of the magnetic oscillations in 1844 and 1848, deduced by him from the colonial observations, as well as of the epochs obtained by Dr LAMONT for the diurnal range, with those which SCHWABE had previously discovered in his persevering observations of sun-spot frequency.*

The coincidence in SCHWABE's and LAMONT's decennial periods was also remarked independently in the same year by Dr R. WOLF of Bern (now of Zurich)† and M. GAUTIER of Geneva.‡ The former, who has devoted himself with great zeal for years to the collection and discussion of observations on sun-spots made during the last two centuries, has deduced a mean period of $11\frac{1}{9}$ years, differing widely from 10·43 years, the mean interval last obtained by Dr LAMONT.§ The care bestowed on this investigation, and the wide interval covered by it, have given to Dr WOLF's result so great a weight that it has been accepted by many men of science as the true duration of this inequality.

While there can be little doubt that continuous series of magnetical observations are better fitted for determining the true epochs of maxima and minima than observations of sun-spots, which cannot always be made, and which before this century were noted by different observers without system, yet when no magnetic observations have been obtained, any epochs of sun-spot frequency which Dr WOLF may have shown to rest upon sufficient data should have a great value in this investigation.

The first part of this paper is occupied in the determination of the mean duration and variable length of the "decennial period" from the earliest systematic observations of the magnetic needle till now, employing for this end a somewhat more exact method than has been used hitherto. The second part is devoted to the decennial period of magnetic disturbance of the magnetic declination (1854–64) at Trevandrum, and the relation of the changes of magnetic disturbance to those of sun-spot area, as determined by Messrs DE LA RUE, STEWART, and LŒWY.

7. *Trevandrum Observations.*—The following table contains the ranges of the monthly mean diurnal variations of magnetic declination for each year from 1853 to 1875; these are obtained from hourly observations from February 1853 to February 1865, and from observations made during the following years eight times daily at the hours—$6\frac{1}{2}$, $7\frac{1}{2}$, $10\frac{1}{2}$, $11\frac{1}{2}$ A.M., and $0\frac{1}{2}$, $2\frac{1}{2}$, $4\frac{1}{2}$, and $5\frac{1}{2}$ P.M. The ranges were also deduced from the observations made at these hours in the years 1853 to 1865, and found to be on the average 0′·03 less than from the whole series of 24 hourly observations; this quantity was therefore added to the ranges after February 1865.

* On the Periodical Laws, &c., received Mar. 18; read May 6, 1852.—Phil. Trans., p. 183, 1852.
† Berner Mittheilungen, No. 245, 1852. ‡ Bibliotheque Universelle, Juillet et Août, 1852.
§ Ueber die zehnjährige Periode; Sitz. b. der k. Akad. z. München, 1862, Bd. ii. Heft 2.

TABLE I.—*Range of the Mean Diurnal Variation of Magnetic Declination for each Month, 1853 to 1876, at Trevandrum.*

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1853	[2′·40]	2′·57	1′·08	1′·35	2′·90	3′·55	3′·51	4′·10	4′·29	1′·07	2′·15	2′·38
1854	2·18	1·65	0·57	1·81	3·20	3·04	2·90	3·30	3·21	1·29	2·26	2·31
1855	2·10	1·65	0·67	2·05	2·76	2·88	2·71	3·19	3·23	1·14	1·52	1·38
1856	1·93	1·40	0·85	1·32	1·97	2·47	2·71	3·20	2·72	1·07	2·18	2·28
1857	1·82	1·69	0·95	1·34	3·05	2·32	2·54	3·18	3·79	0·99	1·78	2·26
1858	2·41	1·59	1·26	1·95	2·86	2·92	3·36	3·78	2·83	1·09	2·71	2·20
1859	2·24	1·69	1·02	1·96	3·29	3·44	3·44	3·62	3·71	1·62	2·59	3·22
1860	2·73	2·03	1·69	2·27	3·54	3·64	3·35	5·32	3·59	1·73	2·46	2·32
1861	1·79	1·08	0·78	1·33	2·77	3·32	3·43	4·51	3·72	1·48	2·03	2·67
1862	2·32	1·28	0·85	1·60	2·79	3·36	2·72	2·84	2·98	1·25	2·36	1·83
1863	2·48	1·49	0·85	1·56	3·00	2·86	3·34	3·18	2·64	1·36	2·19	2·43
1864	1·99	1·65	0·35	1·54	3·09	3·22	2·86	3·73	3·40	1·44	1·68	2·48
1865	2·45	1·94	0·75	1·96	2·81	2·91	2·49	3·18	2·67	1·45	2·01	1·97
1866	1·51	1·95	0·62	1·38	2·27	2·59	2·09	2·30	2·59	1·36	2·07	2·20
1867	1·44	1·51	0·73	1·95	2·36	2·46	2·62	2·75	1·81	1·51	2·18	2·13
1868	1·64	1·60	0·97	2·16	2·67	2·94	2·32	3·58	2·94	1·13	2·05	2·74
1869	3·26	2·19	0·69	1·79	3·29	3·81	3·82	3·89	2·84	1·58	2·47	2·15
1870	2·15	1·34	0·91	1·82	3·08	4·27	4·33	4·96	4·09	1·14	2·03	2·41
1871	2·81	2·24	0·74	2·60	3·01	3·92	4·01	4·65	3·39	1·59	2·43	3·01
1872	2·42	2·23	0·61	2·08	3·49	3·37	3·62	3·97	4·00	1·19	2·47	2·38
1873	2·26	1·96	0·63	1·39	3·27	2·91	3·50	3·75	3·15	1·16	1·68	1·71
1874	1·97	2·24	1·00	1·79	2·57	2·60	2·20	2·91	3·02	1·20	1·54	2·13
1875	1·70	1·80	0·76	0·96	3·02	2·54	2·10	2·88	2·93	1·35	1·48	1·56
1876	1·56	1·82	0·68	1·20	2·66	2·96	...	...	...	...	...	...

8. In order to obtain the epochs of maximum and minimum oscillation it has been usual to employ only the yearly means corresponding (for their middle points) to the 1st July. I have sought more complete results by taking the yearly means corresponding to the 1st of each month;* these are given in Table II., where the quantity under each month is the mean range deduced from the oscillations for six months before and six months after the first of that month. These quantities are projected in Plate XXXIX.

9. From Table II. we obtain at once the following epochs with the corresponding values of the yearly mean range.

Minimum, 1856·3. Range, 1′·88.	Maximum, 1860·3. Range, 2′·98.
„ 1866·5. „ 1·90.	„ 1870·86. „ 2·92.

From the nearly constant and low value (2′·01 to 1′·97) for the 12 months, corresponding to 1st November 1874 to 1st April 1875, I have concluded that the minimum is again nearly attained.†

* This method was employed by me previously for the daily and monthly means of horizontal magnetic force.—Trans. Roy. Soc. Edin., vol. xxii. plates xxv., xxvi., and xxviii.

† I have been able, before printing, to add the means for several months later than those in my possession when this paper was written.

TABLE II.—*Yearly Means of the Ranges of Monthly Mean Diurnal Variations of the Magnetic Declination corresponding to the 1st of each month.—Trevandrum*, 1853–1875.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	′	′	′	′	′	′	′	′	′	′	′	′
1853	...	...	...	...	...	...	2·61	2·59	2·52	2·47	2·51	2·54
1854	2·50	2·45	2·38	2·29	2·31	2·32	2·31	2·30	2·30	2·31	2·33	2·30
1855	2·28	2·27	2·26	2·26	2·25	2·19	2·11	2·09	2·07	2·09	2·03	1·96
1856	1·93	1·93	1·93	1·88	1·88	1·93	2·01	2·00	2·02	2·03	2·03	2·12
1857	2·11	2·10	2·10	2·18	2·18	2·14	2·14	2·19	2·18	2·21	2·26	2·24
1858	2·29	2·36	2·41	2·33	2·34	2·42	2·41	2·40	2·41	2·39	2·39	2·42
1859	2·47	2·47	2·46	2·53	2·58	2·57	2·65	2·69	2·72	2·78	2·80	2·82
1860	2·84	2·83	2·98	2·96	2·97	2·96	2·89	2·81	2·73	2·66	2·58	2·51
1861	2·49	2·49	2·43	2·44	2·42	2·38	2·41	2·45	2·47	2·48	2·50	2·50
1862	2·50	2·44	2·31	2·24	2·22	2·25	2·26	2·28	2·30	2·30	2·29	2·31
1863	2·27	2·33	2·36	2·33	2·34	2·32	2·29	2·25	2·26	2·22	2·22	2·23
1864	2·26	2·21	2·25	2·32	2·32	2·28	2·29	2·32	2·35	2·38	2·42	2·39
1865	2·37	3·34	2·29	2·23	2·24	2·26	2·22	2·15	2·17	2·16	2·11	2·06
1866	2·04	2·00	1·95	1·92	1·91	1·92	1·94	1·93	1·90	1·91	1·95	1·96
1867	1·95	1·99	2·03	1·97	1·98	1·99	1·97	2·00	2·01	2·03	2·04	2·07
1868	2·11	2·08	2·15	2·25	2·22	2·21	2·26	2·39	2·44	2·42	2·39	2·44
1869	2·51	2·63	2·66	2·65	2·69	2·72	2·68	2·58	2·51	2·53	2·53	2·52
1870	2·55	2·60	2·69	2·79	2·75	2·72	2·74	2·79	2·87	2·85	2·92	2·91
1871	2·88	2·86	2·83	2·78	2·81	2·84	2·89	2·86	2·86	2·85	2·81	2·85
1872	2·80	2·77	2·71	2·76	2·73	2·73	2·68	2·67	2·64	2·65	2·59	2·57
1873	2·53	2·52	2·50	2·43	2·43	2·36	2·31	2·28	2·31	2·34	2·37	2·31
1874	2·29	2·18	2·11	2·10	2·10	2·09	2·12	2·10	2·07	2·07	1·97	2·01
1875	2·01	2·00	2·00	1·99	2·00	2·00	1·95	1·94	1·94	1·93	1·95	1·93

10. The earliest systematic series of observations of the magnetic declination showing a maximum or minimum of diurnal range are those of CASSINI (Paris, 1784–88),* GILPIN (London, 1786–1805),† BEAUFOY (London, 1813–20),‡ and ARAGO (Paris, 1820-30).§ Of the later series I have employed the observations of GAUSS (Göttingen, 1834–41),‖ LAMONT (Munich, 1840–50),¶ LLOYD (Dublin, 1840–50),** BROUN, (Makerstoun, 1842–46),†† YOUNGHUSBAND and

* De la déclinaison et des variations de l'Aguille aimantée, Paris, 1791.

† Phil. Trans. 1806, p. 416.

‡ THOMSON's Annals of Philosophy.

§ Œuvres de F. ARAGO, t. iv p. 501.

‖ Resultate des Mag. Vereins im Jahre 1836, 1839.

¶ Resultate d. Mag. Beob. 1840–42; Result. d. Mag. Observatoriums in München, 1843–45; POGGENDORFF Annalen, B. 84. In obtaining the yearly mean range, 1′·1 has been added to the yearly means from 1841.1 (Feb. 1), to 1842·0, derived from two-hourly observations, to reduce to the means from 1842·0 to 1845·5 from hourly observations, after 1845·5, 0′·6 is added. The object of these corrections has been to obtain comparative results from different places for the epochs of minimum (37).

** Observations made at the Mag. and Met. Obs. Dublin, vol. i. p. 89; vol. ii. p. 53. The ranges are taken from observations at 7 A.M. 1 P.M. or $\frac{9+11}{2}$ P.M. in 1840–43, and from the three-hourly observations after 7 A.M. in the following years.

†† Observations in Magnetism and Meteorology (Trans. Roy. Socy. Edin. vol. xvii., xviii., and xix.), 1′·2 has been added to the yearly mean ranges fron January 1, 1843, to June 1, 1844.

Lefroy (Toronto, 1841–48),* Kay (Hobarton, 1841–48),† Smythe (St Helena, 1841–47),‡ Wilmot and Clerk (Cape of Good Hope, 1841–46).§ As the different results obtained by Drs Lamont and Wolf depend wholly on the value to be given to the first series of observations, and the interpretation especially of Gilpin's series, these require the most careful consideration. The essential details relative to the other series will be found in the notes below.

11. The yearly mean ranges corresponding to each month have been obtained for each place in the same manner as for Trevandrum, and they have been projected in the curves, Plate XXXIX. From these means we obtain the following epochs and ranges;—

Munich,	Maximum,	1848·9	= 11′·53	Minimum, 1844·25 = 6′·56
Dublin,	„	1848·9	= 13′·31	„ 1844·25 = 8′·76
Makerstoun,	. . . . .			„ 1844·25 = 7′·86
Toronto,	. . . . .			„ 1843·9 = 8′·47
St Helena,	. . . . .			„ 1843·4? = 3′·44
Hobarton,	. . . . .			„ 1843·4 = 7′·57
Cape of Good Hope,	. . . . .			„ 1843·5 = 4′·52

The hourly observations were not continued at the last five stations sufficiently long to give the time of maximum. We deduce from the southern stations the minimum in 1843·43 nearly, and from the southern stations 1844·25, or we have nearly for the mean epochs, maximum 1848·9, minimum 1843·85.

12. The period preceding this depends on Gauss's observations; they give a double maximum of nearly equal value in 1836·9 and 1838·3 (see Plate XXXIX.), the former of which, however, has the greatest weight; we may perhaps be able to take with no great error the maximum in 1837·5 = 13′·04.‖ The minimum occurred before 1834·7, probably near 1833·5, if we may judge from Schwabe's observations, to which reference will be made hereafter.

13. From Arago's observations, we derive the well-marked epochs, maximum 1829·7 = 13′·74, minimum 1824·3 = 7′·75. (See Plate XXXIX.)

14. *Beaufoy's Observations.*—All the preceding series of observations were

* Observations made at the Magnetical and Meteorological Observatory at Toronto, in Canada, edited by Lt. Col. E. Sabine, vol. i. p. xii; vol. ii. p. 10. 0′·5 has been added to the yearly mean ranges from July 1, 1841, to June 1, 1842, derived from two-hourly observations, to reduce to hourly observations made afterwards.

† Observations, &c., at Hobarton, edited by Lt. Col. E. Sabine, vol. i. p. xxvi.

‡ Observations, &c., at St Helena, edited by Lt. Col. E. Sabine. For 1841–1845, see vol. i. p. 24; for the remaining years the ranges are taken from the tables of hourly observations, vol. ii.

§ Observations, &c., at the Cape of Good Hope, edited by Lt. Col. E. Sabine, vol. i. p. xvi. From April to September 1841, the observations were two-hourly, afterwards they were hourly. Corrections of +0′·55, +0′·08, +0′·61, and +0′·49, have been applied to the mean ranges, for the months of April, May, June, and August 1841 respectively, on account of the observations wanting at the hour of minimum.

‖ This epoch is confirmed very nearly by the Milan observations. Schiaparelli's table of the mean diurnal ranges, from 1836 to 1873, given in his memoir, "Il periodo undecennali," &c., has come to my knowledge only after this paper was written; they confirm very nearly the epochs obtained from other observations.

made with magnets suspended by silk fibres; BEAUFOY observed with a needle and agate cup suspended on a steel point; the imperfections of such instruments are so well known, that observations made with them are always regarded with suspicion (2). BEAUFOY employed two needles, weighing 48 and 65½ grains respectively, making always seven observations direct, and seven observations inverted, with each needle.* While the consistent results obtained by this observer for the monthly and annual mean positions of the needle are much in favour of its sensibility, we have fortunately other evidence in the simultaneous observations made by ARAGO during the eleven months, February to December 1820; the mean ranges, according to both observers, during this period are as follow:—February to December 1820, BEAUFOY, 7′·45; ARAGO, 10′·07.

BEAUFOY'S observations were made near $8^h\ 40^m$ A.M., and $1^h\ 25^m$ P.M., and the ranges are derived from the means at these hours. ARAGO made generally an average number of eleven observations daily, beginning at 7 A.M. and ending at 11 P.M.†, and the ranges are the means of the largest observed oscillations in each day; they are therefore the mean daily ranges, and not the ranges of the hourly means for each month, as in other cases. An approximation to the ratio of the two kinds of ranges may be obtained from the Makerstoun observations for the years 1843 and 1846, during which observations were made two-hourly, from 5 A.M. to 9 P.M. From these we obtain the mean movement from $8^h\ 40^m$ A.M. to $1^h\ 10^m$ P.M. (nearly, as in BEAUFOY'S series) = 6′·98. The mean of the daily ranges for the same two years = 11′·51, and the ratio is—

$$\frac{11\cdot51}{6\cdot98} = 1\cdot65.‡$$

If we multiply BEAUFOY'S range by this ratio we obtain $7'\cdot45 \times 1\cdot65 = 12'\cdot3$, which is the quantity to be compared with ARAGO'S range of 10′·07. Allowing for the irregularity in the number of observations made daily by the French astronomer, by which the daily ranges may have been somewhat diminished, and for any difference of the ranges for Paris and London, the London needle seems to have been sufficiently sensitive.

BEAUFOY'S observations were made from April 1813 to September 1815, and from April 1817 to December 1820. The yearly mean ranges corresponding to

* THOMSON'S Annals of Philosophy, vol. iii. p. 372.

† Œuvres de F. ARAGO, t. iv. p. 427.

‡ A similar comparison of the Greenwich two-hourly observations for the four years, 1843 to 1845, gives the range from $8^h\ 40^m$ A.M. to $1^h\ 20^m$ P.M. = 7′·4, while the mean of the daily ranges = 12′·2, whence the ratio—

$$\frac{12'\cdot2}{7'\cdot4} = 1\cdot65$$

is exactly that found from the Makerstoun observations. The ratio varies somewhat with the year and the amount of disturbance, the values for the four years at Greenwich being 1·53, 1·76, 1·64, and 1·67. As 1820·5 was upwards of two years from the epoch of maximum disturbance, the ratio found cannot be far from the truth.

the beginning of each month are projected Plate XXXIX. From this we obtain the maximum 1818·2 = 8′·79; the minimum 1813·7 = 6′·96 ?

15. As the yearly mean range is so nearly constant from 1817·7 to 1818·5, the epoch of maximum is probably near the truth. The minimum may have occurred earlier.*

16. *Cassini's Observations.*—The earliest systematic series with which we are acquainted showing a maximum or minimum is that made in the years 1784 to 1788 by CASSINI, who employed COULOMB'S silk fibre suspension The astronomer of Paris, with the aid of three assistants, followed the movements of the needle from morning till evening between 1783 and 1792;† the maximum westerly position was obtained for each day from the observations between noon and 3 P.M., the mean of these positions was found for each eight days, corresponding to the 4th, 12th, 20th, and 27th or 28th of each month.‡ In a similar manner the minimum westerly positions were deduced from the morning or evening observations; the differences of four pairs of means thus calculated give the monthly mean ranges. The yearly means, corresponding to the 1st of each month, are projected Plate XXXIX. From this curve we derive the maximum 1787·25 = 15′·29; the minimum 1784·8 = 9′·21 ?

As in the case of BEAUFOY'S observations the minimum noted is so near the beginning of the series that the exact epoch is by no means certain.§

* The ranges at Greenwich from $8^h\ 40^m$ to $1^h\ 20^m$, and at Makerstoun from $8^h\ 40^m$ to $1^h\ 10^m$, about the minimum year 1844 were approximately as follows—

	Greenwich.	Makerstoun.
1843	7′·05	6′·54
1844	6·19	6·01
1845	7·28	7·13

We must conclude then, that if the minimum occurred at 1813·7, it had a greater value than in 1844·5 at Greenwich; or, if the value of the former minimum was nearly the same as the latter, that the former occurred probably near 1813·0.

It may also be remarked that the mean of the daily ranges for Greenwich in 1847 was 17′·8, for which year the range from $8^h\ 40^m$ A.M. to $1^h\ 20^m$ P.M. was approximately 8′·66, which is less than the maximum of 1818. The mean of the daily ranges in 1818 was therefore between 18′ and 19′. From omitting the consideration of the hours to which BEAUFOY'S ranges refer, it has been supposed that the diurnal oscillations was very small in 1818.

† Observations Astronomiques et Physiques faites à l'Observatoire en l'année 1791, p. 345 and note p. 350. Only the observations from 1783 to 1788 were published in CASSINI'S memoir "Sur la déclinaison et les variations de l'Aiguille aimantée lu à l'Académie Royale des Sc., Août 1871. While the results obtained by CASSINI for the mean position of the needle were vitiated by different causes, the deduced ranges are probably approximately true.

‡ Different authors have supposed erroneously that CASSINI observed only on these four days in each month.

§ It is difficult in this instance to seek the epoch from the value of the range, as it is not quite certain whether CASSINI'S ranges are those of the means for each week at the hour of least and greatest declination, or simply the means of the daily ranges. If the former, then the minimum was probably less than that noted, and occurred earlier. It should also be pointed out that we are not acquainted with any marked period, such as that of CASSINI, in which the minimum occurs only $2\frac{1}{2}$ years before the maximum.

17. *Gilpin's Observations.*—Although this series (1786–1805) began after CASSINI'S, I have considered the latter first for reasons which will soon be obvious. The whole difference between the conclusions of Drs LAMONT and WOLF depends on the interpretation of the magnetical and sun-spot observations made between 1787 and 1818. GILPIN employed a needle with an agate cap (carefully turned by the well-known maker NAIRN) resting on a steel point. The instrument was made under the supervision of CAVENDISH,* and every precaution was taken by GILPIN to obtain the true position of the needle at rest by attracting it frequently to both sides. He observed from 6 A.M. to 10 P.M. in all the months of the years 1877 and 1793, but only during from two to seven months in the other years.

In order to deduce the best possible results the means of the ranges for the months on which observations were made in each year were compared with the mean of the ranges for the same months in the years 1787 and 1793; the differences applied to the mean of the ranges for all the months in these two years gave the approximate mean range for each year;† the results are as follows:—

Year.	No. of Months.	Ranges.	Year.	No. of Months.	Ranges.
1787·2	12	14′·84	1796·5	5	7′·21
1787·5	12	14·97	1797·5	5	7·53
1788·5	4	13·83	1798·6	5	7·29
1789·5	2	11·88	1799·5	5	7·17
1790·5	2	11·93	1800·5	5	6·75
1791·5	3	11·41	1801·5	5	7·69
1792·5	7	9·13	1802·5	5	8·23
1793·5	12	8·43	1803·5	5	9·19
1794·5	4	7·01	1804·5	5	8·13
1795·5	5	6·95	1805·5	5	8·17

These quantities are projected Plate XXXIX., and from the curve we obtain the following epochs and ranges—

Maximum 1787·5 = 14′·97	Minimum 1795·1 = 6′·90
„ 1797·7 = 7·60	„ 1800·5 = 6·75
„ 1803·5 = 9·20	

18. Dr WOLF considers that GILPIN'S observations show a minimum in 1796, whereas Dr LAMONT cannot conceive how a minimum in 1796 can be derived from these numbers; he adds, "In fact, they show no period whatever, which is easily understood when we remember that a needle on a steel point was

* See the description by CAVENDISH, Phil. Trans. 1776, p. 385.

† Phil. Trans. 1806, p. 416. The ranges for each month were determined from a mean of observations made at those times of the day when the declination was considered greatest and least. Generally 600 observations were made in each month.

employed, which needle was so insensible that, according to GILPIN's exp statement, the accidental deviations could amount to 8′ or 10′ or even more."*

An attentive consideration of GILPIN's observations induces me to conclude that Dr LAMONT's view cannot be accepted without a considerable qualification; for although the needle *sometimes* did not return to its previous position within 10′ or more, yet we have the evidence of GILPIN's monthly and yearly means, showing with considerable exactness the small westerly movement of the magnet then taking place, from 23° 50′ in 1793, to 24° 9′ west in 1805, as proof of the general exactness of his observations.† At the same time, there is no reason to believe that the *occasional* inconsistencies, which GILPIN sought to correct by frequent observations, would diminish the range more than increase it. We have, however, as in the case of BEAUFOY's instrument, the means of verifying the sensibility of the London needle, by the simultaneous observations made at Paris at the commencement of GILPIN's series; the comparative mean ranges are as follow:—

	CASSINI.		GILPIN.
1787·2, . .	15′·27	. . . . .	14′·84
1787·5, . .	15·14	. . . . .	13·97
1788·5, . .	13·48	. . . . .	13·83

19. These quantities give no reason to doubt the sensibility of GILPIN's needle at these periods. The principal objections to the results of GILPIN are to be found later, in the small ranges for the maximum of 1797·7, to which I shall refer immediately. It should be observed that, as far as I am aware, no one has suggested that GILPIN's observations show a maximum in the year just mentioned; but a consideration of the facts allow, it seems to me, little doubt that a maximum occurred at that time.

A second maximum appeared in 1803·5; but GILPIN's ranges for the last four of the five months of observations in 1805, show an average increase of 0′·52 on the ranges for the corresponding four months of 1804; it is by no means certain then that the second maximum did not occur after 1805·5. We may now compare the results obtained from the whole series of observations.

* "Einige Bemerkungen über die zehnjährige Periode," &c., Sitz. b. der k. Akad. d. W. 1864. GILPIN's statement is—"Sometimes the needle would be extremely consistent with itself, so as to return exactly to the same point, however often it might have been drawn aside; at other times it varied 2′ or 3′, sometimes 8′ or 10′, or even more."—*Phil. Trans.* 1806, p. 416.

† ARAGO has also indicated the large diurnal oscillation obtained by GILPIN and its varying amount with the season (as elsewhere) as evidence of the free movement of GILPIN's needle. His chief difficulty has reference to the small *annual* variation of the *mean position* compared with that found by CASSINI, a result which I believe to be wholly in favour of GILPIN's observations, since no such large annual movement as that found by CASSINI has been shown by any careful series of observations since his time. See "Œuvres de F. ARAGO," t. iv. p. 482. I regret that I have not been able to find the original observations which were made in the Royal Society's Apartments, Somerset House, from 1786 to 1808 by GILPIN, and continued thereafter by Mr LEE, the librarian (See BEAUFOY, Annals of Philos., p. 339). It is not improbable, however, that they may yet be discovered.

20. *Summary of the Epochs if Maxima and Minima, deduced from the Magnetic Observations.*—These are as follow :—

Epochs of Maxima.	Interval Years.	Epochs of Minima.	Interval Years.
		1875·7?	
1870·85			9·20
	10·55	1866·50	
1860·30			10·20
	11·40	1856·30	
1848·90			12·45
	11·40	1843·85	
1837·50			10·35
	7·80	1833·50?	
1829·70			9·70
	11·50	1824·20	
1818·20			.....?
	.....?	.....?	
.....?			.....?
	.....?	1800·5	
1797·7			5·5
	10·45	1795·0	
1787·25			.....?
		.....?	

It will be seen that the maximum interval occurred between the minima of 1843 and 1856, being 12·45 years ; the diminution in the intervals afterwards induced me to conclude in September 1875, that we were approaching a minimum period like that experienced between 1829–1837.* This is more evident when the intervals are arranged in succession as below.

Interval of Minima, 9·20? years.	Interval of Maxima, 11·40 years.
„ Maxima, 10·55 „	„ Minima, 10·35 „
„ Minima, 10·20 „	„ Maxima, 7·80 „
„ Maxima, 11·40 „	„ Minima, 9·30 „
„ Minima, 12·45 „	„ Maxima, 11·50 „

21. Should the next maximum occur about 1879·5, the series will be completed within nearly 42 years. Thus we have the intervals—

1875·7?−1833·5? = 42·20 years?	1866·50−1824·20 = 42·30 years.
1870·85−1829·70 = 41·15 „	1860·30−1818·20 = 42·10 „

22. If with an approximate interval of 42 years, we seek to determine the epochs before 1818, we shall find them as follows :—

1. 1856·3 −42·0 = 1814·3 Minimum.	6. 1829·70−42·0 = 1787·7 Maximum.
2. 1848·9 −42·0 = 1806·9 Maximum.	7. 1824·20−42·0 = 1783·2 Minimum.
3. 1843·85−42·0 = 1801·85 Minimum.	8. 1818·2 −42·0 = 1776·2 Maximum.
4. 1837·50−42·0 = 1795·5 Maximum.	9. 1814·3?−42·0 = 1772·3? Minimum.
5. 1833·50−42·0 = 1791·5 Minimum.	

* Comptes Rendus, t. lxxxi. p. 752, 26th Oct. 1875.

The first epoch is probably one year too late; the second may not be far from the truth (19.); the third is 1·3 years too late; the fourth, 2·2 years too soon; the fifth, 3·5 years too late; the sixth is 0·3 years too late; and the remaining epochs are probably not far from the truth (Arts. 30–32). The most marked differences are those for the otherwise irregular period 1795–1801.

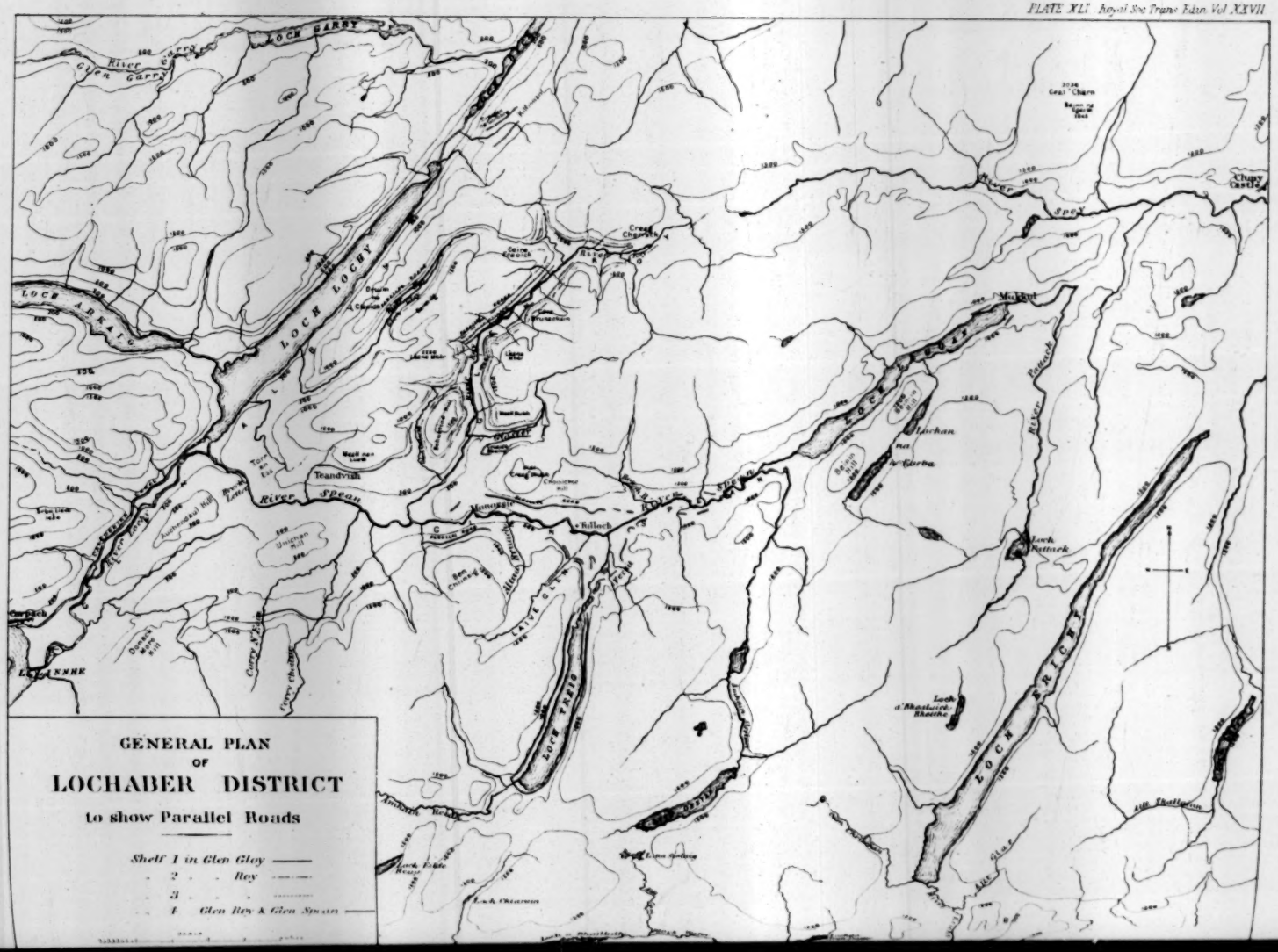
23. *Does the movement shown by Gilpin's observations from* 1795 *to* 1801 *belong to a true period?*—While the very different and sometimes absolutely opposite results, obtained by many celebrated observers, from the needle, supported on a steel pivot, in the period 1770–1780, immediately before the commencement of GILPIN's observations, have thrown doubt on all observations made with the same kind of instrument since, I have already noted some proofs of the accuracy of the conclusions deduced from GILPIN's series, and the agreement in the first instance at least with CASSINI's observations, showing nearly the same epoch of maximum diurnal oscillation. The doubt, however, is not wholly removed by these other proofs, and it is increased by the fact, that since systematic observations have been made with more perfect instruments, we have seen no appearance of so short a period, and especially have seen no period in which the whole variation of the diurnal oscillation was less than 1′, as in the case in question.

On the other hand, we must also observe that *we have no case in which two minima of nearly equal value, belonging to the same epoch, are separated by an interval of* 5·5 *years like those for* 1795·0 *and* 1800·5. It acquires only a glance at the curves projected, Plate XXXIX., to see the weight of this fact. We have, however, another method of determining whether a period probably existed of such a character as that shown by GILPIN's observations.

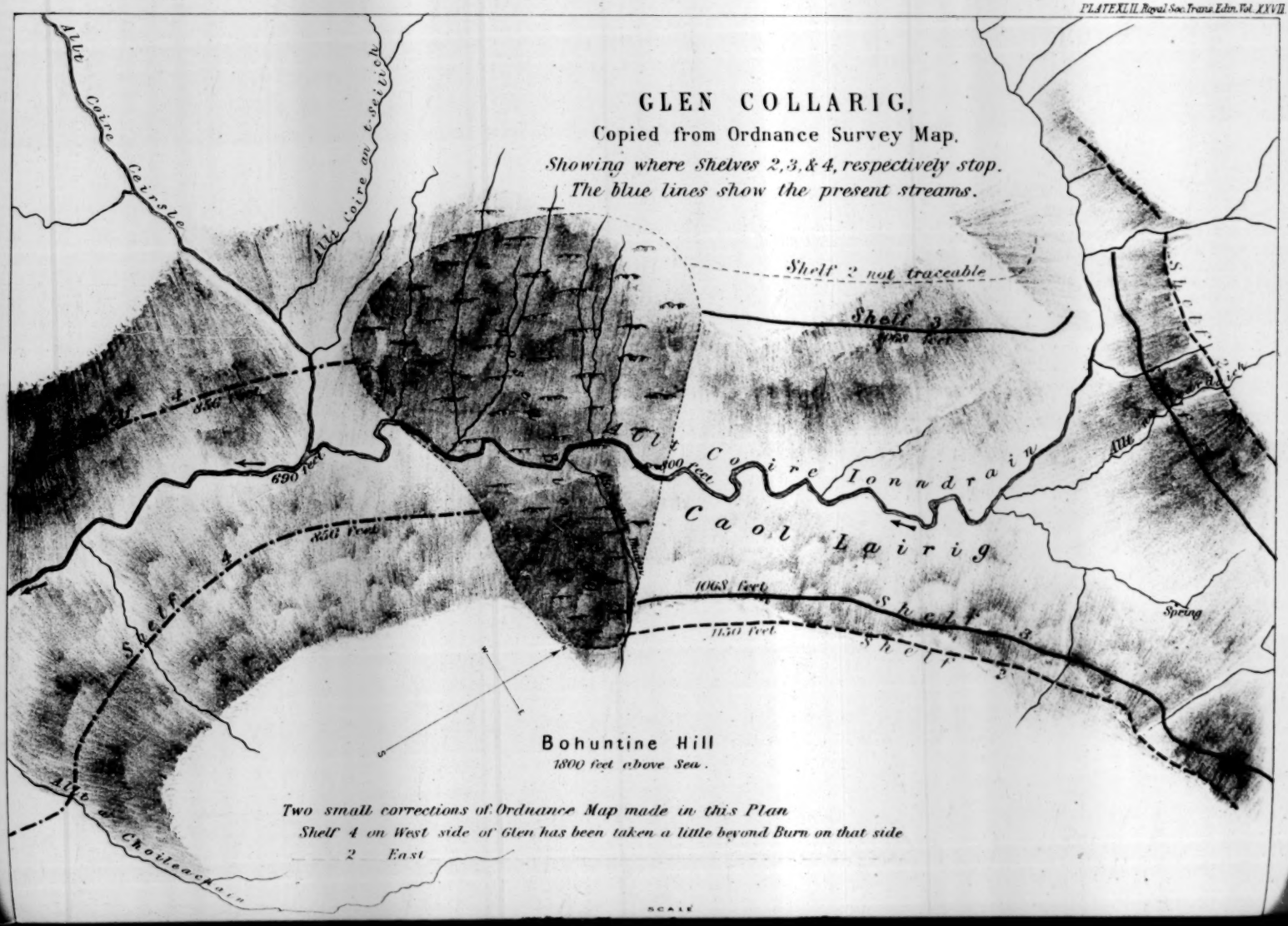
24. The general agreement between the epochs of maximum and minimum sun-spot frequency and diurnal range of the magnetic needle has been already referred to (6.): we shall leave at present the evidences of this agreement during the last half century, and consider what is known of the former phenomenon in the periods immediately preceding. Our knowledge of sun-spot observations during the last as well as in a great part of this century is due to Dr WOLF. Unfortunately, it is just at the time when we have the most need for complete series of sun-spot observations that they are most wanting. From 1790 to 1815 was a period when men seem to have had their attention turned too strongly towards the earth's surface for them to be able to examine with care that of the sun. Dr WOLF's "relative" spot numbers for this period can therefore be considered only as rude approximations in some cases, and perhaps as doubtful guesses in others. They cannot, however, I think, be considered without any value on account of these defects; they contain all the information we possess on spot frequency for the time.*

* Dr LAMONT has criticised some of the epochs which Dr WOLF considers certain ("sicher"), and has

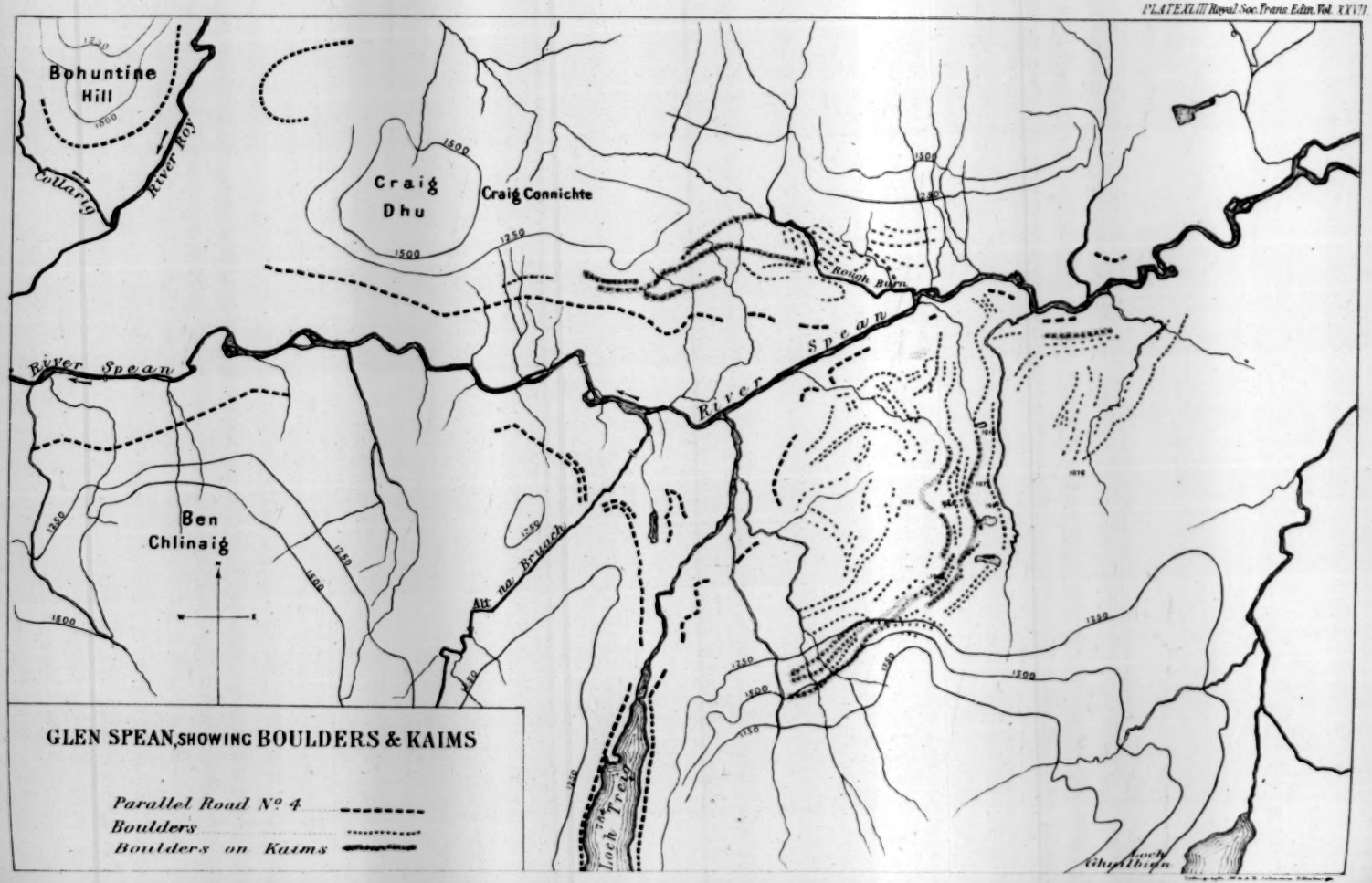
BRITISH MUSEUM
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NATURAL


GENERAL PLAN
OF
LOCHABER DISTRICT
to show Parallel Roads
Shelf 1 in Glen Gloy
2 Roy
3
4 Glen Roy & Glen Spean
LOCH GARRY
River Garry
LOCH LOCHY
LOCH ARKAIG
River Spean
Treandvish
Kinnaside
Rilloch
Allen Burnside
LOCH TREIG
LOCH ERICHT
Loch Rannoch
Loch a'Bhealaich Bhealtha
Loch Ossian
Chapy Castle
Mulroy
River Pattock
Ben Nevis
Aonach Mor
Aonach Beag
Carn Mor Dearg
Meall Cumhann
Meall Fuar-mhonaidh
Stob Coire Easain
Stob Coire Sgriodain
Stob Ban
Mullach nan Coirean
Sgurr a'Mhaim
Binnein Beag
Binnein Mor
Creag Meagaidh
Geal Charn
Beinn Eibhinn
Beinn Heasgarnich
Beinn a'Chaorainn
Beinn Teallach
Beinn Eagach
Beinn Bhreac
Beinn Bhreac-liath
Beinn Udlamain
Beinn Dubhchraig
Beinn Chaorach
Beinn Mhic Mhonaidh
Beinn Mhic Chasgaig
Beinn Mhic Cedidh
Beinn Mhic Cedidh
Beinn Mhic Cedidh

BRITISH
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MUSEUM


GLEN COLLARIG.
Copied from Ordnance Survey Map.
Showing where Shelves 2, 3, & 4, respectively stop.
The blue lines show the present streams.
Allt Coire Ceirste
Allt Coire an t-Seilich
Shelf 2 not traceable
Shelf 3
Allt Coire Ionndrain
Caol Lairig
Shelf 4
Shelf 3
Shelf 2
Spring
Bohuntine Hill
1800 feet above Sea.
Two small corrections of Ordnance Map made in this Plan
Shelf 4 on West side of Glen has been taken a little beyond Burn on that side
2 East
Allt a' Choilleachain
SCALE




Bohuntine Hill
Craig Dhu
Craig Connichte
Collarig
River Roy
River Spean
Rough Burn
Ben Chlinaig
Alt na Bruaich
Loch Treig
GLEN SPEAN, SHOWING BOULDERS & KAIMS
Parallel Road No 4
Boulders
Boulders on Kaims

25. The following are the "relative" numbers (r) of sun-spots given by Dr WOLF for the years which we are now considering:*—

Year.	r.	Year.	r.	Year.	r.	Year.	r.	Year.	r.
1770	79	1780	73	1790	84	1800	18	1810	0
1771	43	1781	68	1791	53	1801	39	1811	1
1772	49	1782	33	1792	47?	1802	58	1812	5
1773	40	1783	22	1793	40?	1803	65?	1813	14
1774	48	1784	5	1794	34	1804	75?	1814	20?
1775	28	1785	21	1795	22	1805	50?	1815	35?
1776	35	1786	87	1796	15	1806	25?	1816	45
1777	63	1787	105	1797	8	1807	15?	1817	44
1778	95	1788	108	1798	4	1808	7	1818	34
1779	90	1789	111	1799	10	1809	3	1819	22

I do not think it possible to conclude, from the numbers which Dr WOLF has obtained and employed, that a maximum certainly happened in the year 1804 rather than in the years 1805 or 1806, or a minimum in 1810 rather than in 1813 or 1814. Dr WOLF considers that the minimum occurred for 1798·5 +0·5.† Mr THIELE, on the other hand, finds 1796·5,‡ and Sir JOHN HERSCHEL has remarked that 1800 was a year of minimum.§ On the whole, it seems extremely probable that there were comparatively few spots on the sun from 1795 to 1800. This conclusion agrees with that which may be deduced from GILPIN's observations, since the increase in the number of spots, which should have corresponded to the increase of the diurnal oscillation of the needle in 1797, must have been one which would have been shown distinctly only by a careful system of accurate sun-spot observations.

26. We have still another phenomenon, the aurora borealis, which can be related to the magnetic disturbance. I have indeed shown "that the laws of the aurora borealis may be concluded from those of magnetic disturbance, and *vice versa*.‖ That this should hold for the decennial period was at once evident, and different discussions have been performed to prove it. One of the most careful and satisfactory is that by Professor LOOMIS, who has considered only

shown that they depend on few observations. He remarks that old observers directed their attention chiefly to large sun-spots; so that FLAUGERGUES (one of the principal observers during the period in question) saw the sun frequently without spots, when many were seen by other observers ("Einige Bemerkungen über die zehnjährige Periode," S. 23). In an interesting investigation on the decennial period of auroræ, sun-spots, and magnetic variations, Professor LOOMIS has also pointed out the fewness of the observations employed by Dr WOLF, especially those for the years 1802 to 1806 ("American Journal of Science," April 1873). The numbers for some of these years Dr WOLF has himself marked (?) as doubtful, as in the above table.

* "Astronomische Mittheilungen," xxxv.; "Vierteljahrsschrift d. Natur. Gesellschaft in Zürich," 1874, S. 231.

† Ast. Mit. xxiv. ‡ De Macularum Solis. Ast. Nach. No. 1193, 1859.

§ Phil Trans., 1870, p. 397. The word is "maximum" at the place cited; but this is evidently a clerical error.

‖ Trans. Roy. Soc. Edin. vol. xviii. p. 402, 1848.

those auroræ seen below a line of equal frequency passing through St Petersburg, Makerstoun, and Boston.* The following are the numbers of auroræ given by Professor Loomis for the period 1790 to 1820 :—†

Years.	Auroræ.	Years.	Auroræ.	Years.	Auroræ.
1790	82	1800	6	1810	1
1791	79	1801	5	1811	0
1792	66	1802	8	1812	0
1793	23	1803	10	1813	2
1794	11	1804	12	1814	9
1795	9	1805	22	1815	3
1796	3	1806	11	1816	3
1797	15	1807	5	1817	11
1798	1	1808	3	1818	18
1799	7	1809	2	1819	13

Professor Loomis's numbers show in general a variation which agrees to a considerable extent with that of the numbers of solar spots and the ranges of the magnetic variations; they can be considered, however, in many cases as only rough relative approximations for limited periods, depending as they do on the attention that may have been bestowed on this phenomenon by different observers at different times.‡ Whatever weight they may possess is in favour of the conclusions at which I have already arrived. We see that in 1797 more auroræ were observed than in any year between 1793 and 1805, in which latter year there is also a maximum. With these facts before us, we may now determine the mean duration of the decennial period.

27. *Mean Duration of the Period.*—If the difference of the intervals for successive periods follows no law, we can obtain the best approximation to the mean interval by including the greatest space of time possible, consistent with a knowledge of the number of periods, and the exact epochs at the commencement and end of the time considered. If, however, as seems not improbable, the durations of the different periods vary in such a way as to repeat themselves at equal intervals, we can obtain the mean duration accurately only by comparing epochs in the same phase of variation. This is a consideration which can be attended to only when the law of variation is known. Dr Wolf appears

* The principle on which this limitation is founded had been already indicated by me. See "Trans. Roy. Soc. Edin.," vol. xix. pt. ii. footnote, p. lxxxii. 1850.

† "American Journal of Science," April 1873, pp. 249, 256.

‡ I may point out the very few auroræ noted for the maximum of 1818, yet as I have shown (15, footnote) the diurnal range of the magnetic needle for that year was probably as great (or nearly as great) as at any other maximum. It is possible also that the limiting line of equal auroral frequency is variable. In any case little confidence can be placed in the relative magnitude of the number for sun-spots and for auroræ at the epochs under consideration, when compared with those for the half century 1826 to 1876.

to have found long periods of 56 years and of 80–90 years.* From the preceding discussion, the best known epochs seem to show a period of 42 years; as twice this period is within the limits of the last of Dr WOLF, we should obtain the mean duration, if either be true, by employing an interval of about 84 years. We have, fortunately, two very satisfactory epochs for this determination, that of 1787·25 deduced from CASSINI's observations and confirmed by GILPIN's, and the last maximum of 1870·85. From these two epochs, supposing they include eight periods between them, we have

$$\frac{1870{\cdot}85 - 1787{\cdot}25}{8} = \frac{83{\cdot}60}{8} = 10{\cdot}45 \text{ years.}$$

28. If, however, we suppose with Dr WOLF that there were only seven periods, then we have

$$\frac{83{\cdot}60}{7} = 11{\cdot}80 \text{ years.}$$

As there can be no doubt as to the accuracy of the earlier epoch (confirmed as it is also by Dr WOLF's spot numbers and Professor LOOMIS's aurora numbers), we may take for comparison with it one of Dr WOLF's epochs of maximum spot frequency upwards of 80 years before; we have then

$$\frac{1787{\cdot}3 - 1705{\cdot}5}{8} = \frac{81{\cdot}8}{8} = 10{\cdot}23.$$

The number of periods here employed is that given by Dr WOLF. We have then, according to the astronomer of Zurich—

By 8 periods, 1705 to 1787, mean period 10·23 years.
7 „ 1787 to 1871, „ 11·80 „

These two means differ twice as much from each other as Dr WOLF's mean of 11·11 years does from that originally found by Dr LAMONT, and confirmed by this discussion.†

* See "Astr. Mitt. xv. Vierteljahrss." 1863, S. 97, for a notice of a 56 years' period of the auroræ by Professor FRITZ; also of OLMSTED's period of 65 years. For the period of 80–90 years see "Ast. Mitt." xxxviii. S. 378, July 1875. I am not sure that I am acquainted with all the periods which Dr WOLF has discovered, nor do I know if one excludes the other.

† It is not possible to reconcile the two results deduced from Dr WOLF's epochs and periods; if we take any of the epochs of maximum given by him before that here employed (1705) we always fail to reach his period of 11·11 years; thus we have

9	periods,	1787·3	−1693·0 =	94·3, mean	10·43
10	„	„	−1685·0 =	102·3, „	10·23
11	„	„	−1675·0 =	112·3, „	10·21
12	„	„	−1660·0 =	127·3, „	10·61
13	„	„	−1649·0 =	138·3, „	10·64
14	„	„	−1639·5 =	147·8, „	10·56
15	„	„	−1626·0 =	161·3, „	10·75
16	„	„	−1615·5 =	171·8, „	10·74

Little weight can be given to these earlier epochs; but whatever weight they may possess, if we start from the sure epoch of 1787 no interval including more than 80 years will be found to satisfy the

29. I believe then that I have shown, in opposition to Dr WOLF's conclusion, that there was a maximum in 1797; in opposition to Dr LAMONT, that GILPIN's observations are in all probability trustworthy; that there was probably a period of small variation in the amplitude of the diurnal oscillation; which, should it recur, may aid in the discovery of the cause of the variations of sun-spot and auroral frequency, as well as of the magnetic disturbances; and that the mean durations of the period is about 10·45 years.*

30. *Maximum of* 1776.—It will have been seen (Art. 22) that the long period of 42 years gives 1776·2 as the epoch of *maximum* before 1787·2. Dr WOLF finds from his spot numbers that 1775·8 was the epoch of *minimum*, and that 1779 was the year of maximum; this also agrees with Professor LOOMIS's numbers of auroræ. We have, fortunately, two series of magnetical observations, which may aid in deciding whether 1776 was a year of maximum or of minimum. I have already referred to VAN SWINDEN's result (Art. 2); the following are the numbers of days of disturbance observed by him in each year, as well as the numbers of irregular days on which the north end of the needle attained its most westerly position before noon or after 4 P.M.†

Year.	Numbers of Days of Disturbance.	Of Irregular Maximum.
1771	8	104
1772	20	71
1773	33	83
1774	35	111
1775	27	120
1776	16	137
1777	34	113
1778	17	108
1779	21	81
1780	11	91
1781	14	...

It has been stated that VAN SWINDEN does not seem to have had any exact limit to define his days of disturbance, and all that can be deduced from the first column of numbers is, that the maximum appears to have occurred

period of the Zurich astronomer. It should also be remarked that the longest interval for two successive periods given by Dr WOLF before 1787 is 26 years; since 1818, the longest is 23 years; while from 1787 to 1818 gives 31 years, which, for any other *two* periods, is an interval unknown in Dr WOLF's sun-spot history. It is obvious that if 10·4 years be near the mean duration, the last result of 11·8 years, obtained on the supposition that there was no maximum in 1797, will go on diminishing, passing through the mean of 11·1 years about 1960.

* It will be remarked that this is very nearly the period obtained in 1862 by Dr LAMONT (Art. 6), I would therefore repeat that his result was founded on two hypotheses:—1*st*, That the length of the period should always have been within the limits observed since 1818. 2*d*, That GILPIN's observations and the sun-spot numbers of Dr WOLF (which did not satisfy the first hypothesis) were worthless. These hypotheses seem inadmissible, and Dr WOLF's result has been in consequence very generally accepted. The whole discussion induces me to believe that a maximum occurred near 1797, and the only point on which any doubt can remain is as to its magnitude,—whether it was really so small as all the observations indicate.

† "Analogie de l'Élect. et du Mag." t. iii. pp. 85, 129.

between 1773 and 1777. The last column of numbers has, however, definite limits, and they show very markedly the maximum in 1776, from which VAN SWINDEN concluded the possible existence of a period of eight years.

31. The observations of COTTÉ made at Montmorency have been employed by Dr WOLF and Professor LOOMIS. They give the following quantities for the diurnal range of the magnetic needle:—

1777 . . .	11′·2	1779 . . .	8′·5
1778 . . .	10·0	1780 . . .	5·5

These show very distinctly that the maximum was not later than 1777, and that it occurred probably earlier. They thus confirm, as far as possible, VAN SWINDEN's epoch.

32. VAN SWINDEN's numbers show, also, a minimum in 1772, which agrees with the epoch found Art. 22. The epoch of the next following minimum is not certain, as the series ends with 1780. The Manheim observations of the magnetic needle show a minimum at 1782·5, and the principal minimum at 1784·5; Dr WOLF, by his spot numbers, finds 1784·8; Professor LOOMIS's numbers of auroræ are very irregular from 1779 to 1786, showing a minimum at 1780 and 1784·5. The forty-two year period gives 1782·2; and I think the determination of the preceding maximum gives some weight to this result.

There are no means of testing the earlier epochs of Dr WOLF; but no long period given by him will be satisfied by them. If I have already shown good grounds for substituting a maximum in 1776 for Dr WOLF's minimum, a similar change in some of the epochs of the preceding century and half may be quite possible.

33. *Diurnal Ranges of the Magnetic Needle in the years of Maximum and Minimum, and their Ratios.*—There is considerable difficulty at present in arriving at any satisfactory comparison of the ranges during different periods, derived, as they are, from observations made at different places and at different hours. We may, however, determine approximately the ratios of the greatest to the least oscillations in each case. They are as follows:—

Place.	Observer.	Years.		Ranges.		Ratios.	Observations.
Paris,	CASSINI ;	$\frac{1787{\cdot}25}{1784{\cdot}8\,?}$	;	$\frac{15{\cdot}29}{9{\cdot}21\,?}$	=	1·66 ? *	During the day.
„	ARAGO ;	$\frac{1829{\cdot}7}{1824{\cdot}2}$	;	$\frac{13{\cdot}74}{7{\cdot}75}$	=	1·77	„
Göttingen,	GAUSS ;	$\frac{1836{\cdot}8}{1833{\cdot}5\,?}$	;	$\frac{13{\cdot}04}{7{\cdot}50\,?}$	=	1·74 †	8^h A.M. to 1^h P.M.

* As the minimum probably occurred earlier than 1784·8, the ratio here found is too small.
† The minimum at Göttingen is estimated from the curve, Plate XXXIX.

Place.	Observer.	Years.	Ranges.	Ratios.	Observations.
Munich,	Lamont ;	$\frac{1848{\cdot}8}{1844{\cdot}5}$;	$\frac{11{\cdot}53}{6{\cdot}56}$	= 1·76 *	8 A.M. to 1 P.M.
,,	,, ;	$\frac{1848{\cdot}5}{1856{\cdot}5}$;	$\frac{11{\cdot}15}{7{\cdot}08}$	= 1·57	7 A.M. to 1 P.M. / 8 A.M. to 2 P.M.
,,	,, ;	$\frac{1859{\cdot}5}{1856{\cdot}5}$;	$\frac{11{\cdot}17}{7{\cdot}08}$	= 1·58 ;	,, ,,
,,	,, ;	$\frac{1859{\cdot}5}{1867{\cdot}5}$;	$\frac{11{\cdot}16}{6{\cdot}44}$	= 1·73 ;	8 A.M. to 2 P.M.
Dublin,	Lloyd ;	$\frac{1849{\cdot}0}{1844{\cdot}25}$;	$\frac{13{\cdot}31}{8{\cdot}76}$	= 1·52 ;	3-Hourly.
Hobarton,	Kay ;	$\frac{1848{\cdot}9\,?}{1843{\cdot}2}$;	$\frac{11{\cdot}90}{7{\cdot}57}$	= 1·57 † ;	Hourly.
Toronto,	Younghusband and Lefroy	$\frac{1849{\cdot}0\,?}{1843{\cdot}8}$;	$\frac{12{\cdot}81}{8{\cdot}47}$	= 1·51 ‡ ;	,,
Trevandrum,	Broun ;	$\frac{1860{\cdot}3}{1856{\cdot}3}$;	$\frac{2{\cdot}98}{1{\cdot}88}$	= 1·59	,,
,,	,, ;	$\frac{1860{\cdot}3}{1066{\cdot}5}$;	$\frac{2{\cdot}98}{1{\cdot}90}$	= 1·57	,,
,,	,, ;	$\frac{1870{\cdot}85}{1866{\cdot}5}$;	$\frac{2{\cdot}89}{1{\cdot}87}$	= 1·55	8 observations daily.
,,	,, ;	$\frac{1870{\cdot}85}{1075{\cdot}7\,?}$;	$\frac{2{\cdot}89}{1{\cdot}90}$	= 1·52	,, ,,

It is difficult to say, from the quantities found, that there has been any marked difference in the amount of the maximum diurnal range of the magnetic needle at any given station since the commencement of the magnetic series of observations, with the exception of that for 1797. All the differences shown for the other epochs of maximum may be explained by the different methods of determining the amount of the oscillations.

It will be observed, that the ratio of the yearly mean range at the maximum to that at the minimum, is nearly constant for stations so widely apart and so differently situated on the earth's surface as Dublin, Toronto, Trevandrum,

* This ratio for Munich is that for the true maximum and minimum; the three following ratios are from the greatest and least ranges, corresponding to the middle of each year. They are also not from the same hours.

† The greatest yearly mean was the last from the hourly observations, 1848·25=11′·69, as the yearly mean range at Dublin increased from that epoch to 1849·0 by 0′·33, and at Munich to 1848·8 by 0′·03 ; the mean of these two increases has been added to the last yearly mean at Hobarton.

‡ The last yearly mean from the hourly observations at Toronto (1848·0) was 11·65 ; the correction to the maximum was derived from the Dublin observations, which showed very nearly the same amount of increase from year to year. Thus, the differences of the yearly mean ranges for the two places, Dublin *minus* Toronto, were—

1844·0	1845·0	1846·0	1847·0	1848·0
+0′·54	+0′·37	+0′·54	+0′·46	+0′·58.

The mean excess, 0′·50, has been subtracted from the maximum range at Dublin, 1849·0 =13′·31, to obtain the approximate maximum for Toronto.

and Hobarton. It will be remarked, however, that the ratios for the European Continent are markedly greater than for the other parts of the earth.*

It appears, also, from the results for Trevandrum, that the ratio is not quite constant, but is diminishing gradually. This fact, if followed through successive periods, will probably lead to the knowledge of a periodic variation. I believe Trevandrum is a station particularly suited for the determination of laws depending on small differences, since there the irregular effect of magnetic disturbance is much less felt than in high latitudes.

35. *Ratios of the Ranges derived from the Movements between different Hours.*—If the diurnal movement of the magnetic needle follows the same law, at the same place, in years of maximum as in years of minimum disturbance, we may conclude that the ratio of the movements between any two hours for these years will be constant. The following are the ratios of the yearly mean movements for 1848·25 and 1843·5, between different hours and the hour of maximum at Hobarton :—

From all hours.		20^h to 2^h		21^h to 2^h	
$\frac{11'\cdot69}{7\cdot68}$	$= 1\cdot59$;	$\frac{10'\cdot31}{6\cdot97}$	$= 1\cdot48$;	$\frac{11'\cdot25}{7\cdot43}$	$= 1\cdot51$
22^h to 2^h		23^h to 2^h		0^h to 2^h	
$\frac{9'\cdot32}{6\cdot64}$	$= 1\cdot40$;	$\frac{7'\cdot71}{4\cdot86}$	$= 1\cdot59$	$\frac{4'\cdot49}{2\cdot71}$	$= 1\cdot66$

* This fact is confirmed by the observations made at Lisbon, at 8 A.M. and 2 P.M., by Mr CAPELLO. These give, for the middle of the years, of maximum and minimum—

$$\frac{1859\cdot5}{1867\cdot5} \; ; \; \frac{10'\cdot54}{6\cdot15} = 1\cdot71$$

$$\frac{1870\cdot5}{1875\cdot5} \; ; \; \frac{10'\cdot83}{6\cdot09} = 1\cdot78$$

("Nature," April 1876, p. 448.) The first ratio is almost the same as that obtained from the Munich observations for the same hours and years. We may conclude, also, that had the means for the exact epochs of maximum and minimum been obtained, the ratios would have been still greater. On the other hand, the Milan observations do not agree with those at Munich and Trevandrum in showing a nearly constant ratio for different periods; and the ratios for Milan differ considerably from those for Munich and Lisbon. Thus, the mean ranges for the years 1859·5 and 1866·5, give the following ratios :—

	Munich.		Milan.		Lisbon.	
$\frac{1859\cdot5}{1866\cdot5}$,	$\frac{11'\cdot16}{6\cdot50} = 1\cdot72$;	$\frac{10'00}{4\cdot24} = 2\cdot36$;	$\frac{10'\cdot54}{6\cdot19} = 1\cdot70$.

The Milan ratios for the true maxima and minima also vary from

$$\frac{1848\cdot3}{1844\cdot2} \; , \; \frac{11'\cdot64}{6\cdot77} = 1\cdot72 \; , \text{ to } \frac{1870\cdot8}{1859\cdot3} = \frac{12'\cdot00}{4\cdot01} = 2\cdot99.$$

We can explain these great differences of results from two places so near as Munich and Milan, while stations so far apart as Munich and Lisbon agree so well, only on the supposition of some remarkable *local* cause. We have here another instance of the necessity of having two instruments at the same station to control each other, and to decide what results are instrumental and what not.

The above quantities for Milan are taken from Dr WOLF'S Astron. Mitth. xxxviii. p. 382 (July 1875), where I have found, since this paper was written, that the Zurich astronomer has adopted the method of yearly means corresponding to each month.

We find also from the Dublin observations between 7 A.M. and 1 P.M. for 1849·50 and 1844·0:—

$$\frac{12'\cdot28}{7\cdot93} = 1\cdot55,$$

which is nearly the same ratio as that already obtained (33) from all the observation hours.*

36. If we represent the mean diurnal range of the magnetic needle at any place in years of maximum and minimum disturbance by r_m and r_0 respectively, and at any other place by r'_m and r'_0, we have seen that for stations in the northern and southern hemispheres, and near the equator.

$$\frac{r_m}{r_0} = \frac{r'_m}{r'_0}$$

There is every reason to believe that the cases for which slightly different ratios have been found during the same period can be explained by the superposed variations, due to local laws, of the magnetic disturbance, as they affect the mean position of the needle at different hours.

If, then, the diurnal oscillations increase from the minimum value to the maximum in a constant ratio for different stations, or if

$$r_m : r_0 = r'_m : r'_0 ,$$

We arrive at the very probable conclusion that the varying amount of the oscillation is due to the variation of intensity of the *same* cause; or that the cause of the diurnal movement of the magnetic needle is the same in years when the sun is without spots, as in the years when the spots are most numerous.†

* The generality of this result may also be shown by the equations representing the diurnal variations in different years. These for Hobarton in the minimum year 1843·5 (January to December 1843), and the maximum (observed) year 1848·25 (October 1847 to September 1848) are as follow:—

1848·25. $y = 1'\cdot48 \text{ Sin } (\theta + 200°) + 1'\cdot30 \text{ Sin } (2\theta + 0°) + 0'\cdot56 \text{ (Sin } 3\theta + 201°) + 0'\cdot20 \text{ (Sin } 4\theta + 8°)$

1843·5. $y = 0'\cdot94 \text{ Sin } (\theta + 209°) + 0'\cdot95 \text{ Sin } (2\theta + 4°) + 0'\cdot37 \text{ (Sin } 3\theta + 203°) + 0'\cdot12 \text{ (Sin } 4\theta + 14°)$

When we remember the greater amount of irregularity produced in the diurnal variations by the disturbances in 1848, it will be seen that the equations for both years show as nearly the same law as could be expected with a cause whose increased action in producing the diurnal variations is accompanied with increased irregularities.

† It is not to be forgotten that near the equator the diurnal variation of the magnetic needle during the equinoctial months is nearly annihilated by the action of two opposing laws, that of the northern and that of the southern hemisphere, one of which prevails more or less in the other seasons. Since both the southern and northern forces vary alike (or nearly so) during the decennial period, it is only the difference of the increments or decrements which are shown at Trevandrum; and it is one of the results most confirmatory of the preceding conclusion that the increase from the minimum to the maximum year bears still the same ratio to the whole diurnal collection at the minimum as for Hobarton, Dublin, and Toronto, where one of the forces always prevails.

37. *Differences in the Decennial Variations at different Stations.*—In order to show these differences, the variations of the yearly mean ranges for different stations have been projected Plate XXXIX. for the years near the minimum 1844. It will be seen there that the minimum appears to have been attained earliest at the stations nearest to, and south of, the equator; that an increase of the oscillation occurred 1845–46, at the two widely separated stations, Munich and Hobarton; while a diminution is shown at Toronto, and the oscillation remains nearly constant at Dublin and Makerstoun. These differences are sufficiently marked to prove that the decennial variation is affected by local causes, a result already shown by the differences of the ratios at the same time.*

38. *Comparison of the Yearly Mean Diurnal Oscillation of the Needle and Sun-spot Area.*—Having treated the sun-spot areas given by Messrs De La Rue, Stewart, and Lœwy, in the same way as the magnetic ranges, so as to have the yearly mean corresponding to the beginning of each month, these were projected above the curves of yearly mean diurnal range, Plate XXXIX.

In the period, 1836–1848, we find that the maximum spot-area occurred at 1836·75, and that it diminished with considerable regularity thereafter; whereas the diurnal magnetic oscillation attained the maximum at 1836·9, which was followed by a nearly equal maximum at 1838·25, when the spot-area had diminished by one-third of its whole value.

In the next period the maximum spot area occurs again earlier than that of the diurnal oscillation; while after 1850·5, the former increases and the latter diminishes.

* Dr Lamont has given the diurnal ranges for different years at several places, deduced from observations at two hours only, which he points out are thus affected by disturbances, and are therefore only "preliminary approximations." He has concluded from the general agreement of the increase from minimum to maximum that—

$$\frac{r_n}{r'_n} = c$$

where r_n and r'_n are the diurnal ranges for the same (n^{th}) year at any two stations. Dr Lamont has not, however, given the ratios which result from his data; these scarcely satisfy the equation even approximately. Thus taking St Helena and Munich, for which the ranges are given by him, we find—

$$\frac{\text{Munich}}{\text{St Helena}},\ 1843,\ \frac{6{\cdot}36}{2{\cdot}55} = 2{\cdot}69;\quad 1844,\ \frac{6{\cdot}34}{2{\cdot}81} = 2{\cdot}26;\quad 1848,\ \frac{11{\cdot}20}{3{\cdot}48} = 3{\cdot}22.$$

I believe, however, that Dr Lamont's conclusion is true, "that the cause of the ten-yearly period is to be found in the sun, or more generally, in a cosmic force acting from a great distance." Indeed, the preceding equation is a general form of that given Art. 36. And when we compare the minimum and maximum at Munich for 1850 and 1860 with those for Trevandrum near the same times, we find—

$$\frac{\text{Munich}}{\text{Trevandrum}},\ 1856,\ \frac{7'{\cdot}08}{1'{\cdot}88} = 3{\cdot}76\qquad 1860,\ \frac{11'{\cdot}17}{2'\,98} = 3{\cdot}75.$$

These ratios are not quite accurate, since only the minimum for 1856·5, and the maximum for 1859·5 at Munich are known to me, but the true minimum and maximum values cannot be very different; these ratios then will satisfy Dr Lamont's equation nearly, and confirm his conclusion. See "Einige Bemerkungen über die zehnjährige Period, Sitzungsberichte der K. Akad. der Wischenschatten," München, 1864, II. S. 21.

In the interval 1850–61, the spot area remains nearly constant from 1859·4 to 1861·5, while the magnetic oscillation attains a sharply marked maximum at 1860·25.

In the next period the spot area diminishes with a series of secondary maxima and minima, which are also shown, but not simultaneously in the magnetic oscillations. Dr WOLF's relative numbers have also been projected from 1860 to 1874,* as the spot areas are given only to 1867. In this period the sun-spot frequency curve shows a sharp maximum, while the magnetic oscillation maximum is much flatter.

39. Though the differences between the curves for spot-area, or frequency, and for the diurnal oscillation, are, in several cases, strongly marked, there are also occasions on which inflexions in the one curve find corresponding inflexions in the other. Obviously, the observations of sun-spots are imperfect and incomplete. We know also that the decennial variation of the diurnal oscillation is not exactly the same for different stations. Making every allowance for these two causes of difference, it still appears most probable that neither the area nor frequency of sun-spots is an exact measure of the magnetic action, but that each is a distinct result due to the same cause.† We may also conclude that any attempt to determine the amount of the diurnal oscillation of the needle at any place, by means of equations depending on the spot frequency, can only give such approximations as follow from the general agreement of the two phenomena within certain limits; thus the very rough parallelism of the curves of spot-frequency and of diurnal oscillation at Trevandrum from 1860 to 1869 is very widely departed from thereafter.

40. *Decennial Period of Magnetic Disturbance at Trevandrum*, 1854·5 to 1864·5.—When we determine the mean diurnal variation of the needle for a month, the mean position of the needle for every hour is obtained; the observed positions for a given hour on different days are generally east or west of the mean. These deviations are due to various causes; to variation of the diurnal law with season, to the sun's rotation, the moon's revolution, and the cause producing the secular and other variations of long period. In order to diminish the effects of these various causes, the four-weekly mean diurnal variation, corresponding to the middle of each week in each of the 11 years, was computed. The observations in each week were then compared with their corresponding hourly means, and the differences were taken; these differences obey two diurnal laws, one depending on the moon's, the other on the sun's hour angle. The latter, due to deviations from the mean solar law, are called solar disturbances. The lunar observations have been shown to depend also indi-

* Astron. Metth. xxxviii. S. 385, July 1875.

† This follows from the conclusion, Art. 36, sun-spots appearing only when the magnetic action exceeds a given value.

rectly on the sun,* and the amplitudes of both the solar and lunar diurnal variations have been found to obey the decennial law.† The means of all the differences may therefore be considered due to direct or indirect solar disturbance. The yearly means are given in the following table:—

TABLE III.‡—*Yearly Means of Solar Disturbance of Magnetic Declination corresponding to the 1st of each Month.—Trevandrum*, 1854–1864.

Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
	′	′	′	′	′	′	′	′	′	′	′	′
1854	...	...	...	...	...	...	0·368	0·358	0·347	0·345	0·338	0·334
1855	0·336	0·331	0·336	0·338	0·335	0·335	0·334	0·339	0·339	0·331	0·327	0·323
1856	0·316	0·315	0·314	0·310	0·305	0·305	0·306	0·300	0·298	0·299	0·304	0·317
1857	0·321	0·325	0·322	0·331	0·337	0·344	0·363	0·380	0·390	0·404	0·419	0·415
1858	0·420	0·424	0·432	0·431	0·437	0·442	0·436	0·434	0·448	0·446	0·445	0·447
1859	0·451	0·460	0·477	0·490	0·514	0·516	0·525	0·523	0·519	0·533	0·534	0·540
1860	0·541	0·551	0·553	0·547	0·524	0·516	0·505	0·503	0·495	0·484	0·478	0·468
1861	0·466	0·450	0·437	0·436	0·441	0·447	0·453	0·450	0·450	0·447	0·443	0·439
1862	0·433	0·433	0·433	0·435	0·439	0·436	0·434	0·447	0·449	0·453	0·457	0·463
1863	0·466	0·462	0·459	0·451	0·440	0·435	0·424	0·406	0·406	0·398	0·398	0·399
1864	0·406	0·408	0·407	0·408	0·408	0·411	0·414	...	...	...	...	...

The means of Table III. are projected (Plate XXXIX.) immediately below the yearly mean sun-spot area. I have also projected below the yearly mean diurnal range, the yearly mean areas of the curves representing the monthly mean diurnal variations; the latter curve agrees to a great extent with that of the ranges.

41. The disturbance curve gives the following epochs,—minimum, 1856·7; maximum, 1860·2. These differ little from the epochs derived from the ranges, the minimum by the latter occurring, however, nearly five months later. The two curves do not resemble each other exactly, and there is a tendency to the maximum, shown by the sun-spot area at 1859·2, which is not shown by the ranges.

42. When we seek the ratio of the maximum to the minimum yearly mean disturbance we find

$$\frac{1860{\cdot}2}{1859{\cdot}7}, \frac{0'{\cdot}553}{0'{\cdot}298} = 1{\cdot}68.$$

The ratio for the disturbance is therefore greater than for the amplitudes of the diurnal variations.§

* Trans. Roy. Soc. Edin., vol xxvi. p. 735. See also Trevandrum Magnetical Observations, Art. 398, p. 133, and Appendix, Art. 200, p. 544.

† Trev. Mag. Obs., vol. i. p. 120.

‡ Derived from Table LI., "Trevandrum Magnetic Observations," vol. i. p. 142. The two following means require correction in the table cited. Yearly means for 1864, for 0·406 read 0·414, and mean for July, for 0·399 read 0·406.

§ It is well known that magnetic disturbances are most felt in high latitudes; it would be of im-

43. *Comparison of the Monthly Mean Sun-Spot Area and Monthly Mean Disturbance.*—The annual law of variation of mean disturbance of magnetic declination for high latitudes, first deduced from the Makerstoun observations, and since confirmed by the results of many observatories, is, that there are maxima at the equinoxes, and minima at the solstices. This law, however, does not hold near the magnetic equator. An examination of the monthly mean sun-spot area also has shown no similar law. The annual law of disturbance of magnetic declination at Trevandrum is a maximum in January, and a minimum in June.* This result is derived from the mean of eleven years' observations, and different years show marked deviations from this law.† As I had deduced a somewhat similar mean law for the sun-spot area, I have sought to compare the monthly means of the two variations for the years 1854 to 1864. Both are projected in the lower curves (Plate XL). The following are the conclusions from these curves.

44. 1*st*, During the period in which there were few spots, 1854 to 1856, there were large variations of the monthly mean disturbance, this difference is very marked in 1855. The minimum disturbance occurred evidently in March to June 1856; the sun-spot area was nearly zero from March 1855 to March 1857.

2*d*, In August 1857 a slight increase of sun-spot area is shown with a cor-

portance to know the ratio of the mean difference (or disturbance) for different parts of the earth's surface; unfortunately this quantity has not been sought in general. Sir E. Sabine has made his valuable investigations on disturbances above certain limits, which vary with the station, so that no exact comparison can be made. The only means from *hourly* observations, in which all the disturbances are included with which I am acquainted, are those for Makerstoun in the years 1844 and 1845; the minimum yearly mean value of the disturbance was that for 1845·4 = 1′·66; if we compare this with the minimum at Trevandrum for 1859·7, we have

$$\frac{\text{Makerstoun}}{\text{Trevandrum}}, \frac{1'{\cdot}66}{0'{\cdot}298} = 5{\cdot}6.$$

If we compare, in like manner, the minimum yearly mean range of the monthly mean diurnal variations for the two places, we find

$$\frac{\text{Makerstoun}}{\text{Trevandrum}}, \frac{1844{\cdot}25}{1856{\cdot}3}, \frac{7'{\cdot}86}{1'{\cdot}88} = 4{\cdot}18;$$

a less ratio than for disturbanees.

Dr Lloyd has given the mean of all the disturbances for the years 1841 to 1850 (Dublin Mag. Obs. vol. ii. p. 88); they are derived, however, from two-hourly observations in 1841–43, and from six three-hourly observations in the following years; and the means are not strictly comparable with those at Trevandrum, we find, however, the ratio of the maximum to the minimum yearly mean disturbance from the three-hourly observations,

$$\frac{1848{\cdot}1}{1845{\cdot}75}, \frac{2'{\cdot}39}{1'{\cdot}50} = 1{\cdot}59,$$

which is slightly greater than the ratio for the ranges (Art. 33).

* Trev. Mag. Obs. vol. i. p. 142.

† There is one remarkably uniform result breaking the regularity of the mean law. The mean disturbance for November in eight years, and for October in the three remaining years is less than that for the months immediately preceding and following.

responding increase of disturbance. From this time till 1861 there is a remarkable similarity in the variations, though they do not bear a constant ratio to each other.

3*d*, In 1861 marked differences begin to appear; and these are seen, especially in the great increase of sun-spot area in June 1862, when there is a minimum of disturbance. There is again, however, a considerable resemblance in the two curves during 1863, which disappears again in 1864.

4*th*, On the whole, these curves show that the disturbances cannot be measured by, and are not due to sun spots; but the coincidences are too numerous to allow us to avoid the conclusion, that they are due to a common cause. This cause, however, may produce great changes of sun-spot area without any corresponding change in the amount of disturbance; and considerable changes of the amount of disturbance exist when there are no sun spots.

45. *Decennial Period of Magnetic Disturbance at different hours.*—We have seen that the mean oscillation of the magnetic needle between the hour when the polar end of the needle attains its maximum westerly position and other hours follows approximately the same law as the total oscillation (Art. 35); it has seemed to me desirable to examine whether the irregularities of position at each hour also follow the decennial law. The following table contains these means for each hour in each year:—

TABLE IV.—*Hourly Means of Disturbance of the Magnetic Needle at Trevandrum*, 1854–1864.

Hour.	1854.	1855.	1856.	1857.	1858.	1859.	1860.	1861.	1862.	1863.	1864.
12½ A.M.	0·240	0·178	0·140	0·200	0·237	0·305	0·204	0·267	0·253	0·202	0·237
1½ „	0·261	0·191	0·151	0·219	0·260	0·311	0·304	0·283	0·255	0·237	0·287
2½ „	0·272	0·209	0·150	0·219	0·282	0·335	0·317	0·311	0·268	0·265	0·322
3½ „	0·279	0·225	0·159	0·232	0·302	0·358	0·349	0·310	0·280	0·290	0·341
4½ „	0·289	0·237	0·182	0·227	0·320	0·377	0·412	0·312	0·285	0·315	0·339
5½ „	0·316	0·272	0·209	0·247	0·374	0·437	0·441	0·354	0·324	0·318	0·362
6½ „	0·376	0·333	0·332	0·372	0·443	0·565	0·579	0·493	0·438	0·440	0·436
7½ „	0·432	0·401	0·388	0·446	0·495	0·612	0·619	0·568	0·485	0·497	0·495
8½ „	0·465	0·468	0·438	0·523	0·567	0·677	0·678	0·606	0·519	0·558	0·518
9½ „	0·507	0·535	0·506	0·597	0·623	0·757	0·758	0·677	0·577	0·589	0·556
10½ „	0·562	0·562	0·566	0·613	0·672	0·765	0·824	0·702	0·648	0·622	0·589
11½ „	0·594	0·560	0·570	0·596	0·677	0·826	0·790	0·722	0·695	0·636	0·579
0½ P.M.	0·590	0·548	0·551	0·608	0·699	0·815	0·784	0·758	0·693	0·650	0·602
1½ „	0·564	0·521	0·517	0·575	0·723	0·764	0·760	0·698	0·693	0·612	0·627
2½ „	0·536	0·483	0·453	0·526	0·662	0·707	0·698	0·626	0·639	0·623	0·592
3½ „	0·503	0·431	0·410	0·487	0·611	0·647	0·642	0·558	0·593	0·590	0·562
4½ „	0·422	0·370	0·359	0·414	0·498	0·572	0·567	0·500	0·534	0·544	0·497
5½ „	0·317	0·293	0·289	0·337	0·272	0·521	0·469	0·417	0·428	0·449	0·432
6½ „	0·250	0·234	0·202	0·264	0·317	0·449	0·371	0·336	0·366	0·365	0·357
7½ „	0·224	0·217	0·182	0·242	0·304	0·394	0·359	0·311	0·334	0·341	0·292
8½ „	0·212	0·208	0·163	0·201	0·274	0·394	0·307	0·295	0·308	0·305	0·256
9½ „	0·200	0·190	0·148	0·196	0·258	0·357	0·298	0·245	0·293	0·270	0·219
10½ „	0·199	0·172	0·137	0·193	0·242	0·347	0·267	0·247	0·249	0·237	0·214
11½ „	0·215	0·168	0·137	0·187	0·237	0·310	0·257	0·257	0·266	0·219	0·202

Note.—The means at the half hours for 1854 and 1855 are interpolated from the observations at the whole hours.

In order to diminish the effects of the larger disturbance, which require longer periods for their repetition according to law, the means for each three hours have been taken; these are projected as means for the middle hours of each three (Plate XL.). It will be remembered, in considering these curves, that the successive points are obtained from observations made at the same hour in *different* years. We conclude from the projections,—

46. That the decennial variation does not follow exactly the same law at different hours. Thus there is some variation in the exact epochs of minimum near 1856·5, and of the maximum near 1860·0, according to the hour considered. The maximum occurs earliest (1859·5) in the curve for 9½ P.M., and latest (1860·2) in that for 9½ A.M. Still more marked differences appear in the descending branches of the curves (after 1860). A secondary minimum appears in the curves for 3½ to 9½ A.M. in the years 1862 and 1863, which disappears in the curves after noon, the minimum occurring then in the year 1861. A secondary maximum occurs in all the curves near 1863·0, being least marked in those for noon and midnight. These differences are not due to large accidental disturbances at any given hour, since they change gradually from hour to hour.

47. When we compare the magnitude of the mean disturbance for each three hours at the maximum and minimum of the curves (Plate XL.), we obtain the following ratios:—

11½ P.M. to 1½ A.M.,	$\frac{0\cdot337}{0\cdot141}$	= 2·40
2½ A.M. to 4½ „	$\frac{0\cdot417}{0\cdot102}$	= 2·58
5½ „ to 7½ „	$\frac{0\cdot586}{0\cdot284}$	= 2·06
8½ „ to 10½ „	$\frac{0\cdot776}{0\cdot472}$	= 1·64
11½ „ to 1½ P.M.,	$\frac{0\cdot847}{0\cdot527}$	= 1·61
2½ P.M. to 4½ „	$\frac{0\cdot696}{0\cdot385}$	= 1·81
5½ „ to 7½ „	$\frac{0\cdot457}{0\cdot222}$	= 2·05
8½ „ to 10½ „	$\frac{0\cdot366}{0\cdot145}$	= 2·52
All the hours,	$\frac{0\cdot554}{0\cdot298}$	= 1·86

48. In a similar way we obtain the ratios of the maximum to the minimum disturbances in the diurnal variation for each year. Taking the maximum and minimum for three successive hours, and for one hour only, we have,—

	Three-Hour Means.		One-Hour Means.	
1854, ...	$\frac{\cdot 583}{\cdot 204}$	= 2·86	$\frac{\cdot 594}{\cdot 199}$	= 2·91
1855, ...	$\frac{\cdot 557}{\cdot 172}$	= 3·24	$\frac{\cdot 562}{\cdot 168}$	= 3·35
1856, ...	$\frac{\cdot 559}{\cdot 138}$	= 4·05	$\frac{\cdot 570}{\cdot 137}$	= 4·16
1857, ...	$\frac{\cdot 606}{\cdot 192}$	= 3·16	$\frac{\cdot 613}{\cdot 187}$	= 3·28
1858, ...	$\frac{\cdot 700}{\cdot 239}$	= 2·93	$\frac{\cdot 723}{\cdot 237}$	= 3·05
1859, ...	$\frac{\cdot 802}{\cdot 309}$	= 2·60	$\frac{\cdot 826}{\cdot 305}$	= 2·71
1860, ...	$\frac{\cdot 799}{\cdot 269}$	= ·297	$\frac{\cdot 824}{\cdot 257}$	= 3·21
1861, ...	$\frac{\cdot 727}{\cdot 250}$	= 2·91	$\frac{\cdot 758}{\cdot 245}$	= 3·09
1862, ...	$\frac{\cdot 694}{\cdot 250}$	= 2·71	$\frac{\cdot 693}{\cdot 249}$	= 2·79
1863, ...	$\frac{\cdot 636}{\cdot 219}$	= 2·90	$\frac{\cdot 650}{\cdot 202}$	= 3·22
1864, ...	$\frac{\cdot 607}{\cdot 212}$	= 2·86	$\frac{\cdot 627}{\cdot 202}$	= 3·10

From all these series, we conclude that, in general, the ratio is greatest when the disturbance is least, and *vice versa*.

49. There is nothing in these ratios which can indicate a connection between the increase of mean disturbance from the hour of minimum to the hour of maximum in different years; nor between the increase from the *hour* of minimum and the *hour* of maximum in the year of minimum, to the same *hours* in the year of maximum disturbance. We do not know the nature of the forces which produce these deviations, nor their mode of action, but we may suppose that the latter will be the same in different years, and that the deviations should be related by some common law. The following result may aid in this determination :—

(50.) Let us represent the maximum and minimum mean disturbance for any year by D_m and D_0 respectively, then we find that the following relation holds very nearly,

$$\sqrt{D_m} - \sqrt{D_0} = \text{constant}.$$

The quantities, Table IV., do not give us the exact values of the maxima and minima of the hourly mean disturbance, since these will not occur in general at the exact hours of observation, but they may be taken as near approximations. We find from the following results the errors of the preceding equation.

Year.	D_m	D_0	$\sqrt{D_m} - \sqrt{D_0}$	Difference from Mean 0′·358.
1854	0′·594	0′·199	0′·330	− 0′·028
1855	0·562	0·168	0·340	− 0·018
1856	0·570	0·137	0·385	+ 0·027
1857	0·613	0·187	0·351	− 0·007
1858	0·723	0·237	0·363	+ 0·005
1859	0·826	0·305	0·356	− 0·002
1860	0·324	0·257	0·401	+ 0·043
1861	0·758	0·245	0·376	+ 0·017
1862	0·695	0·249	0·335	− 0·023
1863	0·650	0·202	0·357	− 0·001
1864	0·627	0·202	0·342	− 0·016

(51.) It has been stated that the quantities D_m and D_0 are not the exact maxima and minima, but the differences from the mean shown in the last column (which are not systematic) are most probably due to another cause, if a perfect agreement with the equation should be expected. If the equation (50) be the expression of a law which relates the deviations from the normal positions and the forces producing them, it is obvious that it must refer to separate disturbances, and not to their means; and that if ΣD represent the sum of the deviations (their number being constant for each year), instead of employing $\sqrt{\Sigma D}$ as has been done above, we should have taken $\Sigma \sqrt{D}$. As the differences between these quantities will depend on the mode of distribution of the positive and negative deviations about the mean in each year, the nearness of the approximation obtained can be due only to the fact that the positive and negative disturbances are very similarly distributed at all hours.

(52.) As, in general, the hourly mean disturbance varies little near the hours of maximum and near the hours of minimum, I have sought the exact values for the hours 11½ A.M., 0½ and 1½ P.M.. and for the hours 11½ P.M., 12½ and 1½ A.M., in the curves for these hours showing the decennial variation (Plate XL.), and have found for the former 0′·847 and 0′·527, corresponding to 1859·9 and 1856·7 respectively; and for the latter 0′·335 and 0′·141, corresponding to 1860·2 and 1856·4 respectively. If D_m and D_0 be the maximum and minimum hourly mean disturbance in the maximum period, and d_m, d_0, the corresponding quantities for the period of minimum disturbance, then by the equation, Art. 50, we may place

$$\sqrt{D_m} - \sqrt{D_0} = \sqrt{d_m} - \sqrt{d_0}$$

and

$$\sqrt{D_m} - \sqrt{d_m} = \sqrt{D_0} - \sqrt{d}$$

we have from the values just given

$$\sqrt{·847} - \sqrt{·527} = 0′·194$$
$$\sqrt{·335} - \sqrt{·141} = 0′·203$$

which satisfy the preceding equation nearly. We have, also, by the original equation, Art. 50,

$$\sqrt{\cdot847} - \sqrt{\cdot335} = \cdot341$$
$$\sqrt{\cdot527} - \sqrt{\cdot141} = \cdot350$$

agreeing nearly with the constant previously obtained. In a similar way, if we determine the exact values of the maxima and minima for the upper curves in Plate XL., showing the decennial variation of the disturbance at any hour, we shall have *approximately*,

$$\sqrt{D_m} - \sqrt{d_m} = \text{constant}.$$

53. We may conclude, then, that the increase of the disturbance from minimum to maximum in the diurnal variation follows the same law of the constant difference of the square roots of the deviations in each year of the decennial period; and that the increase of the mean deviation for any hour in the year of minimum to the same hour in the year of maximum disturbance obeys a similar law.

54. *Hypotheses as to the Cause of the Decennial Period.*—The only periodic movement of the sun with which we are acquainted is that of his rotation on his axis. We are induced then to seek, without the sun, for some phenomenon which may occupy a like period with the variations in the areas of his spotted surface. If any such could be found, we might then inquire whether the former could be the cause of the latter; and if so, how? The planetary motions offered themselves at first as possible causes. The sun and the planets attract each other. The relation of the spot area, the amplitude of the magnetic oscillation, and the frequency of the aurora borealis, indicate an electrical cause in the latter cases; and without any exact knowledge of the forces which might be required so to disturb the equilibrium of the gaseous envelope of the sun, it did not appear impossible that electrical actions passing between the sun and the planets might suffice for the production of the solar spots. The only periodic movement, however, approaching the decennial period is that of the revolution of Jupiter, which occupies nearly $1\frac{1}{2}$ year more than the decennial period.

Dr Wolf has endeavoured to represent the latter period by formulæ which are functions of the masses, distances, and periods of revolution of the planets. Such an effort may have some appearance of success when confined to a single period of eleven years; but it fails completely when extended to two or three periods.

55. The most interesting result with which I am acquainted is that due to the investigations of Messrs De La Rue, Stewart, and Lœwy; from which it appears that the mean area of spots (1854–60 and 1862–66) is greatest on the side opposite to Venus and Mercury, and least on the sun's surface presented

to these planets.* This result is all the more marked that it appears to hold even for different solar meridians, so that the meridian passing through Venus and Mercury shows a lesser spot area (on the side next the planet), and a greater spot area (on the opposite side), than the meridians on either side of this principal meridian.

56. At the same time, I have made, for the purposes of this paper, a variety of discussions of sun-spot area relatively to the planetary periods, including the synodical revolution of Venus, and the conclusion has been that different periods of eleven years (1833–44, 1844–55, and 1855–66), give different results.†

57. When we examine the curves representing the three-monthly mean-spot area given with the valuable memoir of Messrs De La Rue, Stewart, and Lœwy, in the "Philosophical Transactions" (1870, Plate xxxi.), we remark that there is not a regular increase from minimum to maximum, nor diminution from maximum to minimum in the decennial period; but that there are series of well-marked secondary maxima and minima. We cannot fail to perceive, also, that the most marked of these maxima (one of which, in each decennial period, is also the principal maximum) occur generally within the months October to March. Only one marked exception to this rule appears in the period 1834 to 1844, when it occurs in September; no exception appears in the period 1844 to 1856. In the third period (1856 to 1867) there is one marked exception in July 1862 (see Plate XL.), and two irregular years. The variation of spot area during some of these secondary periods is so great that it amounts to nearly four-fifths of the range for the whole decennial period. The exceptional cases repeat themselves in the same month in different periods.

58. It follows, from these considerations, that a discussion for a period of twelve months gives a maximum of spot area when the earth is nearest the sun in all the three decennial periods, but in the last a maximum is also shown when the earth is farthest from the sun.

59. It has, however, been already pointed out, that the great maximum of sun-spot area, which occurred for July 1862 (see Plate XL.), had no corresponding maximum of magnetic disturbance, nor was there any corresponding increase in the amplitude of the diurnal oscillation. If we suppose for the moment, that electrical actions passing from the sun to the earth are connected with spot production, we may also assume that when great variations of sun-spot area are unaccompanied by similar variations of terrestrial magnetic disturbance, the earth itself is not in question; and that such cases as that of July

* Proc. Roy. Soc. London, vol. xx. p. 210, March 1872.

† It should be remarked, however, that the discussion of Messrs De La Rue, Stewart, and Lœwy, is founded on more exact and detailed measurements than those for the other periods included in my own investigations.

1862 may be connected with actions passing between the sun and another planet, such as Venus (the two most marked exceptional cases occupying about seven months).

60. The discussions of sun-spot area relatively to planetary periods have all been undertaken on the hypothesis that the actions of the planets should be the same when they return to the same points of their orbits. No hypothesis has been proposed as to the distribution of the medium by which the solar electrical actions are supposed to be conveyed to the planets. In a paper read to the Royal Society of London, January 27, 1876, I have shown that marked magnetic disturbances repeat themselves at intervals of exactly twenty-six days, in which cases there is a sudden diminution of the earth's magnetic force. This interval is that obtained for the time of the sun's rotation from numerous magnetic observations. The result just mentioned proves that these disturbances cannot be related directly to the sun's spots, whose time of rotation varies from twenty-six to nearly twenty-eight days. It also seems to show that the medium which conveys the solar action is not the same as that which transmits light and heat, since the repeated electrical actions are felt by the earth when the same solar meridian returns opposite the earth.*

61. If we suppose that this electrical medium is derived from the sun, is of limited extent, unsymmetrically distributed around him, and has its own proper motion of rotation, it will be evident that no investigation for a given planetary period could give the same result for the same part of the planet's orbit.

62. Whatever theory of the formation of sun-spots may be adopted, we shall always require it to explain why there are few or no spots in some years of the decennial period? Sir JOHN HERSCHEL supposed that sun-spots may be produced in the same way as terrestrial cyclones. M. FAYE has proposed an exceedingly rational hypothesis for their formation, depending on the diminishing velocities of contiguous zones on proceeding from the equator towards the poles. "Le décroissement," says the distinguished French astronomer, "bien plus rapide sur le soleil qu'il ne le serait en vertu de la seule différence des rayons des paralleles de rotation,† donne naissance çà et là dans la photosphère à des tourbillons verticaux tout a fait analogues à ceux qui se produisent si

* I may remark that M. BECQUEREL has proposed a hypothesis by which the atmospheric electricity is derived from the sun, and has connected this derivation with the actions producing solar spots. —*Comptes Rendus*, Nov. 11, 1872, p. 1126.

† M. FAYE explains the difference between the velocities shown by the spots in different latitudes and those which should result from difference of radii of the circles of latitude by a hypothesis, according to which the stratum where the falling incandescent rain is converted anew into vapour, has a different form than the external surface of the photosphere,—is, in fact, ellipsoidal, and flattened at the poles. It seems to me that were this double form consistent with dynamical laws, we should have here an indication of a different distribution of solar heat at the equator and at the poles, and a cause of currents between the two, which M. FAYE objects to in Sir J. HERSCHEL's hypothesis as non-existent. —*Annuaire du Bureau des Longitudes*, 1873, p. 516.

aisément dans les cours d'eau, partout ou une cause quelconque diminue ou augmente la vitesse des tranches paralleles au sens du mouvement."*

66. It is "*ces causes quelconques*" with which we wish to become acquainted. The difference of velocities of the photosphere for two different latitudes is constant, at least I am not aware that it has been found greater in the years of maximum spot frequency. What has become then of the causes which disturb the equilibrium of contiguous zones in years when there are few or no spots? M. FAYE has another hypothesis to explain this. That, on approaching extinction, the contraction of the sun due to the external radiation of heat may proceed by jerks (*saccades*), which will produce a sudden increase of heat and increased brightness, and that this phenomenon will be repeated till the sun becomes extinguished. "Avant d'arriver à ces oscillations extrêmes, les phénomènes beaucoup moins altérés pourront présenter un caractère de périodicité régulière, et telle est, sans doubt, la raison du fait découvert par M. SCHWABE et mis au pleine lumière par M. R. WOLF de Zurich."† It is this hypothesis which must be compared with that of planetary actions. M. FAYE has said of the latter, "Ces idées ont cours à l'étranger; elles ont peu de chance d'être acceptées en France."‡ I doubt much that the ideas which the learned French academician has substituted will be considered more satisfactory than the others. I believe that no hypothesis yet proposed has sufficient weight to be accepted generally, and more facts are required before a sure basis for a hypothesis can be attained. It is hoped that the results of the preceding paper will be found to be a contribution to this end.

* Annuaire du Bureau des Longitudes, 1873, p. 527. † Ibid. 1873, p. 525.
‡ Annuaire, 1874, p. 460.

XXVII.—*On the Parallel Roads of Lochaber.* By DAVID MILNE HOME, LL.D. (Plates XLI., XLII., XLIII.)

(Read 15th May 1876.)

I. The Parallel Roads of Lochaber have presented to geologists a problem, which is still unsolved. Dr MACCULLOCH, about sixty years ago, when President of the Geological Society of London, first called attention to these peculiar markings on the Lochaber Hills, by an elaborate Memoir afterwards published in that Society's Transactions. He was followed by Sir THOMAS DICK LAUDER, who in the year 1824, read a paper in our own Society, illustrated by excellent sketches. His paper is in our Transactions. The next author who attempted a solution was the present Mr CHARLES DARWIN. He maintained that these Roads were sea-beaches, formed, when this part of Europe was rising from beneath the Ocean. He was followed by Professor AGASSIZ, Dr BUCKLAND, CHARLES BABBAGE, Sir JOHN LUBBOCK, ROBERT CHAMBERS, Professor ROGERS, Sir GEORGE M'KENZIE, Mr JAMIESON of Ellon, Professor NICOL, Mr BRYCE of Glasgow, Mr WATSON, and Mr JOLLY of Inverness. Sir CHARLES LYELL, though he wrote no special memoir, treated the subject pretty fully in his works, giving an opinion in support of the views of AGASSIZ.

I took some little part myself in the discussion, having in the year 1847 read a paper in this Society, which was published in our Transactions.

During the last five or six years, there has been an entire cessation of both investigation and discussion, in consequence probably of a desire to await the publication of more correct maps of the district, which at the request of the British Association for the Advancement of Science, the Ordnance Survey Department undertook.

These Ordnance Maps were not available to the public before last Autumn, when with a copy of these in my hand, I went back to the district, to see whether any more distinct views would occur to me, than those I had obtained thirty years ago, when the late ROBERT CHAMBERS and I examined the Parallel Roads together.

II. Perhaps, before stating the results of my recent visit, it may be convenient for those who happen not to be well acquainted with the subject, that I briefly state the problem to be solved, and the different solutions which have been suggested.

There are 3 or 4 valleys in Lochaber, to the North of Ben Nevis, each from 10 to 15 miles in length, and having a depth of from 500 to 1000 feet, the sides of which are deeply notched by shelves called "*Parallel Roads*"; so called probably, because being horizontal, they are parallel to one another, and consist of spaces, so broad and regular, that a cart might be driven on them.*

In Glen Gluoy, whose mouth opens on Loch Lochy towards the West, there are two shelves, at a height, one of 1165, the other of 964 feet above the sea.

In Glen Roy, whose mouth also opens towards the West, there are on each side of the Glen, for a considerable part of the valley, 3 shelves, at these respective heights above the sea (beginning with the highest)—1149, 1068, and 856 feet.

In Glen Spean, which in its lower part joins Glen Roy, there is only one well-defined shelf, 856 feet above the sea. It is a continuation of the lowest in Glen Roy.

Dr MACCULLOCH was too cautious to offer a decided opinion regarding the origin of the shelves. He gave a minute and correct account of the phenomenon:—but he advanced no positive explanation, confessing his inability to give any. He only suggested general views, and farther inquiry.

Sir THOMAS DICK LAUDER thought that the Roads had been formed by lakes, but he was much at a loss to account for the removal of the Barriers by which the lakes had been retained.

Mr DARWIN, in his paper, thought the Barrier difficulty so great, that he deemed no other explanation possible, than that the "*Roads*" were *sea-beaches*. His theory was adopted by the late ROBERT CHAMBERS, Professor NICOL, Mr WATSON, and others.

Professor AGASSIZ and Dr BUCKLAND accepted the theory of Lakes, and suggested that the Lakes had been dammed up by Glaciers; a view taken by a majority of subsequent inquirers, and in particular by Sir CHARLES LYELL, Mr JAMIESON of Ellon, and Mr JOLLY of Inverness.

III. Such being the nature of the problem and the various attempts to solve it, let me now briefly indicate the view which I have taken. It is much the same as that explained in my former Memoir: But I expect now to be able to rest it on a wider basis of facts, and to support it on stronger grounds.

I concur with the great majority of observers, in atributing the formation of the shelves to Lakes. My reasons are these,—

1st. In three of the Glens, the existence of an old River Channel has been ascertained and traced, by which in each Glen, the surplus Waters of the Lakes were discharged.

* In *Glen Gluoy* (as Sir THOMAS DICK LAUDER states) the highest shelf has a width of 100 yards. In *Glen Spean* Shelf 4, above Inverlair House, has a width of 20 yards. (See fig. 17, p. 51.) In *Glen Glaster*, on the side next Craig Dhu, this same shelf has a width of nearly 100 yards, as is shown on the Ordnance Survey 6-inch Map.

Thus the Lake which filled Glen Gluoy at its highest shelf, reached the summit level between that Glen and Glen Roy; and at that summit level an old River Channel was discovered by ROBERT CHAMBERS and me, leading down to Glen Roy, at the timewhen Glen Roy was occupied by a lake, not only when this lake was at its highest level of 1149 feet above the sea, but also when the Glen Roy Lake sank to the next level of 1068 feet above the sea.*

So also at the summit level between Glen Roy and the Valley of the Spey, there is a flattish hollow, exactly corresponding in level with the highest shelf in Glen Roy, over which the Lake in Glen Roy when at its height of 1149 feet above the sea could discharge its surplus waters.

* As this is a point of some importance, it is only fair to state, that a very experienced observer, Professor NICOL of Aberdeen, does not admit that there is evidence of a River Channel from any one Glen into another Glen. He says ("London Geol. Socy. Journal," May 12, 1869, page 284) "I examined the various passes carefully, and found that whilst in none of them was there the slightest trace of an ancient *river*, in all there were distinct indications of the former existence of a *narrow Sea Strait*." The valley by which the Gluoy Lake is assumed to have drained into Glen Roy is very narrow, and encumbered with detritus from the hills on the sides. The summit level is flat and marshy, and it appeared to me considerably below the level of the line (*i.e.*, the 'Road'?). On the other hand, a line of stones, as if washed out of the detritus, appeared to show that the sea or loch, had extended quite through the Strait. I observed no indication of any stream of water, larger than the present small rivulet, having ever been there."

In answer to these statements, I give the following Extracts from the Notes made by me on the occasion of my visit to the place with ROBERT CHAMBERS, in September 1846.

"At the head of Glen Gluoy, Shelf 1 almost disappears in the Moss. It is however faintly visible on the North side, about 1 or 2 feet above the Moss, running towards Glen Turrit. The Moss I found to be 5½ feet thick above Boulder Clay. About 1½ miles to Eastward, found rocks in middle of Channel, much worn and smoothed,—their rough edges or faces all pointing Eastward, *i.e.*, towards Glen Roy. This is near Glen Turrit, and about the place where traces of Shelf 1 disappear. About 8 or 10 feet above these smoothed rocks there are other rocks which present no appearance of smoothing.

"At the side, there are low grassy knolls or hummocks apparently marking edge of a River Course,—which here 40 yards wide.

"Apparently Gluoy Lake continued to discharge, whilst Roy dropped from Shelf 2 to 3, hence additional cutting power given to Gluoy Stream;—and accordingly there is a deep and rocky ravine, down to Glen Turret.

"Measured with ROBERT CHAMBERS (who first drew my attention to fact) depression of Shelf 2 in Glen Roy below Shelf 1, and found that by *Spirit-levelling*, it was 29 feet; by *Sympysometer* 35 feet; by *Barometer* 11 feet.

"Visited next, head of Glen Roy, in upper Glen Roy. Interesting to observe how uniformly the *smooth* surfaces of rocks are to *West*, the *rough* faces to *East*." (Notes p. 47).

The following applies to the Glen Glaster old River course.

"Discovered debouche of Lake No. 3 in Glen Glaster. A Shelf on both sides runs up to *Col*, at a level coincident with Moor there.

"At this place, rocks appear above moss. These are about middle and lowest part of flat.—Evidently, water has rushed over these rocks from West: for their round and smooth faces are to West, their rough faces to East. They form a Channel sloping down towards Loch Laggan for about a mile. By existing burn, they can't have been rounded, as this burn, very small, and 20 feet below them. These rocks occupy a breadth of from 30 to 40 feet, and have evidently occupied bed of a stream, exceeding that width. Walked along this supposed ancient water-course for about a mile, and saw that it extended down towards Loch Laggan, as far as eye could reach. Probably Loch Laggan then stood at level of Shelf 4, discharging itself at Mukkoul, as it continued to do, when Lake Roy sunk to Shelf 4.

"No sand or gravel in this old Water-course, but numerous granite boulders, very spherical.

"Remnants of sloping haughs on each side, evidently formed by old River when in flood,—with precipitous cliffs beyond these haughs,—indicating height to which floods reached."

So also when this Lake sank to the next "Shelf" or "Road" (1066 above the sea), the surplus waters at that level were discharged by a River Channel discovered in Glen Glaster, which led to Loch Laggan at a place called the Rough Burn, and where there is now a great accumulation of materials having all the appearance of a Delta; the Delta being exactly at the place where one would be formed by a river flowing into a lake, at the height of 856 feet above the sea.

So also, with regard to this lake which formed the shelf at 856 feet above the sea; there is at the N. E. end of Loch Laggan, at a place called Mukkoul, an old river channel, which is at the very spot, and level, suitable for carrying off the surplus waters of the Lake. The River Pattaig now runs here into Loch Laggan. But this river had evidently, at a former period, flowed in a different direction, viz., towards the eastward, when the lake stood about 38 feet higher than at present. Last autumn, I walked up the

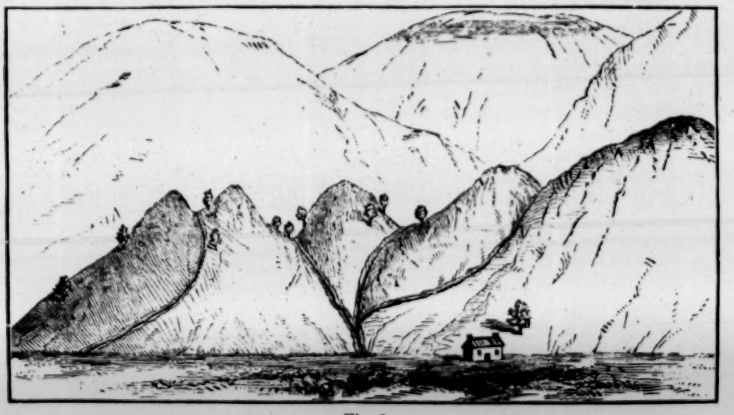


Fig. 1.

Beds of gravel and sand, from 60 to 70 feet high, in Glen Spean (near Tulloch), cut through by small streams from the adjoining hills.

banks of this River, and at about a mile from its debouche into Loch Laggan, I found the old channel, now dry, about 25 feet above the present stream, running Eastward towards the valley, through which Loch Laggan sent its surplus waters into Strath Spey,

From the Ordnance Maps, it will be observed, that this last mentioned shelf runs along the north side of Glen Spean towards the lower end of the Glen, where it encircles some rocky knolls. Between these knolls and the hill called "*Craig Dhu*," there is a summit level or col separating Glen Roy from Glen Spean. As these cols are generally instructive, I examined this one, and was not disappointed. It is about 20 feet below the level of Shelf 4, and con-

sists of a rocky ridge formed by the upturned edges of Mica Slate strata. These strata have been worn down apparently by water flowing over them from the Glen Roy side, their smooth faces being on that side, their rough faces on the side next Glen Spean. On the shelf itself there are multitudes of large well washed pebles and boulders. When the lake stood at the level of the shelf, discharging at Mukkoul, the water then occupying Glen Roy would flow towards Mukkoul, and one of the passages would be the narrow and shallow strait just referred to.

In Glen Roy, and at many other places, the middle shelf is seen to have been deeply cut through by burns, whose sides expose the material of the old beach and of the lake bottom. Thus at Dalrioch and on the N. E. shoulder of Craigh-Dhu, the old lake bottom consists of fine clay, or mud, horizontally stratified and laminated,—evidence of the stillness and depth of the water in which the sediment had been deposited. At the spot last mentioned, there are cliffs of sandy mud from 40 to 60 feet high. These beds of mud are occasionally covered by beds of stratified sand.

In Glen Collarig, the lowest shelf on Bohuntine Hill is crossed and cut through by a burn. The material there also consists of sandy mud with small boulders and pebbles.

At Inverlair and Fersit, (near Loch Treig), there are large portions of the old bottom of the lake still extant, consisting of gravel cliffs from 50 to 60 feet in height. Great beds of sand occasionally occur in these gravel deposits. Fig. 1, page 4, is an attempt to show this old lake bottom cut through by streams from the adjoining hill.

These facts supplied some of the grounds on which the lake theory rested. Other arguments will be noticed, when I refer more particularly to the sea theory.

Meanwhile, I may allude to another fact recently ascertained which greatly strengthens the Lake hypothesis.

Our colleague the Revd. Mr BROWN has in one of the shelves (viz., the lowest of the three in Glen Roy, Shelf 4) discovered fresh water *Diatoms*. They could not have existed where he found them if the shelves were marine ("Roy. Soc. Proceedings," 2d March).

IV. The next point for consideration is by what means the lakes were kept up to the height of the shelves, in the absence of any appearance at present of Barrier or Blockage.

In the lower part of Glen Roy the bottom of the valley is about 800 feet below the highest shelf, and the valley is about a mile wide.

How then were the lakes kept in?

1. To understand clearly this part of the problem, it is proper to observe

in what part of the glens the shelves cease, for there, or near that part, the blockage whatever it was, most probably existed.

In Glen Roy, as will be seen from the map annexed (Plate XLI.), the highest shelf, No. 2, exists in the upper part of the glen, on both sides, but does not come further down the glen than a certain point on each bank. These two points, it will be observed, are nearly opposite to each other. Here, therefore, the lake is generally assumed to have terminated, when it stood at its highest level, and here a blockage of some kind must be sought for. Let it be also noticed, that this shelf, No. 2, enters the side valley, called Glen Collarig, through a hollow or depression called the "Gap," *but only for a certain distance*;—and there, another blockage of some kind must have existed, to account for the stoppage of the shelf.

When the Lake sank to its next, the 1068 feet level (Shelf 3), it formed a beach line not only in the upper part of Glen Roy, but in a lower part, *i.e.*, in a part about one quarter of a mile lower down the Glen than the spot where No. 2 shelf stopped. The blockage, or a part of it, to allow this, must have been lowered 81 feet,* and must have occupied a situation further south. This lower shelf, No. 3, is traceable into Glen Glaster, and approaches the col where there is the old river course, by which the lake, when at that level, overflowed towards what is now called the Rough Burn into Glen Spean.

This lower shelf, continues in Glen Roy on both sides, and, like Shelf 2, stops at certain points, as may be seen on the map, nearly opposite to one another.

This lower shelf also goes through the gap into Glen Collarig;—and it goes a little beyond the place where shelf No. 2 stopped. Here, therefore, as in Glen Roy, something occurred to allow the lake in Glen Collarig to reach a little farther south, and to be kept up there at that lower level. To allow of this extension of the lake, both in Glen Roy and in Glen Collarig, there must have been a scooping away of the blockage. This is an important fact, because the blockage must have been of such a nature as to be capable of being lowered vertically, and of being scooped away horizontally.

We now come to the next subsidence of the lake in Glen Roy, as indicated by Shelf 4, which stands at a height of 856 feet above the sea.

This shelf goes up Glen Roy only for a certain distance, of course stopping at or near the part where the bottom of the valley rises to a level higher than

* It may be proper to explain that the lake did not subside at once 81 feet. Two intermediate shelves are visible between Shelves 2 and 3, in Glen Glaster (east side), in Glen Collarig, in Glen Roy on the south side above Achavaddy, and on Ben Erin—*i.e.*, the hill above the Gap. One of these is about 14 feet below Shelf 2, the other about 32 feet above Shelf 3. So also, when the lake subsided from shelf 3 to shelf 4, there was a halt long enough to allow an intermediate shelf to be formed at a height of 990 feet, which is very conspicuous in Glen Collarig. None of these intermediate shelves are marked on the Ordnance Maps; they were, however, pointed out by me to the Surveyors.

856 feet. This time the lake sank so low, that it could not enter the lateral valley of Glen Collarig through the gap. But what is very important, the lake entered Glen Collarig at its south or lower end, and its beach mark is distinct on both sides of that Glen up to near the middle of it, as is shown on the map.

Some blockage of a permanent character must therefore have existed about this part of Glen Collarig, to prevent the two highest shelves, Nos. 2 and 3, going farther south, and also to prevent the lowest shelf, No. 4, going further north than the points where these shelves respectively stop. To this important discovery I will afterwards advert more particularly.

This lower shelf, No. 4, in Glen Spean runs a certain distance west, towards the glen occupied by the Caledonian Canal. It also runs up the whole length of Glen Spean to the east end of Loch Laggan at Mukkoul, where the lake, when at that level, discharged eastward by the Makkoul valley (now dry) into Strath Spey.

The blockage which caused the waters of the lake to stand at this level, and forced them to flow out at its east end, must have extended across the country, between a spur of Ben Nevis and Teandrish, a distance, as the crow flies, of about 4 or 5 miles. This district, called Unichan, lying between Fort-William and Spean Bridge, consists now of tolerably flat ground composed mostly on its surface, of gravel, clay and sand.

The Author's Theory.

2. The view which I support is, that the lakes at all these different levels were kept in by an accumulation of detritus, which being from time to time lowered in level, caused the subsidence of the lakes, and being scooped away, allowed of the extension of the lakes, beyond the original blockage, to lower parts of the glens.

Mr DARWIN and others represent the impossibility of having barriers several hundred feet in height, and more than a mile long, composed of such loose materials as gravel or clay.

The objection would be well founded, if it was necessary to suppose that the lakes were kept in by barriers resembling *Dam-dykes*, which is the representation given of the blockage by some writers. This, however, is not a correct representation of the nature of the blockage suggested.

It is manifest that all this district of the Highlands was formerly covered by detritus, up to the height of at least 2000 feet above the present level of the sea; and that this detritus filled the valleys, including even the Great Glen which stretches from Fort-William to Inverness. I assume that the detritus had been deposited and spread over the country, when this part of Europe was submerged beneath the sea.

When the sea began to fall in level, so as to expose the land to the agencies of rain, snow, and frost, water would collect wherever there were depressions in the surface of the country, and form lakes. As the sea retired, the streams issuing from these lakes would acquire more power to cut out for themselves deeper channels where the materials were susceptible of erosion.

3. In support of this view, that the whole of this part of Scotland was covered with detritus, the following list of places, with their heights above the sea, is submitted :—

(1.) On the hills which surround *Glen Gluoy*, at a height of 1700 and 2000 feet above the sea, beds of sand and gravel 10 and 12 feet thick at least, are conspicuous in every lateral ravine, and in many of these ravines, the beds of sand and gravel form cliffs or scaurs nearly a hundred feet deep. These beds of detritus abound on *Letter Finlay Hill*, situated between the Great Glen and Glen Gluoy.

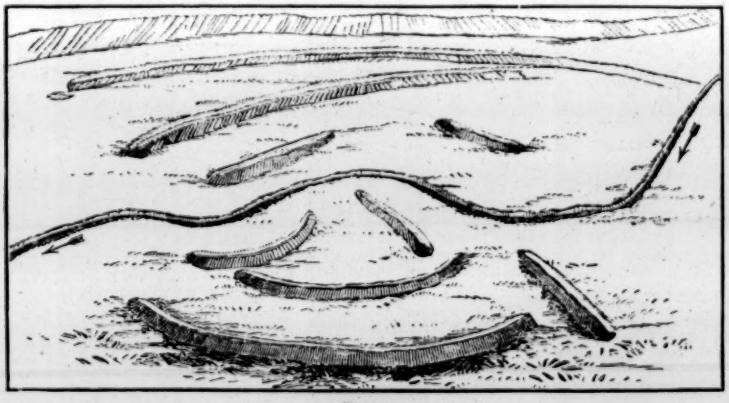


Fig. 2.

Portion of the wide valley of Alt-na-Bruach, showing some of the numerous Escars or Kaims occuring in it. These Kaims consist of gravel and sand, and reach heights of from 30 to 50 feet above the adjoining plain.

(2.) Near the summit of *Craig Dhu*, the hill between Glen Roy and Glen Spean, I found water-worn gravel at a height of 2000 feet above the sea.

(3.) On *Ben Chlinaig*, a hill on the south side of Glen Spean, and nearly opposite to Craig Dhu, at a height of 1700 feet above the sea, beds of boulder clay and gravel are cut through by all the streams flowing down its sides. The beds are rudely stratified, and slope towards the valley.

(4.) In "*Alt-na-Bruach*,"* there are extensive Scaurs or Kaims of gravel and sand, up to a height of more than 1200 feet. Through these deposits the

* I am told that this Gaelic word means "Valley of high heaps or banks."

burns have cut deep ravines, whose banks consist of cliffs or scaurs, at least 400 feet high. Fig. 2 gives a representation of this *Alt-na-Bruach* Valley, as seen from the hill called Ben Chlinaig to the north. Fig 3. gives a section of one of the Escars.

(5.) On the hill situated to the N.E. of the "*Rough Burn*," situated on the east side of Glen Spean Valley, there are knolls of gravel at a height of 1700 feet above the sea. These knolls are the remaining portions of extensive sheets of drift, which have been washed away by rains and streams.

(6.) On the west side of *Loch Laggan*, I found abundance of coarse water-worn gravel upon the "*Bein-in hills*," as far up as I ascended, viz., 1890 feet above the sea.

(7.) In *Glen Collarig*, there is in a lateral valley on the N.E. side, an enormous cliff of grey boulder clay. It is full of boulders and pebbles, and forms a vertical wall from 300 to 400 feet in height, above the highest shelf.*

(8.) To the East of *Loch Treig*, at a height exceeding 1500 feet above the

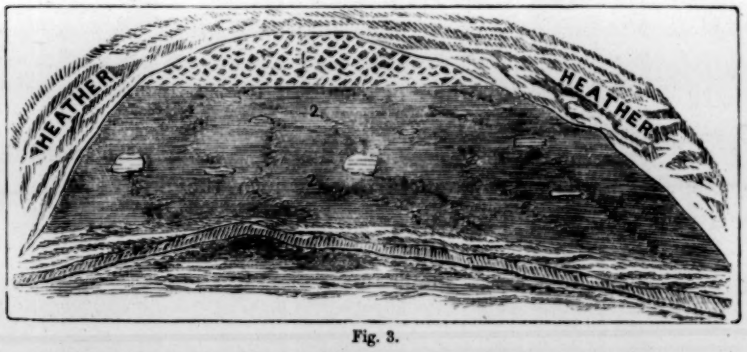


Fig. 3.

Section of an Escar in Alt-na-Bruach Valley, cut through by a stream. No. 1 is a bed of stratified gravel, about 6 feet thick. No. 2 is stratified sand, about 20 feet thick, containing boulders.

sea, a remarkable series of Kaims or Escars occur, consisting of detritus and Boulders.

In corroboration of these observations regarding the enormous accumulation of detritus in these Lochaber valleys, it may be permitted to refer to the testimony of preceding inquirers.

Macculloch, whose precision of description is well known (page 327) describes the *hills* forming Glen Roy as "*covered* with a thick alluvium." The nature of this alluvium he explains as consisting "of deposits of fine sand, gravel, clay, and rolled stones of different sizes, disposed in a manner irregularly stratified, and in a manner more or less horizontal," (page 330).

Darwin in his paper repeatedly alludes to the "enormous accumulation of

* To this cliff I particularly drew the attention of the Government Surveyors, and of Mr Jolly.

perfectly rounded shingle" in all the valleys. "These irregularly stratified beds, near the mouth of the Spean, attain a thickness of several hundred feet, and consist of sand and pebbles." Mr Darwin founds specially on the existence of these deposits at the *cols* of Gluoy and Roy,—1180 feet above the sea,—in support of his theory that the shelves were sea-beaches (pages 43, 53, 65).

CHAMBERS, in his "Ancient Sea Margins" (page 123), referring first to Glen Roy, says "The bottom of the valley is *filled, to a great height*, with these alluvial masses, insomuch as to have appeared to some as in no small degree diminishing the difficulty as to barriers in that glen." Next, referring to Glen Spean, he says, "we find huge protuberances of detrital matter, starting out from the hills, and generally assuming a rude terrace-like form at 534, 627, and 734 feet above the sea."

CHAMBERS refers also to what he calls "the grandest delta of the district," called Unichan, occupying the lower part of Glen Spean. He describes it as "a mass of gravel, 11 miles long, by perhaps 2 broad, and reaching an elevation of 612 feet above the sea" (page 106).

Professor NICOL of Aberdeen (who like CHAMBERS adopts the sea-beach theory), referring to the "col" between Glen Roy and the Spey, at a height of 1150 feet above the sea, alludes to a line of stones left there, where the water had washed away the detritus. "From a lateral corry below Loch Spey,* *great masses of detritus* (he says) project into the main valley. These have been spread out and levelled down, as if thrown into the sea, not as if heaped up in a river valley" ("Proceedings of Geol. Socy. of London," 12th May 1869, p. 285).

He expresses an unhesitating opinion—

"That before the formation of the Glen Roy lines, *the whole region has been submerged in the sea*. This is proved by the uniform coat of detritus covering *the whole surface* in a thicker or thinner sheet, according to the form of the ground. This coat is not the surface waste, but *matter laid down by water*;—it is too wide spread and general in its distribution, and too much mixed in its composition, to have been formed in any mere lake." "It is *in this detrital* cover that the *lines* (meaning the Parallel Roads) are cut" (page 283).

JAMIESON, who adopts the theory of ice barriers, as suggested by AGASSIZ, takes special notice in all his papers of the extraordinary amount of detrital matter in the glens, and allows that there is more in *these* glens than in other Highland glens. In his paper (of 21st January 1863, "London Geol. Society Journal"), he says:—

"*Glen Roy* presents an *exceptional* character to our other mountain glens, not only in respect of its Parallel Roads, but also *on account of its great beds of silt and gravel*, and still more the wonderfully fine deltas at the mouth of its lateral ravines. All these local peculiarities—the lines, the deltas, and *the heavy banks of silt and gravel*, bespeak a *local* cause, such as a fresh water lake, and not a universally present one like the sea" ("Geol. Socy. Pro.," 21st January 1863, page 244).

* Loch Spey is about 1200 feet above the sea.

The foregoing localities for detritus are chiefly at places considerably above the level of the shelves, so that at the required levels, there were materials in abundance suitable to form blockages.

In the bottoms of the valleys, the accumulations of drift are of course much greater than near the tops or ridges of the hills. Scaurs or cliffs of detritus, in some places 80 feet high, are in all the ravines on the sides of the adjoining hills. On the Laire Burn (to the west of Loch Treig) there are scaurs of coarse clay and gravel nearly 300 feet high.

But is it likely that the detritus was deposited only on the tops or ridges of the hills, and in the bottoms of the valleys, and not also in what are now the hollows between the hills? Is it not quite as likely that the detritus, originally filled or occupied these hollows, so that originally no valley existed, at all events valleys of the depth which now exist?

Mr DARWIN argues (page 53, "that the *Valley* was once partly or entirely *filled up*, to the height of the shelves, by drift materials." To a certain extent I concur in this opinion. I think that the more the valleys were filled, the better we can understand how the very tops of the hills, as well as their sides, should have been covered, and should be still covered, by gravel and sand.

On these grounds, I submit, that in the Lochaber district, ample materials existed for forming detrital blockages of the Glens, to keep in and keep up the lakes to the heights which their shelves indicate.

4. When the land began to rise up out of the ocean, so that extensive portions of country became exposed, what would happen? That lakes would be formed at high levels, and be kept at these levels by detritus, is consistent alike with reason and fact.

There are even yet in this district of the Highlands, numerous lakes at high levels, kept in by detrital matter. Last October I visited Loch Earba, a lake about 4 miles in length, situated about 2 miles to the South of Loch Laggan. That lake is at a height of about 1120 feet above the sea, and is *kept up by a mass of detritus*, through which the stream from the lake has cut its way to Loch Laggan. There happens to be a quarry or pit near the point of discharge, for getting from it gravel and boulders, to be broken into road metal, which shews the character and thickness of the detritus at this place. The waters of Loch Earba, however, formerly stood higher than at present, as is shown by a horizontal beach line about 30 feet above the present level of the lake, which runs for several hundred yards at its north end along its west bank. This beach line is traceable also near the south end.

In the same district of Lochaber, there are other two or three lakes at even greater heights, which I was informed, though I have not visited them, are in like manner kept up by detritus.

There are numerous lakes at lower levels, also kept in by detrital

accumulations. Loch Laggan, Loch Treig,* Loch Lochy, and Loch Arkaig are examples. Arkaig is specially interesting, because there is evidence to shew, that at some former period it discharged into Loch Lochy by a channel different from the present one. The old channel is situated at the N. E. end of the lake, and forms a deep dry ravine, called "the Dark Mile." The present channel of discharge is at the S. E. corner. Near the mouth of the lake there is a horizontal terrace, about 90 feet above the present surface, formed on detritus, which suggests that the lake once stood at a higher level.

Loch Laggan now discharges its surplus waters by the River Spean, which has cut for itself a trench about from 30 to 40 feet below what had been here the bottom of the lake. One side of the trench is shown on fig. 4. It

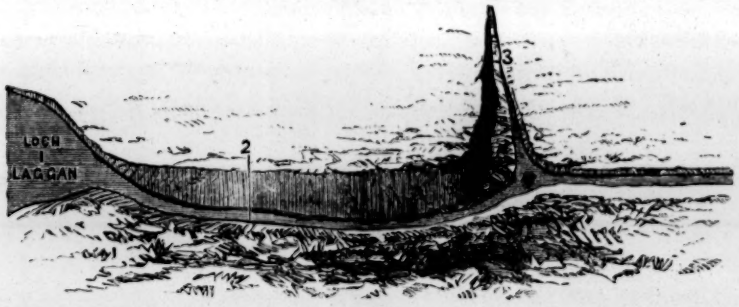


Fig. 4.

Loch Laggan (1) now discharging its surplus waters by the River Spean (2). About a quarter of a mile below the Loch, the Spean is joined by the River Gulbain (3). Both rivers have cut deep trenches through the old bottom of the Lake, viz., the lake which formed Shelf 4.

forms a cliff running for several hundred yards towards the west, till it joins another cliff almost at right angles, which has been formed by the River Gulbain. The material cut through by both rivers consists of sand, and fine clay in horizontal beds, formerly the bottom of the lake which formed Shelf 4. The trench cut by the river would have been deeper, but for rock over which the Spean flows where it runs out of the lake.

In my former paper, I referred to a case somewhat analagous, where a lake had subsided from one level to another, and which is kept up at its present level by detritus. This lake is Loch Tulla, about 3 miles long and 1 mile broad, situated about 40 miles S. W. of Glen Roy. CHAMBERS, after I had pointed out this lake, as a case supporting my theory, visited it, and in his book on Sea Margins admitted the correctness of my statements, as to the horizontality

* Mr JAMIESON (page 250, "London Geo. Soc. Pro.," 21st January 1863), admits Loch Treig, in its outflow, "has not even yet cut its way to the very bottom, for the lake is *still partly retained by banks of gravel.*"

of the several shelves encompassing the lake, and their distance from one another. The surface of the lake is (approximately) 630 feet above the sea. But originally its waters had stood at a height of 1132 feet above the sea; and my position was, that the barrier which kept it in consisted of detritus which from time to time was eroded by the river discharging from the lake. I also stated that there existed still, at and near the point of discharge, "*great heaps of unstratified gravel*," which form the present blockage. This statement Dr CHAMBERS did not question. Nevertheless, he adhered to his opinion, that the shelves round the lake, "*may* all have been produced by the sea" (page 129).*

Another case of the same kind occurs near Kingussie. Loch Gwynac is about a mile in length and 300 yards wide. The present level of the loch is (by Ordnance Survey) 1015 feet above the sea. There are five horizontal terraces, traceable at the following heights above the lake, viz.:—26 feet, 44 feet, 52 feet, 96 feet, and 132 feet. These terraces have been formed on the drift, which here as elsewhere in the Northern Highlands overspreads the whole country. The blockage which kept the waters of the loch up to the heights of these terraces has disappeared, so that there must have been enormous denudation and scouring out of the drift in this valley.

The flat district north of Dunkeld, where the Dalguise and Ballingluig Railway Stations are situated, was formerly a lake. The old Beach line, about 60 feet above the present level of the River Tay, is tolerably distinct. The blockage consisted of a mass of detritus, which was worn down and cut through by the river. A considerable portion of this blockage still subsists, forming a huge embankment transverse to the valley.

Two years ago, when at Inverie, on the west coast of Argyleshire, Mr BAIRD, whom I was visiting, took me to a place a few miles north on the same coast, called Invergussern. There I found a flat valley with a stream (the Gussern) meandering through it. On each side of the valley there are hills, along the base of which a beach line about 50 feet above the present surface was manifest. When I came near to the sea-shore, I found a great ridge of sand and gravel crossing the mouth of the valley, and which, as it impinged on the hills on each side, could, so long as continuous, have effectually kept up the waters in the valley to form a lake. Figures 5 and 6 will assist to make this description intelligible.

The River Gussern had evidently cut through the bank S S, and also a portion of the slate rocks beneath, and so allowed the lake to be drained.

* A few years ago, when at Killin, at the west end of Loch Tay, I made some ascents of the hills adjoining, and saw traces of several lines of terrace up to a height of 890 feet above the lake, and 1240 feet above the sea. Along the north side of the lake there is an extensive flat at a height of about 400 feet, which seemed to have its counterpart along the south side of the lake I mention this only as a suggestion for farther inquiry. I believe that the late Mr M'LAREN described some terraces at the east end of the lake, about 40 feet above it, but I have not his papers to refer to.

The top of the barrier I found to be about 90 feet above the level of the river, and 135 feet above the sea. The bank of sand is about 35 feet deep and 100 yards wide, the depth of the erosion of the rocks is 50 feet.

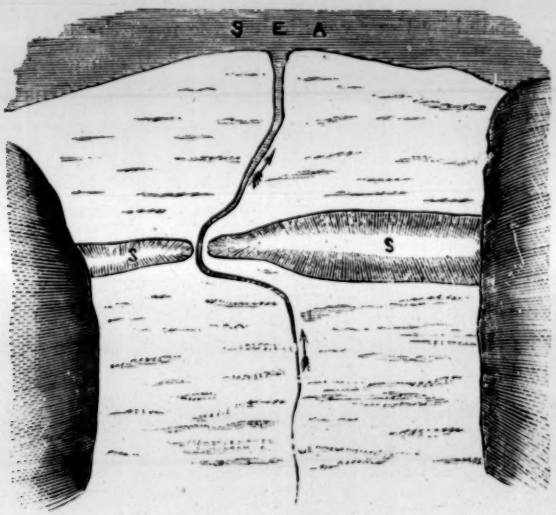


Fig. 5. (See p. 607.)

Ground Plan of detrital bank near the mouth of the River Gussern, crossing the valley. S S represent the two portions of the bank still remaining, and united to the hills on each side.

Glen Spean Valley represents several examples of the same kind. Near the falls of Monessie, detritus had originally filled the valley to such a height as to form a lake, the surface of which reached a height of 520 feet above the sea. The beach line of this lake is distinctly traceable from Auchenleurich Post



Fig. 6. (See p. 607.)

Section of detrital bank in valley of Gussern, showing how the river has cut through it, and the subjacent rocks.

Office and Monessie at its lower or West end, to Inverlair Falls at its upper or East end, a distance of about 3 miles. Figures 7 and 8 are intended to shew how the lake was drained. At Monessie, the channel of the river is about 100 feet below the level of the old beach line. A great mass of detritus still crosses the valley here for about two-thirds of its breadth. The river issuing from the lake, cut through the detritus, and drained the lake.

From the foregoing statements, two points seem established.—(1.) That all

over this district of the Highlands, detritus formerly existed to such an extent, that blockages might have been formed by it, to keep up lakes existing in the

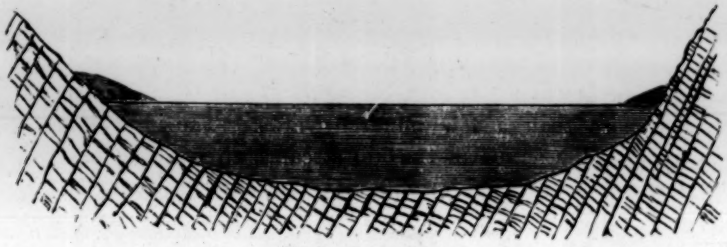


Fig. 7. (See p. 608.)

Section showing lake supposed to have formerly existed near Monessie.

country to the requisite height; and (2.) That there are many cases now in the Highlands, of lakes kept up in this manner; (3.) That lakes have subsided, and even drained altogether, by the wearing down and removal of detrital blockages through the action of rivers.

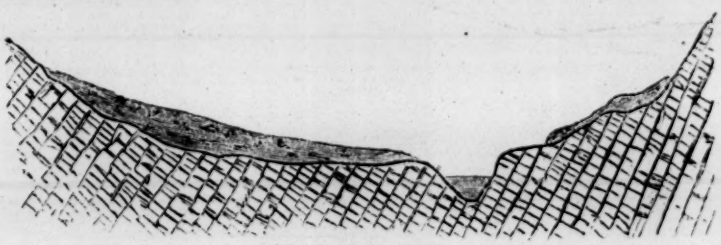


Fig. 8. (See p. 608.)

Section showing the same spot near Monessie after the lake was drained, by the River Spean having cut through the detritus and subjacent rocks.

5. If these views be applied to the Parallel Roads in the respective glens, they will be found sufficient to explain by what means these Roads stopped at the places marked on the map.

(1.) As the two shelves of Glen Gluoy appear only in the upper part of the glen, some blockage must have existed at its mouth. I shall revert to this Glen Gluoy blockage in a subsequent part of the paper when discussing the blockage of the Great Glen. Meanwhile, I would refer to a peculiarity in Glen Gluoy, that the upper shelf extends farther down the glen than the lower shelf. A B in fig. 9 is the hill forming one side of the valley on which the shelves are marked, C C is the highest shelf, and D D the lowest. The greater extension of the upper shelf may be accounted for, by supposing that the detrital blockage E, sloped in the way shown in the figure, which is the usual form of a lake bottom.

(2.) It is not difficult to understand how or why the blockage changed position and level, *if it was detritus*. There are, in all the Glens, multitudes of

streams which now flow down the sides of the hills. These streams must have existed during the period of the lakes. They may have flowed not only into the lake, but also upon the detritus near the lake in Glen Roy at its south end, and have cut out channels through the detritus, which filled that Glen near the present mouth of Glen Glaster. The distance between the highest blockage in Glen Roy (viz. for Shelf 2) and Glen Glaster was small, probably not more than a few hundred yards. The intervening mass of detritus would, by the action of the hill streams, after a time be worn down, and then the outflow of the lake would take the course of Glen Glaster, leaving untouched the remainder of the detritus between Craig Dhu and Bohuntine.

If this was what happened, it is natural that the new line of blockage should be (as shown by the terminations of Shelf 3) right across Glen Roy, and parallel with the new outlet through Glen Glaster.

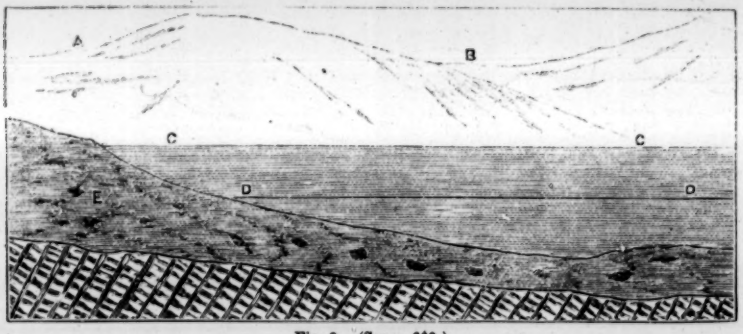


Fig. 9. (See p. 609.)

Section to explain why the upper shelf in Glen Gluoy C C extends farther down the glen than the lower shelf D D.

It is important next to notice what happened in Glen Collarig. It will be seen from the maps (Plates XLI. and XLII.) that Shelf 3, when the lake subsided to it, made its mark along the west side of Bohuntine Hill, and also on the opposite hill. The scooping out of the Collarig blockage need not have happened at exactly the same time as the change in the Glen Roy blockage. But the change was of the same nature, and was probably produced by a similar cause. Strong streams rush down on each side of Glen Collarig, and at or near the very place where the blockage must have originally existed. These streams, descending on the detritus, would wear down and remove a large part of it, and so allow of the extension of this arm of the lake farther south in the glen.

The next change which took place was the entire removal of the blockage between Craig Dhu and Bohuntine, whereby the lake reached the lowest level, viz., that marked by Shelf 4. This change might be effected in the course of the general erosion which had long been going on. In the first place,

there was a strong stream or river flowing through Glen Glaster from the lakes occupying Glen Roy and Glen Gluoy. Then there were streams descending from the Glen Roy hills on both sides. On the south side there was the Bohina Burn, and on the north side the river from Glen Collarig. These numerous streams would act on and annihilate the blockage which kept up the Glen Roy Lake to the middle shelf. The result would be an entrance of lake 4 into Glen Collarig by its south end; and accordingly it will be seen from the map, (Plate XLII.) that Shelf 4 goes up Glen Collarig, but stops on both sides at a point which is *only a few hundred yards from the place where the middle shelf came to, from the north.* What was it which prevented Shelf 4 reaching further north?

The ground plan on Plate XLII., taken from the Ordnance Survey, shows where this blockage must have been situated. It must have been between the points where Shelves 2 and 3 terminate, and the points where Shelf 4 terminates.

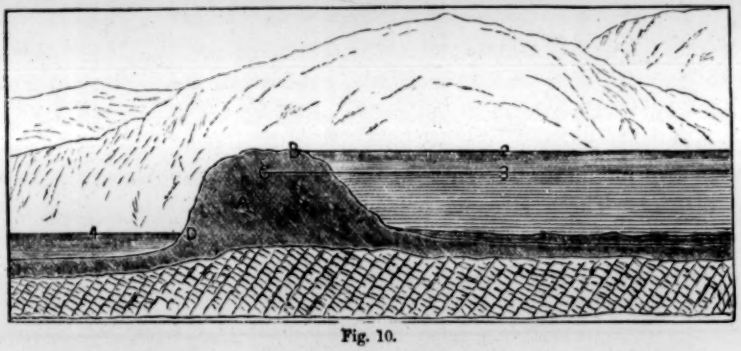


Fig. 10.

Section showing in Glen Coilarig the termination of Shelves 2 and 3, viz., at B and C, and of Shelf 4 in a lower part of the glen, viz., at D, with the supposed detrital blockage A which separated the lakes.

The intervening space must evidently have been occupied by a blockage (A in fig. 10), which answered the double purpose of keeping separate the waters of Shelves 2 and 3 (viz., B C), and the waters of Shelf 4 (D). This intervening space, when measured on the ordnance map, gives an average thickness of blockage or barrier of about 660 yards. The width of the blockage (crossing the glen) need not have been more than 800 yards; and the depth of the blockage, for the highest of the shelves B, even supposing that the glen was as deep then as now, would not have been more than 368 feet. But making allowance for the erosion of the valley, since the time of the Parallel Roads, the probability is that a blockage of considerably less height was sufficient, and existed.

The detrital mass A (fig. 10), at first must have been in such quantity as to reach northwards to where the highest shelf, No. 2, stops, viz., B. It then was scooped out or was undermined on that side for 50 or 60 yards, so that when

the lake fell to Shelf 3, the waters in Glen Collarig reached to C in fig. 10, and thereafter, on the removal of the blockage between Craig Dhu and Bohuntine, the waters of the lowest shelf came up Glen Collarig to the place (viz., D), where the detrital blockage above referred to existed.

When the lake sank to the lowest shelf* (No. 4) in the glens, by the removal of the blockage between Bohina and Bohuntine, the waters of the lake occupied the whole valley of the Spean. Shelf 4 is traceable as far as the east end of Loch Laggan; and towards the west to Teindrish on the one side and Corry Cholzie and Corry N'Eoin on the other side.

And what was the blockage at this point? The intervening space between the two shelves at this their Western termination is no less than five miles in length. Could there have been a detrital barrier here also? There are good grounds for believing that there was. The whole of this district, as Dr CHAMBERS explains, consists of "*a mass of gravel* 11 *miles long by perhaps* 2 *broad, and reaching an elevation of* 612 *feet above the sea.*" I have traversed the district in many directions, and can attest that it presents an enormous accumulation of drift deposits,—not gravel only, but also of sand and clay;—at one spot only does rock come to the surface. Streams cut through this extensive drift plateau from the Aonachmore hills, situated to the south. There are no less than five mountain torrents in the course of two miles in this part of the district. These streams present deep gashes through the detritus, and when they reach the base of the hilly range, unite into considerable rivers, which run, some north to join the Spean, others west towards Fort-William. The scouring out of detritus along the base of these hills has been very great. The result has been a valley deep and wide in an east and west direction. In company with the Rev. Mr CAMERON, minister of the parish, I walked along this valley towards Fort-William. Part of it consists of an elongated marsh, formerly a lake, whose margin had been about 20 feet above the marsh, the surrounding cliffs being detritus. Farther west I came upon a small lake, the banks of which, composed of detritus, are about 170 feet high, showing on their sides two or three terraces, proofs that either the lake had subsided from one level to another, or that the river had eroded first on one side and then on another at the above levels before reaching its present channel. The river has now reached rock, so that further subsidence is arrested.

In different parts of this lower district, knolls and banks of detritus stand up above the general level. It would therefore require no great amount of restoration to supply a detrital blockage sufficient for damming the great lake indicated by the lowest Glen Roy shelf, which is 854 feet above the sea.

* I have already explained, that the subsidence of the lake from Shelf 3 to 4 did not take place all at once. It sank at first only about 78 feet, and formed an intermediate road visible in Glen Collarig, at its north end, on both sides.

When the sides of the hills are examined, on both sides of the valley where this lower shelf terminates, abundance of detrital matter is still traceable. At Teindrish, which is on the north side, there are great cliffs of tenacious clay, full of boulders, reaching to a height of at least 1000 feet above the sea. Similar deposits, and at as high a level, exist on the south side of the valley, viz., at Corry Choilzie and Corry N'Eoin.

But there are other conditions which suggest even a higher and more extensive blockage in this quarter. The Glen Gluoy shelves imply a blockage reaching to a height of 1170 feet above the sea. How long did this Gluoy lake continue even at its lower shelf, which is about 970 feet above the sea? Glen Gluoy opens its mouth on the Great Glen; and the highest shelf in it reaches to a point within a mile of Loch Lochey. The probability therefore is, that if

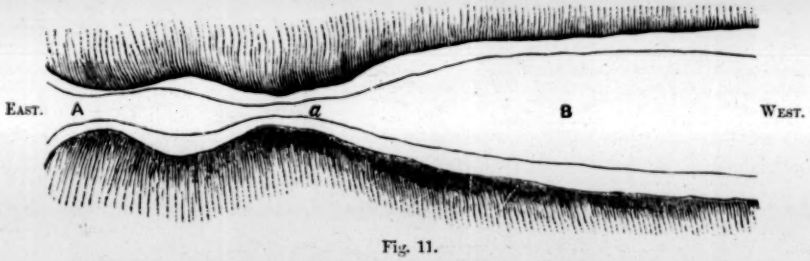


Fig. 11.

Ground plan to show the position of the Kilfinnin shelf, first discovered by Mr Darwin. B is the valley to which Mr Darwin supposed the shelf to be confined, terminating at the summit of the valley *a*. But the shelf passes eastward into the adjoining valley called Glen Buck, marked A.

detritus blocked the Glen Gluoy Lake, this detritus formed part of an accumulation which filled the Great Glen in this part of its course.

Then there is the shelf at the place called by Mr Darwin, *Kilfinnin*, at a height of about 1300 feet above the sea. "Mr Darwin states that, having observed this shelf from many points of view, he is prepared positively to assert that it is in every respect as characteristic as any in Glen Roy; and that its origin must be as carefully attended to in any general theory of the formation of the shelves" (p. 45). In company with the Rev. Mr Cameron, who is probably more than any one familiar with the character of the Glen Roy Parallel Roads, I twice examined this Kilfinnin shelf. It is in a high valley on the south side of the Caledonian Canal, opposite to Invergarry. The valley is called Laggan, and is roughly represented in fig. 11. It runs parallel with the Great Valley. The shelf is visible on both sides of Laggan Valley. But it does not terminate with the head of that valley at *a*, as Mr Darwin seems, from his brief description of it, to have supposed. The shelf passes over the summit level or col which divides Laggan Valley from the valley to the east called Glen Buck, as shown in the diagram.

At the summit level between the two valleys, the hills on each side approach one another. The shelf there is only about 20 feet above the dividing summit ridge, and the space there, between the shelves is from 230 to 250 yards.

On the hills here, as elsewhere, there is a large amount of drift, consisting of huge deposits of sand and rounded gravel. I saw it up to the highest level of the hill reached by me, viz., 1516 feet above the sea.

The shelves in this valley, as elsewhere, have been formed on the detritus. This detritus therefore had come anterior to the formation of the shelf.

I agree with Mr DARWIN that this Kilfinnin shelf is of the same character as the Lochaber shelves. The width of the shelf is not nearly so great as those in Glen Roy. The smallness of the lake would account for this difference. It has another feature of importance, the same as the Glen Roy shelves. Its slope



Fig. 12.

Plan (not to any scale) of part of Loch Ness, to show where deposits of detritus now exist.

down from the hill-side towards the centre of the valley is exceedingly slight—indeed, hardly perceptible.

On this account I am inclined to think that this shelf was made, not by the sea, as Mr DARWIN thought, but by a lake. A sea beach, owing to the action of the tides and waves, never can be so horizontal as the beach of a small lake.

But if a lake, where was the blockage both for Glen Laggan and for Glen Buck?

The most probable explanation seems to be, that *the Great Glen had here also, been filled by detritus* up to *a height exceeding* 1300 *feet.* Indeed, I cannot doubt that the whole of that great valley, from Fort William to Inverness, must have been, at the epoch now referred to, filled with detritus; and that the extensive gravel hills at the East end near Inverness (Tor Vane and Tomnahurich) are remnants of this detritus.

That this was really the case seems plain, from the numerous remnants of detritus which occur along both sides of the Caledonian Canal.

Fig. 12 represents a small portion of Loch Ness near Foyers. The parts marked 1, 2, 3, 4, 5,* represent accumulations of drift, which in some cases

* No. 1 is Ruiske. No. 2, Lein. No. 3, Urquhart. No. 4, Foyers. No. 5, Inverfarrigaig.

have a depth or thickness of hundreds of feet. These accumulations occur, wherever the range of the hills retires from the general axis of the valley, to produce a bay and a depression of surface. These hollows have been and are still filled with drift. Assuming that the whole of the "Great Glen" was so filled; the drift has subsequently been scoured out by powerful agents passing through it, leaving untouched the drift where sheltered in side bays and depressions.

In the ravine of the Inverfarrigaig river (about three miles to the east of Foyers Hotel) there is a very interesting cliff of clay, rudely stratified horizontally, and full of well rounded pebbles and boulders, with thin horizontal seams of sand. The scaur extends for about 250 yards. It is about 50 feet in height. It is 120 feet above Loch Ness. I cannot doubt that this bed of drift has been part of the general accumulation which filled up the Great Glen.*

These deposits, so long as they filled the Great Glen, would facilitate, in the districts adjoining, the formation of lakes, and ensure the retention of these lakes at high levels, because the rivers discharging from them would at first have small power to cut through the detritus. But with the lowering of the ocean level, the removal of detritus everywhere would increase with the steeper gradients of the rivers, and with the greater area on which rain would fall. Nowhere would the removal of detritus be so rapid as in the Great Glen, owing to the height and steepness of the mountains on each side. The removal of detritus would probably be greater at the west end of the Glen than at the east end, because of the greater height of the mountains at the west, and a larger supply there of rain. The reason why portions of the detritus remain at the east end is, that the hills there are lower in height, and more apart from the central line of the great valley.

On these grounds I think that when lakes existed in Glens Gluoy and Roy at their respective heights, exceeding 1100 feet above the sea, there was detritus

* In all the valleys adjoining or near the "Great Glen," there are still enormous deposits of drift, much of which evidently has been removed by rivers. Thus, in a valley called Flichity (about 12 miles S. W. of Inverness) drift covers the hill sides up to the height of 2000 feet; and in that valley there are traces of horizontal terraces, on the sides of the valley, more than 100 feet above the bottom, with a blockage of drift, at the lower or east end of the valley, which before it was cut through by the River Nairn must have been the means of causing the valley to be filled with water.

At Rosemarkie, a few miles N.E. of Inverness, which Mr JOLLY of Inverness took me to visit, there are magnificent sections of boulder clay, forming cliffs about 300 feet high. If the adjoining hills are, as they seemed to be, from their shape, composed of the same material, this deposit reaches to a height above the sea of 800 or 900 feet. How far down below the sea-level this clay deposit goes, it is impossible to tell, but at Fort George on the opposite side of the Firth, about three miles distant, a bore was sunk 100 feet in the same sort of clay, without reaching the bottom. Here then a clay deposit exists about 1000 feet in thickness. Judging from the beds of sand in the clay, and from the pebbles in it being in nearly horizontal lines, I had no doubt that this Rosemarkie clay bed was a water deposit.

exceeding that level, occupying not only the Great Glen, but also the entire district of Unachan towards Fort William.

6. Having in the foregoing remarks explained my reasons for thinking that the whole of this part of the Highlands was covered by drift, the next point to which I would advert is the facility with which this detrital matter can be and has been removed by natural streams and rivers.

As an example, I may refer to a deep gulley near the Old Catholic Burying Ground near Achleuereuch in Glen Spean, called Cillochrill. It is situated on a flat bit of ground, part of the old lake bottom when the water stood at the lowest shelf. This ground consists of an immense mass of gravel, which presents, towards the turnpike road, steep banks of from 300 to 350 feet in height. My guide, Angus M'Master, had been resident in the Glen since boyhood, and he pointed out to me the gully above referred to, which he stated had been formed within his own recollection. He said that it had originated from the plough having formed a "*furr*" on the flat land too near the edge of the bank. The rain scooped out a channel through the detritus and formed the gully. The banks of the gully are from 40 to 50 feet high, and the width in some places about 80 yards.

In Glen Roy there are numerous examples of burns having formed deep gashes through the original beds of clay, which had formed the bottoms of the old lakes. At the road on the east side of Bohuntine, which descends to Cranachan Bridge, there are vertical cliffs of clay, full of pebbles and boulders, 240 feet high, due to the erosion and undermining of the river Roy. It has even cut for itself a passage through the rocks lying beneath the boulder clay, in some places 60 to 80 feet in depth. This stream has enormous power.

Any one who walks along the banks of the River Spean from Spean Bridge to its mouth in Loch Lochy, will not fail to be struck by the height of the detrital cliffs lining its course.

The river channel is in some places at least 300 feet below the adjoining general surface, and there are terraces corresponding with the successive subsidences. The diagram, shown in fig. 13, is a view taken from the left bank of the River about a mile below Spean Bridge. The old Haugh-land next the river is about 35 feet above it, bounded by a detrital cliff about 20 feet high. Above this cliff there is a second haugh about 60 feet above the river, which is bounded by another detrital cliff about 20 feet high. As these banks and haughs slope with the river, there can be no doubt that it produced them. Even still, the old banks are so distinct as to show that the river Spean here ran along a channel more than 100 feet above its present level; so that the quantity of drift removed by it has been enormous.*

* Since this passage was written, I have again examined the Spean. The banks and cliffs referred to in the text, are situated on the east side of the river, about half a mile below Kilmonivaig church.

The *Spey* valley, from its summit at Lochan Spey down to Kingussie (beyond which I have not examined it), shows everywhere similar deposits of drift and similar denudation. Though I did not reach a higher level in my survey than 1161 feet above the sea, I saw that there was detritus on the hill sides at



Fig. 13. (See p. 616.)

River Spean, with old River Cliffs and Haughs, viewed from the left bank at a point 100 feet above the stream.

least 200 feet higher. The power of a river and its tributaries in scouring out a wide district of its detrital covering is well shown in this valley. All along the course of the Spey, there are numerous cliffs, at some distance from the river and approximatively parallel with it, indicating the high levels at which the river formerly ran, and the large amount of materials removed by it. At Garve Bridge, 977 feet above the sea, a tributary called the "*Oig*" joins the Spey. On the right bank of this tributary there are 3 or 4 terraces one above another (to a height of about 120 feet above the stream), being the different levels at which it has successively run.

At "*Gathbeg*" near *Laggan* Church in *Strathspey*, cliffs of detritus face the river, reaching to about 324 feet above it. Banks and terraces on each side of the river continue down to Kingussie.

These terraces show that at a former period the River Spey in its course between Laggan Church and Kingussie, had formerly flowed at least 200 feet higher than now, and must have scoured out the wide valley, at the bottom of which the town of Kingussie stands.

There are similar banks on the west side of the river. I could not make out at this second visit, very clearly, whether the base of the banks sloped with the river or was horizontal. If horizontal, they would of course indicate a lake, with a river running through it. In a higher part of the river, viz., above the Roman Catholic Chapel, there are old river cliffs, which show that the river had formerly ran 180 feet higher than it now runs at that place.

Mr JAMIESON of Ellon ("Lond. Geo. Soc. Journal," Vol. xiv. p. 526) has described a series of terraces extending on both sides of the Spey, in a lower part of its course, viz., at Ballindalloch, Aberlour, Rothes, and Cairnty, over a distance of about 25 miles. These terraces slope down with, and therefore have been made by, the river. He measured their heights very exactly at Rothes, and found them to be there 247 feet above the river. He has described the materials composing part of one of these terraces at Rothes in a cliff, thus—

Loose Sand and Gravel stratified	30 feet.
Stratified Sand and Mud	47 „
Gravel and Sand stratified	15 „
Unstratified Pebbly Clay	15 „
Stratified Sand and Mud	35 „
Base of Bank descending to River	50 „
	192 feet.

Mr JAMIESON has no hesitation in looking on these beds of detritus as having been deposited when the sea prevailed over the country. Since the retirement of the sea, they have been cut through by the river, so that the height of the cliff, at the foot of which the river runs, shows the enormous amount of detritus removed.

Keeping in view these proofs of the action of rivers in eroding and removing detritus, there should be little hesitation in allowing that the blockage of the Lochaber lakes may have yielded to the agencies of the streams which descended upon them from the high and steep hills surrounding the lakes.

7. Two objections have been taken to this theory of detrital blockage which it is right to notice.

(1) It is said by Mr Darwin, that if the subsidence of the lake in Glen Roy, from the highest level to the next, took place by the wearing down of any of the sides of the lake, this wearing down would be at the end of the lake where it was discharging its surplus waters, viz., at the head of Glen Roy; whereas the wearing down is supposed to have been at the opposite end of the lake. If Glen Roy was filled by a lake, which when it stood at the highest shelf, overflowed at its N.E. end into the valley of the Spey, it is contended that the erosion of a deeper channel for the overflow would be at that end where there was already a stream fit to effect the work.

This objection it is not difficult to answer. The erosion and lowering of the blockage would occur at that part of the lake banks where the materials composing these banks were most susceptible of erosion.

The *col* or summit ridge between Glen Roy and the Spey Valley being hard Granite *rock*, it was not so likely to be eroded, as the west end of the lake, if it consisted of detritus, which would be easily undermined, cut into, loosened and carried off by streams from the hill sides.

At the next level of the lake, viz., shelf 3, when its waters were discharging through Glen Glaster, there was also rock at that *col*. But if the blockage between Craig Dhu and Bohuntine for Shelf 3 was detritus, there would be no difficulty in the gradual erosion and ultimate removal of it, to allow another subsidence, viz., to Shelf 4.

What happened in Glen Roy, according to the view now submitted, is not without precedent. Lake Winnipeg, in North America, is about 300 miles long by 60 miles broad. It now overflows at its north end into Hudson's Bay, by the River Nelson. But from a recent survey it appears, that *it formerly discharged from its south end*, into the valley of the Mississipi.

This would be the case, if the waters of the lake stood about 360 feet above its present level, provided there was a *blockage* at the *north* end, reaching to that height above the lake. That the lake had stood at this higher level, is attested by the existence of four or five bluffs and terraces, on each side of the lake, one above another, up to a height of 360 feet above its present level. Some of these terraces run for long distances,—one for no less than 120 miles, forming "*a continuous and horizontal gravel road beautifully arched, about* 100 *feet broad.*"

These may in fact be called the Parallel Roads of Lake Winnipeg. The valley of the Minnesota, through which the lake at a former period discharged southwards, affords by its elevated and continuous bluffs on each side, unmistakable proof that a river, very much larger in size than the stream occupying that valley now, did flow through it from the north. The whole of this region of North America is covered by drift to a great extent and depth. Every lake in it is surrounded by beaches at high levels, all formed on drift deposits. But the peculiarity of Lake Winnipeg is, that whilst it stood at its higher level, it certainly overflowed at its south end, and as it sunk to its present level (which it accomplished by successive stages), it changed its overflow to the north end. Major WARREN, the most recent surveyor of the district, states, after an examination of the River Nelson by which the lake now discharges into Hudson's Bay, that this river "has every indication of being of recent origin," for reasons which he specifies. He adds that "the first material of the bed of the new outlet was probably *loose drift*, so that it was easily removed, and the outlet widened and deepened." From Major WARREN's report, it rather appears that the summit level between Winnipeg and the Minnesota is a ridge of *granite*,—another feature in which it resembles Glens Roy, Gluoy, and Glaster. He adverts to an opinion suggested by some geologists, that a glacier may have moved over the country from the Arctic regions to the lake on the north side, and compelled an overflow towards the south. But this theory he makes short work of, terming it "an unsupported hypothesis, and barren of

any fruit." ("Essay on the Physical Features of the Valley of the Minnesota," 1874. See also "American Journal of Science" for 1875, p. 313.)

2. Another remark to the disadvantage of the detrital theory is, that if there was any blockage of that nature, some remains of it should still be extant.

Dr CHAMBERS says,—

> "If the termination of any of the lines were the proper index to the termination of any of these alluvially confined lakes, we should expect to see some remnants of the blocking matter left at those places, say as hummocks on the sides of the vales with only broken passages between. But the lines vanish gradually on the smooth hill-sides, without any particular mark to distinguish the spots." (See "Margins," p. 113.)

This objection, at the best, is one only of probability. The argument is, that if there was any detrital blockage, fragments of it, in the forms of hummocks, might have been expected to be preserved. But there was little reason to expect, looking to the nature of the "blocking matter," that any of it would be preserved. When this "blocking matter" was undermined, scooped out, and washed away by streams, the probability is that the whole would be removed.

ROBERT CHAMBERS admits that in front of Bohina village (p. 123) "the bottom of the valley is filled to a great height with alluvial masses, which appear to some as diminishing the difficulty as to barriers."

I examined particularly the two places in Glen Roy where the blockages are supposed to have existed on the lake theory.

At the part of Glen Roy, where the highest shelf stops, there it is that the narrowest part of the valley occurs, and it is therefore a spot favourable for a blockage. On Bohuntine hill there is at that spot (I quote from my Notes) "a "cliff of drift which runs down the hill *transversally to the valley*, and may have "been a remnant of the barrier which crossed the valley here." On the south side of the Glen, immediately opposite to the above point, there are "some "detrital mounds on the hill side about 1022 feet above the sea, which may "have formed part of the same barrier." (Notes, vol. i., p. 23.)

Then at the place further west in Glen Roy "on the north side, where "Shelf 3 terminates, there is a large mass of detritus *standing out transversely* "to Glen." (Notes, vol. ii. p. 34.)

I give these quotations from my Notes, to show that there is not, as CHAMBERS and DARWIN state, a total absence of all appearance of detrital matter from the hill sides where the shelves stop. When first I saw these long mounds of drift I looked upon them as remnants of the blockage. But they may be only part of the lake bottom cut up by streams from the hill sides.

It is proper here to advert to the parts of Glen Roy, probably reached by the lakes at its two first subsidences.

It has been assumed hitherto that the uppermost lake at 1149 feet above the sea did not extend further west than Cranachan, because the highest shelf is

represented on the maps as stopping there. But on the occasion of my last visit to Glen Roy, Mr JOLLY and I thought we saw traces of the highest shelf on the north-west corner of Craig Dhu, which forms the south side of Glen Glaster.

If this discovery is confirmed, there need have been no blockage at Cranachan. The blockage must have been at two other places,—viz., first across Glen Glaster, and secondly across Glen Roy, between Bohuntine and Craig Dhu.

A blockage in Glen Glaster would be very easily accomplished, as the width there at the summit level is only one-fourth of the width of Glen Roy at Cranachan, and the lake there might not have been deeper than 81 feet. A very small barrier would be sufficient at that point.

A barrier across Glen Roy between Bohuntine and Craig Dhu would require no change when the lake subsided from Shelf 2 to Shelf 3, so that in that aspect of the case, the blockage is attended with few difficulties.

The objection to the detrital theory which has been most strongly urged, viz., the great height of the required barriers, assumes that *Glenroy and all the valleys of the district had then the same width and the same depth as now.* But if the valleys were still occupied by a large proportion of the detritus which originally filled them, that objection is entirely obviated.

When these Lochaber lakes existed, the sea may still have been from 500 to 600 feet higher than at present. There are in the district, between Fort William and Spean Bridge, a number of remarkable Escars and Flats, at various heights up to even 500 feet, which strongly suggest the prevalence of the sea. There are in like manner extensive plains to the south of Inverness, at about the same level, which have been apparently sea-bottoms. Even in the Great Glen itself, there are on its sides horizontal water-lines at various heights referable to the sea. Now, if when the Glen Roy lakes existed, the sea stood from 500 to 600 feet higher than at present, all the rivers in the country must have been running in channels much higher; and, therefore, at that time the valleys could not have been so deep or wide as they now are.

It is therefore a mistake to assume, that because Shelves 2 and 3 in Glen Roy are about 700 or 800 feet above the bottom of the valley, this affords any indication of what had been the depth of the valley when the lake existed, or of the height of the required barriers. The probability is, that the Glen Roy lake was not 100 feet deep.

Another remark of some weight arises out of this higher level of the sea. If, when the Glen Spean lake existed, having a height of 856 feet above the present level of the sea, the sea was then 600 or even only 400 feet above its present level, the sea must have been close to the detritus which I suppose to have formed a blockage of the lake between Teindrish and Aonach More. This

blockage would thus be subject to the eroding action of the sea, as well as to that of streams issuing from the adjoining hills.

IV. Having in the foregoing part of this memoir explained my own theory, and noticed some of the objections to it, I refer next to the other solutions of the problem which have been suggested.

Darwin's Marine Theory.

1. From what I have said regarding the prevalence of the sea over the highland hills, and the deposit of detritus, by means of sea currents, it might be supposed that I could scarcely refuse assent to Mr DARWIN's views. But whilst I admit that there are traces of the sea at great heights on our coasts and mountains, it is not inconsistent to hold that whilst the sea was subsiding, lakes would be formed in favourable districts.

In the first place, let me state the grounds on which, as it seems to me, the Parallel Roads could not have been sea-beaches.

The principal difficulty which DARWIN and CHAMBERS had to contend with is the circumstance, that in each Glen there is proved to have been an old *river course*, by which *the waters of the lake flowed out to a lower level.*

When Mr DARWIN wrote his memoir, these old river courses had not been discovered. All that he knew was, that the shelves in each of the glens coincided with a col or summit level which separated the glen from some adjoining glen. Mr DARWIN stated, that in the Pacific Ocean, there were many cases where arms of the sea were divided or separated by a spit of sand, which was thrown up by tides or storms. He assumed that the highest shelf in Glen Roy was on the same level as the highest in Glen Gluoy, and that the sea may have stood at the same level in Strathspey. Mr DARWIN, when he made his survey, laboured under two great disadvantages. He had no spirit-level with him, and he did not visit the Pass of Mukkoul, where there is the unmistakable river channel, by which Loch Laggan discharged itself into Strath Spey.

ROBERT CHAMBERS was not under these disadvantages. He and I had each a spirit-level, as well as a barometer and a sympysometer, by means of which we satisfied ourselves that the highest shelf in Glen Roy was at least 12 feet below the highest shelf in Glen Gluoy. In fact, he was the first to detect the appearance of an old river course between these two glens. In his "Ancient Sea Margins," he alludes to the joint survey which he and I made, and also to the Memoir read by me in this Society, and which appeared in our Transactions before his book was published. He takes no notice of the facts stated in my Memoir regarding these old river courses, though so conclusive against the marine theory which he had adopted. He, however, oddly enough makes the following admission regarding one of the lakes:—"The Pass of Mukkul," he says, "the

supposed outlet of the lake of No. 4 shelf, presents the *appearance of an ancient river course*, a flat passage of seventy feet wide, confined by wall-like rocks, *which seem water worn*" (p. 112), and this statement is actually accompanied by a woodcut representing this *ancient river course*.

No doubt he only says that this Pass presents the *appearance* of an ancient river course. But he does not state how the appearance could be explained on any other principle than as the outlet of a lake.

Professor NICOL, as I have already said, expresses an opinion that at none of the cols is "there the slightest trace of an ancient river," adding, that "there are indications of the *passage of water*, as through a narrow sea strait."

But the substitution of "a narrow sea strait" for a "river" is impossible, if the levelling by the Ordnance Surveyors is to be trusted, proving that the water in Glen Gluoy stood twelve feet higher than the water in Glen Roy, these two glens being connected by the passage referred to.

2. Another difficulty with which Mr DARWIN and his followers have to contend is suggested by the flatness of the ground which forms the shelves.

On the sea-shore, where the tide rises and falls, the materials which collect on the beach undergo an action impossible in a lake. The beach must slope more or less from high water towards low water mark. But in the Lochaber shelves there is hardly any slope. ROBERT CHAMBERS in his book (p. 98), gives a woodcut, which shows the flatness of the shelves. (See also fig. 17, p. 51 hereof.)

3. There is another feature which distinguishes the "Parallel Roads" from sea-beaches. There are ancient sea-beaches along our coasts at eleven, twenty-five, and forty feet above the sea, not to speak of others at higher levels. These old "Sea Margins," as CHAMBERS called them, even the lowest and most recent, are *faintly* impressed on the land. But if the Parallel Roads are sea beaches, how much more ancient must *they* be, if height above the sea is any measure of antiquity; and therefore, how *much less distinct and less continuous ought they to be?* Now, what is the fact? Undoubtedly, these "Parallel Roads," at heights exceeding 1000 feet above the sea, are *much more distinct, continuous,* and *perfect, than any ancient sea-margins,* even the lowest and most recent, which can any where be pointed out.

4. But whilst I cannot accept Mr DARWIN'S explanation of the Parallel Roads, I am free to admit that there are other markings on the hills of Lochaber which may possibly be ascribed to the sea.

ROBERT CHAMBERS in his book was the first to indicate certain lines more or less horizontal on Ben Chlinaig, and also on Craig Dhu. He thought that some of those corresponded in level on the opposite sides of the valley, and on this ground he pronounced them to be "ancient sea-levels."

That such lines exist, I can vouch for, having viewed them from various distant points, and having measured the heights of several.

I requested Sir HENRY JAMES to authorise his surveyors to mark on the Ordnance Maps such of these lines as they could recognize as resembling the Parallel Roads. Accordingly, I observe that in the last edition of the Glen Spean six-inch scale map, one of these lines, 1306 feet above the sea, has been represented on "Craig Dhu." It runs continuously for more than half a mile. There is another on the east side of Glen Roy, opposite to the side valley called the "Gap," and Ben Erin, at a height of above 1200 feet.

CHAMBERS, in his book, specifies other lines at 1337 and 1495, which he says "are so bold, that I cannot but wonder at their not hitherto having attracted any special attention."

Mr JOLLY, of Inverness, has within the last three years, not only verified these observations of CHAMBERS', but has discovered on Ben Chlinaig lines of a similar character, exceeding 1700 feet above the sea.

The height of the shelf represented by the Ordnance Surveyors, as occurring on "Craig Dhu," at 1306 feet, does not agree with the heights given by ROBERT CHAMBERS or by Mr JOLLY for any corresponding line on Ben Chlinaig. Therefore, it cannot at present be assumed that these lines on the two sides of the valley, where opposite to one another, correspond in level. CHAMBERS' chief reason for pronouncing them to be sea-beaches, has therefore not yet been verified.

I may mention that there is also a line on Ben Erin (situated between Glen Roy and Glen Gluoy), which my brother the Admiral, who accompanied Mr JOLLY and me on a visit three years ago, was the first to notice. This I judged to be about 1877 feet above the sea, when viewing it as I did repeatedly from Craig Dhu and Ben Chlinaig.

With regard to the character and origin of the lines on Ben Chlinaig, Craig Dhu, and Ben Erin, now referred to, I am unable to give an opinion. They may have been formed by the sea as CHAMBERS supposed; but, whether they were beaches or submarine banks, is a question depending for its solution on further investigation. I regret that more of these anomalous lines, if examined by the Ordnance Surveyors, have not been represented on their maps. Their position on both sides of Glen Spean, and nearly opposite to one another, gives to them peculiar interest in reference to the way in which they could be formed, when the glen was a sea passage.

One thing seems tolerably certain, that these lines are quite different in character from the "*Parallel Roads.*" On that point Messrs JAMIESON and JOLLY have expressed an opinion in which I quite concur, though I do not concur in considering them to be the moraines of glaciers.*

* Since this paper was read, I have had an opportunity of walking along the line, shown on the 6-inch Ordnance Map at a height of 1306 feet on Craig Dhu. It extends for about half a mile on the part of the valley above Achleureuch and Mullaggan. It certainly has none of the characters of a

Besides the lines to which I have adverted on Ben Chlinaig, Craig Dhu, and Ben Erin, and which may very probably be ascribed to sea agency, there are other appearances in the district which may have a similar origin. I allude to the extensive terraces and cliffs at lower levels, which certainly imply the presence and prevalence of large bodies of water, marine or lacustrine.

Thus at and above the Roman Catholic Chapel in Glen Spean, there are terraces, apparently horizontal, at 717 feet, 558 feet, 458 feet, and 409 feet above the sea. On the hills between Spean River and Fort William, there are flats, as ROBERT CHAMBERS points out (p. 94), at a height of about 391 feet above the sea, consisting of gravel and sand, in stratified beds, which strongly betoken sea action. But these flats are manifestly quite different in character from the true Parallel Roads, and (in my view) afford not the slightest argument in favour of the opinion that these "*Roads*" are marine.

I turn now to the

Ice Barrier Theory.

This theory is the one which has received the greatest number of supporters. First suggested by AGASSIZ, it has been accepted by Mr JAMES THOMPSON of Glasgow, Sir CHARLES LYELL, Mr JAMIESON of Ellon, and Mr JOLLY of Inverness.

1. Mr JAMIESON's paper is published at length in the "Journal of the London Geological Society" for 21st January 1863. Mr JOLLY's paper was read before the Edinburgh Geological Society on 17th April 1873, and an abstract of it is given in that Society's "Transactions." I had the pleasure of hearing the entire paper read, and I have since been kindly allowed an opportunity of reading and studying it.

Both of these gentlemen suppose that glaciers were formed in three several places, viz.,—(1.) In the hills situated at the head of Loch Arkaig. (2.) On the north-east flank of Ben Nevis; and (3.) In the hollow between the hills now occupied by Loch Treig.

With reference to the barrier which crossed the Unichan district, between the lowest shelf at Teindrish and the corresponding shelf at Corrychoilzie, Mr JAMIESON makes the following statement;—

"The extension of the lake was cut off here by a glacier issuing from Corry M'Eoin, and on the opposite side by the ice of Glen Arkaig, and the Great Glen, flowing over to near Teandrish. This would close up the mouths of Glen Gluoy and Glen Spean, and so long as the ice exceeded sufficiently the height of the water-sheds at the top of the glens, these cols would determine the level of the water in the lakes (p. 247).

"But in Glen Roy there are three lines;—and this barrier across the mouth of Glen Spean,

Parallel Road, but several features which indicate sea action. There is an accumulation of *debris* from the rocks above, forming a sort of terrace. The terrace is not horizontal towards the valley. It slopes rapidly from the hill.

although it might serve for the lowest, leaves the two higher ones unaccounted for. In order that a lake in Glen Roy could exist at the height of the upper lines, something must have prevented the waters escaping by Mukkoul, and also by the Glen Glaster col.

"In order to explain this, we must go to Loch Treig. Let us suppose a glacier issuing from the mouth of Glen Treig, and let it protrude *across* Loch Spean until it rests on the hills upon the north side of that valley.* This would cut off all outlet to the eastward, both *by Glen Glaster† and Mukkoul*, and so long as the icy barriers maintained a sufficient height, the water filling Glen Roy would have to escape by the col at the top of that glen into the head of Strathspey.

"Now let the Glen Treig glacier shrink a little. This would open the Glen Glaster col, and let out all the water above its level.‡

"Then let the Glen Treig glacier shrink again, until it withdrew out of Glen Spean. That valley being now clear, the water would escape by the outlet at Mukkoul, which would then determine the level of the lake and keep it at the lowest line, so long as the ice stream across the mouth of Glen Spean maintained itself of sufficient height.

"Grant then these two ice streams, one in the Great Caledonian Valley, and the other in Glen Treig, and the problem of the Parallel Roads can be solved, provided we allow that glaciers have the power to dam such deep bodies of water as must have occupied Glen Gluoy and Glen Roy."

Mr JAMIESON explains how he supposes the different ice streams moved. Thus, with regard to the Arkaig glacier, he says that he found on the rocky

"Ridge between Loch Lochy and the Spean, such markings as might be made by ice mounting over it from Loch Lochy. The rocks were bared, where the presence of a glacier wheeling round from the Great Caledonian Valley into the mouth of Glen Spean, would have operated most strongly" (p. 246).

Mr JAMIESON, at the conclusion of his paper, makes the following candid admissions:—

"The greatest difficulty that I find in supposing the parallel roads to have been formed by glacier dammed lakes, arises from *a consideration of the depth of water the ice had to retain*; for it is evident that the moraines were too insignificant to have done much of the duty. One might think that by the hydrostatic pressure of a column of water some hundreds of feet high (the water) would have found an *escape beneath the ice*. If, however, the height or thickness of the glacier were sufficiently in excess of the depth of the water, I imagine there would be pressure enough to keep it in" (p. 257).

Mr JOLLY adopts Mr JAMIESON's views regarding the existence of a glacier in each of the valleys of Arkaig, Corry M'Eoin, and Treig. He, however, suggests an additional glacier from the valley of the Laire (adjoining Loch Treig on the west) which filling Spean valley might flow down towards the west.

"If (says he) a glacier issued from Corry M'Eoin, and added its tributary ice to the Spean

* If Loch Spean existed at this time, of which there can be no doubt, it is difficult to understand how a glacier could protrude across it, in the very middle of the Loch.

† To do this, the glacier must have filled Glen Glaster, and crossed the valley of the Roy.

‡ The level here referred to is that of Shelf 3. If the ice shrunk back from Glen Glaster, there would be no ice barrier across Glen Roy for Shelf 3, which extends down that Glen, beyond the mouth of Glen Glaster.

glacier, the ice would likely push itself across the Spean into the mouth of Glen Roy, and up Glen Collarig" (MSS. p. 25).

The existence of an ice stream which passed from Loch Treig through Glen Glaster, Mr JOLLY thinks is proved by the existence in that glen of what he considers to be moraines. These, he says:—

"Prove the presence of ice at the head of Glen Glaster to dam back the water of the middle shelf in Glen Roy. They also point to an extension of the ice in that glen above the middle shelf to form in conjunction with the glacier extensions from the glens west of Ben Chlinaig, the required dam for the highest shelf in Glen Roy at Cranachan, where that shelf suddenly ends on both sides of the glen" (MSS. p. 29).

2. Having thus explained in what manner the ice dams are supposed to have been formed to suit the different glens and levels, and from what sources the ice came to make these dams, I proceed to notice the difficulties which this theory has to contend with.

(1.) Is it likely that in this Lochaber district, some of the glens should have been filled with solid ice and others with water?

The temperature must have been much the same in all the glens. If there was any difference, one might have supposed that in the glens furthest from the sea, in winter at least, there should have been more cold; and yet Glens Gluoy, Roy, and Spean, which were occupied by lakes, are farther from the sea than Corry M'Eoin and Loch Arkaig, which are supposed to have supplied great glaciers.

There is only one case in the known world of a lake among mountains the waters of which were dammed up by a glacier. The Marjelen See in Switzerland, is a small lake occasionally dammed up by the Aletch glacier, as it flows past one end of the lake. This lake is at a level of 7000 feet above the sea; and the glacier comes from hills having a height of 12,000 feet above the sea. In this case, it is therefore intelligible how there should be a lake at the one place, and a glacier at the other. But in the Lochaber district, there is no such difference of altitude to cause a greatly lower temperature in the glens where Messrs JAMIESON and JOLLY say that glaciers were formed.

Indeed, some of the hills adjoining Glens Gluoy, Roy, and Spean, are quite as high as the hills from which the glaciers are supposed to have come.

(2.) Another difficulty is suggested by the levels of the country.

The barrier for the highest shelf in Glen Gluoy must have been at a height of 1170 feet above the sea. But the valley of Arkaig is only about 200 feet above the sea, and if a glacier emerged from its mouth, it would naturally flow westward towards the sea, instead of climbing 900 feet up a steep hill on the opposite side of the valley.*

The highest shelf in Glen Roy being 1150 feet above the sea, the Treig

* There is a very instructive paper "On the Parallel Roads," by Mr JAMES BRYCE, junior, F.G.S.

glacier would have a less rise to make, because Loch Treig stands at a level of 750 feet above the sea. But its course towards Cranachan, in Glen Roy, would be so obstructed, as to render it impossible that it should ever have reached Glen Roy at the place required, in the condition of a solid body of ice. *First*, it would have to descend into Glen Spean, (which must there have been filled by a lake flowing out at Mukkoul) which is about 200 feet below Loch Treig, and then it would have to climb up the ridge separating Glen Spean from Glen Glaster, requiring a rise of 500 feet, in which case it would have risen above its own fountain head to the extent of no less than 400 feet. After getting over this ridge it would, before it could reach Glen Glaster, have to *wheel round* the S.E. corner of Craig Choinichte, and on passing through Glen Glaster make another bend before it reached Glen Roy. Even if, on reaching Glen Roy, it struck against Bohuntine Hill, and did not flow down Glen Roy, is it possible to suppose that the ice, on reaching Glen Roy, could have formed *a solid body* sufficiently strong to resist "the hydrostatic pressure of a column of water some hundreds of feet high?" This difficulty, suggested by Mr JAMIESON himself, seems to me insurmountable.

(3.) There is a further difficulty. The lake of Glen Roy, at the middle shelf, discharged through Glen Glaster; but if the Treig Glacier came through Glen Glaster to block Glen Roy for Shelf 3, as suggested by Messrs JAMIESON and JOLLY, any overflow of the lake through that glen would be impossible.

(4.) Mr JOLLY thinks that the barrier in Glen Collarig for Shelves 2 and 3 might have been made by the glacier from Corry N'Eoin pushing across the Spean into the mouth of Glen Roy and up into Glen Collarig. To say nothing of the very tortuous course which the ice would have to take before it could reach the middle of Glen Collarig, it is enough to observe that a glacier coming out of Corry N'Eoin must, in virtue of the existing levels of the country, have flowed in an exactly opposite direction. The only use which Mr JAMIESON made of the Corry N'Eoin Glacier was to find a dam for the lake of Shelf 4, which extended down to Unichan. He supposed that this glacier extended across the Unichan Moor in a strait line to Teandrish. But as this barrier required to be four or five miles in length, it is hardly possible to suppose that this Corry N'Eoin glacier could have formed a solid barrier along its whole course. Every winter and every summer it must have changed its position and its internal structure, in such a way as to allow the water to escape through or under it.

(5.) Reference has been made to the Marjelen See. This lake, as I have just observed, is very small in size. Sir CHARLES LYELL, who visited it in the year 1865, says it was only two miles in circumference. Professor FAVRE, of

(published by the Philosophical Society of Glasgow in 1850). Mr BRYCE shows from the geographical position of Loch Arkaig that even supposing a glacier to have been formed in that valley, any moraines which it might have produced, ould not have been so situated as to block Glen Gluoy. He does not think it necessary even to entertain the question of a blockage by the glacier itself.

Geneva, informs me by letter that its greatest length is 1400 yards; its width 430 yards; and depth, next to the glacier, 114 feet.

The Aletch Glacier, in a solid mass, flows past one end of the lake, and presents a wall of from thirty to forty feet above the lake, when it stands at its normal height, forming a solid mass of ice, therefore, of about 114 + 40 = 154 feet altogether. The normal height of the lake is determined by its discharge over a col, at the opposite or east end, into the Viesch Valley.

But though this great Aletch Glacier flows past one end of the lake in a solid mass, does it succeed in damming the waters permanently? Berlepsh, in his "Guide to the Swiss Lakes and Glaciers," describes the Marjelen See particularly. He says (page 484):—"*Tous les ans, en juillet ou en août*, le lac se frage un chemin, sous le glacier d'Aletch, et se verse par le Massa près de Naters sur le Rhone."

The case of the Marjelen See, therefore, presents features which entirely distinguish it from Glen Roy. If even the solid body of the great Aletch Glacier is not sufficient to prevent the water escaping under it, how still more improbable would it be in the case of a dam formed by the tongue of a small glacier formed in a small Lochaber glen? If the solid body of the Aletch Glacier is unable to dam permanently a lake only 114 feet deep and 430 yards wide, how is it possible to suppose that merely the tongue of a glacier from the Treig Valley—a tongue wending its way across valleys and round hills for five or six miles, could dam a lake such as that which filled Glen Roy?

It seems really almost impossible to believe that a glacier formed in any of these Lochaber corries should have produced tongues of ice more than twice as long as the glaciers themselves, and that these tongues should, at their tips, have formed permanent ice barriers.

Mr Jamieson brings together, to form his ice barrier for Shelf 4, no less than three tongues,—one from Corry M'Eoin, one from Arkaig, and one from the Great Glen. The meeting of these three ice tongues, coming from opposite directions, would be a very remarkable conjunction indeed, if it had the effect of producing a solid barrier of ice!

(6.) All the writers who adopt the ice dam theory, feel the necessity of having a glacier from Loch Treig to supply a barrier for the lake in Glen Roy. But what if it should turn out that the Treig Valley, instead of being filled by a glacier at this time, was filled by water? Mr Jamieson says, that had there been any of the "Parallel Roads" in Corry M'Eoin, from which the required glacier was supposed to come, he would have considered it a good objection to Agassiz's ice theory (p. 246, "L. Geol. S. Journal" for 1863).

Now, there is a Parallel Road, or what is equivalent to it, in Loch Treig.

Sir Thomas Dick Lauder says, in his Memoir (p. 43), "Edin. R. S. Trans." vol. ix., year 1823), that the lowest shelf, called by him No. 4, as it approaches

the "River Treig, *sweeps round* in the direction of the mouth of the glen and *lake* of that name. It enters the jaws of the pass *into Loch Treig*, in the manner represented in plate vi."

AGASSIZ and CHAMBERS also attest the existence of a shelf in Loch Treig.

Last October, in company with the Rev. Mr CAMERON of Kilmonivaig, I walked along the south bank of Loch Treig for about two miles from its mouth.

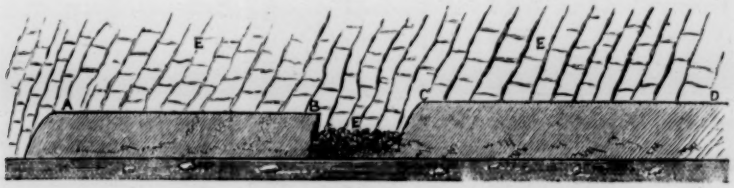


Fig. 14.
Section of north bank of River Treig, to show two terraces or shelves, CD being supposed to be Shelf 4, AB being a separate shelf or water-line, from 25 to 30 feet lower.

We made the following discoveries:—

The lowest shelf, viz., No. 4, we thought was recognizable at one place on the east bank of the loch—two miles from the north end. It was formed on

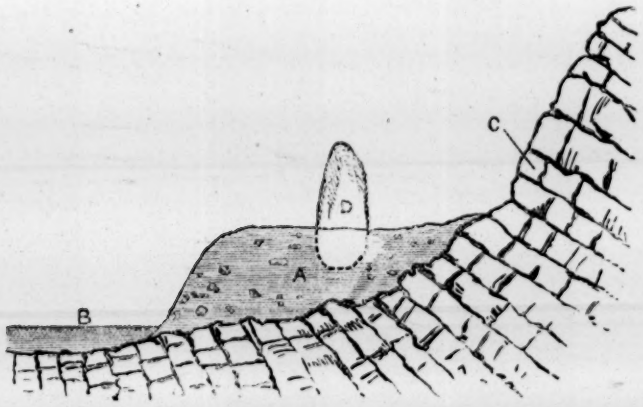


Fig. 15, (pp. 630 and 641)
B is Loch Treig. C are rocks on the south bank, about half a mile from north or lower end of Loch. A is a bed of stratified white sand, much frequented by rabbits. Its surface is about 40 or 50 feet above the lake. A Boulder 5 feet high and 2½ wide stands partly buried in the sand.

a bed of detritus. This, however, was only an inference from the fact of there being a flat bit of detritus at the same height above the lake as Shelf 4,—viz., 91 feet. In looking across the lake we observed a similar flat at a corresponding level on the opposite bank.

At the entrance of Loch Treig I found what is shown on figure 14, where C D is Shelf 4. It stops about 100 yards from the entrance to the

loch. About twenty-five or thirty feet below the level of that shelf, there is another shelf, A B, which is situated between the termination of Shelf 4 and the lake, and of which we found traces on both banks of Loch Treig itself. This lower shelf is indicated by isolated flats consisting chiefly of a sandy mud, from 40 to 50 feet above the level of the lake. These flats have, no doubt, been originally continuous, and have been cut through, and somewhat reduced in level by the action of rain and of streams from the steep hills which surround the lake. One of these flats, consisting of a white sand horizontally stratified, full of rabbits, with a remarkable boulder sticking in it, is indicated by fig. 15.

Between these two shelves, viz., between B and C, rock is visible in the channel of the river and close to its brink, covered by drift E. This lower terrace occurs on both sides of the river, as shown on fig. 16.

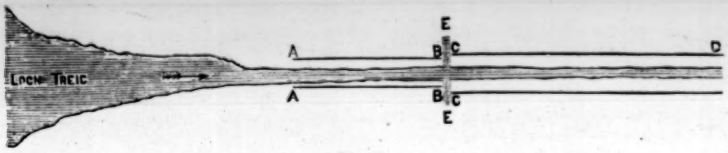


Fig 16.

Ground plan of River Treig, near Loch Treig, showing position of two horizontal water-lines or shelves on both sides of the River. CD is Shelf 4, AB is a water-line from 25 to 30 feet lower in level.

I observe that Mr JAMIESON states ("Lond. Geol. Journ.," vol. xix., p. 250), that at the mouth of Glen Treig he discovered a terrace thirty feet above No. 4 shelf.

There is thus a good deal of more precise observation required to clear up these Loch Treig terraces. But one thing is very clear. So far from there having been a glacier in Loch Treig *when Shelf* 4 *was forming, Loch Treig must*, since there are horizontal water terraces at its mouth, up to 90 feet above its present level, *have been full of water;* and if it was full of water when Glen Roy lake stood at No. 4 shelf, why should it be supposed that it was not full of water when the same lake stood at higher levels?

The probability is, that the mouth of Loch Treig at that time was blocked with gravel, but yet not to such an extent as to prevent the waters of Shelf 4 entering it, and that when the Glen Roy lake fell from Shelf 4, a barrier of some kind, rock or detritus, existed at or near the mouth of Loch Treig, which for some time kept its waters up to the height of shelf A B, *i.e.*, about 40 or 50 feet above the present level of the lake.

I referred to a remark of Mr JAMIESON's, that if any of the Parallel Roads had been found in Corry N'Eoin, he would have considered it a good objection to the theory of AGASSIZ, which required a glacier to come from Corry N'Eoin to provide a barrier at Unichan for Shelf 4.

I examined the mouth of this Corry very carefully. I had not time

to go into the Corry itself; but on each side of its mouth I saw large knolls of detritus. Now, is it possible to suppose that these knolls and hummocks would not have been obliterated by any glacier large enough to protrude a tongue five miles long across Unichan to Teandrish? The fact, therefore, that one of the parallel roads comes close to Corry N'Eoin, and that these knolls and hummocks of gravel are standing at its entrance, suggest to the theory of AGASSIZ an objection quite as strong as if there had been a Parallel Road inside of the Corry.

(7.) The chief proof on which Mr JAMIESON and Mr JOLLY rely for supposing the occupation of Loch Treig by a glacier, is the existence of long curved banks of gravel and boulders in the Valley of the Spean, which they consider to be indubitable moraines. Mr JAMIESON says, in respect of these gravel banks, that ("Lond. Geol. Soc. Journal," 21st Jany. 1863, p. 247): "The evidence for the Glen Treig glacier is probably more complete than for any other glacier in the kingdom."

I confess that when I first saw these banks of gravel and boulders, viewing them as I could then only do from a considerable distance, I adopted the idea, and expressed it to Mr JOLLY, who was then with me, that they were moraines, and probably formed by a glacier in Loch Treig.

I have now changed my opinion on this point, in consequence of several more minute surveys of the ground.

The gravel banks alluded to are situated on both sides of the Spean Valley, a little to the east of Glen Treig.

The plan (on Plate XLIII.), copied from the Ordnance Survey Map, shows these banks in red colour.

In the first place, it will be observed that portions of Shelf 4 have been formed upon these banks. The banks, therefore, must have existed before the shelf was formed. As this fact is of importance, I may observe that, three years ago, I satisfied myself that it was so; and the Ordnance Surveyors, judging only by the appearances on the ground, came to the same conclusion. The shelf is, no doubt, much broken in continuity, owing to the rough character of the materials on which it has been impressed, and to the number of streams which have cut through the shelf since its lake was drained off. But the fragments of the shelf impressed on the banks are sufficiently numerous to leave no doubt that these banks must have existed previously to the formation of the shelf.

Mr JAMIESON admits that Shelf 4 *has been formed on these moraines*, as he terms them. Referring to "the *terminal moraines* of Glen Treig," he says that "at Inverlaire and around the large rocky knoll called Tom-na-Fersit," they have "*been finely terraced by the action of the water, when at the level of the lowest Glen Roy line*" ("L. Geol. Soc. Proc.," vol. xix. p. 249.

Therefore, if these banks are the moraines of a glacier from Loch Treig, *that glacier could not have existed contemporaneously with the lake of Shelf* 4. *The lake must have been formed after these so-called moraines were formed, and after the glacier which produced them ceased to exist.*

The idea that these banks were moraines was suggested, probably, by their curved forms. When glaciers protrude from a valley, they often carry rubbish and boulders on their surface, which, on reaching the lowest end of the glacier, are projected over its surface as the glacier melts; and should the glacier at some future period push forward again, they are, by the pressure of the glacier, formed into a bank, which has somewhat of a crescent or horse-shoe form, the concave side being, of course, towards the glacier.

In examining the Ordnance Map, it will be seen that there are some very remarkable curved lines dotted over with boulders, but that only a few of the curves face Loch Treig.

If any inference is to be drawn from the aspects of the lines, they point rather to some agency which has come up Glen Spean Valley. Now, there is strong evidence to show that an agent of some kind did come up this valley, and on its way up acted with tremendous power. The hard rocks on both sides of the valley, where the valley is narrowest—viz., near the Roman Catholic Chapel and at Murlaggan—have been ground down, smoothed, and polished. These are well seen, at five or six places between the chapel and the junction of the high roads to Laggan and Loch Treig.

Most of the surfaces of rock so polished, slope *down*, and, as it were, *face down* the valley, and the striæ upon them run W.N.W. and E.S.E.—being a direction parallel with Glean Spean valley. There are very few smooth surfaces of rock which face or look *up* the valley, and *these few are not striated.*

Near Achleureuch Post-Office a clay slate rock, sloping down the valley, presents very large smoothed surfaces. It so happens that the rock there contains many nodules of white quartz, which stand up above the softer rock, having better withstood the grinding. These quartz nodules have been beautifully polished on their sides or surfaces *which face down the valley*, indicating clearly that some body or bodies passed up the valley, pressing severely its sides, and grinding, smoothing, and striating the rocks.

Other localities may be mentioned in Glen Spean. Thus, near Inverlair Bridge there are several rocks with striæ on them, running N.N.W. At the junction of the Inverlair and Loch Laggan roads, there is a rock showing striæ W.N.W.

These directions are quite at variance with the notion of an outflow from Loch Treig, and indicate a movement *through, parallel with, and up* Glen Spean valley.

As the facts now referred to are important in their bearings on this

question, I may refer, in confirmation of the foregoing statements, to the testimony of Professor NICOL of Aberdeen, and the more so that he does not agree with me in my way of solving the problem. He says—

"The detritus has *not* come *down* Loch Treig, as might at first sight be imagined, but from the Corry Laire, to the west; and has then been *swept eastward* by tidal currents, and *even up into Loch Treig*, on which it abuts, with a bold, almost vertical end. In many other places in this part of Glen Spean, there is similar evidence of a *current from the west flowing up the valley*."—("Lond. Geol. Socy." 21st May 1869, p. 288.)

Higher up Glen Spean (as at Carbeg and Moy) there are several places where smoothed rock surfaces occur, sloping towards the west, with numerous parallel striæ, running at one place W.½N.; at another, W. by S.

There is, on the north side of the road, a mass of hard granite rock, rough on its east side, ground down and beautifully smoothed on its west side.

At the Rough Burn there is a rock on the north side of the road, with a smoothed surface, dipping W.N.W., at an angle of 22°. Striæ run obliquely across its face from W. by S.

On the hill north-east of Rough Burn, 1600 feet above the sea, the rocks are smoothed on north-west sides, and are rough on opposite sides. There are quartz nodules in the rocks about 8 inches square, which are smoothed only on their west sides.

All these facts tend to show a movement up Glen Spean, and not down from Loch Treig.

Mr JAMIESON, whilst he thinks he saw evidence that ice had passed down the Spean valley in its lower part, admits that in the upper part of Glen Spean Valley it is

"The *west* side of the rocky masses that has sustained most abrasion. Far away, even to the end of Loch Laggan, I traced the same appearances. Beyond the Pass of Makoul the low rocky eminences show evident traces of the passage of ice, *going out towards the valley of the Spey*."—("Lond. Geol. Socy. Journal," 26th February 1862, p. 173.)

The places before referred to of rocks ground and striated in Glen Spean, are along the whole course of the valley near the bottom. But higher up on its sides similar appearances exist.

Thus Craig Dhu, on the north side of Glen Spean, near the foot of the valley, reaches to a height of 2161 feet above the sea. On the *west* and *south* aspects of the hill, I found the rocks bared and smoothed to a height of about 1800 feet. The rocks on *north* aspects of the hill were rough. At two places, striæ were observed by me, and their direction in both cases was W. by N. At the part of the hill next Glen Glaster, the rocks were smooth on the *west* side; rough on the *east*. There was a large surface of rock, dipping at

an angle of 50° degrees towards west. Striæ were on this rock, and *these striæ* were *horizontal*. Within a few yards of the top of Craig Dhu, the edges of the strata running north and south were all ground down on the *west* sides, and remained rough on their east sides.

Ben Chlinaig is opposite to Craig Dhu, being on the south side of Glen Spean. Mr JOLLY found, on the side facing the valley, striated rocks at a height of 1750 feet above the sea, the striæ running S.E. and N.W., or parallel with the valley at this place.

About two miles further south, in the Laire valley, at a height of 1600 feet above the sea, Mr JOLLY found, on a rock surface, striæ 50° W. of north.

Mr JAMIESON examined the rocks at the outlet of Loch Treig, and found the rocks smoothed at a height of 1000 feet above the Loch, and 1740 feet above the sea.

"One bare flat surface of gneiss, thirty yards long, beautifully smoothed, and covered with parallel scratches, scores, and flutings, running straight from end to end." Elsewhere,—"*Striæ running horizontally along the faces of the rock*, were traced up to 2000 feet. Not that I affirm even this to be their upper limit." He mentions other appearances, even as high as 3055 feet above the sea, which "raise a suspicion, that some denuding agent has flowed over it at a period geologically recent."—("Lond. Geol. Socy. Journal," 26th February 1862, p. 172.)

Unfortunately Mr JAMIESON does not mention the direction of the parallel scratches and flutings described by him. But it is not unimportant to observe his statement, that "along the faces of the rocks" they were "running horizontally."

In connection with Loch Treig, I may mention that in company with Mr JOLLY and my brother, I walked from its mouth due east along the side of the hill, at a height of about 1600 feet above the sea, and at a distance of about a mile, we came on a large mass of rocks, projecting from the hill side. These rocks, presenting high vertical faces, at once arrested our attention. On their *west* sides they were rounded and smoothed, as if by some heavy body or bodies which had rubbed and pressed against them. On their *east* sides they were rough, as if these sides had been protected, *i.e.* not subjected to the action of the rubbing agent. At the foot of these rocks there was an enormous accumulation of large boulders. The agent, whatever it was, which had been rubbing the rocks, seemed to have brought the boulders and dropped them there. These rocks are at a height of about 1500 feet above the level of the sea, as shown by the O.S. contour lines.

The foregoing remarks apply to markings on the *rocks* in Spean Valley. But it is also proper to refer to the *banks of gravel and boulders* on the hill-side to the east of Loch Treig.* They run along the hill-side in an east and west direction. Near Loch Treig there are eskars, at the several heights (by

* See Plate XLIII.

Aneroid) of 1120 feet, 1165 feet, 1175 feet, and 1489 feet above the sea. With regard to the uppermost, I extract the following from my notes. "It is composed of detritus, with boulders on top. At west end it is a terrace nearly horizontal, but towards east it separates from hill side and becomes an eskar or kaim. The rains and rivulets descending on it from the higher part of the hill, may have scooped out the trench along the south side, which lies between it and the hill. Towards east, this kaim ceases to be horizontal, and slopes down rapidly towards the plain, still retaining the form of a bank." On the top of this highest flat, as well as on the lower flats, there are several large boulders. But above the highest flat we saw other boulders on the hill at a distance, which we had no time to visit, apparently about 1800 feet above the sea.

On a lower part of Spean Valley there are similar banks of detritus up to even greater heights. CHAMBERS characterized them as "sea beaches." Messrs JAMIESON and JOLLY look on them as the lateral moraines of a glacier which descended from the valleys of the Treig and Laire. I examined several of these banks, and agree that they are in no respect "Parallel Roads." Mr JOLLY in his paper admits that they have a great resemblance to "kaims,"—that is to the well-known long banks of gravel which abound elsewhere in Scotland. On Ben Chlinaig, at the lower part of Glen Spean, where the valley is exceedingly narrow, there are several of these kaim-like banks running along the side of the hill. Mr JOLLY describes them correctly when he says that these banks are continuous only "for short distances." "They are not absolutely horizontal, but rise and fall on the hill sides (JOLLY MSS. p. 15). One of these "kaim-like banks" on Ben Chlinaig, opposite to Craig Dhu, I found by aneroid to be 1253 feet above the sea. I crossed several others up to a height of about 1700 feet above the sea. When not horizontal, they seemed to slope down slowly towards the east."

Between Ben Chlinaig and Loch Treig, there is the wide valley of the Bruach, in which there are many eskars or kaims running in various directions. Some of them are straight, others are curved. A view of these is attempted to be given in fig. 2 (p. 668). There is another long eskar, immediately above Tulloch House, which runs on an elevated plateau in a direction north-east and south-west. At its lower end, next the River Spean, there is on it a huge boulder, at a height of about 1057 feet above the sea. As the general surface of the district rises towards the south-west, this eskar rises also and forms a continuous bank, with steep sides, for about a quarter of a mile in length. Its south-west end is 1200 feet above the sea. The eskar is composed of fine sand and small gravel.*

* Mr JAMIESON admits that in this valley the gravel is water worn. He says "at a deep section of one of these banks at the mouth of Corry Laire, I noticed that the mass consisted of coarse water worn gravel, very pebbly; the pebbles indicated a certain amount of water rolling."

Author's Views.

In reference to the phenomena last described, which Messrs JAMIESON and JOLLY have sought to explain on Glacier principles, I venture to propose an explanation of a different character.

It occurs to me to suggest whether these phenomena rather do not indicate the action of ice floating in a sea at least 2000 feet above the present sea-level? If Glen Spean then formed a Kyle or submarine valley, the floating ice would grate upon the sides of the hills, and plough through the detritus covering them, and occasionally form troughs and banks, more or less parallel with the valley. The markings on the rocks where the valley is narrow evidently indicate some powerful agency, such as an iceberg. At the Bruach Valley, where there would be a bay in this ancient sea between Ben Chlinaig on the north and the rocks of Meal Laire and Loch Treig on the south, there might have been eddies, which would account for the large accumulation there of gravel and sand in the form of long embankments.

Farther east, Glen Spean becomes somewhat more contracted by the projecting rocks at Loch Treig on the south side, and the Coinnichte Hill rocks on the north side. But beyond that point, it will be observed from the Ordnance contour lines * that the valley expands greatly, especially towards the east. A current flowing eastward through the narrow part, between Ben Chlinaig and Craig Dhu, when it reached the part beyond Inverlair and Loch Treig, would become almost stagnant, or form slow eddies. The probability is, that any current coming up the valley from Ben Chlinaig would strike on the projecting rocks east of Glen Treig, and if there was ice floating in the current it would impinge on these rocks and throw down at the foot of the rocks the boulders now lying there in heaps. (See p. 641.)

The foregoing remarks apply to Glen Spean, but similar facts are supplied from other adjoining districts.

In the upper part of Glen Roy there are hard grey granite rocks in the middle of the valley, which in my first memoir I noticed as affording evidence of some agent which had ground them down to a remarkable smoothness of surface. These rocks have been smoothed on their *west* sides, and are rough on their *east* sides. Had a glacier ever existed in Glen Roy, these rocks would have been smoothed on their east sides, and been rough on their west sides. This fact favours the theory I have advanced of a sea laden with ice, which, besides flowing up through Glen Spean had flowed also up through Glen Roy, then a submarine valley.

In another part of Glen Roy, viz., on the south side, nearly opposite to the Gap, and at a height of 1238 feet above the sea, I found rocks with smoothed

* See Plates XLI. and XLIII.

surfaces facing the west, indicating a movement also over them from that quarter.

Mr JAMIESON ("Lond. Geol. Soc. J.," 26th Feb. 1862, p. 176), had observed the appearances in Glen Roy now referred to, and admits his perplexity with reference to their bearing on his glacier views. He says, "I was not a little surprised, to find it quite apparent, that the ice had come from the south-west *up* Glen Roy, and gone out in a stream towards the wide valley of the Spey." He also admits having, in the same part of Glen Roy, discovered other rocks, "so blunted and rubbed on their *south-west exposure*, as plainly to show that the movement came from *that* quarter."

In Glen Gluoy above Altnaharry Farm-House, at a height of about 1300 feet above the sea, there are rocks presenting vertical faces towards the north-west, from 30 to 40 feet in height. These faces have evidently been smoothed by some agent moving past them from the west, the faces fronting that quarter being *smooth*, and the faces fronting the east, *rough*. If the sea stood at a height of say 3000 above its present level, there would be a passage several hundred feet deep at the head of Glen Gluoy towards Strath Spey.

On the hill between Glen Fintec and Glen Gluoy, at a height of about 1700 feet above the sea, a rock there is smoothed and striated,—the striæ running about W. by N. No glacier coming down either of these Glens could have touched this rock. It was acted on by some agent which came from an opposite direction.

In connection with these facts, reference may be made to observations by Mr JAMIESON on the direction of striæ on the hills lying between the mouths of Glen Spean and Glen Gluoy. He says, on the ridge between these Glens (called Strone-y-Var), he found that the rock-surfaces indicated a "pressure of ice *from the west*."

"At an elevation of 800 or 900 feet, glacial scoring occurs, pointing W. 5° N.; a little lower, W. 20° N., running not horizontally but up and down the slope, as if by ice mounting from Loch Lochy. Again W. 26° N., W. 45° N., W. 15° N., and W. 45° N. The *western* sides of the rocks being most worn, showed that the action had come from that side, and *not down* the Spean Valley." "At a place called Brackletter, on the south side of the River Spean, near its junction with the Lochy, glacial scores point due west, but still inclining a little to the north." ("Lond. Geol. Soc. Journ.," 21st Jany. 1863, p. 246.)

At the mouth of Corry N'Eoin, I found several places where there were rocks smoothed and striated. Thus at one place, the surface of a smoothed rock sloped down to N. ½ W. at angle of 42°. The direction of the striae upon this rock was N.N.W. Some of the striae were 2 to 3 feet in length. From this place the projecting shoulder of the mountain called Aonach More, situated 2 or 3 miles to the west, bore W.N.W., so that it would not intercept any agent moving or floating towards the striated rock.

At another spot near the mouth of this Corry, outside of it, I found a rock with two sets of striæ. One set were in the same direction, N.N.W., the others were running E. and W.

These observations, as regards the markings on the rocks, seem to be irreconcilable with any glacier theories which have yet been proposed. A sea current flowing from a north-westerly direction, but with local variations caused by the direction of the hills, seems to me a more likely solution.

As still bearing on the question, there is another set of facts which deserve notice. I allude to the position of some of the boulders.

(1.) The height at which many are found is interesting.

I extract the following from my notes.—"On the summit of the hills at the head of Glen Roy, about 169 feet above the highest shelf (*i.e.*, 1320 feet above the sea), there are enormous (granite) boulders. Some rest on surfaces of bare rock; but traces of clay and gravel about them suggest that they may originally have been imbedded in drift, which has been since mostly washed away from under and about them."

Mr Darwin was much struck by the number, size, and composition of some of the boulders he saw between Glens Gluoy and Roy. One of the spots where he found several granite boulders was what he calls a "*hillock*" of gneiss, 2200 feet above the sea. The hillock was *separated from all other hills*, so that the boulders could not have rolled down from them. He refers also to the *summit* of a very sharp peak of gneiss (1600 to 1700 feet above the sea), on which he found a block of syenite with pink felspar. This peak is wholly separated from other hills. He did not see anywhere another boulder of this syenite, or any rock *in situ* resembling it. ("Phil. Trans.", Vol. 129, p. 69.)

On the Wester Beinin Hill, situated close to Loch Laggan, I found a *grey granite boulder* about 4 feet in diameter, resting on a flat shelf, about 1516 feet above the sea. The rocks of the hill here, are a *coarse red granite*. The side of the hill on which the boulder lies points W.S.W.

Mr Jamieson mentions several large boulders on or near the top of Craig Dhu, which is 2100 feet high. I was also at the top of Craig Dhu, and saw the boulders which Mr Jamieson examined. This hill is of *gneiss* rock, whilst many of the *boulders* are "of *syenitic granite*."

Mr Jamieson adds:—

"What is remarkable is, that the largest and most angular are more numerous, *high up on the very brow of the hill*, than they are *farther down*. Thus one 12 × 9 × 6 feet lay only 130 feet below the hill summit; another was a magnificent block 15 × 10 × 6 feet."

Mr Jamieson also found granite boulders on the *top of Bohuntine*, a hill 2000 feet high, situated between Glen Roy and Glen Collarig.

Mr Jolly states (MSS., p. 10) that on the west side of Craig Dhu, he found

a huge granite boulder some 60 feet round and 12 feet high at a height of more than 1800 feet above the sea.

Professor NICOL of Aberdeen says:—

"I found huge blocks of *black granite* and smaller masses of *red porphyry* within a few yards of the *summit of Craig Dhu*, a conical mountain of *mica slate*. One block must weigh 40 tons. They are evidently ice-born masses, probably floated from far in the west." ("Lond. Geol. Soc. Jour.", August 1869, p. 283.

Sir JOHN RAMSDEN of Ardverikie informs me, that he has seen granite boulders on the tops of two contiguous hills, called Ben Sguth and Gealcharn, the latter exceeding 3000 feet above the sea. They are situated to the north of Loch Laggan.

The Ordnance Surveyors, when examining the Stratherrick Hills, noticed that the *largest boulders* were often on *elevated* sites. In a Report by Captain White to the Royal Society Boulder Committee, it is stated that the highest hills in that district are about 2900 feet above the sea, and that large boulders were most abundant above a level of 2250 feet. ("Roy. Society Proc.," 2d June 1873, pages 141 and 157.)

(2.) The following facts indicate the peculiar position of boulders.

On the west side of Bohuntine Hill there are numbers of gneiss boulders, at from 800 to 1100 feet above the sea, Those whose length is greater than their width, generally lie east and west, their smooth end, which is also the narrowest, being towards west. They probably came from the west, as the lowest ground bears from the boulders west ½ south.

Mr DARWIN takes special notice of the circumstance that the Ben Erin Hills of *gneiss* (1600 feet above the sea), on which he saw numbers of *granite* boulders, are completely isolated by deep valleys on each side of them, leading to the conclusion that the boulders must have been transported to their sites by "floating ice." It struck him that the boulders were "*most frequently on the summits* of little peaks—such as Meal Dherry," from the numerous cases of that kind which he observed.

At the head of Glen Glaster, on the N.E. part, there is a sort of amphitheatre from 1000 to 1200 feet above the sea, on which there are gathered a large number of gigantic grey granite boulders mostly angular. One is 18 × 12 × 4½ feet. Another is 12 or 13 feet wide, 9 feet high, and 23 paces in girth. These blocks are not near any rocks, craggs, or hills, from which they could have fallen. They appear to have been somehow *transported* to their present position. The lowest ground, and the widest opening, by which they could reach it, bears from it W. by S. and W.S.W., the direction of Fort-William. It appeared to me, that the most probable explanation was, that ice had floated to this spot, which, surrounded by hills, except towards the west,

formed a sort of *cul de sac*, where the ice, arrested in its farther progress, and melting, dropped the boulders.

On Craig Dhu, the boulders, in so far as not rounded, have their longer axis, most frequently east and west in direction. The following notes from my Field Book (p. 41) refer to these:—"A little above the 1391 level, found boulder lying on bare rock here forming a flat surface, glaciated like the rest from W. by N. (At this time I was inclined to glacier views.) The boulder, therefore, must have come here after glaciation of rocks. Longer axis of boulder E. and W. Standing at it, and looking *towards west*, I see that a line *from that direction* clears all the hills, and that there is an opening for a current to have flowed from west towards and upon Craig Dhu. Found masses of white quartz rock glaciated from *west*. A boulder lying on top of this rock, with longer axis W. by S. At a level of 1751 feet, found boulder with longer axis W. by S. Near Glen Glaster Col, 1445 feet above sea, boulder with longer axis W. by S. Another, 8 × 4 × 4 feet with longer axis W. by S. Another of grey granite a little below summit level (1075 feet) with longer axis W.N.W. (7 × 6 × 3 feet). It stands very oddly, not resting on its wide surface of 6 feet, but on its narrow edge of 3 feet. Another boulder 12 × 6 × 6 feet at 918 feet level, with longer axis N.W. by W. Another 12 × 6 × 5 feet, with longer axis N.N.E."

The boulders just mentioned are situated along the flattish valley lying between Craig Dhu and Craig Coinnichte on the south side, and Craig Willeim on the north side. This valley runs in an east and west direction, so that it is most probable that the boulders, almost all of which lie with their longer axis in the same direction, have been put into that position by the agency of a stream of some kind which flowed through the valley from the west.

It was through this valley that the Lake of Glen Roy, when it stood at the middle shelf, discharged. But it is not probable that the stream from that lake would have had power to bring these boulders, or put them into the positions they occupy. The boulders, moreover, are above the level of the stream.

In Glen Gluoy there are several boulders which deserve notice. Glen Gluoy is the narrowest of all the valleys, with sides high and steep. There is a boulder 9 × 8 × 6 feet, on the hill which separates Glen Fintec from Glen Gluoy. It is at a height of 812 feet above the sea. The hill here slopes towards W.S.W. at an angle of about 45°. There is a thick covering of drift on the hill above, and immediately below, there is a striated rock. This boulder must have been brought here from the west by some agent, such as floating ice, which stranded on the hill.

At another spot there are several large boulders, also on a steep slope, at a height of about 866 feet above the sea. They stand up somewhat conspicuously above the drift in which they are partially imbedded.

How came these boulders into this position?

I can see no better explanation than this, that originally these valleys had been filled up with drift, and that the same agent which spread the gravel, brought the boulders. At this time there was an open kyle or passage at the head of Glen Gluoy, and through which a current could pass. After the sea subsided to a level below 1000 feet, a lake was formed here, as in other glens; and as these lakes subsided from one level to another, much of the drift was washed away, leaving the boulders exposed, and putting them into positions which they had not originally.

Several boulders were noticed by me, standing in a very *unstable* position. Thus, there is one on the south bank of Loch Treig, as shown in fig. 16 (page 630). A is a bank of detritus, the top of which is about 40 or 50 feet above the lake B. The upper part of A consists of white sand, horizontally stratified, formerly a bed reaching back to the rock of the hill C, distant about 50 feet. The breadth of this sandy plateau may be about 30 feet. It is broken down by rabbits next the lake, whereby a depth of several feet of sand is exposed. The boulder D stands upright in the sand bed, about 5 feet above its surface, and is about 2 feet thick How much of the boulder is buried in the sand-bed I had not time or means to discover. Its appearance suggested the idea that it had been floated on ice, and slipping off with one end foremost, sunk into the sand, then the bottom of a lake or sea.

In reference to the direction of that general movement which seems to have passed over this district, causing a transport of boulders and gravel, one or two specific facts may be added.

In the Ault Laire valley, situated about a mile to the west of Loch Treig, I found a number of red porphyry boulders. Several boulders of the same rock I found on Craig Dhu and Craig Coinnichte. I see that Mr JOLLY supposes that these may have been brought by the Loch Treig glacier. I am not aware of any rocks of that description being *in situ* in any part of Glen Treig; but they exist on Ben Nevis. I saw them there *in situ* on the east side of that mountain.*

The next fact which I mention was made known to me by the late Mr ANDERSON of Inverness, a geologist of great intelligence and experience. He informed me that among the gravel of the hill of Torvane, situated at the east end of the Great Glen, there are numerous pebbles and boulders from a conglomerate rock existing *in situ* near Fort Augustus (30 miles to the west).

There is a huge boulder on the hill situated on the north side of the Linnhe Loch, opposite to Fort-William. It is on the west side of the hill, and at a height of 1494 feet above the sea. It appears to have been brought from the

* Mr JOLLY informs me that red porphyry rocks occur on the hill called Ben Dearg, so called from its red colour. Ben Dearg is distant some miles from Glen Treig, to the west.

N.W. across a deep valley. ("Proceedings of the Royal Society of Edinburgh" for 1872–3, p. 162.)

At the east end of the Great Glen of Scotland, on the hills and elevated plateaus west and south of Inverness, there are large boulders of a coarse granitic conglomerate, whose parent rocks are situated to the W.N.W. In the district where Lochan Clachan is situated (about 8 miles S.W. of Inverness), the striations on the rocks are due E. and W. On one of these rocks so striated (at a height of 1259 feet above the sea) lies a large boulder, with its sharp point towards the *west*. Its broad end lies against a portion of the rock, which has prevented its further progress *eastward*.

On the hills adjoining, there are numerous boulders, mostly on gravel drift. One boulder, of large size, and visible at a great distance off, is situated on the ridge of the highest hill in the district, about 1100 feet above the sea. It is called the "Watch Stone." It is also a coarse granitic conglomerate. Its position could not have been reached except by coming in a direction between W.N.W. and W. by N. (See Appendix.)

A very difficult question, as it appears to me, remains to be solved, regarding the agent which affected the smoothing and striation of the rocks in Lochaber and elsewhere in Scotland. Those who have preceded me in this "Parallel Roads" inquiry have referred to it, and I cannot pass it unnoticed.

On one point most geologists are agreed, viz., that the detritus spread over the country, and reaching to our highest hill ranges, is *m rine*.

Even Dr Macculloch, in whose day so little was known about these matters, inferred from the facts which he saw, that "portions of the lines (or roads) have been formed in a *rounded* and *transported* alluvium of pebbles, sand, and gravel. We suppose that a *rounded alluvium had been by previous causes accumulated in the glens. If this took place from the action of former waters flowing through the valleys* (*and to what other causes can we assign it*"), &c., p. 389.

Darwin, who maintained the marine origin of the Parallel Roads, founded his strongest argument on the undoubtedly marine character of the detritus, on which these "Roads" had been impressed.

Professor Nicol, of Aberdeen, in his paper on the Parallel Roads, refers particularly to the "detrital cover," in which he says "the lines are cut;" and adds, "that it is a *marine deposit*, seems beyond doubt," p. 283.

Mr Jamieson describes the thick beds of stratified clay, sand, and gravel, in various parts of Perthshire and Aberdeenshire up to about 2000 feet above the sea, as *apparently marine* ("Lond. Geol. Soc. Jour.," vols. xvi. and xxi.). He considers that it was "during this submergence that the brick clays containing arctic shells were deposited, and that boulders were drifted here and there, by floating ice," p. 194, vol. xxi. In a previous paper (vol. xviii. p. 164), the same author was on this point still more explicit. "At the bottom

of all the drift beds, there is in our northern latitudes a phenomenon, which, if rightly understood, would dispel much of the obscurity that still envelopes the history of that period—I mean that *curious scoring and polishing of the rocky bed, on which the drift is found so frequently reposing.*"

On this point, therefore,—viz., the marine origin of the detritus,—there is a general agreement. But how about the smoothing and striation of the rocks beneath the detritus?

Professor NICOL and Mr JAMIESON ascribe these effects to the action of land ice,—not so much to local glaciers, as to glaciation of a more general character.

Professor NICOL thinks that the rocks were smoothed and striated during "*the general glacial striation of the land.*"

Mr JAMIESON thinks that these effects were produced during "*the great land glaciation of Scotland.*"

CHAMBERS entertained the same view.

These authors adopt the idea that, before the submergence of the land, the rocks had been ground down, smoothed, polished, and striated, under the operation of a great ice sheet which covered the country, as such a sheet now covers Greenland.

According to that view, there could have been no detritus when the rocks were thus acted on. Professor NICOL on this point is explicit. He says that the "detrital cover has been formed, *since* the general glacial striation of the land." This (he says) "is also proved by the fact that it *spreads over the rocks* marked in this manner." "In Glen Roy, these striated rocks occur immediately under the lines." "The old line or parallel road now *passes over the rock surface,* that in a former period was worn and striated by the glacier."

This point, that the detrital cover, in which the Parallel Roads are cut, lies directly upon the striated rocks, is one of some importance with reference to the cause of striation. I therefore add one or two extracts from my own notes. "Found a flat on which Roman Catholic Chapel stands, about 100 feet above Bridge of Roy Inn. At this place, smoothed rocks with scratches on them. Some of the faces of these scratched rocks horizontal, others nearly vertical. They rise up abruptly from the flat. *They are covered with sand and gravel*" (Notes, vol. i. p. 3). About 500 yards to the north of the summit of Craig Dhu, found *smoothed rocks, covered* partially by detritus (Notes, vol. ii. p. 42).

In farther illustration of this point, I give a diagram of a smoothed rock (fig. 17.) atthe sid e of Shelf 4, covered by drift, except where exposed. The idea suggested on the spot was that the boulder, as well as the rock, had been originally entirely covered by detritus; and that the detritus being removed by the water of the lake, the boulder and rock were left exposed. This rock

had formed an angle or projecting point when the lake existed. The old beach, viz., Shelf 4, goes round the rock.

JAMIESON, in like manner as Professor NICOL, maintains that whilst the "drift beds" *repose* on the scored and polished rocks, the rocks were scored and polished while the land was covered with ice, the drift beds being deposited during the subsequent submergence of the land. (Vol. xviii., p.164.)

The question is, whether the ice-sheet alone could have polished and scored the rocks in the peculiar positions in which they are sometimes found. Had these smoothed and scored rocks been generally on the tops or ridges of hills, there would have been less difficulty in ascribing these effects to a general ice-sheet. But instead of being in these exposed positions, the smoothed and striated rocks are most frequently in valleys :—and the *narrower* the valley, the

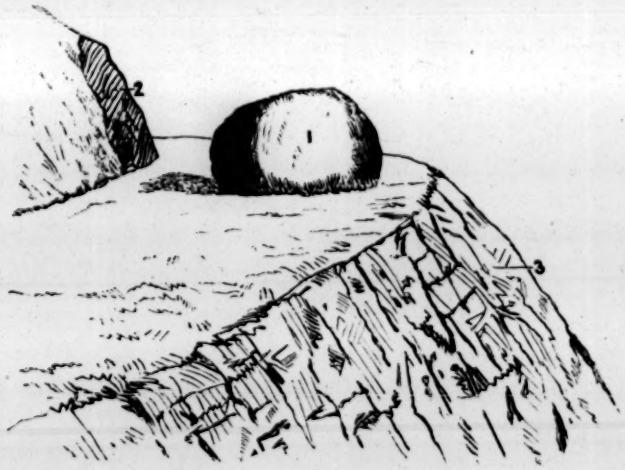


Fig. 17.

1. Gneiss Boulder, 8 feet high, 48 feet round, on Shelf 4. above Inverlair House ; shelf here about 60 feet wide. 2. A rock smoothed by some agent. 3. Gravel stratified, on which the shelf was formed, forming a steep scaur about 50 feet high.

more remarkable is the smoothing and polishing. This is undoubtedly the case in Glen Spean, at its narrowest parts between the Roman Catholic Chapel and Inverlair.

It appears also that the heavy body which effected the striations was, when obstructed in its passage by a solid rock, capable of rising over it.

Now what would happen, when a body of land ice was so obstructed ? If the rock did not break, the under part of the ice would probably be arrested, and the upper part would, under the influence of the propelling force, move on.

On the other hand, in the case of a body of ice floating, the buoyancy of the ice, when obstructed, would enable it to rise up and over the obstruction.

There is another difference between the two cases, favorable to the theory of floating ice. The detritus covering the sea-bottom, would afford to floating ice means of smoothing and scoring. But as the ice cake period is supposed to have *preceded* the detritus period, the land ice in passing over the rocks would have no such help.

There are other circumstances which favour the idea of floating ice, suggested by some observations by Mr. JAMIESON himself.

He mentions having noticed, at considerable heights both at Loch Treig (vol. xviii. p. 172) and in Glen Roy (p. 177) striæ running *horizontally*, on the rock *faces*.* I made a similar observation on Craig Dhu.† Mr JAMIESON also takes notice of the circumstance that on Craig Dhu, "the largest and most angular blocks are more numerous high upon the *very brow of the hill*, at a level of from 130 to 400 feet from the top, than they are farther down" (vol. xviii. p. 175).

Similar observations were made by Mr DARWIN and others.

There is another feature in these Glens, pointing to floating ice. I allude to the discovery of exceptional lines of detritus, running nearly horizontal on the sides of Craig Dhu and Ben Chlinaig, where they face each other, and also on the sides of Glen Roy. These lines are at different heights, from 1200 to 1700 feet above the sea. Mr JAMIESON says of the Ben Chlinaig lines, "I ascertained that these short lines were neither quite horizontal nor perfectly parallel. I therefore think they "have arisen from *some other cause* than what formed the Parallel Roads."

Mr JOLLY concurs with Mr JAMIESON in stating "that these lines are not absolutely horizontal. They rise and fall in short distances on the hill side, gradually but perceptibly" (MSS. p. 15).

Mr JOLLY adds, that these banks "form a rounded curve like that of a mound or ridge, laid upon the hill. They resemble "kaims" in the outline, and present the appearance which ridges of detritus would have if deposited on a slope. Between them and the hill, there exists a hollow, often deep enough to form a kind of valley. This moundy character is well seen on all the lines on Ben Chlinaig, and Craig Dhu" (MSS. p. 13).

Mr JOLLY is decidedly of opinion that these banks are moraines; Mr JAMIESON had previously indicated an inclination towards the same view. ROBERT CHALMERS felt assured that they were sea-beaches, and entirely of the same class as the Parallel Roads.

I have the misfortune to differ from all these views. I think that Mr

* See p. 41 hereof.

† See p. 640 hereof. For an interesting account of the agency of Pack Ice, not only in transporting boulders, but in making "horizontal grooves and scratches" on cliffs of rock, see a paper by Professor JOHN MILNE, F.C.S., in the *Geological Magazine* for September 1876.

JAMIESON suggested a theory nearer the truth when he asked "Might not some of these curious accumulations known as *Eskars*, *Osar*, and *Kaims*, have been formed by a re-extension of the ice ploughing into the old marine beds, and forcing them up into long narrow mounds? In some regions, these may have arisen from glaciers terminating in the sea." (Vol. xix. p. 253,)

The only modification I would presume to make on this suggestion is, that instead of glaciers pushing into the sea, and "ploughing the marine beds into long narrow mounds," I would suppose that icebergs, floating through Glen Spean and grating on Ben Chlinaig and Craig Dhu, might have produced the long ridgy kaim-like mounds referred to.*

If it be generally agreed, that the land was *submerged;* that the beds of gravel and sand spread over the country are *marine*, and that many of the boulders (especially those on hill tops) cannot have been transported except by floating ice, it seems to me more philosophical to use that acknowledged agency, for the explanation of other phenomena of the same class (I mean the smoothed and striated rocks), than resort to a different agency altogether, whose existence is, to say the least, very problematical.

Before concluding, let me shortly state the views I have taken in this Memoir, both on the local question of the Parallel Roads, and on the more general questions into which I have been led.

1*st.* The valleys in which these roads occur have been occupied by lakes which subsided from one level to another, as the blockages of the lakes were worn down.

2*nd.* These blockages consisted of detritus (clay, sand, and gravel) which had been spread over the country when it was submerged, and which filled all the valleys, up to considerable heights.

3*rd.* The blockages were from time to time worn down, and the materials composing them removed by the action of rivers, the cutting power of which would increase as the sea sank from its original high level to its present level.

With regard to the more general question it would appear—

1*st.* That before these Lochaber Lakes were formed, the whole country had been under the sea, and that during this submergence, currents with floating ice spread gravel, sand, and clay over what was then the sea-bottom, filling hollows on what is now the land, and causing rocks to be smoothed and scratched by the passage and pressure upon them of stones and pebbles.

2*nd.* That the sea prevailed to a height of at least 3000 feet above the present sea-level.

* It is not unimportant to observe that all these abnormal lines, in Glen Spean and Glen Roy, are in level above the highest of the "Parallel Roads." The sea may have formed these lines at a period antecedent to the formation of the lakes. The lakes, of course, could not be formed till the sea had sunk to a level below the highest shelf.

3*rd*. That the direction in which this current flowed over this part of Scotland was from the W.N.W. (magnetic), judging by the transport of the gravel and boulders, and also by the markings on the hills and rocks; but that this direction was modified by the hill ranges and submarine valleys over or through which the current passed.

APPENDIX.

The statements in the text as to the existence of a sea with floating ice, in which a current from the N.W. prevailed, receives remarkable confirmation from the direction of the striations on the hills of Ross and Argyleshires, as observed by the late ROBERT CHAMBERS. In a paper read by him before this Society, in December 1852, he gives the following observations as made by himself :—

1. On Cuineag and Canish hills (in Ross-shire) at a height of from 1700 to 1800 feet, the striations run about N. 60° W.

To this general rule there are certain exceptions, caused, as Dr CHAMBERS shows, by the contour of the hills.

2. On a summit running S. from Ben More, fully 1500 feet high, four or five miles S.E. of Cuineag, there are streakings on the quartz, observing the normal direction, viz., about N. 60° W.

3. On the Gneissic platform, between Coal More and Suilvean, there are polished surfaces striated between N.W. and W. To the west and north of the latter mountain there are similar markings.

These are situations where, in Dr CHAMBERS' opinion, *no local glaciers could have existed*.

4. Streaking, precisely the same, exists at an elevation of at least 2000 feet on the quartz mountain named Ben Eay, south of Loch Maree.

5. On free ground, between Gairloch and Poolewe, there is similar marking with a direction from W.N.W.

6. So also is there, in the great elevated valley of passage across the island in Ross-shire, the Derry More.

7. North at Rhiconnish, there are tsriæ *coming in from the coast*, viz., *from the north-west*, and passing across a high moor, with no regard whatever to the inequalities of the ground.

8. A little further north, at Loch Laxford, a fine surface is marked with striation from the N.W., being *across the valley in which it occurs*.

9. At an opening in the bold gneissic coast which looks out upon the Pentland Firth, there is strong marking from N.N.W.

10. The high desolate track called Moen, between Loch Eribol and Tongue Bay, where there is nothing that could restrain or guide the movement of the ice, exhibits striation from N. 28° W.

11. Striæ N. 25° W. occur four miles to the east of Tongue Bay.

12. In Caithness there are traces of striation from points between N. and N.W., being *directly transverse to a line drawn from the neighbouring hills*.

13. In the Island of Kerrera, opposite Oban, and in Mull, striations are noticed pointing N. 60° W.

In all the above cases, the agent which produced the striations, came, in the opinion of CHAMBERS, *not from the hills in the interior of the country*, but from *seaward*.

Mr J. F. CAMPBELL of Islay, in a paper read by him in the London Geological Society (25th June 1873), mentions that the perched boulders on the hills of Tiree and Barra, as well as the striations of the rocks in these islands, indicate transport from the N.W. and N.N.W. He suggested an Arctic current from Greenland.

Dr BRYCE, LL.D., and Mr JOLLY, Inverness, have given similar testimony as to the direction of the agents which smoothed the rocks on the Long Island,—viz., that they came from the west, and not from the mainland of Scotland.

EXPLANATION OF PLATES.

Plate XLI. General Plan of Lochaber, showing all the Parallel Roads.
Plate XLII. Plan of Glen Collarig, where Shelves 2, 3, 4 terminate.
Plate XLIII. Part of Glen Spean, where Escars and Boulders abound.

LIST OF DIAGRAMS.

Figure 1. Sketch from Turnpike Road at Tulloch, looking towards north side of Glen Spean, showing mounds of detritus which had formed part of bottom of the lowest Lake (Shelf 4), cut through by mountain streams. Referred to p. 598.
Figure 2. Plan of Escars in Alt-na-Bruach. (Text, p. 602.)
Figure 3. Section of Escar in Alt-na-Bruach. (Text, p. 603.)
Figure 4. Lower end of Loch Laggan, showing old bottom of Lake (Shelf 4) cut through by River, now discharging from it. (Text, p. 606.)
Figures 5 and 6. Plan and Section of Invergussern (in Text, p. 608.)
Figures 7 and 8. Plan and Section of old Lake near Monassie, (in Text p. 609.)
Figure 9. Glen Gluoy, its two Shelves, to explain why higher Shelf reaches farther down Glen than lower Shelf. (Text, p. 610.)
Figure 10. Glen Collarig; the blockage in it. (Text, p. 611.)
Figure 11. Plan showing the position of the Kilfinnin Shelf. (Text, p. 613.)
Figure 12. Loch Ness, part of, showing where detritus prevails most on its banks. (Text, p. 614.)
Figure 13. Old River Cliffs on right bank of Spean. (Text, p. 617.)
Figures 14 and 16. Plan and Section of two Shelves or Roads at mouth of Loch Treig. (Text, pp. 630, 631.)
Figure 15. Section showing Terrace on South Bank of Loch Treig, with tall Boulder sticking in it. (Text, p. 636.)
Figure 17. Shelf 4, near Inverlair, with Boulder on it. (Text, p. 645.)



From Nature Prints by J Stark

XXVIII.—*On the Shedding of Branches and Leaves in Coniferæ.*

By Dr James Stark of Huntfield. (Plate XLIV.)

(Read 3d January 1876.)

In referring to the tendency to bilateral ramification on the branches and even on the main stem in Cupressineæ, such as *Cupressus*, *Thuja*, and *Libocedrus*, Professor Sachs, in his Text-book (Bennett's translation pp. 444–5), thus expresses himself: "Branch systems of three or four orders of shoots are developed in one plane in such a manner that a system of this kind assumes a definite contour, and somewhat the appearance of a pinnate leaf. In Taxodium the foliage-leaves are formed in two rows on slender branches a few inches in length; in *T. distichum* these fall off in autumn, together with their leaves, thus presenting a still greater resemblance to pinnate leaves."

The shedding of leafy twigs (or *cladoptosis*, as it has been termed) in various Cupressineæ is not unknown to botanists, and the passage above quoted is sufficient to show that the analogy, in behaviour, of such twigs to leaves, has not been overlooked. Indeed, this singular phenomenon is one of the most striking cases of one morphological structure (the shoot) behaving physiologically like another, and wholly distinct one (the leaf).

Although the subject of this paper is not to be regarded as a new one, yet it does not appear to have attracted the attention which its importance merits; and hence I venture to believe that in laying a few observations and experiments before the Society, my work may not be considered superfluous.

My attention was specially directed to this subject by an incident which happened a few years ago. The spring frosts in May blighted three of the branches of a fine Nootka-Sound cypress which stood on the lawn in front of my house. In the hope that these blighted branches would partially recover, they were not removed that year; but as they died up to the stem, I resolved to cut them off the next spring. As all are aware, the branches of this cypress have strong buttresses of bark at their junction with the main stem, as if to aid in supporting the branch. When the knife, however, was applied close to the stem, to my surprise it passed through the bark without resistance, and I had only to cut through the thin quill of woody fibres when the branch dropped into my hands.

On examining the place from which it had been removed, it was found that the bark of the tree under the part where the branch had adhered was wholly cicatrised, so that the branch had only been adhering by its quill of woody fibre;

and the cicatrix presented much the appearance which the horse-shoe mark does in the bark of the Horse-chestnut when the large leaf drops from the branch. The other branches were removed without a knife by merely bending down the branch so as to break the quill of woody fibre, when the branches fell off, leaving the bark cicatrised, or healed.

The above incident set me a thinking. The mode in which these branches had been prepared to be thrown off, and the apparent provision of an articulation between the branch and the main stem, seemingly so analogous in general mechanism to that by which leaves drop off, excited my interest, and led me to examine, more closely than I had hitherto done, the defoliation in Coniferæ, and that process which, in Cupressineæ especially, often replaces it, viz., the shedding bodily of leafy twigs.

In the following remarks I shall, in the first place, refer to members of the sub-order Cupressineæ, in which I have examined plants belonging to six genera, viz., *Thuja*, *Libocedrus*, *Cupressus*, *Juniperus*, *Sequoia*, and *Wellingtonia*, in all of which "cladoptosis" occurs. I shall then turn to the sub-order Abietineæ, commencing with the genus *Pinus*, where, in the periodic shedding of the stunted shoots bearing the leaf-fascicles, we are reminded of the analogous phenomenon in Cupressineæ; and concluding with remarks on the defoliation, &c. of the genera *Abies*, *Araucaria*, *Larix*, and *Cedrus*.

The deciduous leafy twigs may be designated as *ramuli decidui;* but, for shortness' sake, I shall speak of them as *ramules.*

Sub-Order Cupressineæ.

Thuja occidentalis (Common Arbor-vitæ).—Here the ramules are cast off annually, and may be collected in great quantities under old trees in September and October. Every one must recognise the fallen ramule, for it is the first thing which falls in the autumn and litters the shrubbery. In this species, the ramule generally remains for three seasons on the tree before it is dropped. During the *first year*, it is developed from a bud, and forms a central stem with short lateral secondary shoots. During the *second year*, these lateral shoots become compound from the formation of tentiary ones; and, if the plant is in flowering condition, the extremities of a variable number of the secondary and tertiary axes become developed as female cones; male flowers appearing only exceptionally the second season. During the *third year*, the extremities of a large number, sometimes nearly the whole of the remaining branches, become developed as male flowers or catkins; and, at the close of the season, the ramule drops off bodily, generally at the end of September or beginning of October, the exact time depending on the season.

This is the general succession of events with the deciduous ramules; but

those shoots which develope into permanent branches exhibit, as might easily be supposed, irregularities as to the succession of fructification. Figs. 1–5 are from "nature-prints" or impressions of fallen ramules. Figs. 1 and 2 show a few of the old adhering cones of the previous year, with the male catkins of the current year at the terminal point of nearly every branchlet. Figs. 3, 4, and 5 show the fallen ramule with the male catkins only. As the above process of "cladoptosis" goes on from year to year, it so happens that in old plants there is an annual shedding of the ramules of the third previous season; while the ramules of two years' growth remain on all the winter. From a cause afterwards to be explained, young plants of Arbor-vitæ shed no ramules; neither do the young plants of any of the other of the Cupressineæ to be described.

Libocedrus decurrens.—As this plant has not yet flowered with me, I am not able to speak with certainty as to the number of years its ramules remain attached to the tree before they are cast off; but, like *Thuja*, the older trees of *Libocedrus* annually cast their ramules in October and November. Fig. 6 is from an impression of one of these.

Cupressus (*Chamæcyparis*) *Lawsoniana* and *C.* (*Chamæcyparis*) *Nutkænsis.*—These also throw off ramules annually; but I have not yet been able to ascertain how long they remain on the tree before they are thrown off.

Figs. 7, 8, and 9 are from shed ramules of *C. Lawsoniana*, and figs. 10 and 11 of *C. Nutkænsis.*

Juniperus communis, suecica, hibernica, chinensis, thurifera, Oxycedrus, Sabina, and *virginiana,* all throw off ramules every year, which may be picked up in great numbers under old plants during September and October. So far as my observations have yet gone, they seem to indicate that the effete ramules, like those of the Arbor-vitæ, are thrown off at the end of the third year. The fallen ramules of the junipers are so very brittle that I have only succeeded in taking good impressions of those of *J. virginiana* and *J. chinensis.* Figs. 12–14 are from fallen ramules of *J. virginiana,* illustrating the two kinds of foliage exhibited by that species; the spreading juniper form of leaf on the one hand, and the closely appressed and scale-like cypress-form on the other.

*Sequoia sempervirens** (Californian Red-wood). Here the ramules are annually cast off during October, November, and December. These are so very peculiar in shape, and so resistent of decay after their fall, that they are not easily over looked. One distinguishing peculiarity of this beautiful tree is, that the central leaves of each year's growth are longer and larger than those produced early in

* The position of the genus *Sequoia* is not very well determined. Sometimes it is placed among Abietineæ, sometimes among Cupressineæ. With Abietineæ it agrees in the pendulous or inverted position of the "seeds." PARLATORE, in DECANDOLLE'S "Prodromus," associates it with *Taxodium, Cryptomeria,* &c., in his sub-tribe Taxodieæ.

spring or at the close of summer; consequently each year's growth can easily be traced in the ramule. The cast ramules exhibit sometimes *two*, sometimes *three*, sometimes *four* seasons' growth; and in one of the specimens figured, the growth of *five* seasons is seen. The great majority, however, show that they have only been *three* years upon the tree. These ramules are generally simple, but occasionally they are compound. Figs. 15, 16, 17, 18, and 19 illustrate the above.

Wellingtonia (or *Sequoia*) *gigantea*.—Here ramules are cast off annually; this process continuing during October, November, and December. Figs. 20, 21, and 22 are from impressions of some of the fallen ramules. Very much larger ones are thrown off, but it was found impossible to take impressions, because they were so brittle that even the gentlest handling caused all the smaller branchlets to become detached. I have as yet quite failed to trace the number of years these ramules remain attached to the tree before they are thrown off.

It is thus seen that in six genera of Cupressineæ, leafy twigs or "ramules" are annually cast off, and it seems probable that when the other genera belonging to that sub-order are examined they will be found to present the same peculiarity.

The mechanism by which the ramule is thrown off appears to be strikingly similar to that by which the leaves of ordinary deciduous trees are shed. The ramule is attached to the branch by a joint or articulation, which, in so far as the bark-structures are concerned, is very similar to that which exists at the base of the leaf. The chief difference lies in the more pronouncedly woody characters of the fibro-vascular elements of the ramule. As the autumn advances, the bark of the branch forms a fine corky layer under the point of adhesion of the ramule; while at the same time a change takes place in the woody structure, which seems to undergo a kind of ulcerative process which breaks up the tissue at the point of junction, so that it snaps at the slightest touch or even by the simple weight of ramule. When the ramule falls (as I have already stated regarding the Nootka-Sound Cypress) the whole bark surface to which it had adhered is seen to be perfectly healed, excepting the small round aperture in the centure through which the quill of wood passed.

It appeared to me of interest to ascertain whether there was any essential difference between the ramules in the Cupressineæ and the permanent shoots. For several years past, therefore, I have been making experiments to ascertain whether these ramules could be developed into permanent shoots by making cuttings take root and tracing what became of the ramules; and also by endeavouring to raise plants from pure ramules, which if left on the branches would undoubtedly have been cast off as effete within the year. I have thoroughly succeeded in both of these attempts. *Thuja occidentalis* and *Sequoia sempervirens* were found best adapted for making these experiments; but they

equally succeeded with *Wellingtonia gigantea*, and *Cupressus Lawsoniana*. Every one knows that cuttings of the common Arbor-vitæ root freely and grow into plants. Every cutting contains several ramules, which in the course of the season would have been cast off by the old tree. The cutting, however, takes root, and every ramule, instead of being cast off, developes into a true branch, which remains as a permanent part of the new plant. In fact, in all young rooted plants the change from ramules into branches goes on for several years, so that during that time no ramule whatever is thrown off at the end of autumn. The same in the case with all seedling plants. It is, in fact, only when the plants have attained some age and have so far acquired their full supply of branches, that they begin to throw off ramules; but after having once begun they continue annually to cast them off. The same is the case with all plants raised from pure ramules.

It thus appears that there is no essential difference between ramules and branches proper. Morphologically they are identical, save in the weaker growth of the former, and their ultimate fall by disarticulation; and, as regards these peculiarities, the foregoing experiments abundantly prove that they are of an adventitious or extrinsic character.

Sub-Order Abietineæ.

Genus *Pinus*.—The Pines consist of trees which in common parlance are termed "evergreen"—that is to say, they remain covered with leaves both summer and winter. The several species, however, show great differences as to the period during which the leaves of a given season are retained; and this, when ascertained for all the species, may prove a valuable aid to botanists in determining alliances and ascertaining species. One great peculiarity, as is known, of this genus is that the green leaves or "needles" are not developed directly from the growing or elongating shoots, but are produced in clusters or fascicles of two or more on shortened and aborted branches. The leaves of the growing shoots, on the other hand, are developed as chaffy scales, from whose axils the aborted branches with leaf-fascicles arise. When the fascicled green leaves are shed, they do not fall off singly, as do ordinary leaves, but in a cluster or mass; the aborted branchlet falling with them. In the genus *Pinus*, therefore, we have a true cladoptosis analogous to that which occurs in the Cupressineæ; but differing in that the leaves are developed on an aborted scarcely visible branchlet, whereas in the Cupressineæ the shed ramule is fully developed, though its vitality may not be so strong as in those shoots which develope into permanent branches.

Pinus Strobus (Weymouth Pine).—Here the leaf-fascicles of the previous year are thrown off in October; so that this species has two years' growth of

leaves on its branches during summer, but only one year's growth during the winter.

Pinus excelsa (Bhotan Pine), in my locality shows the same peculiarity: retaining on its branches during winter the leaves of that year's growth, and throwing off the leaves of the previous year during October.

Pinus sylvestris (Scotch Fir).—This species, unlike the preceding, retains on its branches during the winter the leaf-fascicles of two years' growth. During the whole summer, therefore, it presents three years' growth of leaves; but towards the end of September or beginning of October, according to the season, the third previous year's growth of leaves assumes a yellow colour and drops off, leaving only two years' growth of leaves to withstand the colds of winter. All the varieties of *P. sylvestris* show this peculiarity; and trees raised from seeds brought from the Black Forest of Germany, from Russia, Holland, Switzerland, and the Highlands of Scotland, alike retain on their branches, during the winter, two years' growth of leaf-fascicles.

Pinus Laricio (Corsican Pine).—This resembles the Scotch Fir in retaining during winter two years' growth of leaves, and shedding in October the leaves of the third previous season's growth.

Pinus Cembra (Swiss Stone Pine).—In general, this species retains on its branches, during the winter, the leaf-fascicles of three years' growth, throwing off in October those of the fourth previous season's growth. In my high locality, nearly a half of the trees retain during the winter only two years' growth of leaves; but, as yet, I have failed to trace the cause of this.

Pinus Austriaca (Austrian Pine).—This differs from all the preceding in that it retains on its branches, during the winter, four seasons' growth of leaves; throwing off in October the leaves of the fifth previous season's growth. As the whole plant is robust, the branches heavy and spreading, and the leaves long and coarse, this tree, from retaining so many years' growth of leaves, gives a greater amount of shelter than any other of the genus: and in a landscape it is, in *spring*, one of the most beautiful of the pines, as its very large white silky shoots offer such a fine contrast to its dark heavy foliage.

Pinus pumilio, *P. uncinata*, and *P. Mugho*, agree with the Austrian pine, in retaining during winter the leaves of four seasons; throwing off, early in October, the leaves of the fifth previous season's growth.

In order to show how valuable this character of defoliation (or, rather, cladoptosis) is in enabling us to ascertain the analogies of the species of *Pinus*, may be mentioned the disputed point as to whether *P. pumilio* and the allied forms *P. uncinata* and *P. Mugho* are merely varieties of *Pinus sylvestris*, or are wholly distinct. Loudon, in his "Encylopædia of Trees and Shrubs," has written of these trees—"They bear such obvious marks of belonging to the

Pinus sylvestris in their foliation, habits, and locality, that we cannot for a moment hesitate about their connection with that species."—P. 956.

These researches, on the other hand, seem to show that the alliance of these species is with *Pinus austriaca*. Instead of retaining on their branches during winter only two years' growth of leaves, like the Scotch Fir, they retain four years' growth, like the black Austrian. Moreover, they have the same spreading habit as the Austrian Pine, throwing out limbs nearly as heavy as the main stem. They have, besides, another very striking character in common with the Austrian Pine, in the young shoots in spring being clothed with conspicuous white silky bud-scales, very unlike the dull greenish brown ones clothing the young shoots of the Scotch Fir.

In consequence of the high elevation of my property above the sea, viz., 900 feet, and the consequent destruction of all the more tender pines by spring frosts, I have not had an opportunity of studying with sufficient minuteness the defoliation of the more recently introduced species of the genus *Pinus;* but other observers will easily supply the deficiency.

Abies (Spruce Fir), *Picea* (Silver Fir).*—These genera differ widely from the Pines in that they retain their leaves so long on the branches that the exact number of years which elapse before they are thrown off cannot be accurately ascertained. Moreover, they exhibit *defoliation* properly so called, as distinguished from the *cladoptosis* of the Pines and Cypresses, the leaves being shed singly. In most of the trees, leaves are dropping off all summer; but a far higher proportion are cast off during November than any other month of the year. On healthy free-growing trees I have often counted from seven to nine annual growths still thickly covered with leaves, so that from seven to nine years may elapse before the leaves drop from these trees. On very old trees, however, the leaves seem to be cast off after having been only five years on the tree, and chiefly during the month of November.

In *Araucaria imbricata*, the leaves do not seem to be united to the stem by an articulation. From this circumstance it does not shed its leaves, which seem only to be destroyed through the increasing diameter of the stem, and the increasing thickness of the bark probably impairing their vitality.

Larix europæa (Common Larch).—This, like all species of the genus, is a deciduous tree; the leaves developed in spring being thrown off in November. The Larch has two kinds of shoots; one consisting of actively elongating branches with the leaves solitary and scattered in a loose spiral; the other consisting of shortened stunted branches, with undeveloped internodes, so that the leaves come to be crowded in bundles or fascicles. The stunted branches producing the leaf-fascicles I shall term *spurs*.

* The Continental botanists transpose these generic names.

The spur first occurs as a bud in the axil of one of the solitary leaves on the elongated shoot of the year's growth; and, in general, is still a bud when the leaf drops off in November. Only about every fourth of the solitary leaves developes a bud in its axil; the rest have none. Next spring the bud enlarges, assumes the form of a spur, and pushes out from its crown a tuft of leaves. The spur once formed, remains attached to the branch for about eight years, and (when its growth is not brought to a premature conclusion by its terminating in a male catkin or female cone) *annually* developes from its crown the beautiful tuft of linear leaves which gives such a marked character to the foliage of the Larch. So long as growth goes on during the summer, the spur continues to push out fresh leaves from its crown; so that the leaf-fascicle gets larger and the foliage of the tree fuller as the season advances. It not unfrequently happens that a spur, after producing a tuft of leaves one year, becomes elongated the next season into an ordinary branch with scattered leaves. It is from the spurs, the aborted branches, alone that the male catkins and the female cones are developed.

Cedrus, Libani C. Deodara, and *C. Atlantica*.—In the arrangement of their leaves these do not differ from the Larch; and, just as in the Larch, there is produced, in the axil of about every third or fourth leaf on the elongated shoot, a bud which becomes developed as a spur, throwing out a tuft of leaves, often during the same season. During the following spring, when these tufts of leaves are well developed, the scattered leaves on the previous season's growth of the elongated shoots wither and gradually drop off during the months of June and July. At the same time the older trees on each fascicle wither and drop off, one by one, as new leaves sprout from the crown of the spur; but this shedding of effete leaves occurs so gradually, that, unless carefully looked for, it would never be noticed.

The genus *Pinus*, as resembling *Larix* and *Cedrus*, on the one hand, in the production of stunted or aborted shoots, and the Cypresses, on the other hand, in the phenomenon of cladoptosis, may be regarded, so far, as a connecting link between these two genera and the Cupressineæ.

Strange as it may at first sight appear, that leafy branches should periodically be thrown off by certain coniferous trees; yet the process must not be considered as anomalous, but rather as a particular case of the operation of a very general law in the vegetable kingdom, at least where perennial plants are concerned. Thus, almost all the spikes and clusters of flowers we see in trees and shrubs, such as the Horse-chestnut, Maples, Laburnum, Ash, Azalea, Rhododendron, Elder, Lilac, Rowan tree, &c., are in reality metamorphosed branches, and yet they are cast off at the end of the season, when the fruit or seeds are ripe, by a process of disarticulation exactly similar to that leading to the fall of the ramules in Cypresses and Pines. *Cladoptosis* then, in the widest sense of the

term, is not confined to a small section of the vegetable kingdom, but is to be found, in one form, or another, almost everywhere.

It may be that in the foregoing I have gone over already trodden ground. I have not been able to find any special work on the subject, by which, if there is such, I may have been forestalled. I trust, however, that, as one who is not a professional botanist, I may be excused for any failing in that regard.

NOTE to the above by Professor ALEXANDER DICKSON, Glasgow.

From a morphological point of view, the matters treated of in Dr STARK'S interesting paper suggest considerations of the highest importance.

It can hardly be doubted that the *ramuli decidui*, of more or less definite form and duration, so common in the Cupressineæ, are approximations to the more highly differentiated structures we find in the phylloid shoots or *cladodia* which exclusively perform leaf-functions in *Phyllocladus*, and in the still more aberrant *Sciadopitys*. In these two genera, it may be mentioned, the leaves on the permanent shoots are developed, as in *Pinus*, exclusively as bud-scales; and it is interesting to observe a tendency to a similar rudimentary development of the leaves on the permanent shoots of the deciduous cypress (*Taxodium distichum*), which affords the most remarkable known instance of cladoptosis, since, on the approach of winter, it loses the whole year's crop of leafy twigs or ramules.

Reference here may also appropriately be made to the circumstance that in Coniferæ we have perhaps as striking examples as any afforded by the vegetable kingdom of groups in which the forms at an early stage closely resemble each other in certain characters; but where, in the subsequent stages, the primary or juvenile characteristics may become completely lost in one form, may be completely retained in a second, or only partially retained in a third. The greater or less persistence of juvenile characteristics has been termed "Stasimorphy" by teratologists, as expressing a *stasis* or arrest of development. This subject, as illustrated by the Coniferæ, has already been treated of by Dr MASTERS in his "Vegetable Teratology" (pp. 217, 218); but as it is one of great interest and importance, it may not be out of place to present it again, and perhaps in somewhat different form. In the group including the genera *Pinus*, *Abies*, *Picea*, *Larix* and *Cedrus*, we have plants which, in the seedling condition, all exhibit the green leaves scattered upon elongated shoots. In *Pinus*, as development proceeds, this juvenile characteristic is completely lost; the green leaves ultimately being developed exclusively upon aborted shoots, the elongated shoots producing only bud-scales. In *Abies* and *Picea* the juvenile characteristic is completely retained; all the green leaves being scattered upon elongated shoots. In *Larix* and *Cedrus* we have the intermediate condition; some leaves being scattered upon elongated shoots, others being clustered together upon contracted shoots.

Again, in the Cupressineæ we have the genera *Cupressus* and *Juniperus*, in both of which the leaves of the young seedlings are invariably needle-shaped and spreading like the leaves of the common Juniper. In almost all Cypresses this juvenile characteristic becomes wholly lost; the leaves on the older plants being exclusively scale-like and appressed to the stem. In the section *Oxycedrus* of the genus *Juniperus* the primitive needle-like character of the leaves is completely retained. In the section *Sabina* of the same genus (to which *J. virginiana*, referred to by Dr STARK, belongs), we have the intermediate condition, some branches exhibiting juniper-like, others cypress-like foliage; and in the genus *Cupressus*, *C. funebris* exhibits the same phenomenon. Among other orders, we have an exact parallel to these cases in the well-known one presented by the Acacias. In all of these, the young seedlings have pinnate leaves. In

some species, the juvenile characteristic is completely lost, the leaves being subsequently developed exclusively as phyllodia: in some, the original condition is completely retained; while in others we have the intermediate condition, the plant, throughout life, producing both kinds of leaves. The case of the phyllodineous Acacias having when young the compound leaves so characteristic of the order Leguminosæ to which they belong, is one of those specially referred to by Mr DARWIN "(Origin of Species," 4th edition, p. 519), in illustration of what he terms "the law of embryonic resemblance," expressing thereby the fact that in a given group the resemblance between the different forms is found to be greater the younger the stage or phase of development:—in other words, that the structure is more general in type in the younger stages, and more specialised in the older ones.

Lastly, Dr STARK'S observation as to the ramules of *Thuja* developing for the most part *female* cones the second season and *male* flowers the third season, is of interest as suggesting the existence of a condition (possibly not uncommon in the group to which the genus belongs), which is at once curiously like and curiously unlike that of Dichogamy, as seen in hermaphrodite flowers; for it is evident that in *Thuja* the female flowers of one generation of ramules would thus almost necessarily be fertilised by pollen from male flowers developed upon ramules of an older generation.

EXPLANATION OF PLATE.

The figures, about two-thirds of the natural size, are photo-lithographic reductions from "nature prints" of the fallen ramules.

Thuja occidentalis (Common Arbor-vitæ).

Figures 1, 2. Fallen ramules with the still adhering female cones of the second season; and the third season's male catkins terminating most of the smaller twigs.

Figures 3, 4, 5. Fallen ramules, showing numerous male catkins of the third season.

Libocedrus decurrens.

Figure 6. Fallen ramule.

Cupressus (*Chamæcyparis*) *Lawsoniana.*

Figures 7, 8, 9. Fallen ramules.

Cupressus (*Chamæcyparis*) *Nutkænsis.*

Figures 10, 11. Fallen ramules.

Juniperus Virginiana.

Figure 12. Fallen ramule, exhibiting spreading needle-like foliage, like that of ordinary Junipers.

Figures 13, 14. Fallen ramules, exhibiting appressed scale-like foliage, like that of Cypresses.

Sequoia sempervirens (Californian Redwood).

Figure 15. Fallen ramule of two season's growth

Figure 16. Do. do. three do. do.

Figure 17. Do. do. four do. do.

Figure 18. Do. do. five do. do.

Figure 19. Fallen ramule of two seasons' growth, and branched.

Sequoia (*Wellingtonia*) *gigantea.*

Figures 20, 21, 22. Fallen ramules.

APPENDIX.

TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH,

NOVEMBER 1876.

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LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH,

AS REVISED 31ST OCTOBER 1871.

LAWS

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5.), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

Title.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

II.

The fees of Ordinary Fellows residing in Scotland.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.*

III.

Payment to cease after 25 years.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from farther payment.

IV.

Fees of Non-Resident Ordinary Fellows.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any farther payment. In the case of any Resident Fellow ceasing to reside

Case of Fellows becoming Non-Resident.

* At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s., to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

A modification of this rule, in certain cases, was agreed to 3d January 1831.

in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of election, and shall lie upon the table during that time.

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall

* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

XI.

Royal Personages.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X.

XII.

Recommendation of Honorary Fellows.

Mode of Election.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election.

XIII.

Election of Ordinary Fellows.

The election of Ordinary Fellows shall only take place at the first Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote.

XIV.

Ordinary Meetings.

The Ordinary Meetings shall be held on the first and third Mondays of every month from November to June inclusively. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make.

XV.

The Transactions.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which

* We hereby recommend_____
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

_____

_____

_____

To the President and Council of the Royal Society of Edinburgh.

they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

XVI.

How Published. The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council. There shall be elected annually, for conducting the publications and regulating the private business of the Society, a Council, consisting of a President; Six Vice-Presidents, two at least of whom shall be resident; Twelve Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors. Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers. An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

XX.

Special Meetings; how called. Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties. The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor. At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society. General Secretary's Duties.

XXIV.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the procedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty. Secretaries to Ordinary Meetings.

XXV.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows. Curator of Museum and Library.

XXVI.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint. Use of Museum and Library.

XXVII.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date. Register Book.